POLITECNICO DI TORINO
Repository ISTITUZIONALE

The Transient Spectrum of an Unstable Resonant System

Original
The Transient Spectrum of an Unstable Resonant System / Galleani, L.. - In: ELECTRONICS LETTERS. - ISSN 0013-5194. - ELETTRONICO. - 62:1(2026), pp. 1-6. [10.1049/ell2.70715]

Availability:
This version is available at: 11583/3017898 since: 2026-11-13T16:55:17Z

Publisher:
Wiley

Published
DOI:10.1049/ell2.70715

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)

18 November 2026

Check for updates

Electronics Letters

LETTER OPEN ACCESS

The Transient Spectrum of an Unstable Resonant System

Lorenzo Galleani

Department of Electronics and Telecommunications, Politecnico di Torino, Turin, Italy

Correspondence: Lorenzo Galleani (galleani@polito.it)

Received: 28 April 2026 | **Revised:** 29 July 2026 | **Accepted:** 25 August 2026

ABSTRACT

We investigate the phenomenon of instability in resonant systems, a critical condition which produces the divergence of the system output. To this end, we derive the closed-form transient spectrum of a key unstable resonant system, the undamped harmonic oscillator driven by causal white Gaussian noise. The obtained transient spectrum reveals how the frequency content of the system output evolves in time and eventually diverges at the resonant frequencies. We show that this divergence is quadratic in time, and that it is caused by the collapse of the time–frequency poles, the roots of the system in the time–frequency domain. The derived transient spectrum clarifies the spectral mechanisms leading to instability, and supports the development of improved analysis, design, and identification methods for resonant systems.

1 | Introduction

Instability is a critical condition in oscillatory systems [1, 2], because it leads to a divergence of the system output over time. In electrical and electronic circuits, this growth is typically limited by nonlinear effects, resulting in saturation of voltages and currents. In other systems, such as vibrating structures (e.g., buildings or bridges), the increasing amplitude of oscillations may instead cause severe damage or catastrophic failure. Understanding the mechanisms underlying instability is therefore of primary importance.

In this article, we consider the most fundamental prototype of an unstable resonant system, namely, the undamped harmonic oscillator, defined as [3]

$$\frac{d^2x(t)}{dt^2} + \omega_0^2 x(t) = f(t), \tag{1}$$

where ω_0 is a constant parameter, $f(t)$ is the input, and $x(t)$ is the output, or response. This system is bounded-input bounded-output (BIBO) unstable [1], since its impulse response [3]

$$h(t) = \frac{1}{\omega_0} u(t) \sin(\omega_0 t), \tag{2}$$

with the Heaviside step function $u(t)$ defined as $u(t) = 0$ for $t < 0$ and $u(t) = 1$ for $t \geq 0$, is not absolutely integrable. Equivalently, this instability is associated with the singularities at the resonant frequencies $\omega = \pm\omega_0$ in the transfer function

$$H(\omega) = \frac{1}{\omega_0^2 - \omega^2}, \tag{3}$$

where the Fourier transform of a signal $x(t)$ is defined as

$$X(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt. \tag{4}$$

To investigate this instability, we consider a causal white Gaussian noise (WGN) input, obtaining

$$\frac{d^2x(t)}{dt^2} + \omega_0^2 x(t) = u(t)\xi(t), \tag{5}$$

where $\xi(t)$ is a WGN with zero mean and autocorrelation function $R_\xi(\tau) = E[\xi(t)\xi(t+\tau)] = N_0\delta(\tau)$, with E denoting expectation. Moreover, to focus on how the system processes the stochastic input, we consider zero initial conditions, namely $x(0) = x'(0) = 0$. Once the WGN input is applied at $t = 0$, the system enters

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.
© 2026 The Author(s). *Electronics Letters* published by John Wiley & Sons Ltd on behalf of The Institution of Engineering and Technology.

a transient phase, and the output $x(t)$ becomes a nonstationary random process. As a result, its statistical properties, including its variance and frequency content, evolve with time.

