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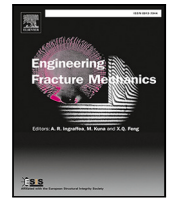
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Tooth root bending fatigue: An open source framework to model crack's nucleation and propagation

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ABSTRACT

This work presents a validated modeling approach to predict fatigue failure of gear teeth, both crack nucleation to final propagation, using the Extended Finite Element Method (XFEM) within an open-source framework. Rooted in classical formulations and modern standards, the methodology integrates analytical stress evaluations based on ISO 6336 with advanced numerical simulations, focusing on multiaxial stress states in the critical tooth root region. Through the use of XFEM, the model successfully simulates crack initiation and growth paths under realistic cyclic loading, enabling the reconstruction of the stress-life curve for a tested gear configuration. The framework was validated through comparison with experimental results from controlled fatigue tests conducted under two different load scenarios, complemented by ISO 6336-based theoretical predictions. Although some limitations in experimental data constrained the full quantitative validation, the model demonstrated solid agreement in trend and propagation behavior. The visualization and interpretation of the simulation outputs has been enhanced by specific data processing. The key innovation of this study lies in its combination of analytical theory and enriched finite element techniques in an open-source environment, providing a versatile tool for researchers and engineers. The proposed workflow supports fatigue life estimation with practical relevance for structural health monitoring, and gear design optimization. While currently tailored for spur gears, the methodology can be extended to other gear types and components subjected to cyclic mechanical stress.

1. Introduction

Gears are among the most fundamental components in mechanical engineering, designed to transmit torque and rotational motion efficiently [1]. The meshing of teeth enables precise control of torque conversion, speed variation and motion direction, which explains their widespread use in critical applications such as automotive drivetrains, aerospace propulsion systems and heavy industrial machinery [2]. The working principle of gears relies on the repeated engagement and disengagement of meshing teeth, a process that subjects them to complex cyclic loading conditions [3]. Although the performance of a gear system depends on the collective action of the meshing pair, the mechanical response of individual teeth under load frequently determines the overall durability and reliability not only of the single component but of the entire mechanical system. Multiple failure modes can compromise gear performance. In conditions of high contact pressure combined with insufficient lubrication, surface damage becomes dominant, giving rise to phenomena such as wear [4], scuffing [5], pitting [6,7], and micropitting [8]. These failure modes

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Nomenclature

Nomenclature table

Symbol	Description	Units
a, b	Initial crack semi-major and semi-minor axis lengths	mm
C, m	Paris law material propagation constants	mm/cycle, $\text{MPa}\sqrt{\text{m}}$
E	Young's modulus	MPa
K_I, K_{II}, K_{III}	Mode I, II, and III stress intensity factors	$\text{MPa}\sqrt{\text{mm}}$
$K_{I,C}$	Fracture toughness (critical stress intensity factor)	$\text{MPa}\sqrt{\text{mm}}$
ΔK_{eq}	Equivalent stress intensity factor range	$\text{MPa}\sqrt{\text{mm}}$
ΔK_{th}	Threshold stress intensity factor range	$\text{MPa}\sqrt{\text{mm}}$
$\sigma_{n,max}$	Maximum normal stress on the critical plane	MPa
$\Delta\tau/2$	Maximum shear stress amplitude on the critical plane	MPa
$\Delta\varepsilon_1/2$	Principal strain range amplitude	–
ν	Poisson's ratio	–
N_f	Number of fatigue cycles to failure	–
a	Crack length	mm
b	Fatigue strength exponent	–
c	Fatigue ductility exponent	–
n_{reff}	Effective mesh refinement level	–
σ_{res}	Residual stress field/tensor	MPa
G	Strain energy release rate	J/m^2
Subscripts & Abbreviations		
c, a	Critical plane angle	
eq	Equivalent	
I, II, III	Crack opening modes (Mode I, Mode II, Mode III)	
I, C	Mode I Critical condition (Fracture Toughness)	
max/min	Maximum/Minimum value	
STBF	Single Tooth Bending Fatigue	
SWT	Smith–Watson–Topper fatigue parameter	
th	Threshold	
XFEM	Extended Finite Element Method	

typically evolve gradually, degrading gear performance through increased noise, vibration, and transmission error. In contrast, Tooth Bending Fatigue (TBF) at the root is recognized as one of the most dangerous and catastrophic failure mechanisms [9]. Unlike surface-related failures, TBF does not provide early warning: cracks nucleate at the tooth root surface due to high localized bending stresses and propagate sub-surface under cyclic loading until the remaining cross-section is unable to sustain applied forces, resulting in the instantaneous detachment of the tooth [10]. This sudden failure makes TBF especially important to be considered in the design stage. From a structural perspective, TBF arises due to the particular loading condition of the gear teeth. Gear teeth can be idealized as cantilever beams [11], fixed at the root and loaded at the tip, which leads to maximum bending stresses being concentrated in the root fillet region [12,13]. The cyclic repetition of these stresses accelerates the nucleation of microcracks at or near the root surface. Once nucleated, cracks propagate along paths dictated by the local stress field until failure occurs. Because of this, the tooth bending strength has become a primary design criterion in the gear industry [14].

To facilitate safe design against TBF, widely adopted standards such as ISO 6336 [15] and ANSI/AGMA 2001 [16] provide analytical frameworks to estimate bending stresses at the root (σ_F) and define permissible stress levels (σ_{FP}) for standard materials and geometries. These standards rely on tabulated data and correction factors to account for geometry, size, and loading conditions. However, as specified in the ISO 6336-5 [17], for non-standard materials or components subject to special surface treatments, the permissible stress must be experimentally determined. In both cases, σ_{FP} is ultimately related to the fatigue limit of the material (σ_{Flim}), making the accurate determination of σ_{Flim} essential for reliable gear design. Several experimental methodologies have been developed to determine σ_{Flim} . The Meshing Gear (MG) test [14,18] is considered the most representative, as it uses complete gear pairs manufactured from the target material and, if needed, subjected to the same surface treatments as in service. This approach reproduces the real stress cycles experienced in operation, including the effects of geometry, manufacturing techniques, lubrication, and surface finishing [19,20]. However, MG tests are costly and time-intensive: each gear pair provides only a single point on the stress-life (S-N) curve, since specimens become unusable after one single test since one tooth detaches but also all the others have been damaged. To reduce testing costs, some researchers employ notched specimens [21–23], which can be tested on standard uniaxial fatigue machines. While economical and versatile, this method sacrifices realism: the absence of actual gear geometry and lubrication means that significant influences are excluded. Correction coefficients are therefore introduced [24] to

reproduce real conditions, but results remain less reliable compared to MG testing. A widely adopted compromise is the Single Tooth Bending Fatigue (STBF) test [25–27]. In this method, two opposing flanks of an isolated tooth are cyclically loaded between anvils positioned at a given distance, usually calculated according to the Wildhaber length [28], or with other methods (asymmetric configuration) [29], ensuring equilibrium of applied forces. STBF tests retain many of the benefits of MG testing, such as representing actual gear teeth geometry, while also reducing cost and time requirements. Indeed, they allow multiple tests to be conducted on a single gear specimen (since only two teeth per test are damaged), they eliminate the need for lubrication during testing [30], and the time required for the test is directly correlated to the pulsator machine operating frequency. These advantages make STBF [31] an attractive experimental configuration for estimating bending fatigue strength. However, challenges remain in converting applied force levels into material fatigue limits. This process typically relies on the inverse application of standards, which requires redefinition of certain parameters and may introduce significant uncertainty.

