Summary

This thesis investigates multidimensional Bernoulli distributions through algebraic and geometric representations, which prove to be effective tools to address open issues in the theory of negative dependence. Bernoulli distributions play a central role in many applied contexts, such as statistical physics, finance, and insurance, yet characterizing their joint behavior under negative dependence remains a significant challenge. By focusing on Fréchet classes, i.e., sets of joint distributions with fixed univariate marginals, we treat these structures as convex polytopes. This setting allows for a study of dependence properties independently of marginal effects. In this analysis, we employ two main algebraic frameworks: the classical probability generating functions and a recent representation for identically distributed marginals based on ideals of polynomials.

A primary focus of this work is the characterization of extremal negative dependence. Unlike positive dependence, which is bounded by the comonotonic distribution, negative dependence does not have a unique, well-defined lower bound in dimensions $d > 2$. We concentrate on Σ-countermonotonicity, a property of extremal negative dependence that generalizes the concept of countermonotonicity to high dimensions. Our first major theoretical result proves that, in the Bernoulli setting, Σ-countermonotonicity coincides with the minimality of aggregate risk in the convex order. This simplifies the task of finding minimal risk bounds, which is essential for credit risk and insurance portfolios.

Furthermore, we integrate the strongly Rayleigh property into the framework of extremal dependence. As a form of stochastic repulsion based on the geometry of zeros of probability generating functions, the strongly Rayleigh property is stronger than negative association. Our second main result demonstrates that every Bernoulli Fréchet class contains at least one distribution satisfying both Σ-countermonotonicity and the SR property. We identify this distribution through entropy maximization within the class of Σ-countermonotonic Bernoulli distributions.

These theoretical findings are applied to specific subclasses, namely symmetric Bernoulli and Conway–Maxwell-binomial distributions, where we explicitly characterize extremal negative dependence distributions and parameter ranges that guarantee strongly Rayleigh, respectively. Finally, we extend our results to copula theory. Using stochastic representations, we construct extremal mixture and Farlie–Gumbel–Morgenstern copulas, and we also consider a generalization of the latter. This extension allows us to move beyond Bernoulli marginals and provide analytical tools for determining lower bounds in high-dimensional and continuous settings for risk measures, such as Value-at-Risk, expected shortfall, and the entropic risk measure.