Raising galaxy rotation curves via dressing

Letter

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We present a manifestly diffeomorphism-invariant simple model of galaxy dynamics obtained by applying the dressing field method (DFM) to a general relativistic system comprising the metric and four scalar fields, phenomenologically representing the four-velocity of a cosmological fluid or dust field. The DFM, a systematic tool for extracting the gauge-invariant content in general relativistic theories, provides a physical coordinatization that yields corrective terms to the rotational velocity profile. These corrections produce galaxy rotation curves that combine Keplerian and constant velocity terms, effectively emulating a dark matter contribution. We compare DFM-derived rotation curves to observed data for spiral galaxies, from the Spitzer Photometry and Accurate Rotation Curves database, showing that the DFM allows one to fit them well.

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Introduction. Spiral galaxies rotate around their vertical axes, and, by measuring the Doppler shift of atomic lines, one can determine the circular velocity of stars and other tracers as a function of their distance from the galactic center, obtaining a rotation curve. This produces galaxy rotation curves, which thus describe the orbital speeds of stars and gas in galaxies as a function of their distance from the galactic center. Typically, Newtonian gravity predicts that these speeds should decrease with increasing distance, with a Keplerian decline. However, observations reveal that rotation curves remain flat—or even rise—at large radii, suggesting the presence of "unseen" mass, thus associated with dark matter (DM), which provides additional gravitational influence. This discrepancy between theoretical predictions and observed velocities, first noted by astronomers like V. Rubin in the 1970s, supports the hypothesis that DM halos surround galaxies, contributing significantly to their gravitational potential. See, e.g., [1] and the comprehensive reviews [2–6]. The latter provides also a classified collection of literature on various topics related to DM, galaxy rotation curves, and alternative theoretical scenarios (such as, famously, the one of MOND).

In this letter, we establish that a manifestly invariant formulation, obtained through the application of the dressing field method (DFM) [7–10] to a system whose field content is given by the metric and four scalar fields—representing, e.g., the phenomenological four-velocity of a cosmological fluid or dust field—yields "raised" galaxy rotation curves, effectively emulating DM contributions.

The DFM is a systematic mathematical tool to exhibit the gauge-invariant, relational [11] content of general-relativistic (gauge field) theories, whereby physical field theoretical degrees of freedom (d.o.f.) codefine each other and coordinatize the *physical spacetime*.

In this paper, we apply the DFM to galaxy rotation curves, exploring whether a gauge-invariant reformulation of gravitational and matter d.o.f. can naturally produce flat rotation curves, reproducing the effects traditionally attributed to DM. Our approach thus tests an alternative theoretical framework for galactic dynamics. In this cosmological application, the DFM generates a physical scalar coordinatization, which, under specific conditions for the perturbative expansion of the scalar fields, yields a constant corrective term alongside the Keplerian contribution to the rotational velocity squared, therefore raising galaxy rotation curves.

The remainder of this paper is structured as follows: In the Dressing for diffeomorphisms section, we briefly review the basics of the DFM in the case of diffeomorphisms in general relativistic physics, following [9]. In the section on Physical coordinatization corrections to rotational velocity, we apply the DFM to the four-dimensional general-relativistic case in

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which the field content is given by the metric field and matter, described phenomenologically as a fluid supplying scalar fields φ. The latter, in fact, provide the dressing field to obtain the manifestly diffeomorphism-invariant formulation, acting as a reference physical system. We consider the (bare) Schwarzschild metric to provide the baseline geometry for a central mass M. The scalar fields—more precisely, the fields appearing in the perturbative expansion of the latter—contribute to the effective mass $M_{\rm eff}$. These corrections are external to the original Schwarzschild solution—in the sense that, in principle, they arise from solving the (dressed) Einstein equations with a dust field contribution—but are incorporated into the dressed metric to describe the *relational*, dressed gravitational field. We show that the scalar profile introduced allows for corrective terms that raise galaxy rotation curves, i.e., provide a constant correction to the squared rotational velocity. In the Scalar profile and four-velocity of dust field section, we discuss the interpretation of the scalar profile in terms of the four-velocity of a dust field. In the Phenomenological comparison of DFM to observed rotation curves section, we compare DFM-derived rotation curves to observed data for the spiral galaxies NGC 3198 and NGC 2403, using rotation curve data extracted from the Spitzer Photometry and Accurate Rotation Curves (SPARC) database [12]. In Supplemental Material [13] accompanying this paper, we also provide further fits of DFM velocity profiles for other high-resolution galaxies. This phenomenological analysis shows that the DFM allows to fit rotation curves, particularly well at large radii. The Conclusions section is devoted to final remarks and future developments. While this paper focuses on galaxy rotation curves, the DFM framework is extremely versatile, and can be extended to other cosmological phenomena, such as perturbation theory and large-scale structure formation, which we plan to explore in future work.