Alongside experimental approaches, numerical simulations have become increasingly important for evaluating gear bending fatigue, especially when the reference standards struggle to keep up with advancements in gear design and the new challenges that are brought up. The Finite Element Method (FEM) is particularly powerful, as it enables detailed stress and strain calculations in the tooth root region through fine discretization of the geometry [32]. FEM not only provides insights into stress distributions during complex loading cycles but can also be used to replicate the boundary conditions of experimental configurations such as in STBF tests. Moreover, FEM can be integrated with multiaxial fatigue criteria to predict fatigue damage. Among these, critical plane models have gained prominence for their ability to predict crack's nucleation point and orientation. These models assume that fatigue damage is governed by stresses or strains acting on a specific material plane, the critical plane [33]. By rotating stress and/or strain tensors at each material point, the location and orientation of the critical plane can be identified, allowing predictions of crack nucleation sites, initial crack paths, and fatigue life. Research has shown the potential of these combined approaches. For instance, FEM simulations of STBF tests on AISI 9310 gears demonstrated that stress-based criteria, particularly Findley's model [34], offered accurate predictions of crack nucleation sites and initial orientations under High-Cycle Fatigue (HCF) conditions, as presented in [35] and in [36]. Complementary studies using strain-based criteria revealed that models emphasizing maximum normal strain range outperformed shear-based criteria, especially for reproducing crack trajectories [37,38]. Together, these findings confirm the effectiveness of critical plane approaches in predicting bending fatigue behavior. Such approaches allow for a more accurate characterization of fracture mechanisms and facilitate the extraction of detailed data from simulations, including the number of cycles to crack nucleation and subsequent propagation.

Up to date numerical techniques, such as the Extended Finite Element Method (XFEM), enable the possibility to develop more advanced numerical models capable of considering various factors to better study and develop fail-safe designs to mitigate damages due to TBF. The crack nucleation point is of great influence [38], since it gives an indication of the possible followed path. 2D simulations are enough to predict it with the due approximations in terms of initial nucleation, determined by the maximum Von-Mises stress, and initial length. However, 3D simulations not only better represent the evolution of the entire phenomena but increase the accuracy of the acquired data in terms of cycles and propagation rates in correlation with the specific nucleation site [39]. Furthermore, still throughout XFEM simulations, it is possible to study the crack's path in correlation with the rim and tooth thickness [40] marking the possibility to develop a fail-safe design and to avoid catastrophic failures. Other works in literature investigated the influence of residual stresses and hardness gradients on carburized gears [41] exploiting XFEM ability to deal with cracks crossing a multi-material layered mesh.

Nevertheless, open challenges remain. In particular, in this work, a combined numerical–experimental framework is proposed to improve the assessment of gear tooth root bending fatigue. Rather than relying solely on the inverse use of standards to translate applied forces into material fatigue limits, FEM simulations of the STBF configuration are employed to directly compute the stress state throughout the loading cycle. The resulting stress tensors are then evaluated using a comparison between multiaxial fatigue criteria. Findley's [34] and Smith Watson Topper's (SWT) [42] critical plane approaches are able to derive Damage Parameters (DP) from stress and/or strain tensors and relate them to fatigue life to crack initiation through relationships of the Basquin-Coffin-Manson [43] type. Moreover, these criteria can be applied to successfully predict the point of crack nucleation and the initial trajectory of the crack. Subsequently, XFEM based numerical simulations, in combination with a local refinement algorithm, will be used to produce reliable results in terms of propagation cycles and path, crack shape and stress evaluation along its bottom. The approach presented in this article, starting from the limitations and approximations present in literature, aims to address and provide a comprehensive structure able to describe the entire tooth root breaking phenomena.

While existing gear fatigue modeling workflows rely heavily on expensive, proprietary software suites (e.g., ABAQUS integrated with FE-SAFE and FRANC3D), the primary contribution of this work is the establishment of a fully integrated, open-source toolchain based on Code_Aster and Salome_Meca. Rather than proposing foundational modifications to underlying finite element or fracture mechanics solver kernels, this study focuses on the architectural coupling and end-to-end automation of the gear fatigue life prediction process. By developing custom Python scripts to bridge 3D contact FEA, multiaxial critical-plane nucleation search (Findley and SWT), automated sub-millimetric seed crack insertion, and adaptive XFEM propagation, this framework offers a accessible and scriptable alternative for tooth root bending fatigue analysis.

2. Materials and methods

In order to develop a simulation tool able to completely describe the TBF failure, by taking advantage of the strengths and accuracy of XFEM, the study has been divided in three parts:

Table 1
Characteristic dimensions of the gear.

Parameter	Symbol	Value	Units
Module	m	3	mm
Number of teeth	z	20	/
Pressure angle	α	20	°
External diameter	D_e	68	mm
Primitive diameter	D_p	60	mm
Base Diameter	D_b	52.5	mm
Key	CH7	6	mm
Internal diameter 1	B	22.8	mm
Internal diameter 2	AH7	20	mm
Tooth width	H2	28	mm
Core width	H1	30	mm
Core diameter (tolerance)	$D_m(h8)$	30	mm

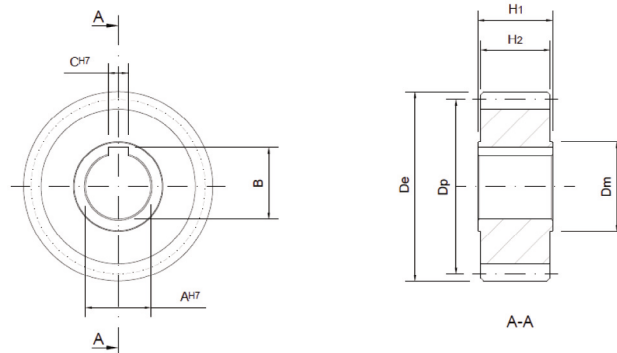


Fig. 1. Gear geometry, lateral and frontal view.

1. an experimental part to produce points suitable for the validation of the developed fatigue modeling framework;
2. a first simulation part that focuses on identifying the nucleation site of the crack and the corresponding required number of cycles;
3. a second simulation part, that starts from the outputs of the nucleation simulation and aims at evaluating the propagation of the crack until reaching a critical point where it causes a sudden fracture.

The two simulation parts faithfully replicate the STBF setup in order to reconstruct the same stress and strain states developed during the tests.