This letter derives a closed-form expression for the transient spectrum of the undamped harmonic oscillator (5), that is, the time–frequency spectrum [4] of its output during the transient phase. Consistently with the singularities in the transfer function (3), the transient spectrum diverges for $t \to \infty$ at the resonant frequencies $\omega = \pm\omega_0$, while for non-resonant frequencies it remains bounded and oscillatory (except for the zero frequency). We show that this divergence is quadratic in time. To the best of the author's knowledge, the derived transient spectrum has not been previously reported in the literature.

Our result is significant for two main reasons. First, it reveals the spectral mechanisms underlying the instability of the undamped harmonic oscillator, providing a time–frequency interpretation of the singularities in the transfer function (3). This insight cannot be gained through the standard simulation-based approach, namely, computing the output $x(t)$ and its time–frequency representation through numerical algorithms alone. Second, the availability of a closed-form transient spectrum enables improved system design, analysis, and identification methods based on joint time–frequency representations, rather than on separate time- and frequency-domain approaches.

The present result extends the one derived in [5] for the harmonic oscillator driven by a causal WGN, defined as

$$\frac{d^2x(t)}{dt^2} + 2\mu\frac{dx(t)}{dt} + \omega_0^2 x(t) = u(t)\xi(t), \tag{6}$$

where the analysis is restricted to the stable case $\mu > 0$. Under this stability assumption, the transient spectrum of the undamped harmonic oscillator (5) cannot be obtained by setting $\mu = 0$ in the transient spectrum of (6) reported in [5]. However, it can be obtained by following the same approach, namely, by first transforming the time-domain system into a time–frequency domain system, and then by computing the transient spectrum as its output.

This formulation highlights the role of the poles of the time–frequency system, here referred to as time–frequency poles [6], in governing the behavior of the transient spectrum. In particular, instability arises from a specific configuration of these poles, and is ultimately caused by the collapse of a pole to zero, providing a direct spectral interpretation of the divergence.

2 | Time–Frequency Representation of Nonstationary Random Processes

The Wigner distribution is defined as [7]

$$W_x(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x^*(t-\tau/2)x(t+\tau/2)e^{-i\tau\omega}d\tau, \tag{7}$$

where the star sign indicates complex conjugation. Originally introduced in quantum mechanics, the Wigner distribution is the prototype of many time–frequency representations [8]. By taking its expected value, we obtain the Wigner spectrum, an effective time–frequency representation for nonstationary random processes [9],

$$\overline{W}_x(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E[x^*(t-\tau/2)x(t+\tau/2)]e^{-i\tau\omega}d\tau. \tag{8}$$

The Wigner spectrum has a straightforward connection to the autocorrelation function of the random process $x(t)$,

$$\overline{W}_x(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_x(t+\tau/2, t-\tau/2)e^{-i\tau\omega}d\tau. \tag{9}$$

When $x(t)$ is a real-valued random process, the Wigner spectrum is even with respect to frequency, namely

$$\overline{W}_x(t,\omega) = \overline{W}_x(t,-\omega). \tag{10}$$

If $x(t)$ is causal, with $x(t) = 0$ for $t < 0$, it is

$$\overline{W}_x(t,\omega) = 0, \text{ for } t \le 0. \tag{11}$$

Moreover, if $x(t)$ is a wide-sense stationary (WSS) random process, then

$$\overline{W}_x(t,\omega) = \frac{1}{\sqrt{2\pi}}S_x(\omega), \tag{12}$$

where $S_x(\omega)$ is the power spectrum, defined as

$$S_x(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} R_x(\tau)e^{-i\tau\omega}d\tau. \tag{13}$$

A consequence of this property is that the Wigner spectrum of the WGN $\xi(t)$ in (5) is given by

$$\overline{W}_\xi(t,\omega) = \frac{N_0}{2\pi}. \tag{14}$$

Similarly, by using (9), for the causal WGN $f(t) = u(t)\xi(t)$ in (5), we obtain

$$\overline{W}_f(t,\omega) = \frac{N_0}{2\pi}u(t). \tag{15}$$

The Wigner spectrum provides a meaningful description of nonstationary random processes. For example, the Wigner spectrum of a WGN is constant, as in (14), consistently with the fact that WGN is expected to have constant power at all frequencies and at all time instants. Similarly, the Wigner spectrum of a WGN starting at $t = 0$ is zero for negative times and constant in time and frequency for $t \ge 0$, as in (15), which is again consistent with physical intuition. Moreover, the Wigner spectrum has many mathematical properties, and therefore, similarly to the power spectrum for WSS random processes, it is an ideal tool for the theoretical description of the time–frequency spectrum of nonstationary random processes. For these reasons, the Wigner spectrum is adopted as the time–frequency representation of the transient spectrum.

