

Turbulent Dispersion in cloud-mixing layer

A DNS study under different stratification conditions

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Motivation and Objectives

- Droplet dispersion in cloud-mixing layers is governed by turbulence, stratification and buoyancy effects.
- Classical Richardson theory [1] predicts scale-dependent dispersion, but cloud flows rarely satisfy ideal homogeneous assumptions.
- Here we use direct numerical simulation (DNS) to investigate how vertical temperature gradients modify relative dispersion and the observed scaling laws.

Methodology

- DNS setup:** the turbulent cloud-clear air mixing layer is simulated using an incompressible Boussinesq DNS, including temperature, water vapor and inertial Lagrangian droplet (Fig.1).
- Initial conditions:** different stratification regimes are obtained by varying the temperature jump ΔT and the turbulent kinetic energy ratio between the two layers (Table 1).

Droplets are tracked as Lagrangian particles transported by the DNS velocity field. Their motion is described by the evolution of particle position and velocity:

$$\frac{d\mathbf{X}_i}{dt} = \mathbf{V}_i,$$
$$\frac{d\mathbf{V}_i}{dt} = \frac{\mathbf{u}(\mathbf{X}_i, t) - \mathbf{V}_i}{\tau_i} + \left(1 - \frac{\rho_a}{\rho_w}\right) g \mathbf{e}_z.$$

$\mathbf{u}(\mathbf{X}_i, t)$: Eulerian velocity interpolated at droplet position;
 τ_i : droplet response time of droplet i.

Pair-dispersion analysis [1]:

For each droplet pair (i, j) , the separation distance is defined as

$$\ell(t) = |\mathbf{X}_i(t) - \mathbf{X}_j(t)|.$$

Then the ensemble average $\langle \ell(t) \rangle$ over different initial group of particle is computed to have a comparison with respect to each other and the 3/2 Richardson's law.

The time increments $\Delta t = \ell(t + \Delta t) - \ell(t)$ are then used to estimate conditional moments of the separation dynamics.

