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Novel Modeling Technique of Metamaterials Using One-Dimensional Finite Elements

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Abstract. Geometry-driven metamaterials offer negative Poisson's ratios, vanishing shear stiffness, and programmable multi-stability, but their slender struts and large rotations overwhelm conventional finite-element meshes. A streamlined framework that couples classical finite elements with the Carrera Unified Formulation (CUF) resolves cross-sectional deformation through higher-order beam expansions, reproducing three-dimensional behaviour while retaining a one-dimensional model. Contrary to the conventional beam definition, in this study the structural beam axis is aligned with the thickness of the sandwich panel, while the cross-section extends across its width. This particular application is useful to highlight the flexibility of CUF in modelling any structural configuration as an advanced 1D element. Representative static linear analyses are presented of the hexagonal metamaterial design.

Introduction

In the past decade, lattice-based metamaterials have risen to prominence as engineered architectures whose macroscopic behaviour is dictated far more by geometry than by chemistry. Built from periodically replicated unit cells (UCs), these structures are meticulously tuned to deliver unprecedented mechanical responses. For instance, by judiciously sculpting the UC, targeted properties can be achieved—from auxetic systems with a negative Poisson's ratio [1, 2] to pentamodes that suppress shear almost entirely [3], or non-linear responses can be elicited through large bending deformations and elastic instabilities. Because the global behaviour of a lattice hinges on its microscopic architecture, many studies have sought closed-form links between UC geometry and effective properties. Pioneering work based on Bernoulli–Euler and Timoshenko theories yielded analytical expressions for the elastic moduli and Poisson's ratio of cuboid microlattices [4].

Transitioning to three dimensions unlocks richer behaviours: 3D NPR lattices have been conceived through both additive manufacturing [5] and mechanical assembly. Li's 3-D re-entrant lattice, experimentally validated in compression [6], and its augmented counterpart with extra struts, capable of wide-range tunability [7], are prime examples. The interlocking-assembly route further reduces fabrication overheads compared to layer-wise printing [8]. Accurately simulating the rich mechanics of such structures requires numerical tools capable of resolving rigid-body rotations, large strains, and steep stress gradients without prohibitive cost. Conventional FEM, though ubiquitous, struggles: reproducing the fine architectural details typically forces excessively dense meshes and long runtimes. A general solution is presented by integrating classical FEM with the Carrera Unified Formulation (CUF) [9]. CUF enriches one-dimensional finite elements through tailored expansion functions that faithfully recover cross-sectional deformation modes. When Lagrange expansions are employed, the method accommodates arbitrarily intricate cross-sections—making it especially suited to the convoluted geometries of metamaterials [10].

Metamaterial structure

A metamaterial structure is used as a benchmark to demonstrate the CUF-based modelling capabilities. The geometry of a generica metamaterial is reported in Fig. 1. In this study,

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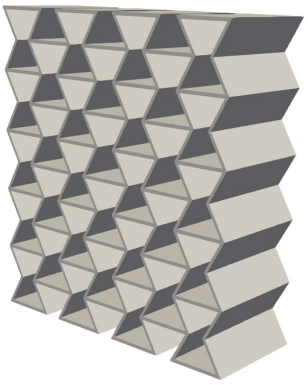


Figure 1: Re-entrant metamaterial structure.

exploiting CUF capabilities, the whole structure is modelled as 1D beam elements. The beam axis is aligned with its width, leaving the conventional beam definition, as reported in Fig. 2. The cross-sections of the metamaterial is discretized using high-order Lagrange polynomials;

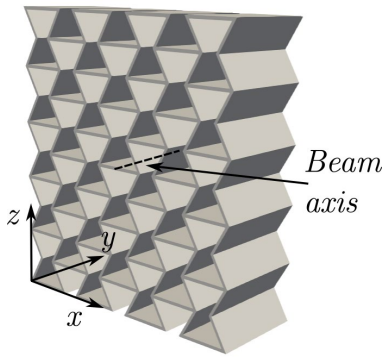


Figure 2: CUF model of the re-entrant metamaterial structure.

this discretization ensures detailed representation of geometry and its mechanical behaviour. To maintain displacement continuity between the domains, shared Lagrange points are defined at their interfaces, as reported in Fig. 3. The precise alignment of shared Lagrange points eliminates any potential discontinuities at the interfaces, enabling an accurate representation of the coupling the elements. The use of high-order Lagrange polynomials enables

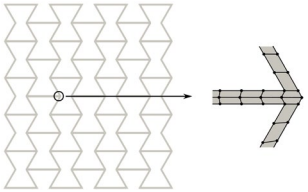


Figure 3: Lagrange elements for the discretization of the re-entrant metamaterial structure.

enhanced cross-sectional resolution, capturing thoroughly the displacement field. This is achieved by significantly reducing DOFs without compromising accuracy with respect to the high fidelity 3D FEM simulations, as widely discussed in the following. To perform comparisons, the same metamaterial structure has been analysed by means of commercial FEM

software, i.e. Abaqus. In CUF, the displacement field u(x, y, z) for a 1D beam is represented as:

$$\boldsymbol{u}(x,y,z) = F_\tau(x,z)N_i(y)\boldsymbol{u}_{\tau i} \qquad \tau = 1,2,\dots,M \quad i = 1,2,\dots,N_n$$

Materials Research Proceedings 69 (2026) 1379-1383 https://doi.org/10.21741/9781644904251-240

where $F_\tau(x,z)$ denotes the expansion functions in the cross-sectional plane, $N_i(y)$ the shape functions along the beam axis, and $\boldsymbol{u}_{\tau i}$ the nodal unknowns. As earlier mentioned, Lagrange polynomials are used as $F_\tau(x,z)$.

