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Self-contact-induced stiffening in curved-ligament re-entrant lattice metamaterials

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ABSTRACT

This work investigates the compressive response of curved-ligament re-entrant lattice metamaterials in which internal self-contact acts as a geometry-controlled stiffening mechanism. Under compression, neighbouring ligaments progressively come into contact within the central neck region, introducing additional constraints and modifying the load-transfer paths.

Two complementary numerical analyses are performed. First, the effective elastic response of the undeformed periodic cell is characterized through first-order computational homogenization with periodic boundary conditions. This provides a reference description of the initial open-cell regime and of the elastic anisotropy before contact activation. Second, nonlinear compression simulations with frictionless self-contact are used to investigate the response beyond the initial open-cell regime.

The results show that contact activation is directly associated with the departure from the initial compliant regime and with a progressive increase in the slope of the force–displacement response. Within the selected set of representative geometries, ligament spacing mainly affects the displacement range over which the cell remains open, whereas aspect ratio and ligament number influence the spatial distribution of the active contact regions.

Overall, the results indicate that geometry-dependent self-contact can be exploited to tailor the nonlinear compressive response of curved-ligament lattice architectures. The observed stiffening originates from contact activation rather than from material nonlinearities or additional internal components.

1. Introduction

Architected materials derive their mechanical response from the interaction between the properties of the solid material, the geometry and the connectivity of the internal architecture (Sridhar et al., 2018; Surjadi et al., 2019; Wu et al., 2019; Wang et al., 2023). Periodic and nearly periodic lattice microstructures can exhibit tailored combinations of stiffness, strength, deformability, energy absorption and wave-control capabilities that are not directly attainable in the corresponding bulk material. This has motivated extensive research on metamaterials and mechanical metamaterials across several fields, including lightweight structures, impact mitigation, adaptive systems, elastodynamics and wave manipulation (Xiao et al., 2020; Wu et al., 2021; Bacigalupo et al., 2020; Elefante et al., 2023; Misseroni et al., 2019; Ning et al., 2020; Yao et al., 2021).

Within mechanical metamaterials, a central research direction concerns the design of lattice architectures whose response evolves during deformation. In this class of systems, the effective stiffness is not

fixed by the initial topology alone, but changes as the microstructure undergoes rotations, bending-dominated deformation, local instabilities, snap-through events, prestress activation or internal contact. Such mechanisms can be used to obtain strain-dependent stiffness, energy dissipation, shape adaptation and multi-stage force–displacement responses (Bertoldi et al., 2017; Yu et al., 2018; Barchiesi et al., 2019; Jiao et al., 2023; Coulais et al., 2018; Wenz et al., 2021; Wu and Pasini, 2023; Roback et al., 2025). Examples include architected lattices with high stiffness-to-weight ratios, enhanced toughness, ultra-light structural response, stiffness modulation and programmable deformation paths (Al-Ketan and Abu Al-Rub, 2019; Jia and Wang, 2019; Mohsenizadeh et al., 2018; Zangeneh-Nejad and Fleury, 2019; Hashemi et al., 2021; Li et al., 2020; Zhang et al., 2021). These studies show that substantial changes in mechanical response can be achieved through geometric reconfiguration of the unit cell, without requiring changes in the solid material itself.

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Among the possible mechanisms for stiffness modulation, internal self-contact provides a purely geometric source of nonlinear response. Contact activation can introduce additional internal constraints, alter the active load-bearing network and redirect the load-transfer paths during deformation. Self-contact-based stiffening has been investigated in mechanical metamaterials where deformation-induced contact produces contact-induced kinematic reconfiguration and tunable nonlinear stiffening under different loading modes (Wagner et al., 2022). Related shape-changing systems have shown that contact events can guide multi-step deformation pathways (Coulais et al., 2018), while recent designs have exploited self-contact as a geometry-controlled internal event to trigger abrupt changes in mechanical response at prescribed deformation levels (Schwarz et al., 2023). Other contact-based architectures have used internal stops to switch deformation modes and adjust the response by changing the initial contact configuration (Tang et al., 2024). In parallel, computational design frameworks have incorporated contact modelling within inverse homogenization, topology optimization and shape optimization to target prescribed macroscopic responses (Dalkint et al., 2023). These studies identify self-contact as a design mechanism for controlling nonlinear stiffness evolution and deformation pathways. In most of these architectures, however, self-contact is exploited as a threshold-type event — a single transition, a switch between deformation modes, or an abrupt change in response triggered at a prescribed deformation level (Wagner et al., 2022; Schwarz et al., 2023; Tang et al., 2024) — or it is embedded a posteriori within a computational design framework (Dalkint et al., 2023). Comparatively little attention has been devoted to self-contact regarded as a spatially distributed set of unilateral constraints that is recruited progressively, and in a geometrically controllable order, during compression, and to the continuously evolving multi-branch stiffening response that such progressive recruitment produces.

Re-entrant lattice architectures provide an effective geometric framework for contact-driven stiffness modulation. Their inclined or curved ligaments can undergo significant relative motion under compression, and small variations in the initial geometry may affect the formation, sequence and spatial distribution of contact zones. Curved ligaments further enlarge the geometric design space, allowing local curvature, ligament spacing and internal gaps to be tuned. Curved and geometrically anisotropic lattice architectures have been shown to enhance structural integrity and mechanical performance in architected materials (Vangelatos et al., 2020). Recent works on modified re-entrant architectures have also reported that replacing straight struts with curved members can improve stiffness, deformation patterns and energy-absorption capacity, confirming the relevance of curvature as a design parameter in this class of lattices (Etemadi et al., 2024). In re-entrant and curved lattice architectures specifically, contact has so far been considered mainly in connection with large-deformation collapse, cell locking and densification, rather than as a geometrically ordered constraint mechanism governing post-onset stiffening within the elastic regime.

From an engineering perspective, this mechanism is particularly relevant for passive architected components requiring a two-stage compressive response. In the initial open-cell regime, the structure can remain compliant and limit force transmission; beyond a prescribed deformation threshold, internal contact progressively increases the stiffness and helps prevent bottoming-out under larger compression. This type of response is of interest for lightweight protective cores, impact-mitigation layers, programmable bumpers and load-limiting inserts. Since the onset and spatial evolution of contact can be tuned through ligament spacing, aspect ratio and ligament number, the force-displacement response can be adjusted without changing the base material or introducing active elements. The same principle may also be relevant for additively manufactured components and biomedical devices requiring graded or deformation-dependent stiffness.

In this work, we consider a family of re-entrant lattice cells composed of two horizontal flanges connected by initially non-contacting

curved ligaments. Under compression, the initially separated ligaments come into mutual contact in the central neck region, introducing additional internal constraints that modify the load-transfer mechanisms and produce a geometry-driven stiffening response. Because the ligaments are curved and arranged symmetrically about the central neck, the facing surfaces are closest near the mid-height of the cell and the initial gap between them widens towards the flanges; under compression, contact therefore does not occur as a single simultaneous event, but nucleates at the innermost ligament pair, where this gap is smallest, and the active contact set is then progressively enlarged towards the adjacent ligaments and along inclined ligament segments as compression proceeds. The macroscopic force-displacement response is therefore a multi-branch hardening response rather than a one-step transition, whose onset and post-contact tangent stiffness are governed by distinct geometric parameters. Within the representative configurations examined here, the ligament spacing primarily sets the displacement at contact onset, the number of ligaments governs whether the active contact set grows in a localized, step-like fashion or in a smooth, distributed one, and the cell aspect ratio controls both the initial elastic anisotropy and the orientation of the contact zones, from predominantly vertical to inclined. On this basis, the present study characterizes progressively recruited self-contact in curved-ligament re-entrant cells as a purely geometric route to a tunable post-contact stiffening response, distinct from the threshold-type contact mechanisms reported so far.

Two complementary numerical analyses are carried out. First, the small-strain effective elastic response of the periodic cell in its initial configuration is evaluated through first-order computational homogenization with periodic boundary conditions. This provides a reference mechanical characterization of the undeformed periodic architecture before contact activation. Second, finite-cell compression simulations with frictionless self-contact are performed to study the nonlinear response associated with progressive contact development. The latter simulations are interpreted through contact onset, force-displacement curves, and the corresponding changes in the slope of the global response. This slope increase is viewed as the macroscopic manifestation of the incremental constraints introduced by the growing active contact set. The results show that relatively small geometric variations can substantially modify the onset and evolution of internal contact, leading to markedly different post-contact stiffening responses and load-transfer mechanisms. The selected configurations are treated as a representative set rather than a full factorial study, so the parameter roles outlined above should be read as dominant tendencies rather than as a strict isolation of the individual parameters.