Code_Aster natively provides standard 3D elastic solvers, contact mechanics algorithms, and built-in XFEM level-set propagation routines (PROPA_FISS). The custom framework developed in this study provides the high-level automation, scripting, and inter-module coupling: auto-extracting 3D stress tensors, executing Findley critical-plane and SWT calculations at tooth root notches, generating the 3D seed crack geometry at the critical tensor location, and managing adaptive h -mesh refinement along the evolving crack front.

2.1. Experimental setup

Two STBF tests have been performed on spur gears made of case hardened 20MnCr5 steel (SAE5120, elastic modulus $E = 194$ GPa [44]) to generate a set of reference values for the validation of the simulations and validate them. In Table 1 and Fig. 1 the geometrical data are reported, meanwhile, in Fig. 2 the in-depth hardness profile is reported. The tooth trochoid has been measured using a profilometer (Fig. 3 - left). Subsequently, the acquired point have been interpolated and used to recreate the tooth profile through a dedicated script and CAD routine (Fig. 3 - right).

A symmetrical setup similar to the one shown in Fig. 4 was adopted. Two sinusoidal loading scenarios, of 34.5 [kN] and 36.5 [kN], have been applied at a frequency of 174 [Hz]. Also, to avoid undesired movements, a locking pin and a load ratio of $R = 0.1$ have been used to keep the gear in place. The highest point of single tooth contact, according to a Wildhaber's length [28] of 3 teeth, has been identified by the tooth geometry.

2.2. Crack nucleation simulation

The crack nucleation simulation focused on reproducing the load and stress conditions characteristic of the contact between the punch and the tooth flank to find the nucleation point and critical plane's orientation. This involved meshing the geometry

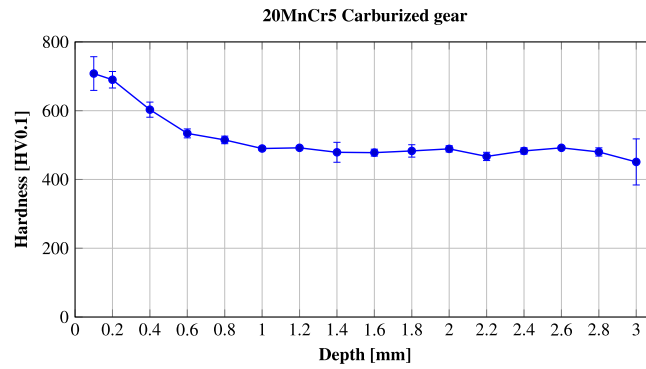


Fig. 2. Hardness' evolution.

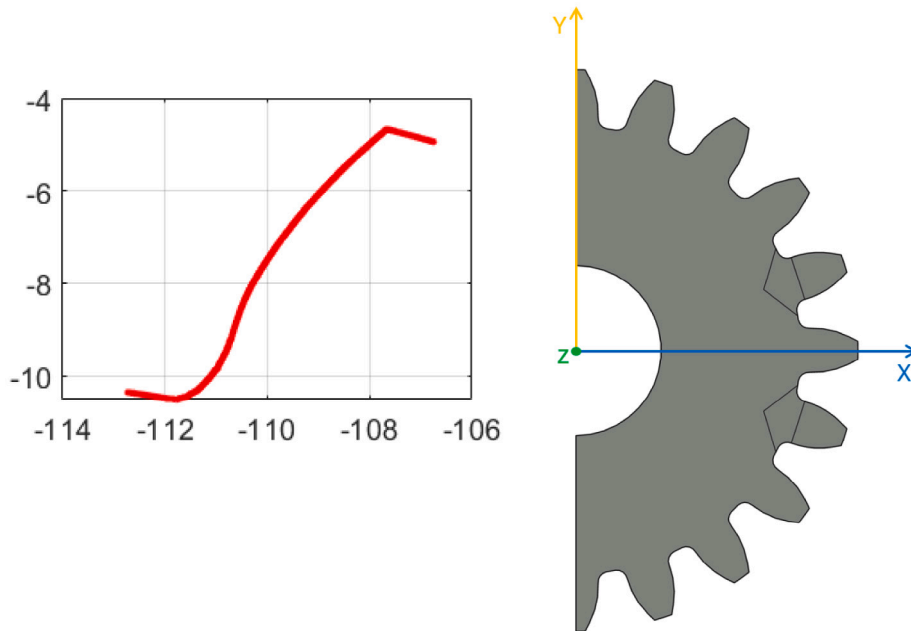


Fig. 3. Tooth and gear reconstruction from Profilometer data to CAD.

reconstructed in Fig. 3 and applying the same boundary conditions characteristic of the STBF. As shown in Fig. 5 only half gear was modeled to reduce the number of elements and improving the computational effort, without invalidating or approximating the full contact between the support punch and the forcing punch. A compressing pulsating nodal force has been applied at the top surface of the upper punch. It was bound to move only along the Y -axis to avoid spurious moment that might cause incorrect stress' distribution. Therefore, the dx and dz directions have been blocked. Since has been taken advantage of the geometrical symmetry of the gear, combined with the peculiar blocked position of the test, the dx , dy and dz displacements have been blocked along the vertical symmetry plane. The lower punch has been blocked by the lower face in all three displacements. The load was introduced as a ramp function, instead of a step function, in order to apply it gradually thus reducing numerical errors and improving model convergence.

In contact area and at the tooth root a mesh refinement was implemented to increase the accuracy of the results in the region of interest, further easing the computational effort and convergence analysis. More precisely, only the roots and flanks of the teeth identified by the Wildhaber length [28] (i.e., the teeth in contact with the punches) were refined. Hence, to model the contact between the punch and the tooth flank has been chosen an hexahedral sub - mesh with an average cell length for slave surface and master surface of $l_{slave} = 0.211629$ and $l_{master} = 0.421096$ respectively, as shown in Fig. 6. This difference between the two mesh sizes is needed to avoid poor approximation of contact pressure distribution, artificial oscillations in the Newton–Raphson convergence and numerical instabilities due to the proximity of nodes between the master and the slave surfaces. A mesh sensitivity analysis was performed to ensure the stabilization of the values. Once the simulation faithfully reconstructed the stress and strain histories developed in the tooth roots during the STBF tests, critical plane criteria for multiaxial fatigue analysis were employed to determine

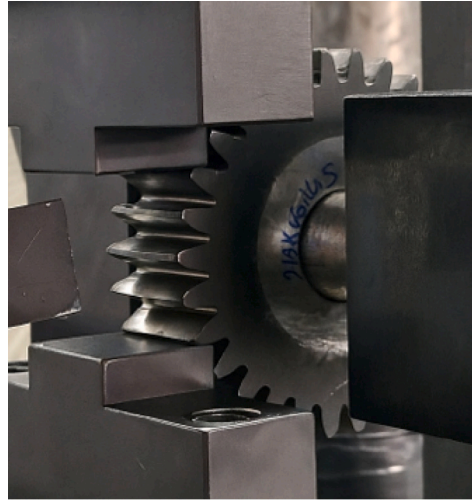


Fig. 4. STBF symmetrical setup.

