Using Explainable Machine Learning to Support the Modeling Strategies of Composite Structures

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The proper modeling of composite structures aims to minimize computational overhead without compromising accuracy. When finite elements are used, modeling requires selecting the structural theory, element type, and mesh; these choices depend on the problem features, e.g., the type of analysis, the outputs of interest, the material system, and others. The aim of the present work is to use Explainable Machine Learning methods to provide guidelines for the proper modeling of composite structures, with a focus on the choice of the set of generalized variables, i.e., the model's degrees of freedom.
One-dimensional (1D) structural theories with higher-order displacement fields are implemented using the Carrera Unified Formulation (CUF) [1], which enables systematic multi-fidelity analyses by varying the order of the kinematic expansion [2]. In this work, the evaluated theories range from first- to fourth-order models, with the full fourth-order theory adopted as the reference.

$$\begin{aligned} u_x &= u_{x1} + x u_{x2} + z u_{x3} + x^2 u_{x4} + xz u_{x5} + z^2 u_{x6} + \cdots + xz^3 u_{x14} + z^4 u_{x15} \\ u_y &= u_{y1} + x u_{y2} + z u_{y3} + x^2 u_{y4} + xz u_{y5} + z^2 u_{y6} + \cdots + xz^3 u_{y14} + z^4 u_{y15} \\ u_z &= u_{z1} + x u_{z2} + z u_{z3} + x^2 u_{z4} + xz u_{z5} + z^2 u_{z6} + \cdots + xz^3 u_{z14} + z^4 u_{z15} \end{aligned} \tag{1}$$

The classical Timoshenko theory is a special case of the expansion that uses five of the 45 degrees of freedom. A Neural Network (NN) [3,4] is trained on input features representing generalized variables [5], material properties, and geometry to predict the accuracy of structural theories for a given problem. Explainable ML [6] tools based on SHapley Additive exPlanations (SHAP) [7] are then used to quantify the contribution of each generalized variable and problem feature to the predicted error, enabling identification of the most influential terms.
Preliminary results focus on a three-layer laminated beam [0°/90°/0°], clamped-free, made of an orthotropic material with $E_L = 250\,GPa$, $E_T = 10GPa$, $\nu_L = \nu_T = 0.33$, $G_L = 5\,GPa$, $G_T = 2GPa$, $\rho = 2700 kg/m^3$, $L/h = 50$. Figure 1 reports, on a logarithmic scale, the errors associated with the best structural theories as their degrees of freedom change. Errors were computed on the first ten natural frequencies. These curves are referred to as Best Theory Diagrams (BTD) [8], as they show the best achievable accuracy for a given number of DOF. The 15-DOF model is the full fourth-order theory of Eq. 1; the actual number of DOF is 45, but the three components of a given expansion term are either activated or deactivated simultaneously, and, therefore, the number of expansion terms is reported. The 21-DOF and 12-DOF theories are highlighted as the best combinations of 21 and 12 generalized variables, respectively, i.e., those that maximize accuracy. The results from a full FEM analysis and those obtained by a Deep Neural Network are compared and a perfect match was found. Table 1 reports all the best theories with black triangles indicating active terms. The SHAP analysis includes Feature Importance bar plots, see Fig. 2, and Beeswarm plots, see Fig. 3. In both cases, generalized variables are reported along one axis and SHAP values (ϕ) along the other, representing each variable's contribution to the predicted error. The bar plot shows the average absolute SHAP value $|\bar{\phi}|$, providing a direct measure of variable importance. The Beeswarm plot displays the distribution of SHAP values across all structural theories, with red points indicating active terms and blue points inactive ones. This analysis provides insight into how generalized variables influence accuracy. Figures 4 and 5 show, respectively, the effects of changing the lamination [0°/0°/90°] and the slenderness, L/h = 10. Assessing the influence of generalized variables on the dynamic response of composite beams supports the extraction of the most relevant terms, facilitating the construction of efficient reduced-order models with minimal complexity and high accuracy.