The paper is organized as follows. Section 2 describes the geometry of the lattice cell and the selected design parameters. Section 3 recalls the continuum formulation of the frictionless contact problem and the corresponding node-to-segment numerical treatment. Section 4 introduces the first-order homogenization procedure used to characterize the small-strain elastic response of the initial periodic cell. Section 5 reports and discusses the numerical results, including the initial homogenized elastic properties, the contact-induced stiffening behaviour, and the influence of the main geometric parameters. Final Remarks are drawn in Section 6.

2. Geometry of the metamaterial

The considered microstructure is a two-dimensional curved-ligament re-entrant lattice cell, schematically shown in Fig. 1. Fig. 1(a) presents a portion of the periodic material with one representative cell highlighted, while Fig. 1(b) reports the selected cell and the geometric parameters used in the numerical analyses.

The cell consists of two horizontal flanges connected by a set of N curved ligaments. The ligaments are arranged symmetrically with respect to the horizontal and vertical midlines of the cell, producing a central neck region where adjacent ligaments are initially separated

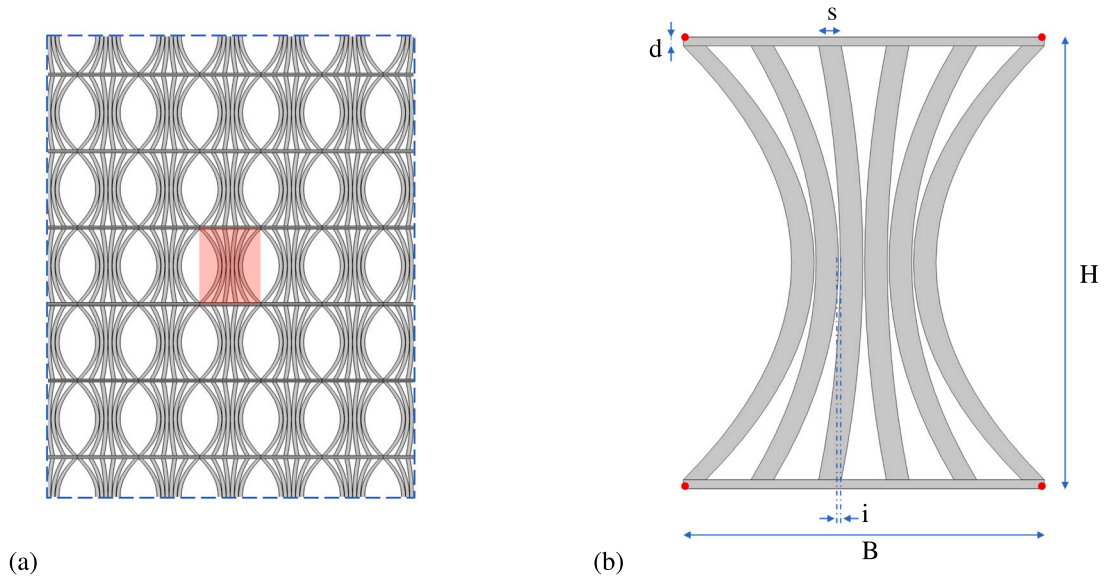


Fig. 1. Geometry of the considered curved-ligament re-entrant lattice. (a) Portion of the periodic microstructured material, with one representative cell highlighted. (b) Selected cell and geometric parameters: cell width B , cell height H , ligament spacing i , flange thickness d , and ligament thickness s .

by finite gaps. In the undeformed configuration, no internal contact is present. Under vertical compression, the progressive closure of the initial gaps may lead to self-contact in the central region of the cell, thereby modifying the load-transfer mechanism.

The geometry is described by the cell width B , the cell height H , the spacing parameter i , the flange thickness d , the ligament thickness s , and the number of curved ligaments N , as indicated in Fig. 1(b). These parameters define the initial layout of the cell and govern the development of internal contact during compression. In particular, the spacing between adjacent ligaments influences the displacement level at which self-contact is first activated, while the cell aspect ratio B/H and the number of ligaments affect the spatial distribution and propagation of the contact zones.

In the finite element model, the entire lattice cell is modelled in plane-stress conditions and discretized with structured bilinear quadrilateral elements. The curved ligaments are discretized with an 80×4 mesh (length $\times$ thickness), while the flanges are discretized with 4 elements through the thickness. The adopted discretization resolves both the local deformation of the lattice members and the contact interaction between neighbouring ligament surfaces. The mesh density was verified to provide converged results for both the homogenized stiffness components and the nonlinear contact response.

The bulk material is assumed to be polylactic acid (PLA), modelled as linear elastic with Young's modulus $E = 3.43$ GPa and Poisson's ratio $\nu = 0.35$. The analysis is restricted to the in-plane response, consistently with the two-dimensional geometry. The out-of-plane thickness is set to $t = 0.01$ cm; this value enters as a scaling factor for the contact area regularization and for the resultant forces in the compression simulations, but does not affect the homogenized stiffness, which is an intensive quantity. The flange thickness is fixed at $d = 0.15$ cm in all configurations. The penalty parameter is set to $\epsilon = 9.81$ GPa ($\approx 2.86 E$), which enforces the impenetrability constraint within a tolerance of order 10^{-4} cm without introducing ill-conditioning of the global stiffness matrix in the Newton iterations.

The same discretization and contact settings are adopted for all configurations, so that the comparisons are based on consistent numerical assumptions. The discussion below therefore focuses on global force-displacement trends, contact-pattern evolution and relative differences among geometries rather than on local contact-pressure values, which are intrinsically more sensitive to penalty regularization and to the local discretization of the contacting surfaces.

Table 1

Geometric parameters of the selected lattice configurations. See Fig. 1(b) for the definition of the symbols.

Geometry	N	B [cm]	H [cm]	B/H	i [cm]	s [cm]
G1	6	5	4	1.25	0.03	0.3
G2	6	6	2	3.00	0.10	0.3
G3	16	4	5	0.80	0.06	0.1
G4	16	6	2	3.00	0.08	0.1
G5	6	6	2	3.00	0.02	0.3
G6	16	6	2	3.00	0.15	0.1

The selected configurations are listed in Table 1. They are not intended as an exhaustive parametric study, but as a compact set of representative geometries used to compare how ligament spacing, cell aspect ratio and ligament number influence contact activation and post-contact stiffness evolution. Accordingly, the comparisons reported below should be interpreted as a mechanically motivated exploration of representative configurations, rather than as a strict statistical isolation of each geometric parameter. This choice keeps the analysis focused on the role of self-contact in changing the load-transfer mechanism while avoiding an unnecessarily large design space.

Geometries G1 and G3 provide two baseline configurations with different ligament numbers and cell aspect ratios. Geometries G2 and G4 correspond to configurations with $B/H = 3.00$, with $N = 6$ and $N = 16$, respectively. Geometries G5 and G6 are introduced to highlight the influence of ligament spacing through comparison with G2 and G4, respectively, for which the aspect ratio, ligament number and ligament thickness are kept unchanged within each pair. This organization allows the subsequent analysis to identify how selected geometric variations influence contact activation, force-displacement evolution and post-contact stiffening.

3. Micromechanical model of the self-contact problem

The interaction between facing ligament surfaces is described as a frictionless self-contact problem. Contact becomes active only when the internal gaps close during compression and is enforced through a node-to-segment finite element discretization with penalty regularization (Wriggers and Laursen, 2006; Zavarise et al., 2006; Zavarise and De Lorenzis, 2009).

For a generic contact problem written in the reference configuration, the local balance equations relating the first Piola–Kirchhoff stress tensor $\mathbf{P}^\alpha$ and the body force $\mathbf{b}^\alpha$ associated with the α -th body read

$$\text{DIV } \mathbf{P}^\alpha + \mathbf{b}^\alpha = \mathbf{0}. \quad (1)$$

The associated Dirichlet and Neumann boundary conditions hold, respectively, as

$$\mathbf{u}^\alpha = \bar{\mathbf{u}}^\alpha \quad \text{on } \Gamma_D, \quad (2)$$

$$\mathbf{P}^\alpha \mathbf{N}^\alpha = \bar{\mathbf{t}}^\alpha \quad \text{on } \Gamma_N, \quad (3)$$

where the overbar denotes assigned quantities and $\mathbf{N}^\alpha$ is the outward normal. On the contact interface Γ_c , the Karush–Kuhn–Tucker conditions for frictionless contact are

$$g_N \geq 0 \quad p_N \leq 0 \quad g_N p_N = 0 \quad \text{on } \Gamma_c, \quad (4)$$

where g_N denotes the normal gap function and p_N is the normal contact pressure.