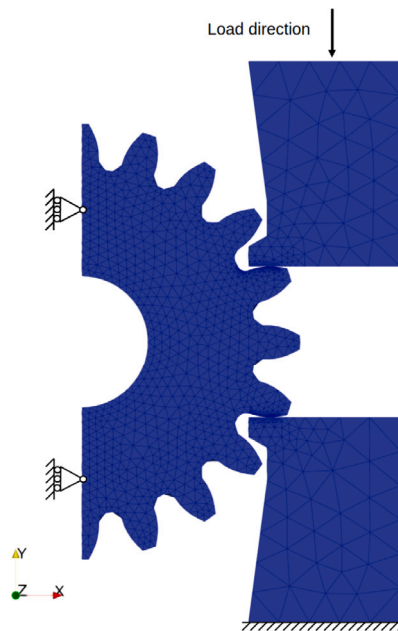


Fig. 5. Boundary conditions in the simulation environment.

the life to crack nucleation, the nucleation position of the crack and the initial crack trajectory. As previously mentioned, these criteria assume fatigue damage to be maximum on a specific material plane. To find such plane, for every material point under analysis, stresses and/or strain histories are rotated according to the reference system of every spatial plane passing through the material point. The plane featuring the worst combination of rotated stresses and/or strains is considered the critical plane. The material point where the critical plane is found is where the crack is supposed to nucleate, while the initial trajectory of the crack is supposed to lie along the critical plane itself. The combination of stresses and/or strains that is used to choose the critical plane depends on the criterion that is employed [45]. In particular, in this work Findley's criterion [46] has been chosen to determine the nucleation position and initial trajectory, because of its reliability and accuracy of the results when considering TBF [35,36].

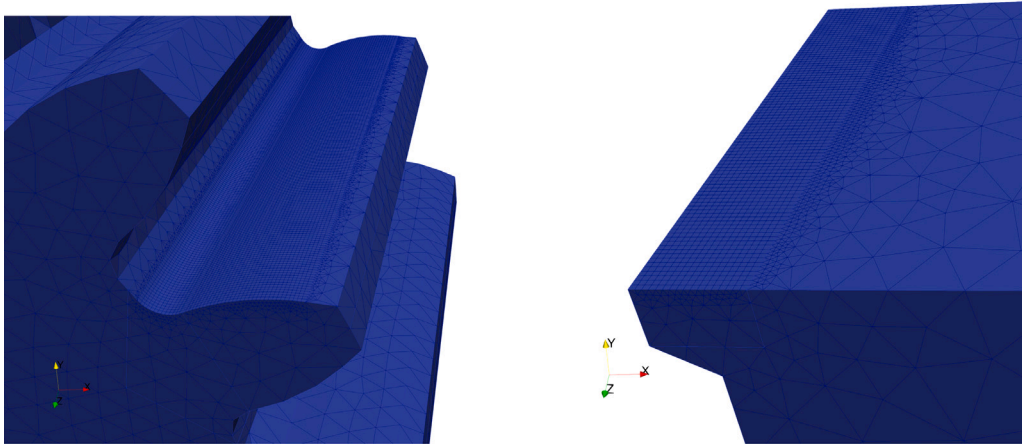


Fig. 6. Detail of the *master* and *slave* sub-meshes on the punch and gear contact regions.

According to Findley (1), the critical plane is the plane where the damage parameter DP_F is maximum:

$$DP_F = \tau_{c,a} + \frac{2r_{\tau/\sigma} - 1}{2\sqrt{r_{\tau/\sigma} - r_{\tau/\sigma}^2}} \cdot \sigma_{c,\max} \leq \frac{\tau_f}{2r_{\tau/\sigma} - 1} \quad (1)$$

where $\tau_{c,a}$ is the alternated shear stress on the critical plane, $\sigma_{c,\max}$ is the maximum stress normal to the critical plane and $r_{\tau/\sigma}$ represents the relationship between the material fatigue limit under fully reversed torsional loading τ_f and the material fatigue limit under fully reversed bending loading σ_f .

To evaluate the number of cycles needed for the crack to nucleate the Smith - Watson - Topper (*SWT*) [42] criterion, later modified for multiaxial fatigue state by Socie [47], has been considered.

This method identifies the critical plane as the plane featuring the maximum shear strain $\Delta\epsilon_{n,\max}$ and quantifies fatigue damage through DP_{SWT} , as shown in Eq. (2):

$$DP_{SWT} = \sigma_{n,\max} \cdot \frac{\Delta\epsilon_{n,\max}}{2} \quad (2)$$

where $\sigma_{n,\max}$ is the maximum stress normal to the critical plane. DP_{SWT} can be related to the fatigue life to crack initiation (N_f) through a relationship of the Basquin-Coffin-Manson [43] type:

$$\sigma_{n,\max} \cdot \frac{\Delta\epsilon_{n,\max}}{2} = \frac{\sigma_f'^2}{E} (2N_f)^{2b} + \sigma_f' \epsilon_f' (2N_f)^{2b+c} \quad (3)$$

The Roessle - Fatemi method [48] was implemented to obtain σ_f' , ϵ_f' , b and c needed in Eq. (3), where:

$$\sigma_f' = 4.25 \cdot HB + 225 \quad (4)$$

$$\epsilon_f' = \frac{0.32 \cdot HB^2 - 487 \cdot HB + 191000}{E} \quad (5)$$

with $-0.057 \leq b \leq -0.14$ and $-0.39 \leq c \leq -1.4$. HB is the Brinnell hardness of the material and has been retrieved from Fig. 2. Both Findley and *SWT* criteria were implemented in the Code_Aster environment through a Python script.

2.3. Crack propagation simulation

In the second simulation phase of the study a code able to follow the predicted propagated path from one tooth flank to the other was developed, reconstructing the complete fracture. Since the challenges to overcome and the requirements to fulfill are different to the ones faced in the first phase of the study, it was possible to pose more simplifying assumptions without influencing the results. Therefore, only one quarter of the gear and the upper punch were modeled and a lower refined mesh was adopted, decreasing the total number of elements from 83337 to 16988. The sub - meshes at the contact interface and at the root have been changed to a *tetrahedral* cell with an average size of $l_{slave} = 0.492095$ and $l_{master} = 0.535241$.

In the position where Findley's criterion predicted the nucleation of a crack, a crack of elliptical shape was inserted in the mesh with $a = 0.5$ [mm] and $b = 0.1$ [mm] as the length of the major and minor axis. In this manner it was possible to model a crack small enough to not create a distortion that could lead to a sudden breakage, but, at the same time, being able to appreciate the stress field at its tips [49].

