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Abstract. Tensegrity systems are structures in equilibrium under a self-stress state prior to the application of external loads. Such self-stress state is the result of the superposition of self-stress basis vectors. Tensegrities have complex nonlinear behavior when subjected to external actions (loads or imposed displacements). In this work, the robustness of this particular structural typology is evaluated by numerical simulations. Robustness is the ability of a locally damaged structural system not to behave in a manner disproportionate to the original failure, i.e., the insensitivity to local damage. This concept and its insights have been thoroughly analyzed for several structural systems, as in the case of frame structures. To date, limited studies have been performed on the robustness of tensegrity systems to unexpected conditions. Among the potential damages, one can consider the settlement of one of the supports, the reduction of prestress and the failure of one of the cables. The present study reports the results of a set of damage simulations on a simple T-3 prismatic tensegrity structure. Several risk factors are considered to study the response of the structure to unexpected damage. In particular, the tensegrity configuration is subjected to different stress loss in the constituent elements. The results are obtained through simulations performed in MSC.visualNastran 4D (vN4D), which run dynamic simulations by merging CAD, motion, FEA, and controls technologies into a single functional modeling system. The models were established to obtain different equilibrium configurations and conditions of loss of equilibrium, with the corresponding large displacements measured for the most significant points of the structure, assumed as markers for the resulting effects on the structure.

Keywords: Tensegrity Systems, Robustness, Dynamic Simulations, Structural Damage, Prestress Reduction

1 Introduction

Buckminster Fuller and Snelson exhibited tensile structures with patents issued in 1962 [1] and 1965 [2], respectively. In the same period, Emmerich was studying similar structures [3]. In the beginning chapters of his 1974 book on geodesic dome geometry, the renowned literary critic Hugh Kenner [4] offered an insightful mathematical study of regular tensegrity prisms and spherical tensegrities. A companion book by Pugh [5] is essentially a how-to guide for constructing hundreds of regular tensegrity structures. Early findings on this topic are based on the works by Calladine, Tarnai, and Pellegrino [6–11], that discuss the basic mechanics of this kind of structures. The characterisation of rigid tensegrity configurations, which generalizes previous findings on the rigidity of pinned frames, has been the main focus of excellent mathematical research by Roth and Whiteley [12], Connelly [13], and Conneley and Whiteley [14]. There is also a corpus of research in cell biology, introduced by Ingber [15], that examines the mechanical behaviour of a tensegrity system that serves as the framework for the ctyoskeleton.

More recently, a few straightforward tensegrity structures, each of which has an analytical model, were analysed by Micheletti [16]. In this studies, a qualitative description of tensegrity system as a first-order mechanism were provided, together with static and dynamic analytical models (see also the literature referenced in [17]).

The focus of this paper's discussion will be on the findings regarding the robustness of simple tensegrity structures for applications in civil engineering and architecture.

In short, robustness is a structural property. According to Eurocode 1 – EN 1991-7 [18], “a localised failure due to accidental actions may be acceptable, provided it will not endanger the stability of the whole structure, and that the overall load-bearing capacity of the structure is maintained and allows necessary emergency measures to be taken”. Hence, a robust structure is designed in order not to behave in a disproportionate way with respect to an initial localised failure. The concept, formulated after Ronan Point Building Accident in 1968 [19], has been applied to a large set of structures, mainly frame structures and bridges. The basic strategy for ensuring robustness in structures characterized by the presence of several elements that are continuously connected (such as in frame structures or in trusses) is to provide alternative load paths to ensure the transfer of forces when one or more elements are removed.

Although robustness studies have accounted for a multitude of structural types, so far, to the knowledge of the authors, extremely limited research has been performed on the robustness of tensegrity systems [20]. A tensegrity structure is a three-dimensional truss that has many tension members and few discontiguous compression members, that are in a pre-stressed condition at a certain geometry. While many patterns of connection may be used to assemble these structures, the simplest fundamental version has no more than one compression members and three tension members connected at each node.