Results

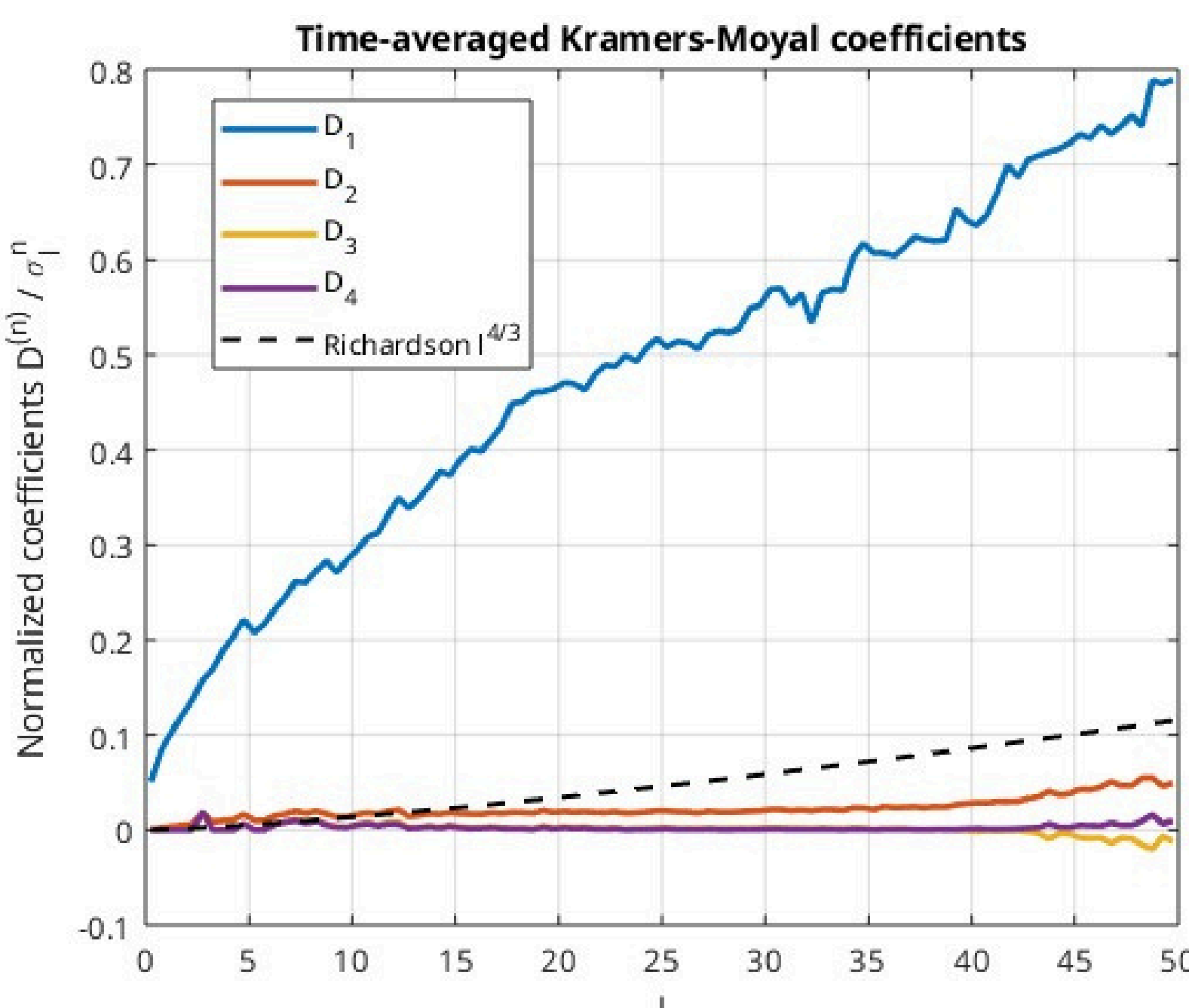
The fig.2 reveal a clear difference between stable and unstable regimes. Stable stratification suppresses particle-pair dispersion, while unstable conditions enhance it.

Regime	Simulation	t_c/τ_0	α_1	α_2
Unstable	MDU7	15.4	0.48	0.98
Unstable	MDUT1	17.5	0.42	0.82
Unstable	MDUT4	7.37	0.48	0.79
Stable	MDS7	7.79	0.41	0.30
Stable	MDSTABT1	6.53	0.42	0.27
Stable	MDSTABT4	5.83	0.42	0.27

Table 2: On this table, for each simulation, the time set of the new regime are indicated, with the corresponding power law exponents. Values found in field observations has higher exponents (see Palomba et al [6] and Zulli et al [7]).

As shown in fig.3 and table 2:

- In the **stable** cases, α is almost independent of ΔT ; larger ΔT mainly anticipates the onset of the observed regime.
- Lower **TKE ratios** reduce the separation magnitude but preserve a similar temporal trend.
- In the **unstable** cases, stronger instability increases the separation magnitude and anticipates the regime onset, while leaving the scaling exponent nearly unchanged.



- The Kramers–Moyal expansion [2] was applied to the distance-neighbour function, with the coefficients estimated from conditional separation increments.

$$\frac{\partial Q(l, t)}{\partial t} = \sum_{n=1}^{\infty} (-1)^n \partial_l^n [D^{(n)}(l, t) Q(l, t)], \quad D^{(n)}(l, t) = \frac{1}{n!} \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \langle (\Delta l)^n | l(t) = l \rangle$$

- When normalized by the standard deviation of the separation distance raised to the power of n, the first Kramers–Moyal coefficient appears as the dominant contribution over most separation ranges.
- Higher-order normalized coefficients remain comparatively small, suggesting that a Fokker–Planck-type closure may provide a reasonable approximation.