The choice of the initial elliptical crack dimensions ($a = 0.5$ mm and $b = 0.1$ mm) to bridge the nucleation and propagation phases is consistent with current XFEM gear literature, where predefined initial sub-millimetric crack lengths ranging from 0.125 mm to 1 mm are widely utilized to evaluate structural trajectories [50,51]. Physically, this 0.5 mm depth represents an engineering boundary marking the transition from a short crack to a distinct macro-crack governed by Linear Elastic Fracture Mechanics (LEFM). However, it must be noted that because the nucleation phase accounts for the vast majority of the total life (75.43% and 91.80%), the overall deterministic output is highly sensitive to this initial dimension, as well as to the Paris constants (C, m) and the threshold (ΔK_{th}) adopted from literature. Since real tooth root cracks originate as short cracks within a steep stress gradient, utilizing long-crack parameters represents a simplification. A rigorous sensitivity study varying the initial crack aspect ratio alongside advanced short-crack closure models is necessary to further map out the exact numerical tolerances of the workflow. The main advantage of XFEM is its ability to model crack propagation without the need for complete remeshing after each crack's advancement or for an auxiliary mesh dedicated to the crack. XFEM is particularly effective when combined with a local mesh refinement algorithm, which progressively thickens the mesh around the crack tip. Specifically to this work, the so called h - refinement method was used to create a correlation between the mesh size and the iteration needed to obtain the desired refinement level, as expressed in Eqs. (6)–(9). The number of refinement steps needed to properly model crack growth (n_{ref}) is:

$$n_{ref} = E(n) \quad (6)$$

where $E(n)$ is the integer part of n :

$$n = \frac{\ln(h_0) - \ln(h_c)}{\ln(2)} \quad (7)$$

where h_0 is the mesh size around the crack tip and h_c mesh size after refinement ($a/10$ of the crack's depth). The number of iterations needed to reach n_{ref} is:

$$n_{iter} = n_{ref} + 1 \quad (8)$$

The average dimension of the most refined mesh h at the latest level of the algorithm is expressed by:

$$h = \frac{h_0}{2^{n_{ref}}} \quad (9)$$

To decide which mesh elements need to be affected by the refinement, an area of interest was set according to Eq. (10):

$$crit = -\max(r_{ref}, 2.0 \cdot \frac{h_0}{2.0^{n_{ref}}}) \quad (10)$$

The mesh refinement around the crack's front is fundamental to evaluate the energy release rate G according to the method Theta [52,53], as expressed in Eq. (11).

$$G = \frac{1 - \nu^2}{E} \cdot (K_I^2 + K_{II}^2) + \frac{K_{III}^2}{2\mu} \quad (11)$$

where E is the Young's modulus, ν is the Poisson's coefficient, $\mu = \frac{E}{2(1+\nu)}$ and K_I , K_{II} and K_{III} are the values of the stress intensity factor respectively for Mode I, Mode II and Mode III crack opening modes. The critical stress intensity factor (fracture toughness) necessary to advance the crack front was evaluated according to (12):

$$K_{I,C} = Y \sigma_{prin,max} \sqrt{\pi a} \quad (12)$$

where $\sigma_{prin,max}$ is the maximum principal stress characteristic of the tooth's bending phenomena, Y is the crack's shape correction factor. When $K_{I,C}$ reaches the threshold value K_{th} , representing the fracture toughness of the material, the crack propagates.

The crack advancement per cycle (da/dn) and the number of fatigue cycles, for each iteration, have been evaluated accordingly to Paris' Eq. (13):

$$\frac{da}{dN} = C \cdot (\Delta K_{eq} - \Delta K_{th})^m \quad (13)$$

where ΔK_{eq} is the equivalent stress intensity factor range at the crack tip, ΔK_{th} is the threshold stress intensity factor, C and m are material parameters.

To model the crack propagation phase via XFEM, the Paris law parameters (C, m) and the threshold stress intensity factor range (ΔK_{th}) for the 20MnCr5 case-hardened steel were adopted from literature calibrated under Single Tooth Bending Fatigue (STBF) testing conditions [54], rather than measured directly for the specific material batch. It must be noted that in gear tooth roots, cracks originate as short cracks within steep stress gradients. In these early stages, microstructurally and physically short cracks can propagate below the conventional long-crack threshold ΔK_{th} at rates higher than those predicted by standard Paris parameters. In the current framework, this early short-crack regime is implicitly absorbed into the critical-plane nucleation phase (Findley/SWT), which accounts for the vast majority of the fatigue life (75.43% and 91.80% for the two tested load levels). Consequently, the total life prediction exhibits high sensitivity to both the chosen transition geometry ($a = 0.5$ mm, $b = 0.1$ mm) and the fracture parameters ($C, m, \Delta K_{th}$). Variations in C and m directly scale the propagation velocity in the LEFM regime, while changes to the initial depth a alter the stress intensity factor range ΔK_{eq} at the onset of XFEM tracking. While the present workflow provides a robust engineering baseline for process integration, incorporating dedicated short-crack closure models and performing a multi-parameter sensitivity

Table 2

Nucleation simulation results. ϕ_{SWT} and θ_{SWT} are the polar coordinates of the critical plane. X, Y and Z are the coordinates of the initial location of the crack.

Load	ϕ_{SWT}	θ_{SWT}	DP_{SWT}	X	Y	Z	N_f
36.5 [kN]	157.5°	72°	3.1698	24.25	10.88	-1.12	85 563
34.5 [kN]	157.5°	72°	2.8361	24.25	10.88	-1.12	139 638

study are essential next steps to refine the transition boundary and quantify prediction tolerances. Meanwhile, the direction of the propagation is determined by recalling the Maximum Hoop Stress criterion [55] to calculate the deviatoric angle according to Eq. (14):

$$\beta = 2 \arctan \left[\frac{1}{4} \left(\frac{K_I}{K_{II}} - \text{sign}(K_{III}) \sqrt{\left(\frac{K_I}{K_{II}} \right)^2 + 8} \right) \right] \quad (14)$$

To monitor, evaluate and justify the development of the propagation in 3D has been taken in consideration the values of the three stress intensity factors for each opening mode (K_I , K_{II} and K_{III})

3. Results and discussion

The two tests described in Section 2.1 returned a total bending fatigue life of 185121 cycles (test at 34.5 kN) and 93203 cycles (test at 36.5 kN). These lives represent the reference values for the validation of the simulation results.

3.1. Nucleation results

The simulations returned a value of the maximum stress at the tooth root of $\sigma_{F,36.5} = 1195.21$ and $\sigma_{F,34.5} = 1131.63$. Only the maximum principal stress ($\sigma_{\text{prin,max}}$) is considered since it is the one that represents better the stress field due to the complexity of the tooth bending phenomena [56]. These values can be compared with the theoretical values of the stress at the tooth root calculated in accordance with Method B of ISO 6336 - 3 [57], as expressed in (15) and (16):

$$\sigma_F = \sigma_{F0} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha} \quad (15)$$

$$\sigma_{F0} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \cdot Y_\beta \cdot Y_B \cdot Y_{DT} \quad (16)$$

K_A , K_V , $K_{F\beta}$ and $K_{F\alpha}$ coefficients are all equal to 1 in case of STBF testing [29], meanwhile, the form factors have been evaluated at $Y_F = 1.8$ and $Y_S = 1.7$ [15], meanwhile Y_β , Y_B and Y_{DT} are equal to 1 in the case of spur gears. It follows that the resulting calculated tooth root stress for the two tested loading scenarios are 1329.642 [MPa] and 1256.785 [MPa], for 36.5 [kN] and 34.5 [kN] respectively. The discrepancy between numerical and ISO 6336 values are 10.1% and 9.95%, respectively. According to [58,59] this discrepancy is in an acceptable range since the ISO 6336 [15] tends to underestimates the strength of teeth due to how Y_F and Y_S are calculated. The results from the nucleation simulation are reported in Table 2 and shown in Fig. 7.