The geometry we refer to as a T-3 prism has nine (tensile) cables and three (compressive) bars, making it the least complex of these spatial structures. There exist corresponding T-n prisms with 3*n* cables and *n* bars, and so on. It is a matter of fact that such a prism is a form-finding structure that becomes a stiff prestressed system as the last cable is pulled in tension. Our conventional use of Maxwell's rule does not adequately describe this structural type [21], and our traditional concepts of and statical determinacy, structural stiffness, and statical stability do not directly apply.

It has long been known that these prisms can be made to change their nodal geometry by appropriately adjusting the cable lengths, and it is because of this behavior that the prisms are compared to variable geometry truss manipulator in many applications.

We point out right away that there is no universal method for specifying the nodal locations that characterize a tensegrity state. For the situation of a right regular T-prism, a beginning state may be determined via symmetry and verified using straightforward statics. A 30 degree relative twist angle exists between the end faces of the T-3's two equilateral triangle end faces, which are parallel to one another and orthogonal to the axis connecting their centers. Every possible T-3 geometry can theoretically be achieved from that recognizable starting configuration by appropriately adjusting member lengths.

We shall demonstrate, however, that determining "suitable" (allowed) changes in member lengths is a difficult task in and of itself. We are motivated to investigate the appropriateness of tensegrity structures for robotic and civil engineering applications by a number of specific properties, including the following one: by tensioning the last cable, tenssegrity prisms may completely be erected from a low-volume bundle. This property is advantegeous for instance for erecting and dismantling temporary buildings.

A tensegrity tower might be quickly erected, designed to be strong and rigid enough to withstand the weight of the workers, who could then climb the tower and install more structural members to make a conventional tower framing. In an analogous way, a self-erecting prism could be remotely moved to a specific point inside a pipe system in its collapsed bundle condition, where it would subsequently take the self-stressed shape to create a crawling robot.

Because tensegrity structures typically include more cables, it is possible to obtain geometries with less total material weight than a traditional three-dimensional truss. Self-erecting capabilities and the anticipated weight economy point to applications for space constructions and robotics. Tensegrity structures present different families of shape-adaptable structures and corresponding machine kinematics when member lengths can be changed or when large elastic elongations are allowed. This has the advantage that actuation can be envisaged only for tension members, with simpler technological solutions than general bi-directional translational actuators.

The present study reports the results of a set of damage simulations on a simple prismatic tensegrity structure. Several risk factors are considered to study the response of the structure to unexpected damage. In particular, the T-3 structural configuration is subjected to different stress loss in the constituent elements. Results are obtained through simulations performed in MSC.visualNastran 4D (vN4D), which run dynamic simulations by merging CAD, motion, FEA, and controls technologies into a single functional modeling system. The models were established to obtain different equilibrium configurations and conditions of loss of equilibrium, with the corresponding large displacements measured for the most significant points of the structure, assumed as markers for the resulting effects on the structure.

2 Robustness of a Simple Tensegrity System to Local Damages

The first structure considered is the structure of the T-3 in its standard configuration. The realized model consists of rods 278 mm long with a diameter of 10 mm (elements a , b, c).

The top and base ropes have a length of 173 mm (u-1, u-2, u-3) and the diagonals have a length of 207 mm (d-1, d-2,d-3). These values correspond to a height *h* of the structure of 200 mm and a radius *r* of the circle circumscribed to the triagular bases of 200 mm. In the present analysis, we assume bars to be rigid and cables to be inextensible. The configuration shown in Figure 1 below represents the stable equilibrium configuration characterized by a symmetrical loading condition at the vertices of rods A, B, C and pretension in all ropes u1-3, d1-3, f1-3. Ropes f1-3 take the same lengths of u1-3.