The contact interaction between neighbouring curved ligaments is discretized through a penalty formulation with area regularization (Wriggers and Laursen, 2006; Zavarise and De Lorenzis, 2009). The corresponding contact potential is

$$\Pi_c = \frac{1}{2} \epsilon A \langle -g_N \rangle_+^2, \quad (5)$$

where $\epsilon > 0$ is the penalty parameter, A is the tributary area associated with the contact element, and $\langle \bullet \rangle_+ = \max(\bullet, 0)$ denotes the Macaulay bracket. Introducing the penetration measure

$$g_N^- = \langle -g_N \rangle_+ \geq 0, \quad (6)$$

the penalty contribution is active only for closed projections, while open gaps do not contribute to the residual or tangent stiffness.

Since the lattice cell is discretized with plane-stress continuum elements, contact is treated through a node-to-segment (NTS) discretization. As illustrated in Fig. 2, the position of a node belonging to one contacting surface is projected onto a segment of the opposing surface, while the open/closed status is monitored through the sign of the normal gap g_N . The discrete contact contribution is obtained from the variation of the contact potential. In the following expressions, g_N denotes the active signed gap after the open/closed projection test, so that only closed projections contribute to the contact terms:

$$\delta \Pi_c = \epsilon A g_N \delta g_N = \delta W = F_N \delta g_N, \quad (7)$$

where F_N denotes the normal contact force associated with the slave node. The nonlinear equilibrium problem is solved through a Newton-type iterative procedure, leading to the tangent contribution

$$\Delta \delta W = \Delta F_N \delta g_N + F_N \Delta \delta g_N. \quad (8)$$

The corresponding residual and tangent contributions are therefore

$$\delta W = \epsilon A g_N \delta g_N, \quad (9)$$

$$\Delta \delta W = \epsilon A \Delta g_N \delta g_N + \epsilon A g_N \Delta \delta g_N. \quad (10)$$

The first variation of the normal gap follows from its definition and, using the orthogonality of the local normal and tangent vectors, reduces to

$$\delta g_N = [\delta]^T [\mathbf{N}_S]. \quad (11)$$

Its linearization involves the directional derivatives of the local frame $(\mathbf{n}, \mathbf{t})$ and of the projection coordinate along the master segment of length l ; collecting these contributions gives $\Delta \delta g_N$ as

$$\Delta \delta g_N = -\frac{1}{l} \left[[\delta]^T [\mathbf{T}_S] [\mathbf{N}_0]^T [\Delta] + [\delta]^T [\mathbf{N}_0] [\mathbf{T}_S]^T [\Delta] + \frac{g_N}{l} [\delta]^T [\mathbf{N}_0] [\mathbf{N}_0]^T [\Delta] \right]. \quad (12)$$

Substitution of Eqs. (11) and (12) into the penalty contribution $\delta W = \epsilon A g_N \delta g_N$ gives the discrete residual

$$\delta W = \epsilon A g_N [\delta]^T [\mathbf{N}_S], \quad (13)$$

and its linearization

$$\begin{aligned} \Delta \delta W = & \epsilon A [\delta]^T [\mathbf{N}_S] [\mathbf{N}_S]^T [\Delta] - \epsilon A \frac{g_N}{l} ([\delta]^T [\mathbf{T}_S] [\mathbf{N}_0]^T [\Delta] + [\delta]^T [\mathbf{N}_0] [\mathbf{T}_S]^T [\Delta]) \\ & - \epsilon A \left(\frac{g_N}{l} \right)^2 [\delta]^T [\mathbf{N}_0] [\mathbf{N}_0]^T [\Delta]. \end{aligned} \quad (14)$$

Since the virtual and incremental nodal quantities are arbitrary, the residual vector $[\mathbf{R}]$ and the tangent stiffness matrix $[\mathbf{K}_T]$ are obtained by factoring the terms multiplying $[\delta]^T$ and $[\Delta]$, leading to

$$[\mathbf{R}] = \epsilon A g_N [\mathbf{N}_S], \quad (15)$$

$$[\mathbf{K}_T] = \epsilon A [\mathbf{N}_S] [\mathbf{N}_S]^T - \epsilon A \frac{g_N}{l} ([\mathbf{T}_S] [\mathbf{N}_0]^T + [\mathbf{N}_0] [\mathbf{T}_S]^T) - \epsilon A \left(\frac{g_N}{l} \right)^2 [\mathbf{N}_0] [\mathbf{N}_0]^T, \quad (16)$$

where the following vectors have been introduced to compact the notation

$$[\mathbf{X}] = \begin{bmatrix} X_S \\ X_1 \\ X_2 \end{bmatrix}, [\delta] = \begin{bmatrix} \delta X_S \\ \delta X_1 \\ \delta X_2 \end{bmatrix}, [\Delta] = \begin{bmatrix} \Delta X_S \\ \Delta X_1 \\ \Delta X_2 \end{bmatrix}, \quad (17)$$

$$[\mathbf{N}_0] = \begin{bmatrix} 0 \\ -\mathbf{n} \\ \mathbf{n} \end{bmatrix}, [\mathbf{T}_0] = \begin{bmatrix} 0 \\ -\mathbf{t} \\ \mathbf{t} \end{bmatrix}, [\mathbf{N}_g] = \begin{bmatrix} \mathbf{n} \\ -\mathbf{n} \\ 0 \end{bmatrix}, [\mathbf{T}_g] = \begin{bmatrix} \mathbf{t} \\ -\mathbf{t} \\ 0 \end{bmatrix}, \quad (18)$$

$$[\mathbf{N}_S] = [\mathbf{N}_g] - \xi [\mathbf{N}_0] = \begin{bmatrix} \mathbf{n} \\ -(1-\xi)\mathbf{n} \\ -\xi\mathbf{n} \end{bmatrix}, [\mathbf{T}_S] = [\mathbf{T}_g] - \xi [\mathbf{T}_0] = \begin{bmatrix} \mathbf{t} \\ -(1-\xi)\mathbf{t} \\ -\xi\mathbf{t} \end{bmatrix}. \quad (19)$$

Here, ξ denotes the local coordinate of the projection point along the master segment; X_S is the slave-node position, while X_1 and X_2 are the positions of the two master-segment nodes. The full derivation of the node-to-segment element is reported in [35, Ch. 6]. Numerical simulations are performed using the nonlinear finite element program FEAP (courtesy of Prof. R.L. Taylor) (Taylor, 2024).

4. Homogenized characterization of the open-cell response

The effective elastic tensor is derived through first-order computational homogenization of the periodic cell (Hill, 1963; Suquet, 1987; Miehe et al., 1999). The homogenization is performed in the initial configuration and within the small-strain elastic regime. Accordingly, all stress and strain measures introduced in this section are linearized quantities referred to the undeformed configuration. In this limit, the nominal stress tensor work-conjugate to the infinitesimal strain coincides, at first order, with the corresponding Cauchy stress tensor. The present linearized setting is therefore consistent with the reference-configuration formulation adopted in Section 3.

At the macroscopic level, $\mathbf{X} = (X_1, X_2)^T$ denotes a material point of the equivalent continuum and

$$\mathbf{U}(\mathbf{X}) = \begin{pmatrix} U_1(\mathbf{X}) \\ U_2(\mathbf{X}) \end{pmatrix} \quad (20)$$

is the macroscopic displacement field. The infinitesimal strain tensor is

$$\mathbf{E} = \frac{1}{2} (\nabla \mathbf{U} + \nabla \mathbf{U}^T). \quad (21)$$

Using Kelvin notation, the corresponding strain vector is written as

$$\underline{\mathbf{E}} = \begin{pmatrix} E_{11} \\ E_{22} \\ \sqrt{2} E_{12} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial X_1} & 0 \\ 0 & \frac{\partial}{\partial X_2} \\ \frac{1}{\sqrt{2}} \frac{\partial}{\partial X_2} & \frac{1}{\sqrt{2}} \frac{\partial}{\partial X_1} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}. \quad (22)$$

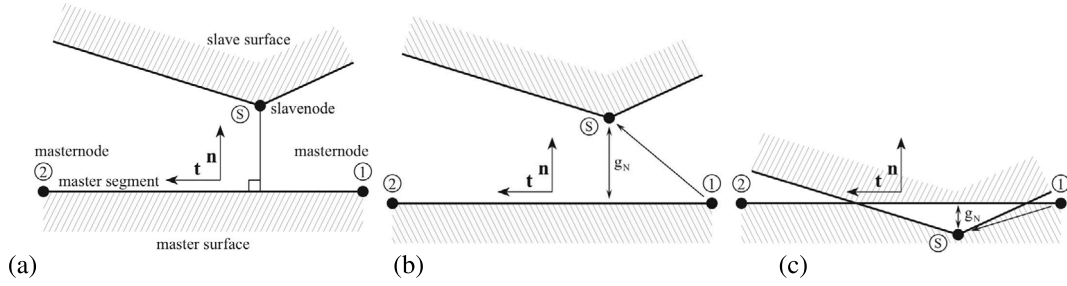


Fig. 2. Node-to-segment contact discretization (Zavarise et al., 2006): (a) geometry assumptions; (b) open gap status; (c) closed gap status.