Dressing for diffeomorphisms. Via the DFM, one produces gauge-invariant variables out of the field space $\Phi = \{\phi\}$ of a general relativistic (gauge field) theory.

Let us briefly revisit the dressing procedure implemented in the DFM in the case of $\mathrm{Diff}(M)$, with M being the manifold on which the field theory is defined. Here we provide a field theoretical presentation, while we refer the reader to [7,8] for a formulation in terms of differential bundle geometry.

Consider a general relativistic theory with field content given by, e.g., $\phi = \{A, \varphi, g\}$, where A is a 1-form gauge potential, φ represents the matter fields, and g is a metric field on M. The field content supports the pullback action of the group of diffeomorphisms,

$$\phi^\psi \coloneqq \psi^*\phi, \qquad \psi \in Diff(M), \\ \text{i.e.}\{A^\psi, \varphi^\psi, g^\psi\} \coloneqq \{\psi^*A, \psi^*\varphi, \psi^*g\}. \tag{1}$$

A dressing field for diffeomorphisms is a smooth map

$$\upsilon : N \to M, \quad \text{such that } \upsilon^\psi \coloneqq \psi^{-1} \circ \upsilon, \tag{2}$$

for any $\psi \in \mathrm{Diff}(M)$. A dressing field should be extracted from the field content of the theory, namely, it should be a field-dependent dressing field $\upsilon = \upsilon[\phi]$, so that $\upsilon^\psi \coloneqq \upsilon(\psi^*\phi) = \psi^{-2} \circ \upsilon[\phi]$, allowing a relational interpretation of the dressed variables.

Given such a dressing field υ, the dressed fields are defined as

$$\phi^\upsilon \coloneqq \upsilon^*\phi, \\ \text{i.e., } \{A^\upsilon, \varphi^\upsilon, g^\upsilon\} = \{\upsilon^*A, \upsilon^*\varphi, \upsilon^*g\}, \tag{3}$$

and are $\mathrm{Diff}(M)$-invariant by construction. Here we are using the DFM rule of thumb for the case of diffeomorphisms: to dress fields or functionals thereof, we compute first their transformation under diffeomorphisms, then formally substitute $\psi \in \mathrm{Diff}(M)$ with the dressing field υ. For $\upsilon = \upsilon[\phi]$, the dressed fields (3) are manifestly relational variables; i.e., they represent $\mathrm{Diff}(M)$-invariant relations among the physical spatio-temporal d.o.f. embedded in ϕ. Observe that they are not objects defined on the "bare" manifold M.

In fact, the dressed fields (3) live on field-dependent *dressed regions*, defined by

$$U^\upsilon = U^{\upsilon[\phi]} \coloneqq \upsilon[\phi]^{-1}(U), \tag{4}$$

with υ^{-1} being the inverse map of υ, such that $\upsilon \circ \upsilon^{-1} - = \mathrm{id}_M$. These are $\mathrm{Diff}(M)$-invariant: indeed, defining the action of $\mathrm{Diff}(M)$ on $U \subset M$ as $U \mapsto U^\psi \coloneqq \psi^{-1} \circ U$, we find

$$(U^\upsilon)^\psi = (\upsilon[\phi]^\psi)^{-1} \circ (U^\psi) \\ = \upsilon[\phi]^{-1} \circ \psi \circ \psi^{-1} \circ (U) = U^\upsilon. \tag{5}$$

The reason for (4), and the justification of the claim that dressed fields ϕ^υ live on field-dependent dressed regions, comes from integration theory; see [9] for details. The $\mathrm{Diff}(M)$-invariant regions $U^{\upsilon[\phi]}$ represent the physical regions of spatiotemporal events, a physical spatiotemporal event being a field-dependent $\mathrm{Diff}(M)$-invariant point $x^{\upsilon[\phi]} \coloneqq \upsilon[\phi]^{-1}(x) \in U^\upsilon$. We may then call $M^\upsilon \coloneqq \mathrm{Im}(\upsilon^{-1})$ the manifold of physical spatiotemporal events. It does not exist independently from the fields ϕ. Hence, the physical spacetime, i.e., the physical Lorentzian manifold, is (M^υ, g^υ).