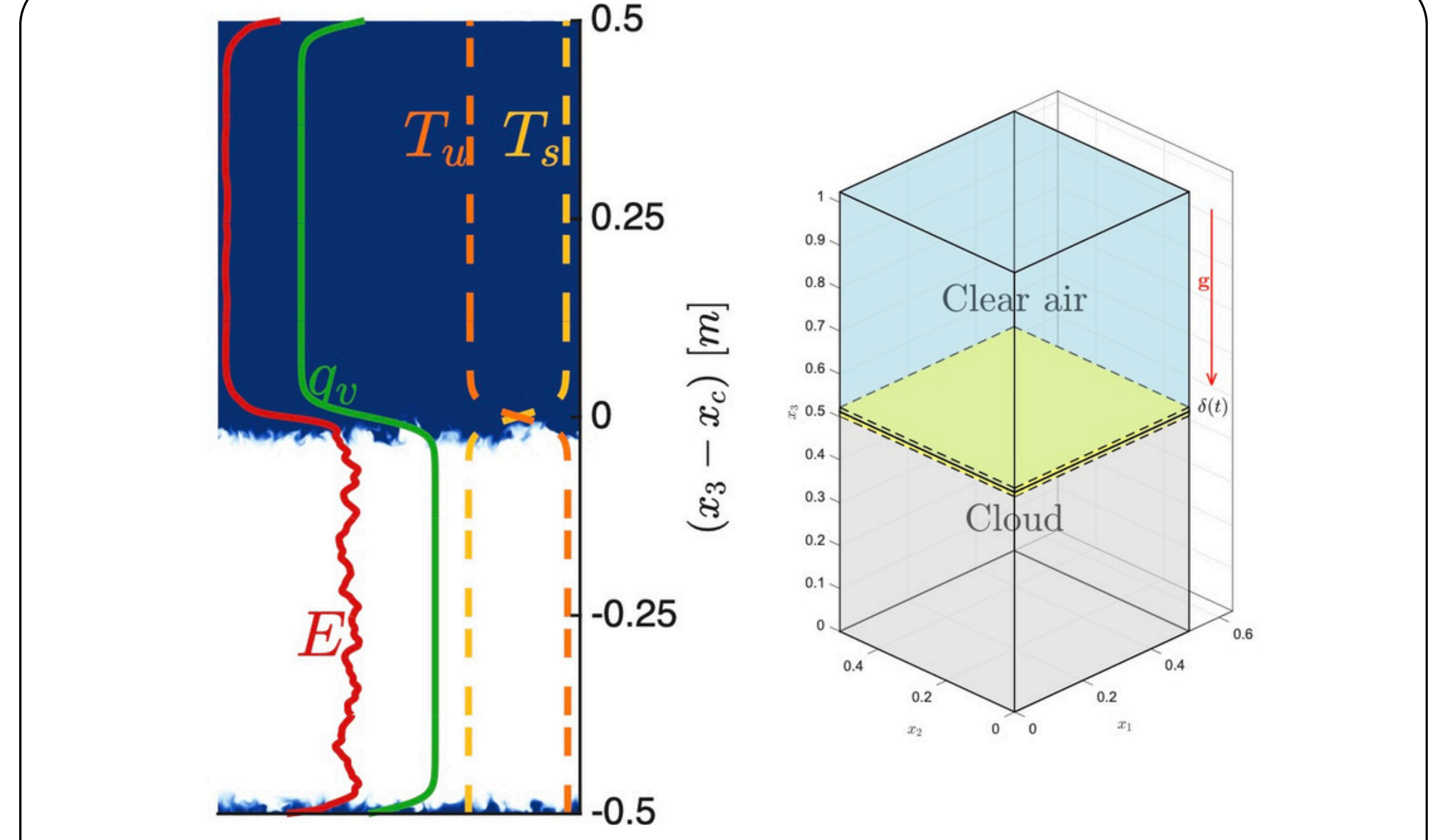


Figure 1: Initial distribution of turbulent kinetic energy (in red), water vapour content (in green), stable (yellow dashed line) and unstable (orange dashed line) temperature configurations. Computational domain representation on the right [3] [4].