The macroscopic constitutive relation is expressed as

$$\underline{\Sigma} = \mathbf{C}^{\text{hom}} \underline{\mathbf{E}}, \quad (23)$$

with

$$\underline{\Sigma} = \begin{pmatrix} \Sigma_{11} \\ \Sigma_{22} \\ \sqrt{2}\Sigma_{12} \end{pmatrix}, \quad \mathbf{C}^{\text{hom}} = \begin{pmatrix} C_{1111}^{\text{hom}} & C_{1122}^{\text{hom}} & \sqrt{2}C_{1112}^{\text{hom}} \\ C_{1122}^{\text{hom}} & C_{2222}^{\text{hom}} & \sqrt{2}C_{2212}^{\text{hom}} \\ \sqrt{2}C_{1112}^{\text{hom}} & \sqrt{2}C_{2212}^{\text{hom}} & 2C_{1212}^{\text{hom}} \end{pmatrix}. \quad (24)$$

No a priori symmetry assumptions are introduced in Eq. (24); the homogenized response is therefore described through the full two-dimensional elastic matrix.

At the microscopic level, the same linearized kinematic and constitutive setting is adopted within the periodic cell. Let A denote the area of the periodic cell and let $A_s \subset A$ denote the portion occupied by the solid phase. For a point $\mathbf{x} = (x_1, x_2)^T$ in the cell, the microscopic displacement field is decomposed as

$$\mathbf{u}(\mathbf{x}) = \mathbf{E}\mathbf{x} + \tilde{\mathbf{u}}(\mathbf{x}), \quad (25)$$

where $\mathbf{E}$ is the macroscopic strain tensor, $\mathbf{E}\mathbf{x}$ is the associated affine displacement field, and $\tilde{\mathbf{u}}$ is a periodic fluctuation field with zero cell average. Periodic boundary conditions are imposed on opposite sides of the cell according to

$$\tilde{\mathbf{u}}(\mathbf{x}^+) = \tilde{\mathbf{u}}(\mathbf{x}^-), \quad (26)$$

while the corresponding boundary tractions satisfy anti-periodicity conditions. In the present elastic characterization, the cell is considered in its initial open-cell configuration; consequently, no contact contribution is active in the homogenization problem.

The microscopic strain and stress fields are denoted by $\underline{\epsilon}(\mathbf{x})$ and $\underline{\sigma}(\mathbf{x})$, respectively, and are related by

$$\underline{\sigma}(\mathbf{x}) = \mathbf{c}(\mathbf{x})\underline{\epsilon}(\mathbf{x}), \quad (27)$$

where $\mathbf{c}(\mathbf{x})$ is the local elastic matrix of the solid phase in Kelvin notation.

The spatial average of a microscopic field over the periodic cell is defined as

$$\langle \bullet \rangle_A = \frac{1}{|A|} \int_{A_s} \bullet \, dA. \quad (28)$$

The Hill–Mandel macro-homogeneity condition imposes energetic consistency between the microscopic and macroscopic descriptions:

$$\underline{\Sigma}^T \underline{\mathbf{E}} = \langle \underline{\sigma}^T \underline{\epsilon} \rangle_A. \quad (29)$$

For admissible microscopic fields satisfying the periodic boundary conditions, the macroscopic strain is recovered as the cell average of the microscopic strain, while the macroscopic stress is obtained as the corresponding area average of the microscopic stress:

$$\underline{\Sigma} = \langle \underline{\sigma} \rangle_A, \quad \underline{\mathbf{E}} = \langle \underline{\epsilon} \rangle_A. \quad (30)$$

For a prescribed macroscopic strain, the solution of the periodic boundary value problem defines the localization relation

$$\underline{\epsilon}(\mathbf{x}) = \mathbf{B}(\mathbf{x})\underline{\mathbf{E}}, \quad (31)$$

where $\mathbf{B}(\mathbf{x})$ is the strain localization matrix. Substitution of Eq. (31) into the microscopic strain energy together with the Hill–Mandel condition gives

$$\frac{1}{2} \underline{\mathbf{E}}^T \mathbf{C}^{\text{hom}} \underline{\mathbf{E}} = \frac{1}{2} \frac{1}{|A|} \int_{A_s} \underline{\epsilon}^T \mathbf{c}(\mathbf{x}) \underline{\epsilon} \, dA. \quad (32)$$

The homogenized elastic matrix is therefore obtained as

$$\mathbf{C}^{\text{hom}} = \frac{1}{|A|} \int_{A_s} \mathbf{B}(\mathbf{x})^T \mathbf{c}(\mathbf{x}) \mathbf{B}(\mathbf{x}) \, dA. \quad (33)$$

To describe the directional elastic response without introducing symmetry assumptions, the compliance matrix in Kelvin notation is defined as

$$\mathbf{D}^{\text{hom}} = (\mathbf{C}^{\text{hom}})^{-1}, \quad (34)$$

Directional elastic quantities are evaluated from the corresponding fourth-order compliance tensor D_{ijkl}^{hom} . Let

$$\mathbf{n}(\theta) = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \quad \mathbf{m}(\theta) = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} \quad (35)$$

be the loading direction and the associated transverse direction. The directional Young's modulus is

$$E_{\text{hom}}(\theta) = \left[D_{ijkl}^{\text{hom}} n_i n_j n_k n_l \right]^{-1}, \quad (36)$$

where summation over repeated indices is implied. The corresponding directional Poisson ratio, associated with the transverse strain along $\mathbf{m}(\theta)$ induced by uniaxial loading along $\mathbf{n}(\theta)$, is

$$\nu_{\text{hom}}(\theta) = - \frac{D_{ijkl}^{\text{hom}} m_i m_j n_k n_l}{D_{ijkl}^{\text{hom}} n_i n_j n_k n_l}. \quad (37)$$

Eqs. (36) and (37) are valid for a general two-dimensional anisotropic elastic response and do not rely on symmetry assumptions.

5. Numerical results

5.1. Equivalent elastic properties

The homogenized elastic tensors reported in this section characterize the periodic cells in their initial open-cell configuration, before the activation of internal contact.

For each geometry, the full two-dimensional homogenized tensor $\mathbf{C}^{\text{hom}}$ is computed as described in Section 4. The directional elastic quantities are then evaluated from the compliance matrix $\mathbf{D}^{\text{hom}} = (\mathbf{C}^{\text{hom}})^{-1}$ through Eqs. (36)–(37).

Although no material symmetry is enforced in the homogenization, the computed tensors turn out to be orthotropic to a very good approximation, with the cell axes acting as principal material directions: the shear–axial coupling components C_{1112} and C_{2212} are negligible in all configurations and are therefore omitted from Table 2. The response is thus orthotropic and, at the same time, strongly anisotropic. The stiffness components are reported in tensor notation and in MPa; hence $C_{1212}^{\text{hom}} = C_{33}^K/2$ in Kelvin notation. The rows are grouped according to the

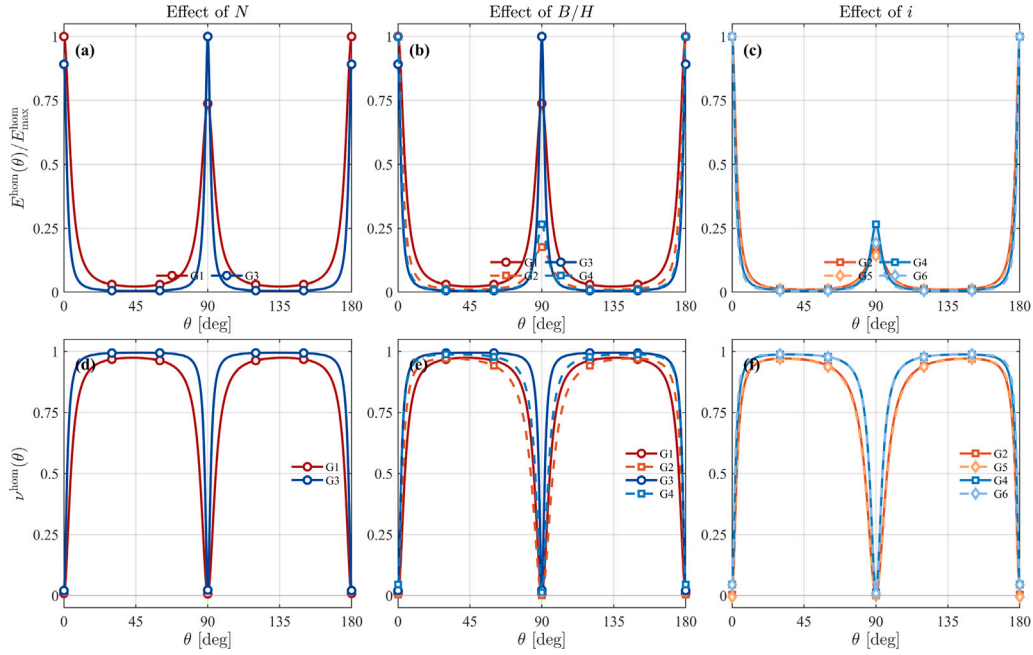


Fig. 3. Directional Young's modulus $E^{\text{hom}}(\theta)/E_{\text{max}}$ and Poisson ratio $\nu^{\text{hom}}(\theta)$ of the homogenized open-cell configurations. Columns illustrate the effects of ligament number (a,d), aspect ratio (b,e) and ligament spacing (c,f).