Let us finally spend a few words about the dynamics. The Lagrangian of the theory is $\mathrm{Diff}(M)$-covariant, and so are the field equations $E(\phi) = 0$ derived from it. Given a dressing field υ, the dressed Lagrangian $L(\phi^\upsilon) \coloneqq \upsilon^*L(\phi)$ is strictly $\mathrm{Diff}(M)$-invariant by construction. The field equations for the dressed fields, $E(\phi^u) = 0$, have the same functional expression as the "bare" ones [8], but have a well-posed Cauchy problem.

a. Scalar coordinatization via dressing. The framework described above encompasses diverse variants of the so-called "scalar coordinatization" in general relativistic physics. For instance, in approaches à la Brown–Kuchař [14–16], if matter is described effectively as a fluid (gas, particles, dust, etc.), it provides scalars from which one gets the dressing field to obtain the manifestly diffeomorphism-invariant formulation. In this case, the dressed metric represents the invariant, *relational* structure instantiated between the d.o.f. of the metric and of the effective matter field. This is the conceptual basis we start from in the present paper. It is also conceptually close to the proposals in [17,18], applied to black hole physics. In the approach à la Kretschmann–Komar [19,20], instead, a dressing field is extracted from the d.o.f. of the bare metric [21].

Physical coordinatization corrections to rotational velocity. We consider a theoretical setup whose field content is given by $\phi = \{g, \varphi\}$, where g represents the metric field and matter is described as a fluid/dust supplying scalar fields $\varphi: U \subseteq M \to N = \mathbb{R}^4$, $\varphi = \varphi^a$, $a = 1, 2, 3, 4$. The $\mathrm{Diff}(M)$-transformations are

$$\phi^\psi = \{\psi^* g, \psi^* \varphi\} = \{\psi^* g, \varphi \circ \psi\}, \qquad \psi \in \mathrm{Diff}(M). \quad (6)$$

We identify a $\mathrm{Diff}(M)$-dressing field as

$$\upsilon = \upsilon[\varphi] \coloneqq \varphi^{-1}\colon \mathbb{R}^4 \to M. \quad (7)$$

Indeed, it transforms under $\mathrm{Diff}(M)$ as a dressing field has to, $\upsilon^\psi = \upsilon[\varphi^\psi] = \psi^{-1} \circ \upsilon[\varphi]$; see (2). Note that this allows one to define dressed regions $U^\upsilon \coloneqq \upsilon^{-1}(U)$, such that $(U^\upsilon)^\psi = U^\upsilon$, yielding a $\mathrm{Diff}(M)$-invariant, relational definition of physical regions of events via (scalar) matter fields as a reference physical system [22].

Dressed metric components: Most importantly for the cosmological application we aim to provide in this paper, the dressing procedure yields the $\mathrm{Diff}(M)$-invariant dressed metric g^υ, which encodes the geometric properties of M^υ—we refer the reader to [9] for the explicit proof of the $\mathrm{Diff}(M)$-invariance of the dressed metric.

The dressed metric can be understood as the physical gravitational field as measured in the coordinate system supplied by the matter distribution φ. In abstract index notation, we have

$$\bar{g}_{ab} \coloneqq g^\upsilon_{ab} = J(\upsilon)_a{}^\mu g_{\mu\nu} J(\upsilon)^\nu{}_b, \quad (8)$$

with the Jacobian of the "physical coordinatization" being

$$J(\upsilon) = J(\varphi^{-1}) = J(\varphi)^{-1} - = \left(\frac{\partial \varphi^a}{\partial x^\mu}\right)^{-1}. \quad (9)$$

We may now consider perturbation theory and write the "dressed coordinates" as

$$\bar{x}^a \coloneqq \varphi^a = \delta^a_\mu x^\mu + \chi^a(x) \quad (10)$$

perturbatively, in terms of the "bare" ones, x^μ, and of a small deformation $\chi^a = \chi^a(x)$, parametrizing the "infinitesimal" dressing. Considering χ^a small enough indeed, we will neglect higher-order terms in χ^a, focusing on the linear, first order, level. One can also write the inverse relation $x^\mu = \delta^\mu_a(\varphi^a - \chi^a)$. Using (8) and (10), we can cast the dressed metric in the following form:

$$\bar{g}_{ab} = g_{ab} - 2g_{\mu(a}\partial_{b)}\chi^\mu, \quad (11)$$

where g_{ab} is the bare metric in components and ∂ denotes the ordinary partial derivative.

For simplicity, we shall now consider a galactic setup with spherical symmetry, which can be described in terms of the (bare) Schwarzschild metric, focusing on the equatorial plane (i.e., with fixed coordinate $\theta = \pi/2$) [23],

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\phi^2, \quad (12)$$

with $f = f(r) \coloneqq 1 - 2GM/r$, with G being the gravitational constant and M being the central mass. We have the bare metric components

$$g_{tt} = -f, \qquad g_{rr} = f^{-1}, \qquad g_{\phi\phi} = r^2,$$
$$g^{tt} = -f^{-1}, \qquad g^{rr} = f, \qquad g^{\phi\phi} = r^{-2}, \quad (13)$$

while all off-diagonal components of the bare metric identically vanish. This provides the baseline geometry.