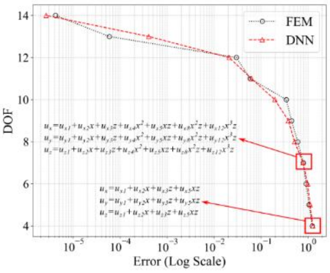

Figure 1: BTD of the [0°/90°/0°] laminated beam with L/h=50

DOF	$const.$	x	z	x^2	xz	z^2	x^3	x^2z	xz^2	z^3	x^4	x^3z	x^2z^2	xz^3	z^4	$\%E_{AVG}$
14	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	▲	△	2.642E-06
13	▲	▲	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	▲	▲	△	4.069E-04
12	▲	▲	▲	▲	△	▲	▲	▲	▲	▲	△	▲	△	▲	▲	2.090E-02
11	▲	▲	▲	▲	▲	▲	△	▲	▲	△	▲	▲	△	▲	△	6.277E-02
10	▲	▲	▲	▲	△	▲	△	▲	▲	△	△	▲	△	▲	▲	1.969E-01
9	▲	▲	▲	▲	▲	▲	△	▲	△	△	△	▲	▲	△	△	3.844E-01
8	▲	▲	▲	▲	▲	▲	△	▲	△	△	△	▲	△	△	△	5.193E-01
7	▲	▲	▲	▲	▲	△	△	▲	△	△	△	▲	△	△	△	7.905E-01
6	▲	▲	▲	△	▲	△	△	▲	△	△	△	▲	△	△	△	9.858E-01
5	▲	▲	▲	△	▲	△	△	▲	△	△	△	△	△	△	△	1.115E+00
4	▲	▲	▲	△	▲	△	△	△	△	△	△	△	△	△	△	1.268E+00

Table 1: Best theories of the [0°/90°/0°] laminated beam with L/h=50 evaluated by DNN

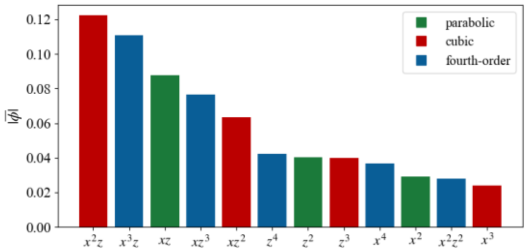

Figure 2: Generalized variables influence bar plot for the [0°/90°/0°] laminated beam with L/h=50

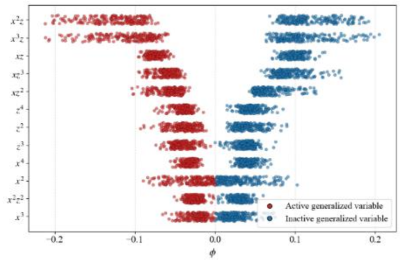

Figure 3: Generalized variables beeswarm plot for the [0°/90°/0°] laminated beam with L/h=50

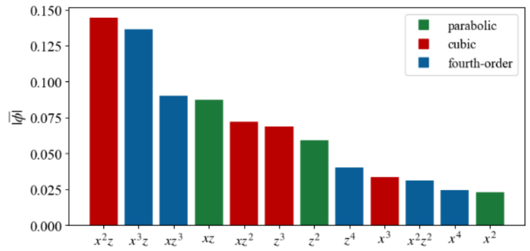

Figure 4: Generalized variables influence bar plot for the [0°/0°/90°] laminated beam with L/h=50

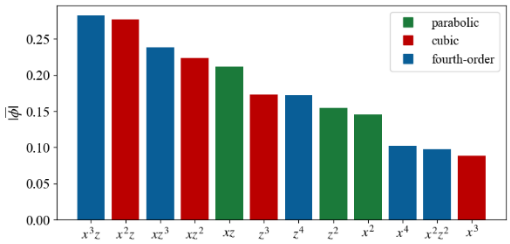

Figure 5: Generalized variables beeswarm plot for the [0°/90°/0°] laminated beam with L/h=10

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