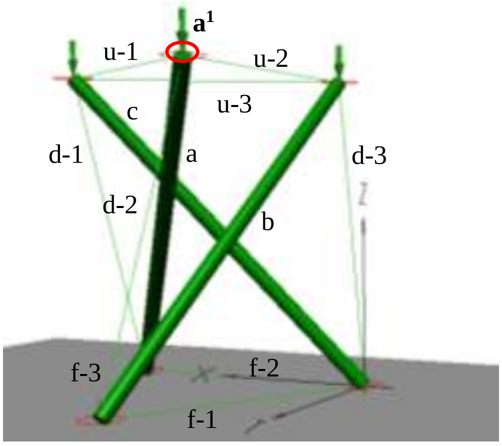
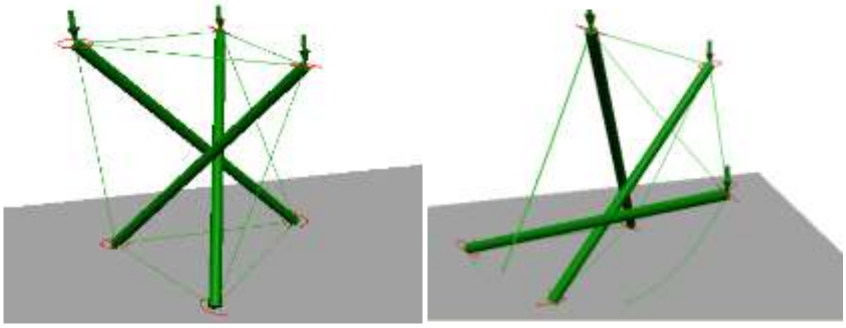


Fig.1: Simple Tensegrity system T-3 configuration 3 rods and 9 ropes.

In order to be able to explore the collapse conditions as a function of reducing the degree of pretension of the ropes and also studying the relative level of associated slacking, we first proceed to increase the length of the rope d-1 passing to an increase equal to 1% from the initial length in pretension condition. The d-1 rope thus increases from a length of 207 mm to 209 mm. The effect is to achieve an equilibrium configuration close to the previous one that does not compromise the overall condition still far from the collapse condition.

By placing a position sensor on the vertex a^1 it is possible to obtains a new configuration with displacements of node a^1 (see Figure 2) equal to 4% 3% and 2.5% of the height *h*, respectively for the x, y, z components. With an increase of the rope length d-1 by 2%, we still obtain a configuration without contact between the rigid elements (rods) a,b,c but with a displacement of node a^1 equal to 6% of 3.5% and 3% of the height *h*, respectively for the x,y,z components. The first configuration involving contact between the rigid rod elements is that correspondent to an increase in the length of the rope d-1 by 7% arriving at an overall length of 221 mm compared to the starting 207 mm. The collapse with overturning of the T-3 scheme occurs for an increase in rope length d-1 of 26% (see Figure 2).

(d-1) – (d-1$_{initial}$)/ (d-1$_{initial}$)=0.07 (d-1)–(d-1$_{initial}$)/ (d-1$_{initial}$)=0.26

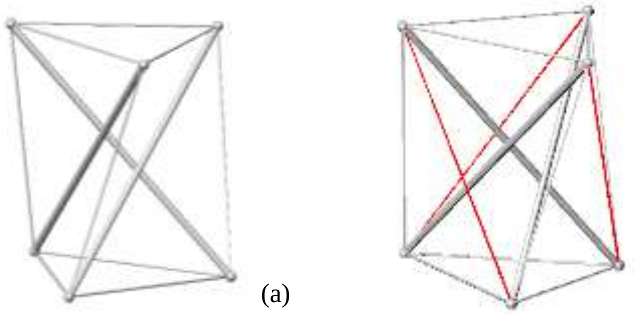


(a) (b)

Fig.2: Configuration of the T-3 basic system with one of the diagonal element d-1 with an increment of the length of 7% (a) and of 26% (b)

3 Robustness of a Reinforced Tensegrity Structure

In the configuration shown in Figure 3, the stiffening ropes shown in red have been added. The effect of these additional ropes will be considered. In particular with respect to the (d-1)-(d-1initial)/ (d-1initial)=0.26 configuration that resulted in the collapse condition in the basic version (Fig.3a), a condition is obtained that does not result in the complete collapse of the system with the addition of the new ropes.



(a) (b)

Fig.3: Simple Tensegrity system T3 (a) and modified T3 with three additional diagonal ropes (b).

In particular, the stiffening ropes come into operation during the kinematics of the main mechanism, preventing tilting. Again, monitoring of the displacement of a^1, values of 5%, 24% and 16% of the height *h* are obtained respectively for the x,y,z components of the point a^1 with respect to the global reference system.