We dress the metric (12) as in (8), taking the bare Schwarzschild metric components as $g_{\mu\nu}$. Note that the dressed metric thus obtained has, in principle, also off-diagonal components. The functional expression of the dressed components, consisting of perturbative deformations of the bare ones, is different with respect to the original Schwarzschild one. In particular, setting, for simplicity and in compatibility with the assumed cosmological setting, $\partial_\phi \chi^a = 0$ and $\partial_t \chi^\phi = 0$, we get the relevant nonvanishing dressed components

$$\bar{g}_{tt} = g_{tt} - 2g_{tt}\partial_t\chi^t = -f(1 - 2\partial_t\chi^t),$$
$$\bar{g}_{\phi\phi} = r^2, \quad (14)$$

while, e.g., under the above assumptions, $\bar{g}_{t\phi} = 0$. Observe that the dressing operation thus yields an effective mass

$$M_{\mathrm{eff}} = M(1 - 2\partial_t\chi^t) + \frac{r}{G}\partial_t\chi^t, \quad (15)$$

where $\partial_t\chi^t$ is a small deformation (one can in fact assume $0 < \partial_t\chi^t < 1/2$), which, if constant, gives a contribution to the effective mass proportional to r. Notice also that, since, from (10),

$$r = \varphi^r - \chi^r = \bar{r} - \chi^r, \tag{16}$$

the effective mass can be rewritten in term of the dressed coordinate $\bar{r}$, neglecting higher-order terms in the χ variables, as

$$M_{\rm eff} \approx M(1 - 2\partial_t\chi^t) + \frac{\bar{r}}{G}\partial_t\chi^t, \tag{17}$$

which therefore has the same functional expression as (15) for the radial coordinate, respectively, bare and dressed.

Using (10), we also rewrite the dressed metric components in terms of the dressed coordinates. This is because the dressed theory is the one to be compared with observations, and it shall be entirely expressed in terms of the dressed variables. We obtain

$$\bar{g}_{tt} = -\left[1 - \frac{2GM}{(\bar{r} - \chi^r)}\right](1 - 2\partial_t\chi^t),$$
$$\bar{g}_{\phi\phi} = (\bar{r} - \chi^r)^2 \approx \bar{r}(1 - 2\chi^r), \tag{18}$$

neglecting $\mathcal{O}^2(\chi^r)$ terms. From these, one also derives

$$\frac{d\bar{g}_{tt}}{d\bar{r}} = \frac{2GM(-1 + \partial_t\chi^t)}{(\chi^r - \bar{r})^2},$$
$$\frac{d\bar{g}_{\phi\phi}}{d\bar{r}} = 2(\bar{r} - \chi^r), \tag{19}$$

which will be used in the following analysis of the dressed galaxy kinematics and test particles dynamics.

Dressed rotational velocity Let us now consider the following (dressed) Lagrangian describing the dynamics of a test particle:

$$\bar{\mathcal{L}} = \frac{1}{2}\bar{g}_{ab}\dot{\bar{x}}^a\dot{\bar{x}}^b, \tag{20}$$

where $\dot{\bar{x}}^a$ denotes the dressed four-velocity. From it we can derive the dressed energy and angular momentum:

$$\bar{p}_t = \frac{\partial\bar{\mathcal{L}}}{\partial\dot{\bar{t}}} = -\bar{E}, \qquad \bar{p}_\phi = \frac{\partial\bar{\mathcal{L}}}{\partial\dot{\bar{\phi}}} = \bar{L}. \tag{21}$$

Thus, considering, for simplicity, circular orbits, $\dot{\bar{r}} = 0$, which implies

$$\bar{E}^2 = \bar{V}_{\rm eff}(\bar{r}), \tag{22}$$

where $\bar{V}_{\rm eff}(\bar{r})$ is the dressed effective potential, together with $\bar{g}_{t\phi} = 0$, we get

$$\dot{\bar{\phi}} = \frac{\bar{L}}{\bar{g}_{\phi\phi}}, \qquad \dot{\bar{t}} = -\frac{\bar{E}}{\bar{g}_{tt}}. \tag{23}$$

We focus on the timelike case, for which we have

$$\bar{g}_{\mu\nu}\dot{\bar{x}}^\mu\dot{\bar{x}}^\nu = -1 = \bar{g}_{tt}\dot{\bar{t}}^2 + \bar{g}_{\phi\phi}\dot{\bar{\phi}}^2, \tag{24}$$

and, using (22) and (23), we find the following expression of the dressed effective potential:

$$\bar{V}_{\rm eff}(\bar{r}) = -\bar{g}_{tt}\left(1 + \frac{\bar{L}^2}{\bar{g}_{\phi\phi}}\right). \tag{25}$$