Table 2

Homogenized elastic descriptors of the initial open-cell configuration. Stiffness components and E_{min} are in MPa.

Geom.	C_{1111}	C_{2222}	C_{1122}	C_{1212}	E_2/E_1	E_{min}	A_E	ν_{min}
G1	1140.8	841.4	7.01	6.46	0.74	25.5	44.7	0.006
G2	1347.6	237.5	0.77	3.43	0.18	13.5	99.9	0.001
G5	1346.1	192.2	-1.07	2.99	0.14	11.8	114.6	-0.006
G3	884.8	992.6	19.88	1.14	1.12	4.5	218.8	0.020
G4	1314.6	348.7	15.80	1.84	0.27	7.3	179.4	0.012
G6	1313.7	252.6	11.05	1.37	0.19	5.4	241.6	0.008

number of ligaments ($N = 6$ and $N = 16$), with successive variations in aspect ratio and ligament spacing reported within each group.

The dominant feature of the homogenized tensors is the contrast between axial and shear stiffness. The horizontal component C_{1111} is about 25%–39% of the bulk-material modulus $E = 3.43$ GPa, while the transverse component C_{2222} is more sensitive to the cell aspect ratio and drops to roughly 5%–10% of E for the high-aspect-ratio cells ($B/H = 3.00$). The shear component C_{1212}^{hom} ranges from 1.14 MPa (G3) to 6.46 MPa (G1), leading to values of $C_{1212}^{\text{hom}}/(2C_{1111}^{\text{hom}})$ between approximately 88 (G1) and 480 (G6). The maximum ratio is obtained for G6 as a consequence of the combined effect of a relatively large axial stiffness and a very low shear stiffness. This behaviour is consistent with the bending-dominated deformation of the curved ligaments.

Fig. 3 summarizes the directional elastic response and separates the effects of ligament number, cell aspect ratio and spacing parameter. In the directional Young's modulus normalized by its maximum value $E^{\text{hom}}(\theta)/E_{\text{max}}$ (top row), the response decays steeply away from the principal material directions, reaching a minimum near $\theta = 45^\circ$. At $\theta = 90^\circ$ the normalized modulus equals the ratio E_2/E_1 , so that it exceeds unity only when the vertical direction is stiffer than the horizontal one, as for G3. The decay is sharper for the $N = 16$ configurations (panel a), where the normalized modulus drops below 0.1 within about 8° of either axis, against roughly 15° for the $N = 6$ group; this reflects the lower C_{1212} of the sixteen-ligament cells, whose thinner members ($s = 0.1$ cm versus 0.3 cm) have substantially higher bending compliance.

The cell aspect ratio sets the balance between the two principal stiffness directions (panel b). For the quasi-square reference cells the ratio E_2/E_1 is of order unity (1.12 for G3, $B/H = 0.80$; 0.74 for G1, $B/H = 1.25$), whereas for $B/H = 3.00$ it drops to 0.14–0.27, indicating a markedly stiffer horizontal response. The ligament number and thickness instead control the angular sharpness of the directional peaks through C_{1212} : thinner, more numerous ligaments lower the shear stiffness and produce the steeper decay observed for the $N = 16$ group.

By contrast, the spacing parameter has a limited effect on the open-cell directional response (panel c). Within the pairs G2–G5 and G4–G6 the curves are nearly indistinguishable, and the relative differences in C_{1111} remain below 0.1%. The vertical modulus C_{2222} is more sensitive, decreasing from 237.5 to 192.2 MPa for G2–G5 and from 348.7 to 252.6 MPa for G4–G6 as the mid-height gap changes the local ligament curvature and effective cross-section; this variation is nonetheless modest in relative terms and does not alter the qualitative character of the response. The influence of the spacing parameter becomes more apparent after contact activation, as discussed in Section 5.2.

The directional Poisson ratio (bottom row of Fig. 3) follows the same anisotropic pattern. Along the principal directions ν_{12} and ν_{21} are very small (Table 2), whereas off-axis loading drives $\nu(\theta)$ towards unity over broad angular intervals (panels d–f). This saturation has the same origin as the off-axis minimum of $E(\theta)$: when the compliance term D_{1212} dominates the response, the axial and transverse strains become comparable in magnitude. The maxima reach $\nu_{\text{max}} \approx 0.97$ for $N = 6$ and $\nu_{\text{max}} \approx 0.99$ –1.00 for $N = 16$. Such values are fully admissible for an anisotropic two-dimensional elastic solid, whose directional Poisson ratio is not restricted by the bounds valid in the isotropic case. The only geometry with a negative on-axis value is G5 ($\nu_{12} = -0.006$, $\nu_{21} = -0.001$, corresponding to $C_{1122} = -1.07$ MPa); the magnitude is close to zero and confined to a narrow angular interval around the axes.

This elastic characterization defines the pre-contact reference response of each geometry. The Poisson ratios reported here are tangent quantities of the undeformed cell and should not be confused with the apparent transverse-to-axial strain ratios measured during compression. Beyond providing this reference, the homogenized anisotropy also anticipates the contact behaviour analysed in Section 5.2: the strong

axial-to-shear stiffness contrast and the associated bending-dominated compliance account for the extended compliant open-cell regime, while the aspect-ratio control of E_2/E_1 — close to unity for the compact cells (G1, G3) and as low as 0.14–0.27 for the elongated ones (G2, G4) — rationalizes the orientation of the contact zones, from predominantly vertical in the neck to inclined when the stiffer horizontal response redirects the load towards inclined ligament segments.

5.2. Self-contact-induced stiffening response

The nonlinear compression simulations are used to examine how progressive internal self-contact modifies the mechanical response of the selected lattice geometries beyond the initial open-cell regime characterized in Section 5.1. The finite-cell compression problem is driven by the prescribed vertical displacement of the upper flange, with the lower flange prevented from moving vertically and the lateral boundaries left unconstrained. A single horizontal constraint is introduced only to remove the residual rigid-body mode. The reported force is the resultant vertical reaction on the loaded flange. The analysis combines the force–displacement response with the evolution of the active contact regions during compression. Particular attention is devoted to the onset of contact, its subsequent propagation and the associated changes in the slope of the force–displacement response. The objective is to interpret how the selected geometric features affect the transition from the initial compliant regime to the contact-constrained response.

The compression simulations are carried out within a small-displacement kinematic description of the bulk continuum. The geometric nonlinearity retained in the analysis is that of the self-contact constraint, enforced through the node-to-segment formulation. The adequacy of this assumption was checked by re-analysing the configurations with a finite-displacement formulation of the continuum elements: over the range of converged solutions the responses were found to be in close agreement. Although the imposed compression produces large overall displacements, these are mainly accommodated by the bending-dominated, nearly inextensional motion of the slender curved ligaments. As a consequence, the local strains in the solid phase remain small and the membrane forces developing along the ligaments are limited. The finite-deformation check therefore confirms that geometric stiffening of the bulk continuum is negligible in the present configurations. The observed post-contact stiffening is instead governed primarily by the progressive activation and evolution of self-contact constraints, which modify the deformation modes and load paths. Consistently with this, the computed deformation patterns and the global force–displacement curves do not exhibit features typically associated with structural buckling, such as bifurcation points, snap-through or load drops; instead, the response remains monotonic, with a progressively increasing slope.

In all configurations, the initial portion of the response corresponds to an open-cell regime in which the curved ligaments deform without mutual interaction. As the imposed compression increases, the gaps between adjacent ligaments progressively close and self-contact is activated in the central neck region. The activation of contact introduces additional kinematic constraints and modifies the internal load-transfer mechanism. Consequently, the force–displacement response departs from the initial nearly linear trend and progressively evolves towards a stiffer regime. The sequence of contact events therefore provides a direct interpretation of the observed slope changes in the global response.

At the structural level, contact-induced stiffening is used here in an incremental sense. Once a gap closes, the active contact constraint adds a normal resistance to the tangent problem, consistently with the contact stiffness contribution in Eq. (15). As further contact pairs become active, these local increments accumulate and appear macroscopically as an increased slope of the global force–displacement curve.