The results from the nucleation simulation shows that the number of cycles needed for the crack to form are the 75.43% and 91.80%, respectively, of the total fatigue life of the gear tooth, in line with the referring literature [60–62]. The choice of the “scanning” angle, used to identify the critical plane in the simulation, influences the returned orientation and consequently the number of cycles needed to the crack to nucleate and propagate.

3.2. Propagation results

The propagation simulation returned 17162.873 cycles at 34.5 [kN] and 14570.643 cycles at 36.5 [kN]. These values are reported with numbers in the decimal order since the crack's advance is a fixed value for each step. Consequently, due to the specific loading and geometrical conditions the ratio $\frac{da}{dN}$ cannot be a Natural number.

Observing Fig. 8 is possible to see the descending trend of the number of cycles needed to propagate the crack of da for each step. This is due to the fact that the effect of (12) is calculated once a crack is inserted and is not recalculated for each step. It is a limit value that sets the required stress to start the propagation. After reaching and surpassing that threshold, the crack is considered to be in a stable growing region, identified by the almost linear trend, until reaching a critical point where it causes a sudden fracture.

In Fig. 9 is possible to see a first form graphical representation of the results. On left side of the picture is shown the evolution of the propagation along the tooth fillet, followed by the local mesh refinement. Meanwhile, on the right side, the propagated crack is shown from inside the gear to better appreciate the evolving 3D front in the depth of the tooth and the variation of the maximum principal stress. (in the software used for the simulation SIEQ_NOEU_PRIN3 correspond the maximum principal stress evaluated at each node)

These stress concentration factor are calculated and combined for each point of the crack bottom by the (14) consequently determining the final shape of the crack.

Fig. 10 shows the evolution of the stress concentration factor K_I corresponding to the main opening mode.

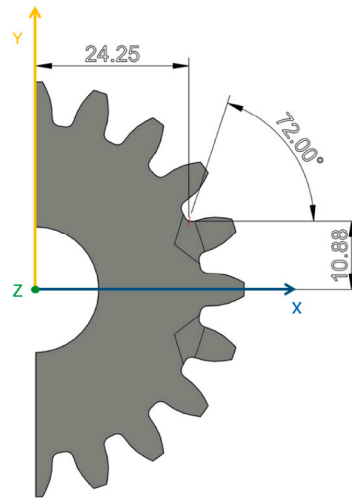


Fig. 7. Crack nucleation point (2D geometric coordinates).

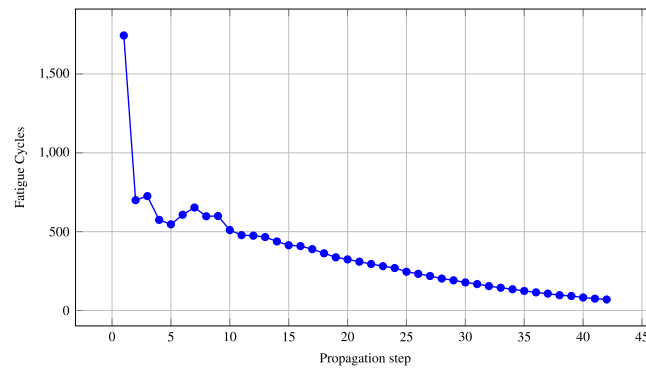


Fig. 8. Fatigue cycles required to advance the crack of da for each propagation step at 36.5 [kN].

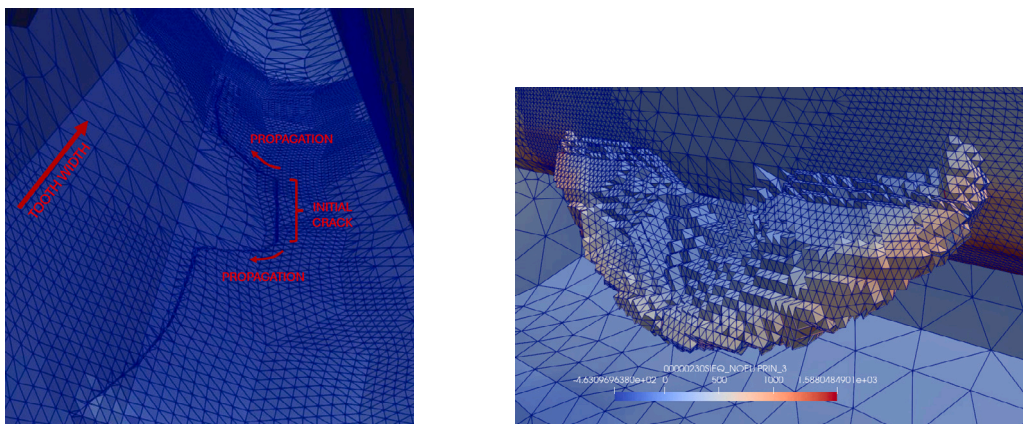


Fig. 9. Inside detail and trajectory of the propagated crack.

In Fig. 11, it is evident how towards the end the crack tends to assume an upwards direction due to the high values of K_{II} . This change in direction is due to the compressive, or the longitudinal, component of the load applied by the punch. The more the crack propagates, the more the tooth deformation increases leading to an higher force along the tooth's axis, causing the characteristic curvature of the crack front.

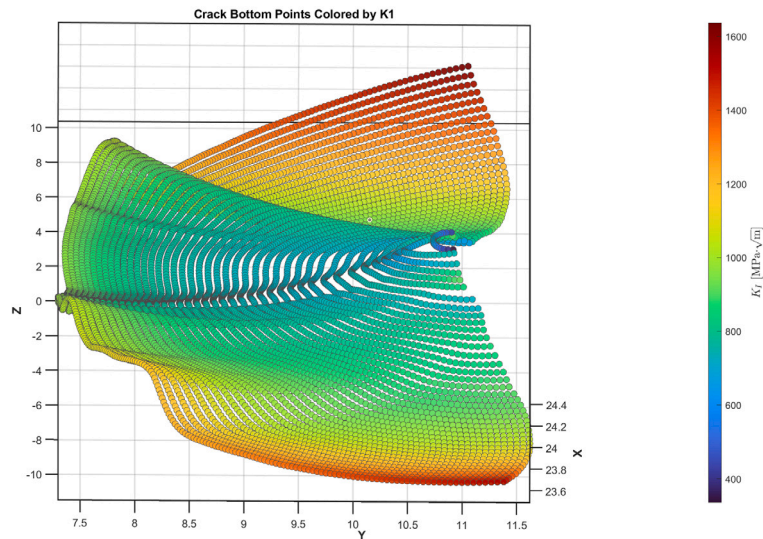


Fig. 10. K_I values for each node at the bottom of the propagated crack.