Then, solving the stability requirement $\frac{d\bar{V}_{\rm eff}}{d\bar{r}} = 0$, we get

$$\bar{L}^2 = \frac{\frac{d\bar{g}_{tt}}{d\bar{r}}\bar{g}_{\phi\phi}^2}{\frac{d\bar{g}_{\phi\phi}}{d\bar{r}}\bar{g}_{tt} - \frac{d\bar{g}_{tt}}{d\bar{r}}\bar{g}_{\phi\phi}}. \tag{26}$$

We can express the *dressed rotational velocity* as $\bar{v} = \bar{r}\,\dot{\bar{\phi}} = \bar{r}\frac{\bar{L}}{\bar{g}_{\phi\phi}}$, that therefore is

$$\bar{v}^2 = \left(\bar{r}\frac{\bar{L}}{\bar{g}_{\phi\phi}}\right)^2 = \frac{\bar{r}^2\frac{d\bar{g}_{tt}}{d\bar{r}}}{\frac{d\bar{g}_{\phi\phi}}{d\bar{r}}\bar{g}_{tt} - \frac{d\bar{g}_{tt}}{d\bar{r}}\bar{g}_{\phi\phi}}. \tag{27}$$

Finally, we substitute in (27) Eqs. (18) and (19) for the dressed metric components, obtaining

$$\bar{v}^2 = \frac{GM\bar{r}}{6GM\chi^r - 3(\chi^r + GM)\bar{r} + \bar{r}^2}, \tag{28}$$

where, again, we have neglected $\mathcal{O}^2(\chi^r)$ terms. Then, in the weak-field approximation ($\bar{r} \gg 2GM$, $1 - \frac{2GM}{\bar{r}} \approx 1$), (28) boils down to

$$\bar{v}^2 \approx \frac{GM}{\bar{r} - 3\chi^r} \approx \frac{GM}{\bar{r}} + \frac{3GM\chi^r}{\bar{r}^2}. \tag{29}$$

We observe that the first term in (29) reproduces the Keplerian behavior, while the second term is a correction due to the presence of χ^r. In particular, if $\chi^r \propto \bar{r}^2$, the second term is positive and constant. We are thus left with

$$\bar{v}^2 = \frac{GM}{\bar{r}} + \bar{v}_0^2, \qquad \bar{v}_0 = \text{constant}, \tag{30}$$

where $\bar{v}_0^2 = 3GMk$, with $k > 0$ being the proportionality constant in

$$\chi^r = k\bar{r}^2. \tag{31}$$

The expression in (30) for the dressed rotational velocity thus contains the (dressed version of the) usual Keplerian term plus a corrective term induced by the dressed formulation which effectively emulates the contribution expected from DM, raising the galaxy rotation curve.

Since $\chi^r = \chi^r(r)$, the condition on χ^r imposed to obtain this result, (31), shall be rewritten, using (16) and neglecting $\mathcal{O}^2(\chi^r)$ terms, as

$$\chi^r = \frac{kr^2}{1-2kr}. \tag{32}$$

As a final step of our analysis, we check the consistency of the conditions implemented on χ^a, giving the scalar profile, and its derivatives with the (dressed) four-velocity of a cosmological fluid or dust field.

Scalar profile and four-velocity of dust field. We define the effective dressed four-velocity of the (comoving) dust field as

$$\bar{u}^a \coloneqq \frac{d\varphi^a}{d\tau}, \tag{33}$$

such that, in terms of the bare four-velocity $u^\mu \coloneqq \frac{dx^\mu}{d\tau} = (1,0,0,0)$, $u^\mu u_\mu = -1$, with τ being the proper time, we write

$$\bar{u}^a \coloneqq \frac{d\varphi^a}{d\tau} = \frac{\partial\varphi^a}{\partial x^\mu}\frac{dx^\mu}{d\tau} = \frac{\partial\varphi^a}{\partial x^\mu}u^\mu. \tag{34}$$

Recall that we are considering motion in the equatorial plane, $\theta = \pi/2$ and $d\theta/d\tau = 0$, and circular orbits. Then, using the fact that $\varphi^a = \delta^a_\mu x^\mu + \chi^a$, we get

$$\bar{u}^a = \left(\delta^a_\mu + \frac{\partial\chi^a}{\partial x^\mu}\right)u^\mu. \tag{35}$$