1350911x, 2026, 1, Downloaded from https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/ell2.70715 by Lorenzo Galkani - Cochraneltalia , Wiley Online Library on [13/09/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

3 | The Transient Spectrum of the Undamped Harmonic Oscillator

We rewrite (5) as

$$(D^2 + \omega_0^2)x(t) = f(t), \tag{16}$$

where $D = \frac{d}{dt}$ and $f(t) = u(t)\xi(t)$. We now transform this stochastic differential equation in the time domain into a deterministic differential equation in the time–frequency domain of the Wigner spectrum [5], obtaining

$$(A^2 + \omega_0^2)(B^2 + \omega_0^2)\overline{W}_x(t,\omega) = \overline{W}_f(t,\omega), \tag{17}$$

where

$$A = \frac{1}{2}\frac{\partial}{\partial t} - i\omega, \quad B \quad = \frac{1}{2}\frac{\partial}{\partial t} + i\omega. \tag{18}$$

Substituting and using (15), we have

$$\frac{1}{16}\frac{\partial^4 \overline{W}_x(t,\omega)}{\partial t^4} + \frac{1}{2}(\omega^2 + \omega_0^2)\frac{\partial^2 \overline{W}_x(t,\omega)}{\partial t^2} + (\omega_0^2 - \omega^2)^2 \overline{W}_x(t,\omega) = \frac{N_0}{2\pi}u(t). \tag{19}$$

This partial differential equation represents the time–frequency system associated with the time–domain system (5). Its input and output are the Wigner spectra of the input and output in the time domain, respectively, while its parameters depend on the frequency ω. Since it involves derivatives with respect to time only, for each fixed value of ω it can be treated and solved as a linear ordinary differential equation with constant coefficients in the variable t. Therefore, it can be factored with respect to the roots of its associated characteristic equation, obtaining

$$\left(\frac{\partial}{\partial t} - p_1\right)\left(\frac{\partial}{\partial t} - p_1^*\right)\left(\frac{\partial}{\partial t} - p_2\right)\left(\frac{\partial}{\partial t} - p_2^*\right)\overline{W}_x(t,\omega) = \frac{8N_0}{\pi}u(t), \tag{20}$$

where, for the factorization, we have used the more convenient form (17), and

$$p_1 = -2i(\omega - \omega_0), \tag{21}$$

$$p_2 = -2i(\omega + \omega_0). \tag{22}$$

We refer to the roots p_1, p_1^*, p_2, and p_2^* as the time–frequency poles [6], since, for each fixed ω, they are poles of the time–frequency system (19) in the standard Laplace-transform sense with respect to time t, rather than poles of the time-domain system (5). Note that the time–frequency poles are purely imaginary and depend on the frequency ω, which is omitted in their notation for conciseness. As a result, the time–frequency system (19) exhibits a different behavior for each value of ω, and consequently so does the transient spectrum, as we now show.

Taking the Laplace transform of (20) and rearranging terms yields

$$\overline{W}_x(s,\omega) = \frac{8N_0}{\pi}\frac{1}{s(s-p_1)(s-p_1^*)(s-p_2)(s-p_2^*)}, \tag{23}$$

where the Laplace transform is defined as [10]

$$\overline{W}_x(s,\omega) = \int_0^{+\infty} \overline{W}_x(t,\omega)e^{-st}dt. \tag{24}$$

We rewrite this equation as

$$\overline{W}_x(s,\omega) = \frac{8N_0}{\pi}Q(s), \tag{25}$$

where

$$Q(s) = \frac{1}{s(s-p_1)(s-p_1^*)(s-p_2)(s-p_2^*)}. \tag{26}$$

This rational function can be decomposed via the partial fraction expansion

$$Q(s) = \frac{A_0}{s} + \frac{A_1}{s-p_1} + \frac{A_1^*}{s-p_1^*} + \frac{A_2}{s-p_2} + \frac{A_2^*}{s-p_2^*}, \tag{27}$$

where the coefficients A_k can be obtained by using the residue theorem [10]. For example,

$$A_0 = \lim_{s\to 0} sQ(s) = \frac{1}{p_1 p_1^* p_2 p_2^*} = \frac{1}{16(\omega^2 - \omega_0^2)^2}. \tag{28}$$