Simulations	ΔT_0 [K]	$E_{\text{cloud}} / E_{\text{clear}}$
MD S7 / MD U7	± 2	6.7
MD ST1E16 / MD UT1E16	± 1	16
MD ST4E16 / MD UT4E16	± 4	16

Table 1: MD stems for monodisperse, S or U stable or unstable and T indicates the temperature gradient, E for the TKE ratio between cloud and clear air.

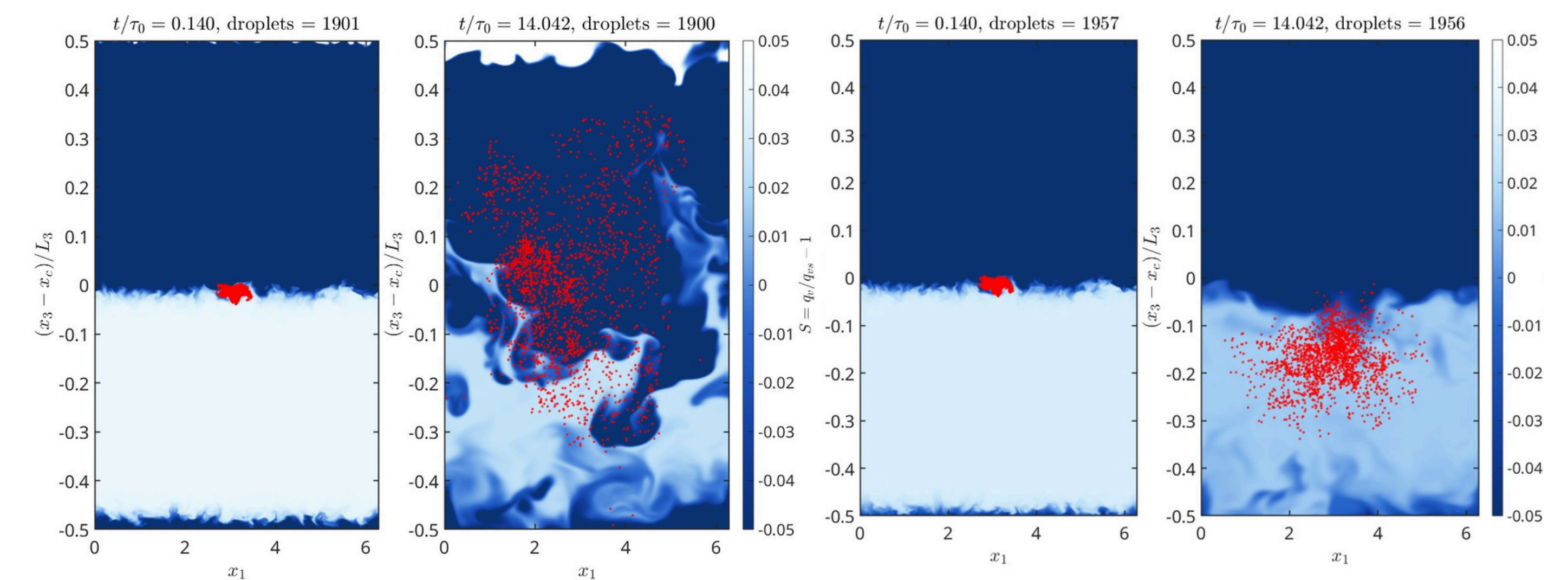


Figure 2: Red points indicate Lagrangian droplets, while colormap represents the supersaturation field. On the left are two snapshots of the unstable case, and on the right the stable case.