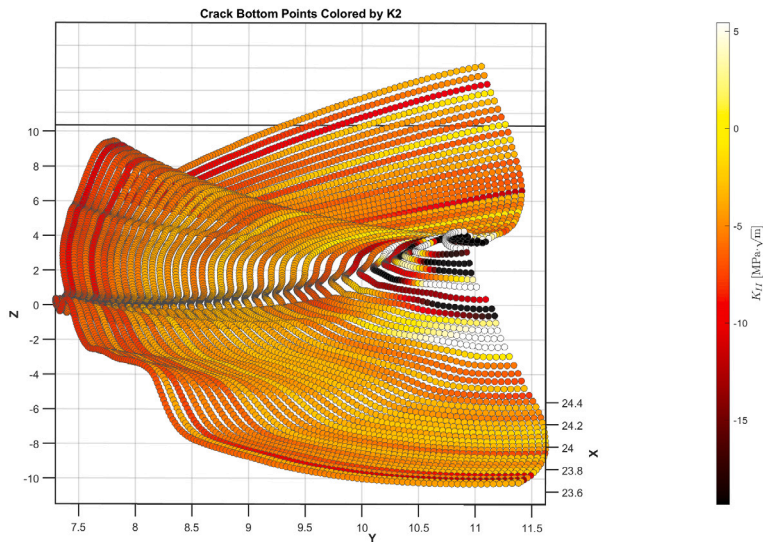


Fig. 11. K_{II} values for each node at the bottom of the propagated crack.

Table 3

Percentage difference statistics for K_I , K_{II} , and K_{III} under the two loading scenarios.

% Difference	ΔK_I [%]	ΔK_{II} [%]	ΔK_{III} [%]
Mean	6.14	7.94	33.04
Max	8.94	10.82	/
Min	1.94	-12.41	/

In Fig. 12 it is possible to distinguish a series of ripples in the surface of the crack. This is due to the out of plane shear evaluated by K_{III} . The percentage differences between the two simulated loading scenarios are reported in Table 3. The presence of very high max and minimum values for K_{II} and K_{III} is due to the fact that they have been evaluated comparing each neighboring point of two slightly different crack surfaces.

The same simulation was performed in 2D to better compare the crack path with the experiments present in literature for similar geometries [60,63], following the same principles as in Sections 2.2 and 2.3 as shown in Fig. 13.

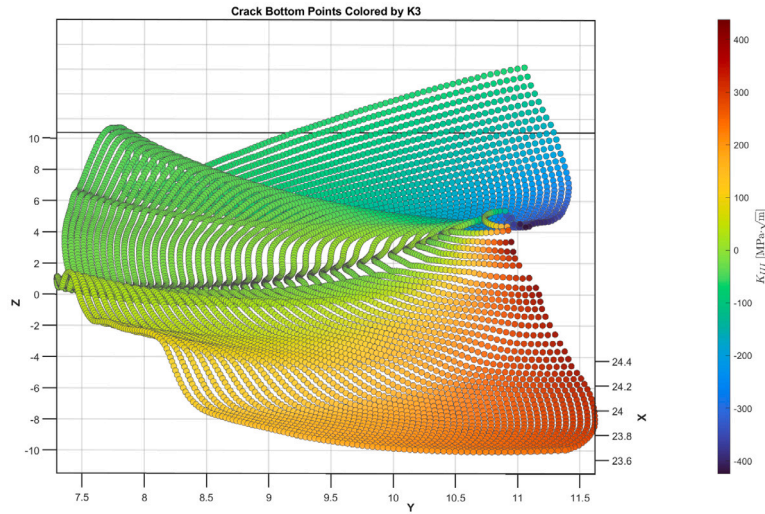


Fig. 12. K_{III} values for each node at the bottom of the propagated crack.



Fig. 13. Predicted propagation path in the 2D analysis.

The trajectory of the propagation path is comparable with the ones found in literature [60,63,64]. Combining the data obtained from the contact and nucleation simulations with the propagation results, it is possible to trace the Wöhler curve that correlates the component response to various stress loads levels in terms of fatigue cycles. In Fig. 14 the curves extracted by the simulations and the experimental tests are compared.

As it is possible to see in Fig. 14, the simulations overestimated the life endured during the test performed at 36.5 [kN] by 6.92% (100134 predicted cycles versus 93203 cycles from tests), meanwhile, the results at 34.5 [kN] return a 15.29% underestimation (156801 predicted cycles versus 185121 cycles from tests). These differences in the outputs can be attributed to: (i) Overall quality and accuracy of the initial data; (ii) Limited number of tests performed in the lab needed to properly characterize the points where to interpolate the S-N curve; (iii) Geometrical approximation and profile reconstruction; (iv) Availability of specific and precise material parameters and constants. In addition to mitigating these factors, energy-based approaches relying on local stored or dissipated energy provide a physically grounded link between cyclic deformation and crack initiation [65]. Likewise, short- and early-crack behavior often dominates the overall fatigue life, underscoring the need for explicit length-scale awareness [66].

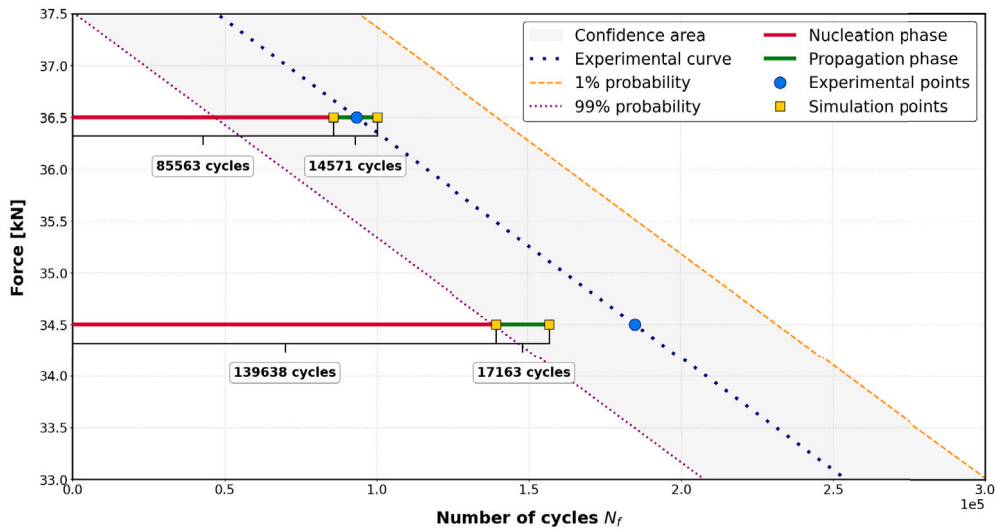


Fig. 14. Load-life curves from experimental tests (dashed line) and numerical simulations (continuous line).