Similarly,

$$A_1 = -\frac{1}{128\omega\omega_0(\omega - \omega_0)^2}, \tag{29}$$

$$A_2 = \frac{1}{128\omega\omega_0(\omega + \omega_0)^2}. \tag{30}$$

Substituting the coefficients into (27), taking the inverse Laplace transform, and substituting the result into (25) yields

$$\overline{W}_x(t,\omega) = \frac{N_0 u(t)}{4\pi\omega\omega_0}\left[\frac{\sin^2((\omega-\omega_0)t)}{(\omega-\omega_0)^2} - \frac{\sin^2((\omega+\omega_0)t)}{(\omega+\omega_0)^2}\right]. \tag{31}$$

This expression is the closed-form transient spectrum of the undamped harmonic oscillator for $\omega \neq \pm\omega_0$ and $\omega \neq 0$. As a basic consistency check, it satisfies the expected symmetry $\overline{W}_x(t,\omega) = \overline{W}_x(t,-\omega)$ in (10). Moreover, it is continuous with respect to time at $t = 0$: the factor $u(t)$ makes it identically zero for $t < 0$, while direct substitution of $t = 0$ yields $\overline{W}_x(0,\omega) = 0$, since both squared-sine terms vanish. Hence, the causality condition in (11) is satisfied exactly.

The oscillatory component of the transient spectrum, that is, the two terms within the square brackets in (31), admits a twofold interpretation. Along the time axis, for a fixed frequency ω, each term corresponds to a bounded oscillation given by a squared sinusoidal function, with period $\pi/|\omega - \omega_0|$ and $\pi/|\omega + \omega_0|$, respectively, and amplitudes proportional to $1/(\omega - \omega_0)^2$ and $1/(\omega + \omega_0)^2$. This behavior is a direct consequence of the time–frequency poles being purely imaginary. Along the frequency axis, for a fixed time t, the transient spectrum is instead given by the difference of two squared sinc functions centered at $\omega = \omega_0$ and $\omega = -\omega_0$, whose zeros are located at $\omega_0 \pm k\pi/t$ and $-\omega_0 \pm k\pi/t$, respectively, for $k = 1, 2, \ldots$

1350911x, 2026, 1, Downloaded from https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/ell2.70715 by Lorenzo Galkani - Cochraneltalia , Wiley Online Library on [13/09/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

At the resonant frequency value $\omega = \omega_0$, the time–frequency poles are given by

$$p_1 = 0, \tag{32}$$

$$p_2 = -4i\omega_0. \tag{33}$$

Therefore, $Q(s)$ becomes

$$Q(s) = \frac{1}{s^3(s^2 + 16\omega_0^2)}, \tag{34}$$

which can be decomposed as

$$Q(s) = \frac{A_0}{s} + \frac{A_1}{s^2} + \frac{A_2}{s^3} + \frac{A_3 s + A_4}{s^2 + 16\omega_0^2}. \tag{35}$$

The solution is

$$\begin{aligned} A_0 &= -\frac{1}{256\omega_0^4}, \qquad A_1 = A_4 = 0, \\ A_2 &= \frac{1}{16\omega_0^2}, \qquad A_3 = \frac{1}{256\omega_0^4}. \end{aligned} \tag{36}$$

By substituting these coefficients into (35) and taking the inverse Laplace transform, we obtain

$$\overline{W}_x(t, \omega_0) = \frac{N_0}{4\pi\omega_0^2} u(t) \left[t^2 - \frac{1}{4\omega_0^2} \sin^2(2\omega_0 t) \right]. \tag{37}$$

Since $\sin^2(2\omega_0 t)$ is bounded, the transient spectrum at $\omega = \omega_0$ exhibits a quadratic growth in time. As $t \to \infty$, $\overline{W}_x(t, \omega_0)$ diverges, consistently with the singularity in the transfer function (3). From a system-theoretic perspective, it is the collapse of the time–frequency pole p_1 to zero which causes the quadratic term in the transient spectrum at $\omega = \omega_0$. By symmetry, the same behavior occurs at $\omega = -\omega_0$, when p_2 collapses to zero.