Numerical results

The numerical study focuses on a metamaterial with hexagonal topology (Fig. 4). Three modeling strategies are considered: (i) a 2D Abaqus model discretized on the surface (inplane section), (ii) a 2D Abaqus model discretized across the width (transverse section), and (iii) a 3D model based on the Carrera Unified Formulation (CUF), which explicitly represents the metamaterial's three-dimensional geometry. The structure is subjected to a uniform transverse pressure and is clamped along one boundary, with all remaining edges free. For each model, a mesh-refinement study was performed. Table 1 reports the total number of degrees of freedom (DOFs) together with the lateral displacement values evaluated at a selected control point. The entries that achieved mesh convergence are highlighted in blue in the table. Compared with the 2D Abaqus reference discretizations, the CUF model reaches the converged response with roughly 20% of the DOFs, indicating a substantial reduction in problem size for a comparable accuracy. This reduction in DOF count translates into lower memory requirements and shorter solution times, while preserving the fidelity needed to capture the characteristic deformation mechanisms of the hexagonal architecture. Finally, Fig. 5 illustrates the deformed configurations (plotted with an exaggerated scale for clarity). The three modeling approaches yield consistent global deformation patterns and confirm the expected stiffness trends under transverse loading, further validating the efficiency and accuracy of the CUF-based 3D representation for this class of metamaterials.

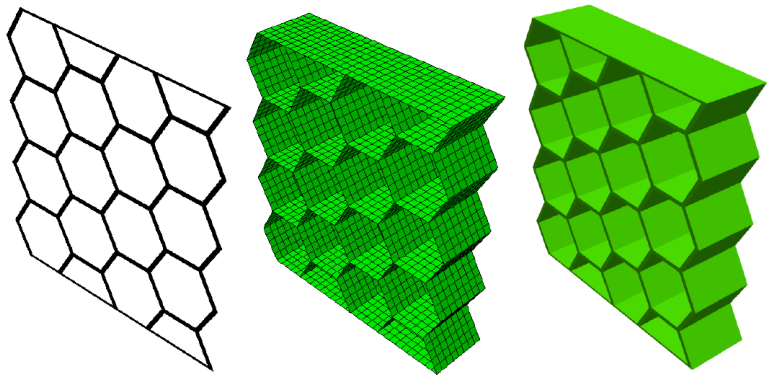


Figure 4: Numerical models of the hexagonal metamaterial. 2D Abaqus models (left and center), CUF 1D model (rigth)

Materials Research Proceedings 69 (2026) 1379-1383 https://doi.org/10.21741/9781644904251-240

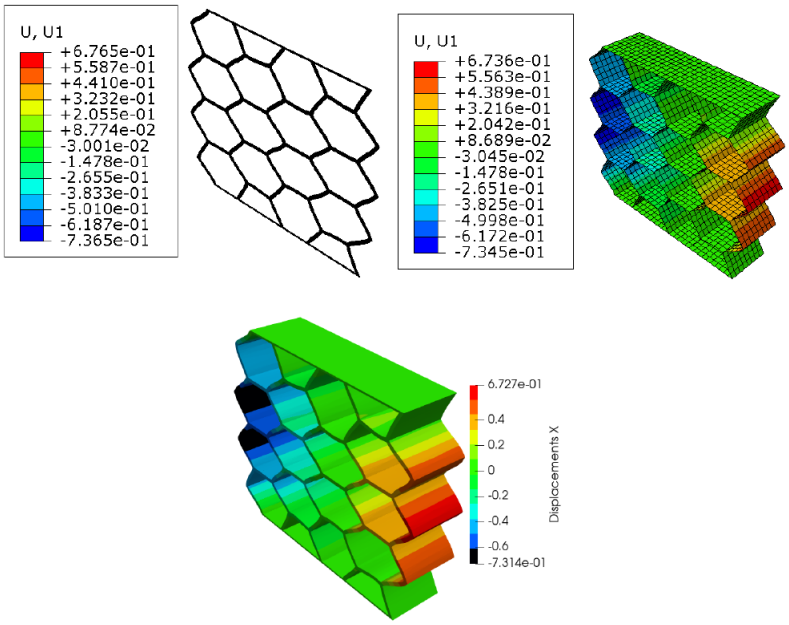


Figure 5: Numerical results of the hexagonal metamaterial. 2D Abaqus models (top), CUF 1D model (bottom)

Table 1: DOFs and lateral displacement of the considered analysis. In blue the converged models.

Model	DOFs	Lateral displacement
Abaqus 2D (Fig. 4 left)	16902	0.6234
	98154	0.6765
	291930	0.6769
Abaqus 2D (Fig. 4 center)	34830	0.6521
	95724	0.6736
	677538	0.6731
CUF 1D (Fig. 4 rigth)	19674	0.6727
	29511	0.6725
	118434	0.6724

Conclusions

This work presented a numerical study of a hexagonal metamaterial structure using the Carrera Unified Formulation (CUF) to achieve accurate and efficient modeling. The key findings and contributions of this research are summarized as follows.

- The use of CUF enabled the development of enhanced one-dimensional finite elements with three-dimensional capabilities. This allowed for precise detection and prediction of deformations across the structure's cross-section, providing a robust framework for analyzing future complex mechanical behaviors.
- The proposed model accurately captured the static behavior of the hexagonal metamaterial, comparing the results with Abaqus results. The gain in terms of number of DOFs is high.
- Metamaterial structures, featuring any cross-sectional geometry across their width, can be modeled easily using CUF. This capability opens up possibilities for adapting the model to further investigate three-dimensional metamaterials.

The findings of this study provide a robust foundation for future research into the design and optimization of metamaterial structures.

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