Despite the mentioned approximations and limitations, the results fall in an acceptable range if the statistical evaluation of the experimental fatigue curves is considered in accordance with the limits prescribed by ISO 6336-5 [17], i.e., if failure probabilities of 1% and 99% are considered. These are represented in Fig. 14 as the yellow and purple lines respectively. The failure probabilities were calculated stating from the experimental curves. Specifically, to transport the reliability from 50% to 99%, since only one test per load scenario was performed, the standard suggests to reduce stresses in fatigue curves by multiplying them by the coefficient $Y_{NT} = 0.85$, obtaining a lower strength. Similarly, to calculate the reliability of 1%, the standard suggests to multiply stresses by the correction factor $Y_{NT} = 1.15$.

4. Conclusion

This work developed and validated an hybrid analytical-numerical methodology for predicting fatigue crack nucleation and propagation in the root of a spur gear teeth. By integrating classical stress analysis approaches, like Lewis formula [11] and the ISO 6336 standards [15], with advanced numerical techniques, such as the Extended Finite Element Method (XFEM), implemented in the open-source Code_Aster framework, the work provides a comprehensive approach for modeling gear fatigue failure.

The proposed simulation workflow is able to reconstructs the gear stress-life (S-N) curve and accurately tracks the progression of fatigue cracks under realistic cyclic loading conditions. The results demonstrated consistency with both standardized analytical predictions and experimental data, confirming the model reliability. In fact, the simulated stress at the tooth root differs only for 8.99% and 9.05%, for the respective loading conditions, with the reference value calculated by the standard [58,59]. The number of calculated cycles need for the crack to nucleate make up for the 75.43% and 91.80% of the total fatigue life of the teeth, in line with the results found in literature [60–62]. Therefore, the use of open-source tools such as Code_Aster and Salome_Meca ensures accessibility and adaptability for both academic and industrial applications.

The novelty of this work lies in coupling a theoretical fatigue theoretical model with a fully numerical crack growth simulation within an unified, open-source environment. This approach can serve as a foundation for further developments in predictive maintenance, twin technology, and fatigue-informed design processes.

It must be acknowledged that the current implementation of this open-source framework operates primarily as a geometry- and loading-driven workflow rather than a fully material-sensitive fatigue model. While the case-hardened state of the 20MnCr5 steel introduces strong microstructural, hardness, and residual stress gradients from the root surface to the core [67], these coupled material-state effects are not yet explicitly integrated into the local fatigue criteria or the crack propagation routine. Consequently, while the framework effectively captures the macro-geometric stress concentrations and loading conditions of the Tooth Root Bending Fatigue (STBF) test, it simplifies the local microstructural resistance to crack nucleation and the gradient-dependent driving forces during propagation. Acknowledging this limitation is critical when transferring this workflow to different surface treatments or alternative gear materials where local microstructure evolution, dislocation accumulation, and severe damage localization heavily influence the early stages of fatigue failure.

A primary experimental limitation of the current study is the lack of quantitative 3D fracture morphology and depth-wise crack growth rate measurements for the tested gears. Experimental monitoring was restricted to recording total cycles to structural failure and macro-crack root break-off. Consequently, while the simulated XFEM propagation paths exhibit qualitative alignment with standard literature trends for spur gear root failure [51], direct quantitative validation of the 3D crack front shape and localized

growth rates (da/dN) remains to be performed. Future experimental campaigns will incorporate 3D optical surface profiling and SEM fractography to rigorously benchmark the numerical crack front evolution. Despite its strengths, the model is limited to spur gears and assumes idealized loading conditions, neglecting other potential failure modes such as surface pitting or scuffing. Additionally, only a limited number of experimental data points were available for validation.

Future work could extend this methodology to different types of gears, components and loading scenarios. Despite the promising alignment between the numerical predictions and the empirical data, several limitations regarding the framework's current predictive scope must be acknowledged. Validating a fatigue model against only two STBF load levels (34.5 kN and 36.5 kN) represents an initial verification rather than a robust reconstruction of an experimental S-N curve, especially given the intrinsic statistical scatter typical of gear fatigue phenomena [68]. In deterministic modeling, local stress and strain fields are highly sensitive to variations in gear profile geometry, hardness gradients, and exact fatigue parameters. A comprehensive uncertainty analysis or a sensitivity study would be required to establish rigorous confidence intervals rather than a single deterministic curve. Furthermore, the transition from local cyclic deformation fields to early crack nucleation is a complex mechanism. While the utilized critical-plane criteria serve as reliable engineering indicators, more physically consistent bridges along with a dedicated length-scale awareness for short-crack behavior could further refine the accuracy and physical representation of the nucleation-to-propagation transition phase.

A key limitation of the present baseline model is the omission of heat treatment residual stress fields (σ_{res}), as specimen-specific X-ray diffraction depth measurements were not performed for the tested batch. Carburizing introduces significant surface compressive stresses that reduce the maximum normal stress ($\sigma_{n,\text{max}}$) in critical-plane criteria and induce crack closure ($\Delta K_{\text{eff}} < \Delta K$) during early propagation. Consequently, the current framework must be interpreted as a geometry- and external loading-driven tool that evaluates macro-scale notch fields rather than full metallurgical states. In open-source software like Code_Aster, residual stress fields can be introduced by mapping depth-dependent stress tensors ($\sigma_{\text{res}}(z)$) via initial field commands (CREA_CHAMP) or equivalent thermal strain fields prior to cyclic loading. Incorporating these initial stress fields remains a vital development path to expand the framework's application boundary to heavily surface-treated or shot-peened gear components.

Finally, it must be acknowledged that the primary contribution of this work lies in the engineering process integration and workflow synchronization within an open-source framework, rather than in the development of new numerical algorithms, XFEM formulations, or performance optimizations inside Code_Aster. The underlying XFEM capabilities and standard fatigue analysis modules are already built-in, existing functionalities of the open-source software. Nevertheless, the original incremental value of this research stems from bridging the gap between standard analytical gear ratings (ISO 6336), multiaxial critical-plane fatigue post-processing routines, and automated mesh-refinement strategies for crack propagation into a singular, cohesive, and entirely accessible pipeline. While this represents a methodological integration rather than an algorithmic expansion, it provides the gear engineering community with a fully transparent, reproducible tool to transition from macro-scale standards to local fracture mechanics without relying on costly proprietary software.

Regarding the methodological scope, it must be explicitly clarified that this study does not introduce fundamental mathematical modifications or algorithmic kernel expansions to Code_Aster's built-in XFEM and fracture mechanics solvers. Rather, the incremental value and primary contribution of this research lie in the end-to-end process integration, critical-plane coupling automation, and experimental validation of a unified gear fatigue pipeline within an open-source ecosystem. By replacing fragmented, proprietary software chains with a scripted, open-source toolchain, this work provides an accessible baseline for tooth root bending fatigue analysis. Further algorithmic expansions — such as integrating depth-dependent material property gradients and dedicated short-crack closure models — represent essential next steps to advance the physical fidelity of this open-source framework.

CRediT authorship contribution statement

Andrea Cittadini: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Lorenzo Fraccaroli:** Validation, Methodology, Formal analysis. **Lorenzo Pagliari:** Writing – review & editing, Writing – original draft, Visualization, Methodology. **Corsaro Luca:** Investigation, Data curation. **Franco Concli:** Validation, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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