We remark that the quadratic divergence at $\omega = \omega_0$ occurs for $t \to \infty$, whereas the transient spectrum $\overline{W}_x(t, \omega)$ is finite for all finite time values t. This can also be seen from the connection between the Wigner spectrum and the autocorrelation function in (9). The output $x(t)$ can be written as the convolution of the impulse response $h(t)$ in (2) with the causal WGN input $u(t)\xi(t)$, and simplifies to

$$x(t) = \frac{1}{\omega_0} u(t) \int_0^t \sin(\omega_0(t - t'))\xi(t')dt'. \tag{38}$$

Therefore, $x(t)$ is causal, namely, $x(t) = 0$ for $t < 0$. Consequently, the term $R_x(t + \tau/2, t - \tau/2)$ in (9) contains the multiplicative factor $u(t + \tau/2)u(t - \tau/2)$, which is nonzero only for $|\tau| \leq 2t$, for $t \geq 0$. Hence, the integral over τ in (9) is effectively computed over the finite interval $-2t \leq \tau \leq 2t$, and is therefore well defined and convergent for any finite time value t. Moreover, we note that the transient spectrum $\overline{W}_x(t, \omega_0)$ at the resonant frequency ω_0 does not represent the variance of $x(t)$, which, since $E[x(t)] = 0$ from (38), corresponds to $\sigma_x^2(t) = R_x(t, t) = E[x^2(t)]$. By using (38), one obtains

$$\sigma_x^2(t) = N_0 u(t) \left[\frac{t}{2\omega_0^2} - \frac{\sin(2\omega_0 t)}{4\omega_0^3} \right]. \tag{39}$$

The connection between the transient spectrum and the variance is instead given by the known property [5]

$$\int_{-\infty}^{+\infty} \overline{W}_x(t, \omega) d\omega = E[x^2(t)] = \sigma_x^2(t). \tag{40}$$

We evaluated this integral using $\overline{W}_x(t, \omega)$ from (31) and recovered (39), thus validating our result.

Finally, at $\omega = 0$, the time–frequency poles reduce to

$$p_1 = 2i\omega_0, \tag{41}$$

$$p_2 = -2i\omega_0. \tag{42}$$

Consequently,

$$Q(s) = \frac{1}{s(s^2 + 4\omega_0^2)^2}, \tag{43}$$

which can be decomposed as

$$Q(s) = \frac{A_0}{s} + \frac{A_1 s + A_2}{s^2 + 4\omega_0^2} + \frac{A_3 s + A_4}{(s^2 + 4\omega_0^2)^2}. \tag{44}$$

The solution is given by

$$\begin{aligned} A_0 &= \frac{1}{16\omega_0^4}, \qquad A_1 = -\frac{1}{16\omega_0^4}, \\ A_2 &= A_4 = 0, \qquad A_3 = -\frac{1}{4\omega_0^2}. \end{aligned} \tag{45}$$

Substituting and taking the inverse Laplace transform yields

$$\overline{W}_x(t, 0) = \frac{N_0}{\pi\omega_0^4} u(t) \left[\sin^2(\omega_0 t) - \frac{\omega_0 t}{2} \sin(2\omega_0 t) \right]. \tag{46}$$

While the $\sin^2(\omega_0 t)$ term is bounded, the amplitude of the $\sin(2\omega_0 t)$ term grows linearly with time, leading to a diverging oscillatory behavior of the transient spectrum at $\omega = 0$ as $t \to \infty$. This diverging oscillatory term arises from the interference between the symmetric diverging contributions at the singular frequencies $\pm\omega_0$, namely, $\overline{W}_x(t, \omega_0)$ and $\overline{W}_x(t, -\omega_0)$.