The conditions we have obtained on χ^a and its derivatives to get a constant corrective term in the rotational velocity squared, raising the galaxy rotation curve, can be collected as follows, together with their relevant implications:

$$\begin{aligned}
&\partial_\phi\chi^a = 0, \qquad \partial_t\chi^\phi = 0;\\
&\partial_t\chi^t = \epsilon, \qquad \epsilon \text{ small constant} \Rightarrow \chi^t = \epsilon t + F,\\
&\chi^r = \frac{kr^2}{1-2kr} \Rightarrow \partial_t\chi^r = \partial_\phi\chi^r = \partial_\theta\chi^r = 0,\\
&\partial_r\chi^r = \frac{2kr}{(1-2kr)^3},
\end{aligned} \tag{36}$$

where, in principle, $F = F(r,\phi,\theta)$ (one can also consider, for simplicity, F to be a function of r only). One can now see that Eq. (36) are consistent with the description of the dressed four-velocity of the dust field. In particular, for consistency with the dust four-velocity description in the bare case, one would expect $\bar{u}^a = (\bar{u}^t,0,0,0)$, which is indeed the case, as we can see from (35), using (36):

$$\bar{u}^a = (1+\epsilon)u^\mu = ((1+\epsilon)u^t,0,0,0) = (1+\epsilon,0,0,0), \tag{37}$$

with ϵ representing the small, constant deformation coming from the dressing operation [24].

Phenomenological comparison of DFM to observed rotation curves. To assess the phenomenological viability of the rotation curve we have derived through the DFM, which can be read from (30), we may compare it to observed rotation curves of spiral galaxies.

We select NGC 3198 and NGC 2403—the latter is less massive than the former—from the SPARC database [12], as both are nearby late-type spirals with extended, high-quality HI rotation curves and well-constrained inclinations. Their kinematic regularity and extensive use as benchmark systems in studies of disk dynamics make them well suited for testing the phenomenological viability of our derived rotation curves. In Supplemental Material [13], we show the plots of the observed rotation curves of NGC 3198 and NGC 2403.

From the SPARC database, we take the rotational velocities of the gas, disk, and bulge components, derived from the surface-brightness distribution, and use them to estimate the corresponding baryonic mass contributions. These are then rescaled by the mass-to-light corrective factors $\Upsilon_{\rm gas} = 1.33 M_\odot/L_\odot$, $\Upsilon_{\rm disk} = 0.70 M_\odot/L_\odot$, and $\Upsilon_{\rm bulge} = 0.70 M_\odot/L_\odot$ [see [25,26]], yielding the physical baryonic mass of each component. Summing these, we get the total baryonic mass of the galaxies. In Supplemental Material [13], we also provide the baryonic mass profile plots of NGC 3198 and NGC 2403.

The DFM model (dressed) rotational velocity, to be compared with observations, is given by

$$\bar{v} = \sqrt{\frac{GM}{\bar{r}} + \bar{v}_0^2}, \tag{38}$$

where $G = 4.302 \times 10^{-6}$ kpc(km/s)$^2 M_\odot^{-1}$ in cosmological units.

In Fig. 1, we present the fit of the DFM velocity profile to the observed rotation curves, using as input mass M the baryonic mass previously derived. For completeness, and to highlight the departure from the purely Newtonian expectation, we also plot the Keplerian velocity profile corresponding to the bare baryonic model (for the same baryonic mass M). The red lines represent the DFM-derived rotation curve with the best-fit parameter $k = 0.0254$ kpc^{-1} for NGC 3198 and $k = 0.0604$ kpc^{-1} for NGC 2403 in both cases with a precision of 10^{-5}. The quality of the fits is good, with coefficients of determination $R^2 = 0.995609$ for NGC 3198 and $R^2 = 0.99794$ for NGC 2403 [27].

In Supplemental Material [13], we also provide a comparison of the residuals (namely, the difference