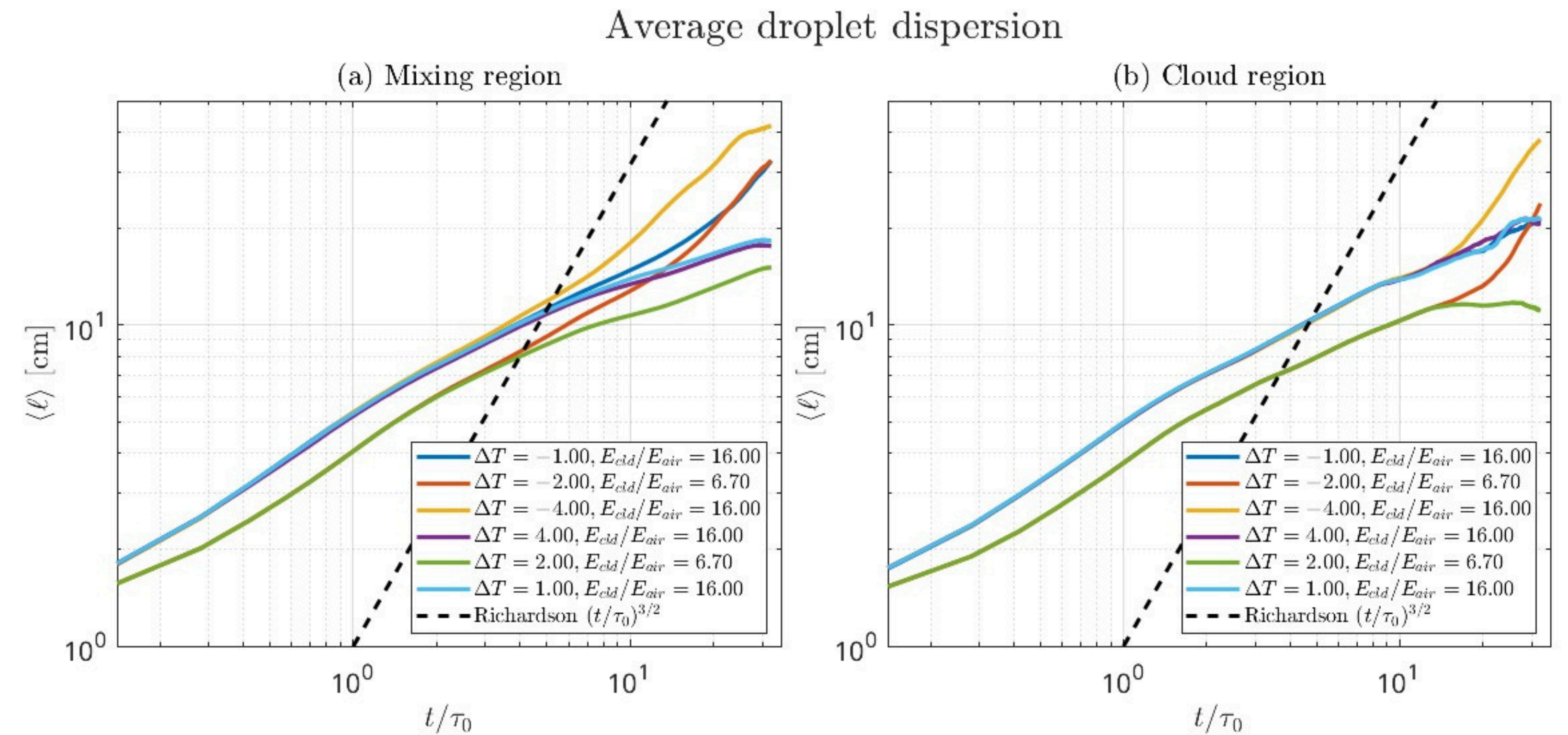


Figure 3: Ensemble averages of the separation distance in time; comparison between different cases. Stratification mainly changes the intensity and the onset time of relative dispersion.

Perspectives:

- DNS data provide a controlled framework to cross-interpret field dispersion measurements from **ultralight mini-radiosondes** as presented by Palomba et al [6] and Zulli et al [7] during this session poster.
- Although limited to an idealized small-scale domain, DNS can help identify the mechanisms behind **regime transitions** in particle/air-parcel dispersion.
- The validity of the **Fokker–Planck** closure can be assessed by comparing the predicted solution with the distance-neighbour function directly measured from the data.

References

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