4 | Example

We consider the case $\omega_0 = 2\pi$ and $N_0 = 1$ (all quantities are dimensionless). Figure 1 illustrates the transient spectrum $\overline{W}_x(t, \omega)$ in (31). For clarity, only non-negative frequencies are shown, exploiting the symmetry property (10). At $t = 0$, the WGN input injects energy uniformly across all frequencies, and the transient spectrum is initially distributed over the entire frequency axis. As time increases, the transient spectrum progressively concentrates around the resonant frequency ω_0. For large times, the transient spectrum diverges, and the thick solid line at $\omega = \omega_0$ highlights the quadratic growth described by (37). At $\omega = 0$, the spectrum exhibits an oscillation at constant frequency with linearly increasing amplitude, as predicted by (46). This behavior provides a direct time–frequency visualization of the instability mechanism.

1350911x, 2026, 1, Downloaded from https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/ell2.70715 by Lorenzo Galkani - Cochraneltalia , Wiley Online Library on [13/09/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

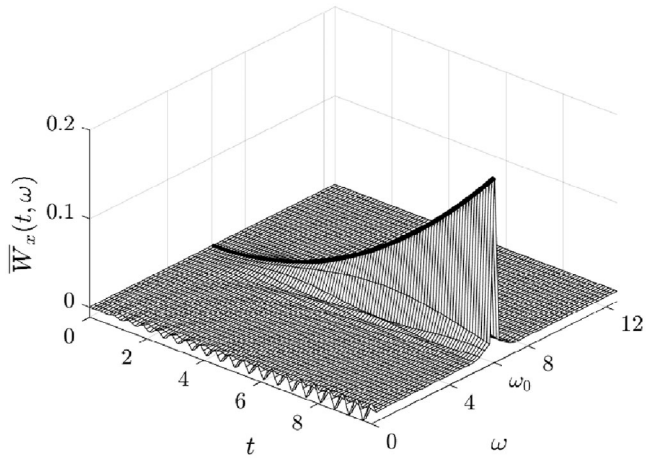


FIGURE 1 | Transient spectrum $\overline{W}_x(t,\omega)$ of the undamped harmonic oscillator (5). The quadratic divergence at the resonant frequency ω_0 is highlighted by the thick solid line.

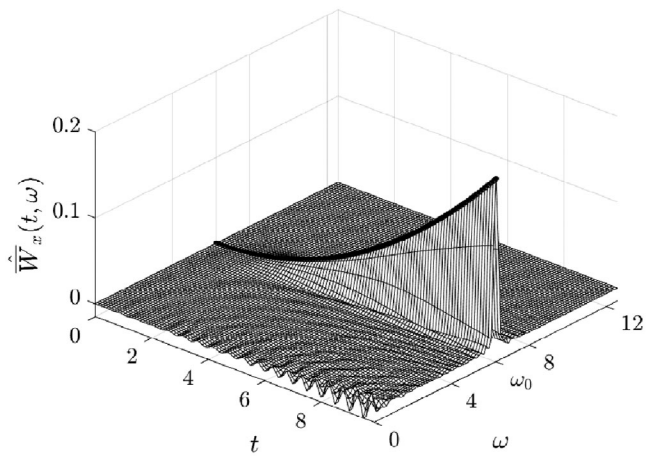


FIGURE 2 | Estimated transient spectrum $\hat{\overline{W}}_x(t,\omega)$ of the undamped harmonic oscillator (5) obtained through Monte Carlo simulations. The agreement with the transient spectrum in Figure 1 is very good. The thick solid line represents the analytical transient spectrum $\overline{W}_x(t,\omega_0)$ at the resonant frequency ω_0, taken from Figure 1 and superimposed for comparison.

To validate the analytical transient spectrum, we generated $M = 1000$ independent realizations of the undamped harmonic oscillator output $x(t)$ using the exact discrete-time equivalent state-space model. We then computed the numerical Wigner distributions for the M realizations, and we used their ensemble average $\widehat{\overline{W}}_x(t,\omega)$ as a Monte Carlo estimate of the transient spectrum $\overline{W}_x(t,\omega)$. As shown in Figure 2, the numerical estimate is in very good agreement with the closed-form transient spectrum in Figure 1, reproducing the growth and progressive concentration around the resonant frequency ω_0.

5 | Conclusion

We have derived a closed-form expression for the transient spectrum of the undamped harmonic oscillator driven by causal white Gaussian noise, providing a complete description of its time–frequency behavior during the transient phase.