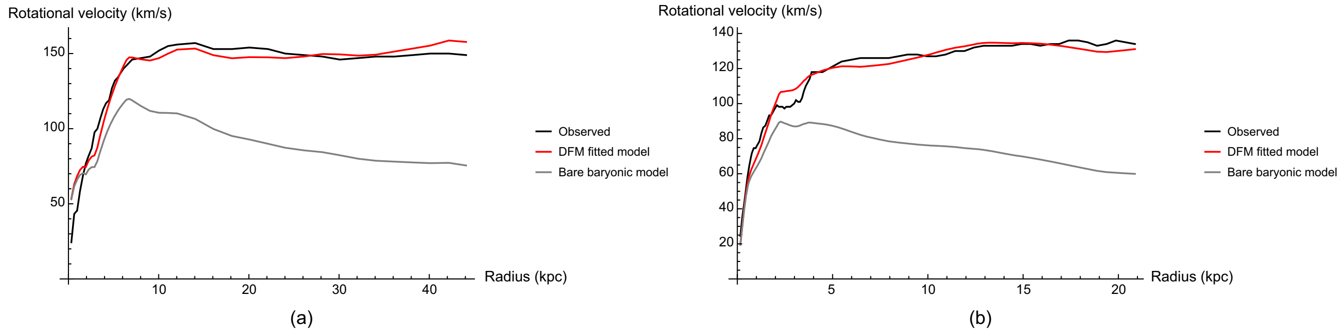

FIG. 1. Rotation curves of NGC 3198 (a) and NGC 2403 (b). The black lines show observed velocities from the SPARC database [12]. The red lines represent the DFM-derived rotation curve. The gray lines show the bare baryonic model prediction, assuming a Keplerian law. (a) NGC 3198 rotation curves. (b) NGC 2403 rotation curves.

between the values of the observed velocity and those provided by the DFM) between the dressed model and the bare baryonic Keplerian model. The DFM residuals are consistently closer to zero across most radii, indicating a significantly better agreement with the observed rotation curves than, e.g., the bare baryonic model.

This analysis demonstrates that the DFM framework can replicate the flat rotation curves observed in spiral galaxies at large radii—with velocities stabilizing at values consistent with typical spiral galaxy observations [2,12,25,26]—through a constant corrective term, without invoking DM in this context.

Conclusions. We have demonstrated that the DFM provides a relational, diffeomorphism-invariant framework that, when applied to galaxy dynamics, yields rotational velocities comprising a Keplerian term and a constant corrective term. This approach produces flat galaxy rotation curves, mirroring the effects typically attributed to DM without requiring additional unseen mass in this context. By constructing physical d.o.f. through DFM, our framework offers a novel perspective on the rotation curve anomaly, highlighting the potential of gauge-invariant formulations to capture observed galactic dynamics. While our analysis is limited to rotation curves and does not address broader DM phenomenology, such as the (baryonic) Tully-Fisher relation, cosmic microwave background fluctuations, or gravitational lensing, these findings pave the way for further applications of DFM in cosmology, including cosmological perturbation theory and other observational tests, as planned in future work.

We have considered the Schwarzschild metric to provide the baseline geometry, and matter described effectively as a cosmological fluid or dust, providing scalars from which we extracted the dressing field to obtain the manifestly diffeomorphism-invariant formulation. The scalars can be thus seen as physical reference frame, whose profile, subject to conditions compatible with the dressed four-velocity of the dust field, allows for a corrective term that raises the galaxy rotation curve.

To assess the phenomenological viability of the rotation curve we have derived through the DFM, we have also provided a phenomenological analysis, comparing our DFM-derived rotation curves to observed data for the spiral galaxies NGC 3198 and NGC 2403, using data from the SPARC database [12]. We have thus shown that the DFM model reproduces well the flat rotation curve at large radii, performing a one-parameter (the free parameter k left of the DFM theoretical analysis) fit to the observed rotational velocities of NGC 3198 and NGC 2403.

Our analysis shows that the DFM model, using the purely baryonic mass as input, provides a good fit to the observed rotation curves of both galaxies, with best-fit parameters $k = 0.0254 \pm 10^{-5}\ \mathrm{kpc}^{-1}$ for NGC 3198 and $k = 0.0604 \pm 10^{-5}\ \mathrm{kpc}^{-1}$ for NGC 2403. The model reproduces the characteristic flattening of the curves at large radii, with velocities stabilizing around 150 km/s for NGC 3198 and approximately 135–140 km/s for NGC 2403, consistent with typical spiral galaxy observations. Residuals analysis [see [13]] further confirms that the DFM predictions closely match the observed velocities, outperforming the bare baryonic Keplerian model.

We have examined several other cases and consistently found agreement with the results of the previous sections, as displayed in Supplemental Material [13]. These results support the phenomenological viability of the DFM framework in describing spiral galaxy rotation curves without invoking DM.

In forthcoming papers, we will further apply the DFM, encompassing the notion of coordinatization through coupling to matter, to develop a manifestly relational, thus automatically Diff(M)-invariant, version of cosmological perturbation theory [28–30], and to study other possible cosmological effects driven by dressing.

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Data availability. The data that support the findings of this article are openly available [12].

[1] V. C. Rubin, W. K. Ford, Jr., and N. Thonnard, Extended rotation curves of high-luminosity spiral galaxies. IV. Systematic dynamical properties, Sa through Sc, Astrophys. J. Lett. **225**, L107 (1978).

[2] Y. Sofue and V. Rubin, Rotation curves of spiral galaxies, Annu. Rev. Astron. Astrophys. **39**, 137 (2001).

[3] G. Bertone, D. Hooper, and J. Silk, Particle dark matter: Evidence, candidates and constraints, Phys. Rep. **405**, 279 (2005).