The six-ligament reference configuration G1, reported in Fig. 4, illustrates the basic stiffening mechanism. Contact is first activated

between the innermost ligament pair and subsequently propagates towards adjacent ligaments as compression proceeds. The corresponding force–displacement curve exhibits a sequence of progressively steeper branches associated with the enlargement of the active contact region. At the largest compression levels, contact is no longer confined to the central vertical alignment and additional contact zones develop along inclined ligament segments.

For selected loading stages, the redistribution of the internal load-transfer paths is further illustrated through the in-plane principal stress fields computed in the solid phase. These plots report the maximum and minimum principal stresses obtained from the same plane-stress finite element simulations used for the corresponding force–displacement curves. The stress values shown in the contour plots are reported in kgf/cm^2 , consistently with the unit system used in the finite element computations; the conversion factor is $1 \text{ kgf}/\text{cm}^2 \approx 0.0981 \text{ MPa}$. The principal stress contours shown in Fig. 5 confirm that the contact-induced constraints redistribute the load transfer through the central portion of the cell, producing a more extended stress-carrying region than in the open-cell configuration.

A different contact sequence is observed for G2 (Fig. 6), which combines a larger aspect ratio and a larger initial spacing. In this case the onset of contact occurs at a larger displacement and the initial compliant branch extends over a wider displacement range. Once contact is activated, the interaction rapidly spreads towards neighbouring ligaments and generates inclined contact patterns already at intermediate compression levels. The corresponding stress field (Fig. 7) reflects this redistribution mechanism, with stresses extending over a larger portion of the ligament network compared with G1.

The effect of increasing the number of ligaments is illustrated by G3 (Fig. 8). Although contact is again initiated in the neck region, the larger number of thinner ligaments produces a substantially different evolution compared with G1. Instead of a sequence of localized contact events, the active contact region expands progressively over a significant vertical portion of the cell. As a consequence, the force–displacement curve evolves more smoothly, without the pronounced slope transitions observed in the six-ligament geometries. The stress contours reported in Fig. 9 show that the load transfer remains distributed over a wider region of the microstructure, consistently with the more gradual development of contact.

A similar distributed mechanism is observed in G4 (Fig. 10), where the larger ligament number is combined with the elongated geometry. Contact is initially activated in the central neck region but subsequently propagates through several neighbouring ligament pairs and eventually involves inclined contact zones extending away from the centreline. The resulting force–displacement response remains comparatively smooth despite the occurrence of multiple contact events. The stress contours of Fig. 11 indicate that the contact-induced load-transfer mechanism extends over a large fraction of the cell, reflecting the distributed character of the contact process.

Taken together, Figs. 4–11 show that the observed stiffening response is directly associated with the progressive enlargement of the active contact region. Contact activation modifies the internal load-transfer paths and progressively restricts the admissible deformation mechanisms of the ligament network. The resulting increase in structural constraint is responsible for the transition from the initial compliant response to the post-contact stiffening regime. Thus, the relevant mechanical quantity is not only the displacement at first contact, but also the way in which the active contact set grows during compression. At the same time, the geometry influences both the displacement level at which contact is first activated and the spatial distribution of the subsequent contact evolution.

5.2.1. Influence of ligament spacing, aspect ratio and ligament number

The comparison among the selected geometries highlights how the contact-induced response can be interpreted in terms of the interplay between ligament spacing, cell aspect ratio and ligament number.

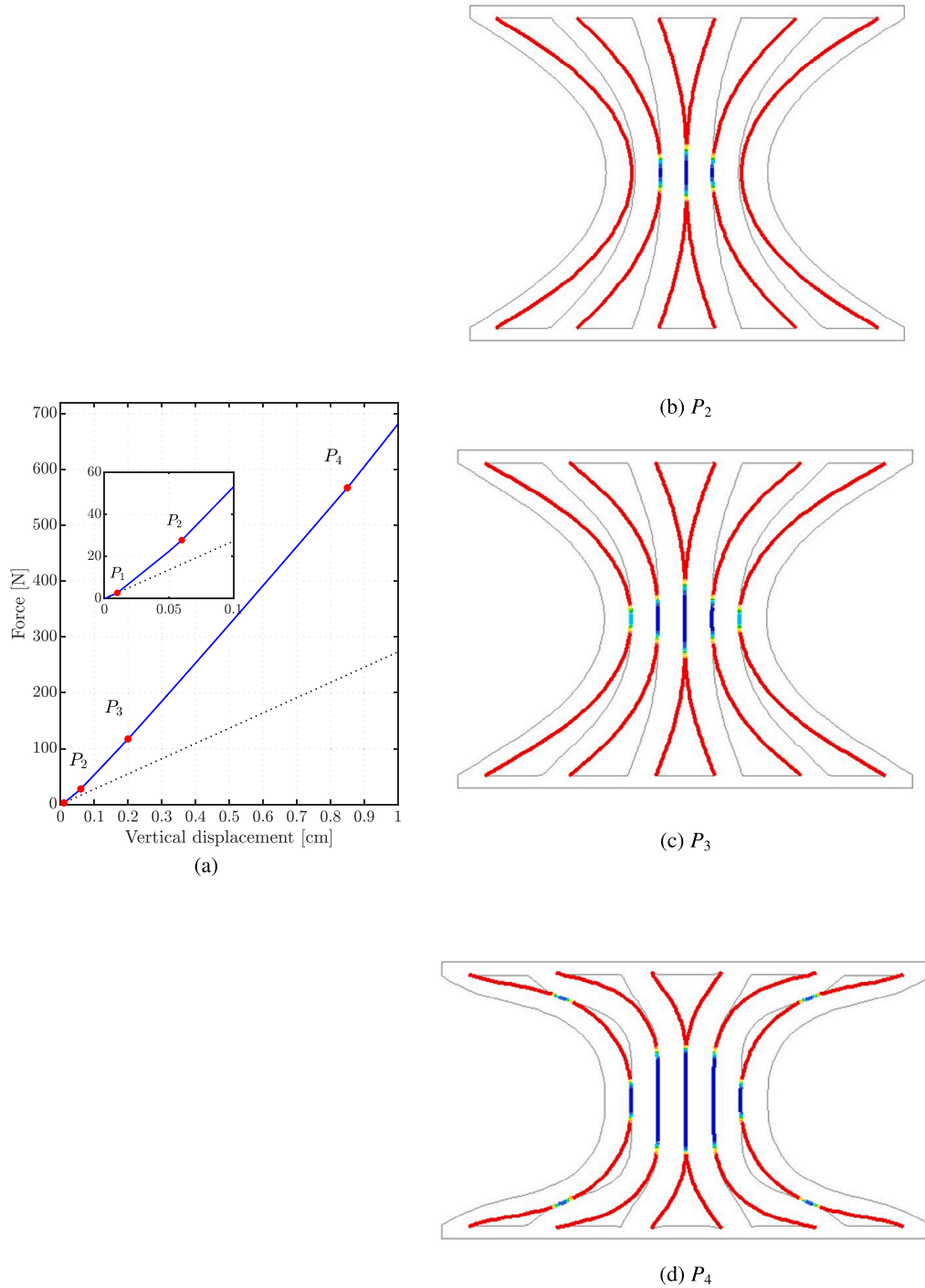


Fig. 4. Self-contact activation in the six-ligament configuration with $H = 4$ cm, $B = 5$ cm and $i = 0.03$ cm. (a) Force–displacement response. (b–d) Contact patterns at stages P_2 , P_3 , P_4 .

Within the pairs G2–G5 and G4–G6, where the aspect ratio, ligament number and ligament thickness are unchanged, the spacing parameter i mainly affects the onset of self-contact. This effect is clearly observed in Fig. 12. Smaller spacing values produce earlier contact activation and therefore reduce the extent of the initial open-cell regime. Conversely, larger spacing delays the first contact event and extends the initial compliant branch of the force–displacement response.

The aspect ratio B/H influences both the initial elastic anisotropy and the spatial distribution of contact zones. Elongated cells ($B/H = 3.00$, G2 and G4) develop inclined contact patterns already at intermediate compression levels, whereas the more compact geometries (G1 and G3) exhibit predominantly vertical contact in the neck region before diagonal zones appear at larger displacements. This reflects the stronger lateral constraint imposed by the wider flanges in the

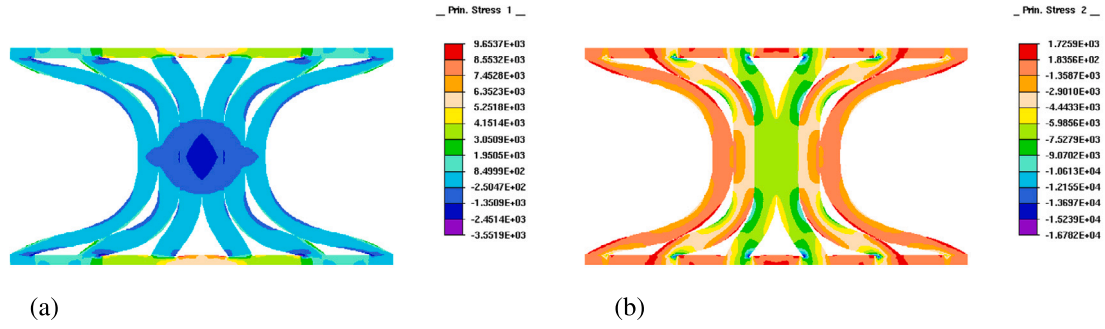


Fig. 5. Principal stress contours at loading stage P_4 of the force–displacement response shown in Fig. 4.