In contrast to classical frequency-domain analysis, the transient spectrum reveals how the frequency content evolves in time and diverges at the resonant frequencies $\omega = \pm\omega_0$, with a quadratic growth. This behavior is explained by the structure of the time–frequency poles: at resonance, one of them collapses to zero, producing the observed polynomial divergence. These results provide a time–frequency interpretation of instability in oscillatory systems and establish a direct link between instability and the structure of time–frequency poles.

The present result concerns the Wigner-spectrum representation of the response of a linear undamped oscillator driven by a causal WGN input, with zero initial conditions. It is therefore distinct from stochastic, chaotic, and vibrational resonance, which rely on nonlinear mechanisms. In particular, stochastic resonance in nonlinear monostable, bistable, tristable, and multistable systems results from the interplay between noise and the system's nonlinear dynamics and, when multiple stable states coexist, from noise-assisted transitions among them [11, 12]. By contrast, the divergence studied here occurs in a linear system and is caused by the collapse of a time–frequency pole at resonance. Moreover, note that for $\mu < 0$ in (6), the time–frequency poles acquire positive real parts and generate exponentially growing contributions. For colored inputs, the same time–frequency equation can be used, but the input Wigner spectrum $\overline{W}_f(t,\omega)$ is no longer proportional to the step function and independent of frequency. Instead, it generally depends on both time and frequency, and the resulting transient spectrum depends on the specific second-order structure of the input. For non-Gaussian white input noise, the present Wigner-spectrum analysis, being based on second-order statistics, remains unchanged provided that the underlying white noise has zero mean and autocorrelation function $N_0\delta(\tau)$. Differences confined to higher-order statistics are not captured by the Wigner spectrum and lie outside the scope of this work. Finally, multiplicative noise would instead require deriving a different time–frequency equation, an interesting and challenging problem which is not considered here.

Author Contributions

Lorenzo Galleani: conceptualization, formal analysis, investigation, methodology, resources, software, validation, visualization, writing – original draft, writing – review & editing.

Acknowledgements

Open access publishing facilitated by Politecnico di Torino, as part of the Wiley - CRUI-CARE agreement.

Funding

The author has nothing to report.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The author has nothing to report.

1350911x, 2026, 1, Downloaded from https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/ell2.70715 by Lorenzo Galleani - Cochraneltalia , Wiley Online Library on [13/09/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

References

1. A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals and Systems* (Prentice Hall, 1997).

2. S. C. Bloch, *Introduction to Classical and Quantum Harmonic Oscillators* (Wiley, 1997).

3. L. O. Chua, C. A. Desoer, and E. S. Kuh, *Linear and Nonlinear Circuits* (McGraw-Hill, 1987).

4. L. Cohen, *Time-Frequency Analysis* (Prentice-Hall, 1995).

5. L. Galleani, "The Transient Spectrum of a Random System," *IEEE Transactions on Signal Processing* 58, no. 10 (Oct. 2010): 5106–5117.

6. L. Galleani, "The Time-Frequency Poles of a Random System," in *Proceedings of the 2011 IEEE Statistical Signal Processing Workshop (SSP)* (IEEE, June 2011).

7. E. P. Wigner, "On the Quantum Correction for Thermodynamic Equilibrium," *Physical Review* 40 (1932): 749–759.

8. B. Boashash, Ed., *Time-Frequency Signal Analysis and Processing: A Comprehensive Reference*, 2nd ed. (Academic Press, 2015).

9. W. Martin and P. Flandrin, "Wigner-Ville Spectral Analysis of Nonstationary Processes," *IEEE Transactions on Acoustics, Speech, and Signal Processing* ASSP-33, no. 6 (1985): 1461–1470.

10. C. J. Savant, *Fundamentals of the Laplace Transformation* (McGraw-Hill, 1962).

11. Z. Qiao, Y. Lei, J. Lin, and S. Niu, "Stochastic Resonance Subject to Multiplicative and Additive Noise: The Influence of Potential Asymmetries," *Physical Review E* 94, no. 5 (Nov. 2016): 052214.

12. C. Nicolis, "Stochastic Resonance in Multistable Systems: The Role of Intermediate States," *Physical Review E* 82, no. 1 (July 2010): 011139.

1350911x, 2026, 1, Downloaded from https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/ell2.70715 by Lorenzo Galleani - Cochraneltalia , Wiley Online Library on [13/09/2026]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License