[4] M. Taoso, G. Bertone, and A. Masiero, Dark matter candidates: A ten-point test, J. Cosmol. Astropart. Phys. 03 (2008) 022.

[5] L. E. Strigari, Galactic searches for dark matter, Phys. Rep. **531**, 1 (2013).

[6] M. Cirelli, A. Strumia, and J. Zupan, Dark matter, arXiv:2406.01705.

[7] J. François, The dressing field method for diffeomorphisms: A relational framework, J. Phys. A **57**, 305203 (2024).

[8] J. T. François and L. Ravera, Geometric relational framework for general-relativistic gauge field theories, Fortschr. Phys. **73**, 2400149 (2024).

[9] J. François and L. Ravera, There is no boundary problem, arXiv:2504.20945.

[10] J. François and L. Ravera, Reassessing the foundations of metric-affine gravity, Eur. Phys. J. C **85**, 902 (2025).

[11] J. François and L. Ravera, On the meaning of local symmetries: Epistemic-ontological dialectics, Found. Phys. **55**, 38 (2025).

[12] F. Lelli, S. S. McGaugh, and J. M. Schombert, SPARC: Mass models for 175 disk galaxies with spitzer photometry and accurate rotation curves, Astron. J. **152**, 157 (2016).

[13] See Supplemental Material at http://link.aps.org/supplemental/10.1103/m9xl-9vvk for further galaxy plots and fits of DFM velocity profiles.

[14] J. D. Brown and K. V. Kuchař, Dust as a standard of space and time in canonical quantum gravity, Phys. Rev. D **51**, 5600 (1995).

[15] C. Rovelli, GPS observables in general relativity, Phys. Rev. D **65**, 044017 (2002).

[16] T. Frankel, *The Geometry of Physics: An Introduction* 3rd ed. (Cambridge University Press, Cambridge, England, 2011).

[17] M. B. Fröb, C. Rein, and R. Verch, Graviton corrections to the Newtonian potential using invariant observables, J. High Energy Phys. 01 (2022) 180.

[18] F. Di Filippo, N. Ogawa, S. Mukohyama, and T. Waki, Soft hair, dressed coordinates, and information loss paradox, Phys. Rev. D **108**, 044034 (2023).

[19] A. Komar, Construction of a complete set of independent observables in the general theory of relativity, Phys. Rev. **111**, 1182 (1958).

[20] P. G. Bergmann and A. Komar, The coordinate group symmetries of general relativity, Int. J. Theor. Phys. **5**, 15 (1972).

[21] This produces what we call an instance of self-dressing.

[22] The fact that the matter distribution is self-dressed (that is, coordinatized with respect to itself) is expressed by $\varphi^{\upsilon} = \mathrm{id}_{U^{\upsilon}}$, meaning that it is (invariantly) at rest in its own reference frame.

[23] We consider the metric signature to be $(-,+,+,+)$.

[24] One can then also show that, in the weak field limit, $\bar{u}^a \bar{u}_a \approx -1$.

[25] F. Walter, E. Brinks, W. J. G. de Blok, F. Bigiel, R. C. Kennicutt, Jr., M. D. Thornley, and A. K. Leroy, THINGS: The HI nearby galaxy survey, Astron. J. **136**, 2563 (2008).

[26] W. J. G. de Blok, F. Walter, E. Brinks, C. Trachternach, S.-H. Oh, and R. C. Kennicutt, Jr., High-resolution rotation curves and galaxy mass models from THINGS, Astron. J. **136**, 2648 (2008).

[27] The coefficient of determination R^2 measures the fraction of the total variance in the observed data that is captured by the model, with $R^2 = 1$ indicating a perfect fit and $R^2 = 0$ meaning the model explains no variance beyond the mean. We quantify the fit quality using the coefficient of determination R^2 rather than a reduced χ^2, because the χ^2 statistic assumes that the observational uncertainties are statistically reliable and Gaussian. In the SPARC data, the error bars vary in precision and are particularly large at small radii for NGC 3198 and at large radii for NGC 2403, violating these assumptions. The R^2 metric provides a more appropriate measure for evaluating the overall agreement in this context.

[28] K. Giesel, S. Hofmann, T. Thiemann, and O. Winkler, Manifestly gauge-invariant general relativistic perturbation theory. I. Foundations, Classical Quantum Gravity **27**, 055005 (2010).

[29] C. Chiaffrino, O. Hohm, and A. F. Pinto, Gauge invariant perturbation theory via homotopy transfer, J. High Energy Phys. 05 (2021) 236.

[30] M. B. Fröb and W. C. C. Lima, Synchronous coordinates and gauge-invariant observables in cosmological spacetimes, Classical Quantum Gravity **40**, 215006 (2023).