These results are obtained by considering the constituent bodies of the rods as bodies characterized by collision with each other and non-penetrating. This realizes an additional constraint of mutual support and friction (coulombian friction) between the three rods that come into contact at the central point of the system. In the case where the latter constraint is not present, an even better condition is obtained. In fact, the monitoring point a^1 yields displacements of 8% by 53% and 13% of the height *h*, respectively for the x,y,z components.

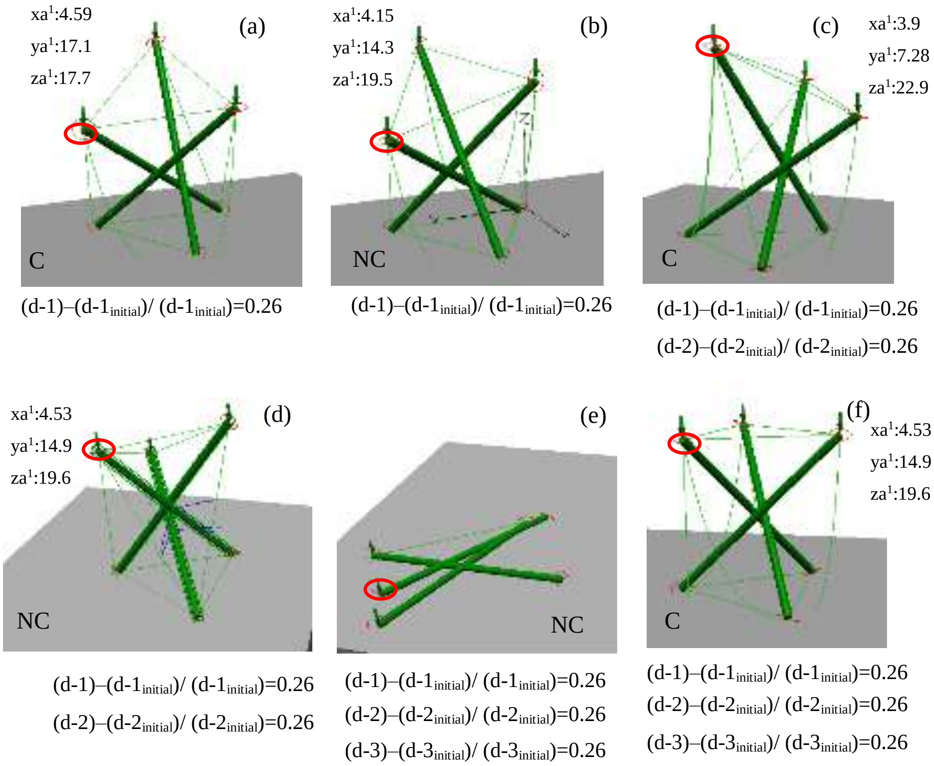


Fig.4: T3 with three additional diagonal ropes: condition with a slack coefficient of 26% for principal rope d-1 in collision modality for the bodies (a), not collision (b). Condition with a slack coefficient of 26% for principal rope d-1 and d-2 in collision modality for the bodies (c), not collision (d). Condition with a slack coefficient of 26% for principal rope d-1, d-2 and d-3 in not collision modality for the bodies (e), and collision (f).

4 Conclusions

The present paper describes a preliminary study on the effects of cable slackening in a tensegrity structure. It is worth to note that the effectiveness of the load paths in a tensegrity structure is determined by the prestress force in the cables and the resulting geometrical configuration. If a cable has no stress it is no more able to transfer force. If a load path is made inactive, a reconfiguration of the elements occurs, possibly leading to the collapse. This is a direct result of the way tensegrity structures are made. Although the total number of elements composing them makes them a “mechanism”, it results that under a specific elements configuration a self-stress state emerges making the mechanism stiff [19]. By changing the length of one of the elements, the self-stress state configuration is modified.

The positive influence of the additional stiffening cables emerges in second set of simulations. Here, the new connection elements provide an alternative path to transfer the forces that are no more balanced by the slacking cable, resulting in a more robust configuration. At present it is better to consider the addition of elements as an integrity-oriented technique to ensure the robustness of tensegrity systems. Additional studies are required to clarify their behavior and define the strategies to foster robustness in this type of structures.

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