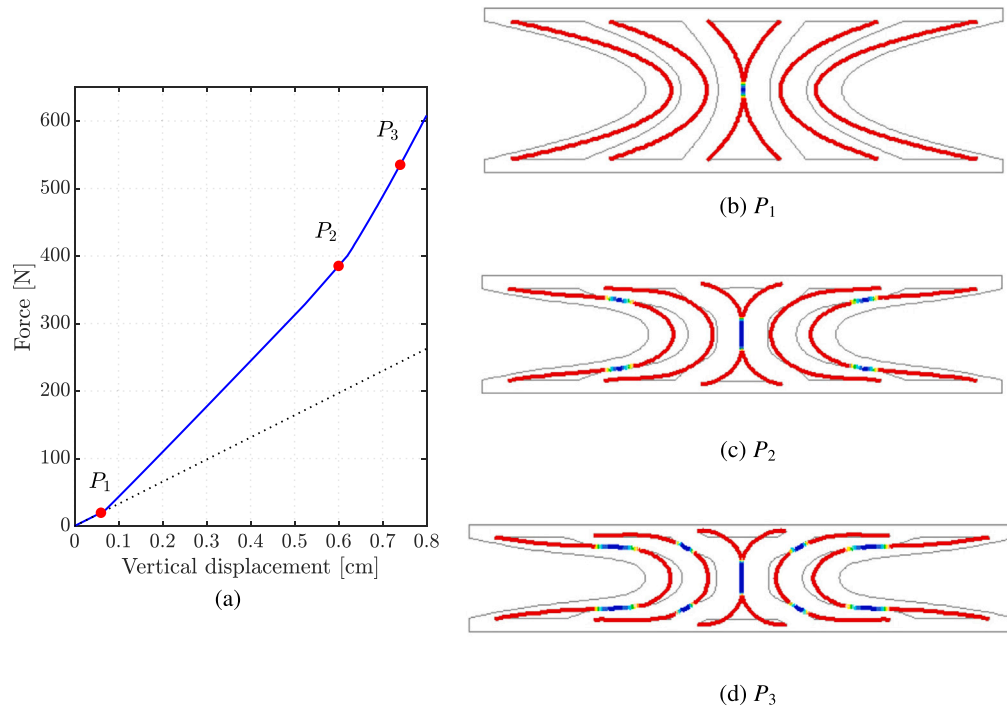


Fig. 6. Self-contact activation in the six-ligament configuration with $H = 2$ cm, $B = 6$ cm and $i = 0.10$ cm. (a) Force–displacement response. (b–d) Contact patterns at stages P_1 , P_2 and P_3 .

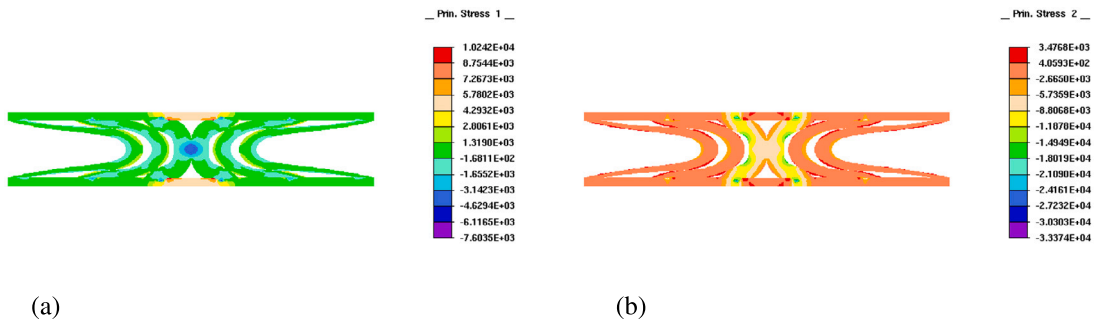


Fig. 7. Principal stress contours at loading stage P_3 of the force–displacement response shown in Fig. 6.

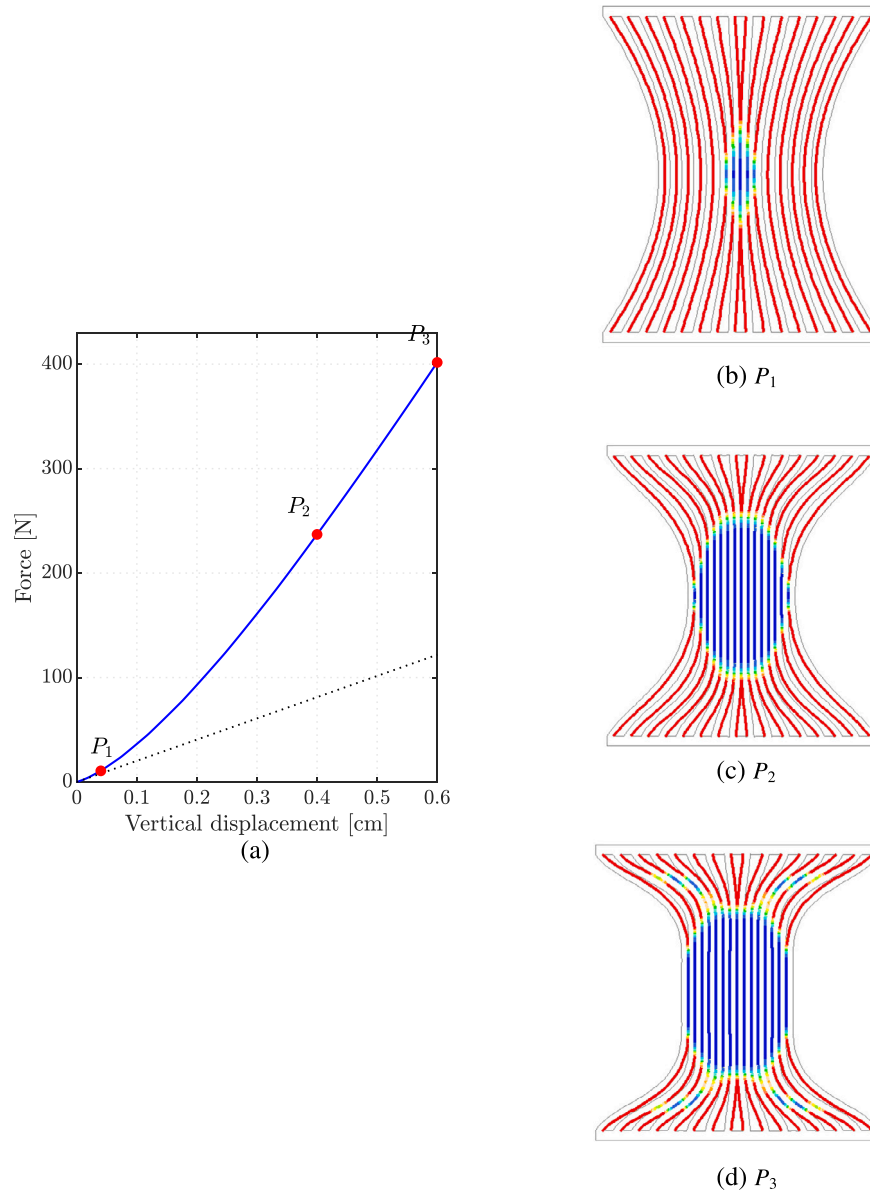


Fig. 8. Self-contact activation in the sixteen-ligament configuration with $H = 5$ cm, $B = 4$ cm and $i = 0.06$ cm. (a) Force–displacement response. (b–d) Contact patterns at stages P_1 , P_2 , P_3 .

