

Abstract

Numerical solution algorithms for electromagnetic scattering and propagation problems constitute an important research topic in computational electromagnetics. The Method of Moments (MoM) based on integral equations offers high computational accuracy; however, it generates dense system matrices, which typically require fast algorithms to improve computational efficiency. These fast algorithms compress the matrix blocks associated with far-field interactions, thereby accelerating the matrix-vector products involving the system matrix and the right-hand side vector, and have been widely employed in iterative solvers. Nevertheless, for electromagnetic problems involving complex objects, the resulting system matrices are often ill-conditioned, which may lead to convergence difficulties when iterative solvers are employed. Moreover, when solving Multiple Right-Hand Side (MRHS) problems, iterative methods generally require the iterative process to be restarted for different right-hand sides, resulting in relatively low computational efficiency. In contrast, direct solvers explicitly construct the inverse of the system matrix and therefore offer higher computational efficiency for MRHS problems, while being largely insensitive to convergence issues. Based on a kernel-independent fast algorithm, namely the Nested Equivalent Source Approximation (NESA), this dissertation conducts the following research:

1. A fast algorithm was developed for electromagnetic scattering problems involving complex objects composed of nonuniform dielectric and metallic materials. For Perfect Electric Conductors (PECs) and nonuniform dielectric media, Surface Integral Equations (SIEs) and Volume Integral Equations (VIEs) were respectively established, and a NESA-based Volume-Surface Integral Equation (VSIE) formulation was developed. The resulting VSIE system was efficiently solved using NESA. In the proposed method, equivalent surfaces and

uniformly distributed equivalent sources are introduced. Based on the equivalence principle, the surface and volume unknowns are mapped onto the equivalent sources, such that the coupling interactions between far-field groups are replaced by the interactions between equivalent sources. Compared with surface discretization, volume discretization generally introduces a substantially larger number of unknowns. In the NESAs-VSIE formulation, however, the volume unknowns are retained only at the finest level, while the coupling interactions involving volume unknowns at higher levels are replaced by the interactions among equivalent surface-source unknowns. Consequently, the dimension of the matrix system to be solved is significantly reduced, thereby improving the computational efficiency of solving VSIE problems.

2. A direct solver was developed for electromagnetic scattering problems in the mid- and low-frequency regimes. Based on the Low-Rank Approximation (LRA) form of NESAs, a system matrix structure similar to that of an H^2 -matrix was constructed, and elimination matrices were introduced. The entire system matrix was factorized using LU decomposition, thereby enabling the explicit computation of its inverse. During the factorization process, dense low-rank matrix blocks arise in the Schur complements. Singular Value Decomposition (SVD) was therefore employed to compress these matrix blocks, maintaining a sparse representation of the far-field matrix blocks. Compared with iterative solvers, the proposed direct solver achieves higher computational efficiency for problems with multiple excitation terms. Moreover, the introduction of equivalent sources further reduces the memory requirements compared with skeletonization-based algorithms. Theoretical analysis and numerical experiments demonstrate that both the time and memory complexities of the proposed algorithm are $O(N)$. Numerical simulations of the monostatic Radar Cross Section (RCS) of complex targets further demonstrate a significant improvement in computational efficiency over iterative solvers.

3. A direct solver was developed for high-frequency electromagnetic scattering problems. For high-frequency problems, the ranks of the matrix blocks increase with the number of unknowns, and uniformly distributed omnidirectional equivalent sources cannot effectively characterize the scattering fields of individual groups. Based on the matrix compression formulation of the Wideband Nested Equivalent Source Approximation (WNESA), the radiation matrices corresponding to different directions of each group are jointly considered during the system matrix factorization. Singular Value Decomposition (SVD) is then employed to extract a generalized radiation matrix, which is used to construct the elimination matrix for each group. The generalized radiation matrix collects the

radiation information of a given group in all directions, thereby ensuring that the resulting elimination matrix can effectively eliminate the far-field coupling blocks in all directions. Since WNESA imposes more stringent admissibility conditions, the indexing scheme for inadmissible blocks in the direct-solution procedure is also modified accordingly. Numerical examples are conducted at different frequencies to compare the computational results and verify the accuracy of the proposed algorithm in high-frequency scenarios. Theoretical analysis and numerical results demonstrate that the proposed direct solver for electrically large problems has a time complexity of $O(N^2)$ and a memory complexity of $O(N \log N)$.

4. An online microwave sensing system was designed for food contamination detection. The proposed direct solvers for electromagnetic scattering problems were applied to the development of a microwave sensing system for the detection of contaminants in food products. An online, real-time microwave sensing system was developed to extract variations in the electromagnetic characteristics inside food products from their scattered-field information, thereby enabling the detection and identification of potential contaminants. An antenna array was employed to focus and scan the electromagnetic beam, thereby improving the detection performance compared with conventional systems based on horn antennas. In addition, a machine learning model was developed to learn and classify the electromagnetic scattering characteristics of food products containing different types of contaminants.

In summary, this dissertation extends the application scope of the NESA-based low-rank approximation algorithm and develops fast algorithms for electromagnetic scattering problems involving PECs and complex nonuniform dielectric media. Direct solvers based on NESA are further developed for electromagnetic scattering problems in both low- and high-frequency regimes, enabling efficient solutions of Multiple Right-Hand Side (MRHS) problems. Finally, an online, real-time microwave sensing system for food contamination detection is developed based on electromagnetic scattering analysis.

Keywords: Volume-surface integral equation, Fast method, Direct solver, Nested equivalent source approximation, Microwave sensing.