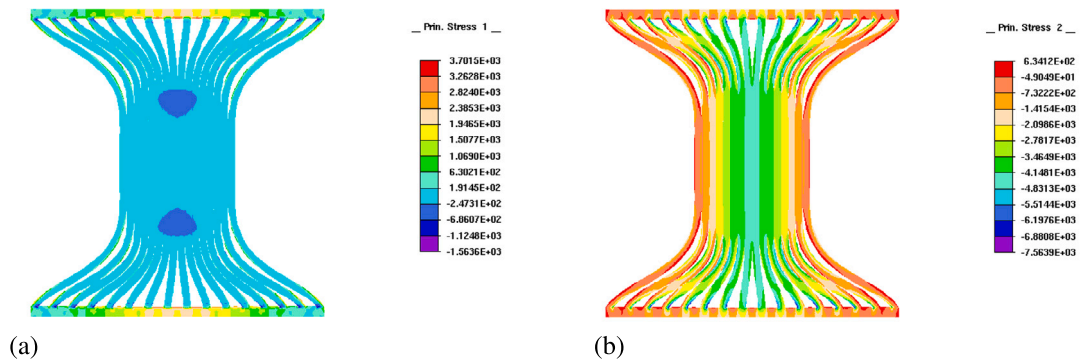


Fig. 9. Principal stress contours at loading stage P_3 of the force–displacement response shown in Fig. 8.

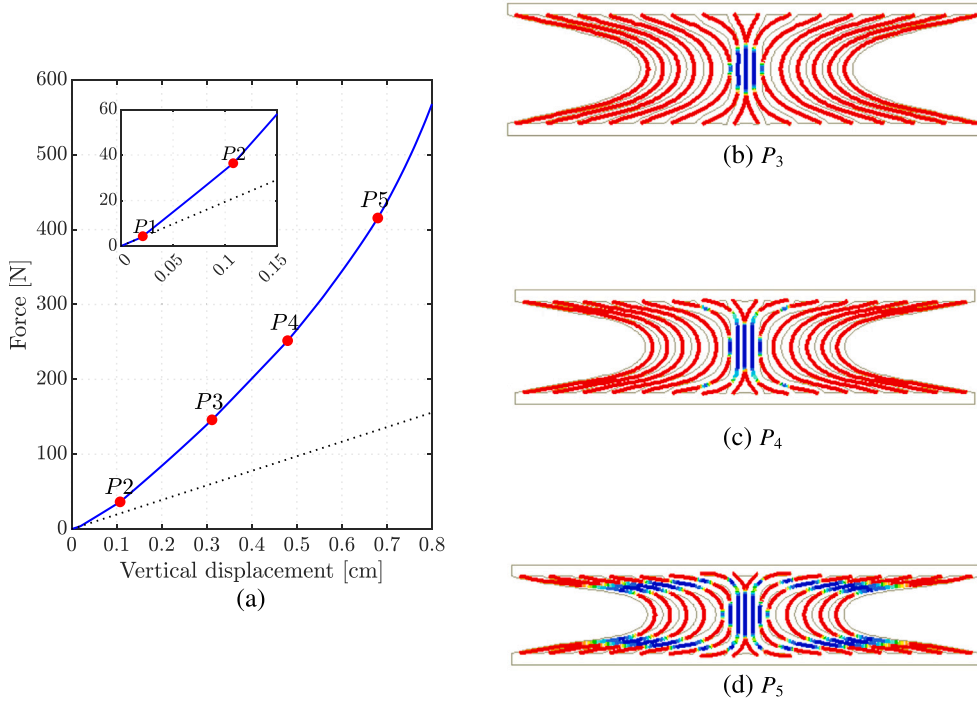


Fig. 10. Self-contact activation in the sixteen-ligament configuration with $H = 2$ cm, $B = 6$ cm and $i = 0.08$ cm. (a) Force–displacement response. (b–d) Contact patterns at stages P_3 , P_4 , P_5 .

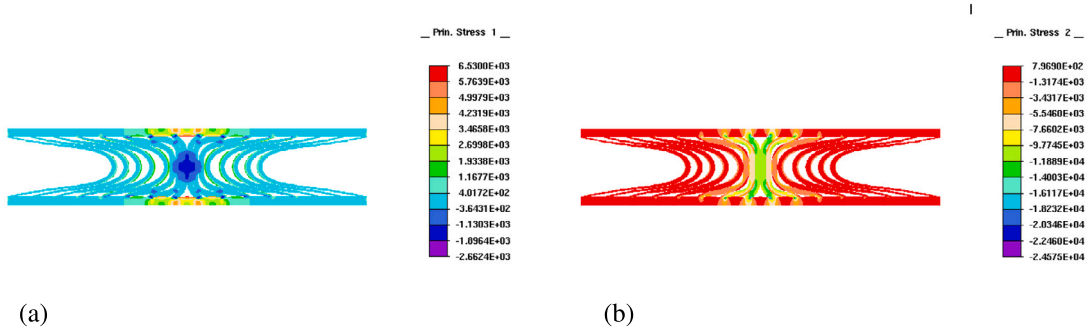


Fig. 11. Principal stress contours at loading stage P_5 of the force–displacement response shown in Fig. 10.

elongated configurations, which redirects the load transfer towards inclined ligament segments at an earlier stage of compression.

The number of ligaments affects the spatial distribution of contact activation. The six-ligament configurations exhibit relatively localized contact events and more abrupt stiffness transitions, whereas the sixteen-ligament configurations develop contact progressively over a wider portion of the cell, producing smoother force–displacement evolution.

Overall, the results indicate that the observed stiffening originates from the progressive activation of internal contact constraints. Contact modifies the admissible deformation mechanisms of the ligament network and promotes the formation of additional load-transfer paths. The transition from the initial compliant response to the post-contact regime is therefore linked to the geometry-dependent evolution of the contact pattern and to the accumulation of incremental contact constraints.

6. Final remarks

This work investigated the compressive response of curved-ligament re-entrant lattice architectures in which internal self-contact acts as

a geometry-controlled stiffening mechanism. The analysis combined first-order computational homogenization of the undeformed periodic cell with nonlinear compression simulations including frictionless self-contact. The main findings can be summarized as follows:

- the homogenized open-cell response is strongly anisotropic and governed by the ligament architecture. The aspect ratio B/H controls the balance between the principal stiffness directions, while the ligament number and thickness influence the angular sharpness of the directional response;
- under compression, progressive gap closure activates internal contact constraints that reconfigure the load-transfer paths. The resulting stiffening is an incremental structural effect, because the growing active contact set adds normal constraints that increase the slope of the force–displacement response;
- within the representative geometries considered, ligament spacing mainly sets the displacement at contact onset, ligament number governs whether the active contact set grows in a localized or distributed manner, and the aspect ratio controls both the initial anisotropy and the orientation of the contact zones;

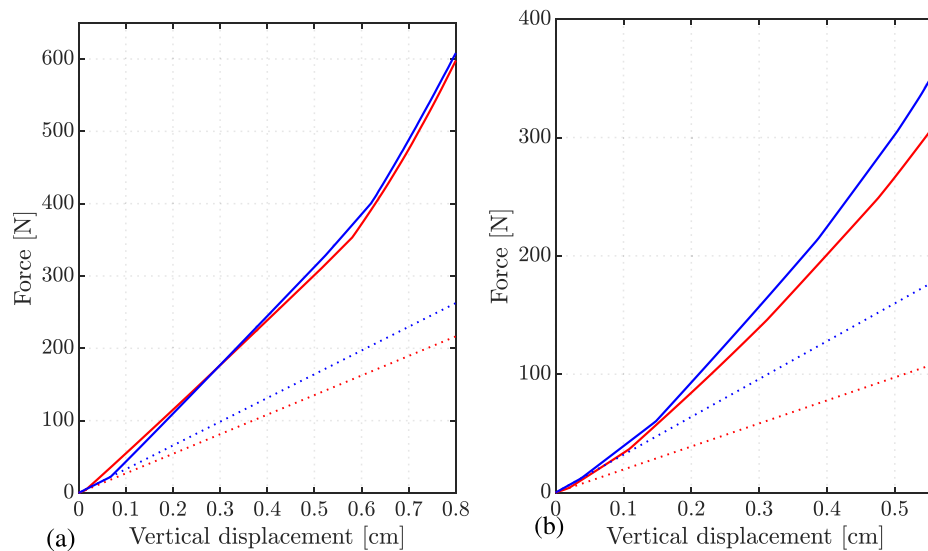


Fig. 12. Force–displacement responses for G2–G5 (a) and G4–G6 (b): G2 (blue) and G5 (red) in (a); G4 (blue) and G6 (red) in (b). Smaller ligament spacing produces earlier contact activation.

- the observed stiffening originates from geometry-dependent contact activation and does not require material nonlinearities or additional internal components.

Overall, progressively recruited self-contact can be exploited as a purely geometric route to a tunable post-contact stiffening response. The present study should be regarded as a focused numerical investigation of representative geometries; a complete design map would require a broader parametric exploration in which spacing, ligament number, thickness and aspect ratio are varied independently. Future work will include the experimental validation of additively manufactured specimens and the investigation of the interplay between contact activation, manufacturing tolerances, friction, material nonlinearity and failure within the proposed lattice architectures.

CRedit authorship contribution statement

Maria Laura De Bellis: Writing – original draft, Software, Methodology, Conceptualization. **Giorgio Zavarise:** Writing – original draft, Software, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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