Spacetime boundaries do not break diffeomorphism and gauge symmetries

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In general relativity and gauge field theory, one often encounters a claim, which may be called the *boundary problem*, according to which "boundaries break diffeomorphism and gauge symmetries". We argue that this statement has the same conceptual structure as the *hole argument*, and is thus likewise defused by the *point-coincidence argument*: We show that the boundary problem dissolves once it is understood that a *physical region*, thus its boundary, is *relationally* and *invariantly defined*. This insight can be technically implemented via the dressing field method, a systematic tool to exhibit the gauge-invariant content of general-relativistic gauge field theories, whereby physical field-theoretical degrees of freedom co-define each other and define, coordinatize, the *physical spacetime*. We illustrate our claim with a simple application to the case of general relativity.

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I. INTRODUCTION

General relativistic (GR) physics, which is based on diffeomorphisms, and gauge field theory (GFT), based on gauge symmetries, form together the broad framework of general-relativistic gauge field theory (gRGFT). A model within this framework is a field theory on an n-dimensional manifold M, whose field content we denote by ϕ and which is acted upon by the covariance group $\mathrm{Diff}(M) \ltimes \mathcal{H}$ as $\phi \mapsto \psi^*(\phi^\gamma)$, with $\mathrm{Diff}(M)$ the diffeomorphisms group and $\mathcal{H} \coloneqq \{\gamma : M \to H | {\gamma_1}^{\gamma_2} = {\gamma_2}^{-1}\gamma_1\gamma_2\}$ the gauge group, and H a Lie group; the field equations for ϕ are covariant under $\mathrm{Diff}(M) \ltimes \mathcal{H}$, hence its name. Considering such a theory over a bounded region $U \subset M$ with boundary ∂U, one frequently encounters in the literature the claim that "diffeomorphism and/or gauge symmetries are broken at spacetime boundaries" [1–6]. We may refer to it as the *boundary problem*. More accurately, what is actually meant is that "field configurations, or functional thereof of special interest, defined at ∂U are not invariant under the action of $\mathrm{Diff}(M) \ltimes \mathcal{H}$." The field configurations in question may be some choice of gauge, the functional of the field at ∂U may be the presymplectic potential or 2-form of the theory, objects of interest notably in the covariant phase space analysis of the symplectic structure of gRGFTs [7–10], see [11].

Depending on the context and aims, various countermeasures are put forth to solve the boundary problem. In the covariant phase space literature, this involves e.g., the *ad hoc* introduction of so-called *edge modes*, degrees of freedom (d.o.f.) typically confined to ∂U, whose gauge transformations are tuned to cancel the terms from the transformation of the symplectic potential and 2-form, thereby "restoring their invariance" [1–3,5,12–14]. Edge modes are sometimes interpreted as "Goldstone modes" resulting from the breaking of $\mathrm{Diff}(M) \ltimes \mathcal{H}$ at ∂U. Some have claimed that edge modes reveal essential new symmetries of gravity (so-called "corner symmetries") and may be key to a new paths to quantum gravity (QG) [4,6,15,16].

However, we show that there is no boundary problem. Indeed, an analysis from first principles reveals it to be logically equivalent to Einstein's famous *hole argument* in GR [17–23], and its generalization to GFT and gRGFT [23–25]. The hole argument was designed to highlight an apparent ill-posedness of the Cauchy problem, and

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attending breakdown of determinism, in $\mathrm{Diff}(M)$-covariant theories. Its generalization shows likewise in $\mathcal{H}$-covariant theories. Einstein only successfully completed GR after he resolved this dual conceptual/technical issue via what came to be known, after Stachel named it thus, as the *point-coincidence argument* (see [17,20,23,24,26] and references therein). A generalized point-coincidence argument establishes *relationality* at the paradigmatic conceptual core of gRGFT; namely, the fact that physical magnitudes, i.e., physical field-theoretical d.o.f., relationally codefine each other, as well as physical *events*, in a $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-invariant way.

This insight ought to defuse the boundary problem. That it did not is owed to the fact that the relationality of gRGFT is only *tacit* in its standard formalism, encoded in its *manifest covariance* under the group $\mathrm{Diff}(M) \ltimes \mathcal{H}$. This makes the physics of gRGFT sometimes tricky to read off from the formalism, and in particular the identification of its observables nontrivial.

A framework allowing a manifestly relational $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-invariant reformulation of gRGFTs would make their conceptual structure transparent, giving ready access to their observables and dissolving the boundary problem. Here we shall present such a framework, based on a systematic approach to reduce symmetries; the *dressing field method* (DFM) [27–33]. We will show how it technically implements the point-coincidence argument [25] via the definition of relational invariant dressed variables, living on relationally and invariantly defined dressed regions, so that the boundary problem cannot get a grip.

The remainder of this paper is structured as follows: In Sec. II we give a dense overview of the conceptual insights stemming from articulating the hole and point-coincidence arguments. In Sec. III we recall the basics of the DFM for gauge symmetries and diffeomorphisms, introducing in particular the notion of *dressed regions*. It is put to use in Sec. IV, where invariant relational *physical spacetime boundaries* are defined, the boundary problem being thereby averted. Our presentation will emphasize for concreteness the explicit example of GR. It will be structured in decreasing levels of abstraction, a progression designed to make the content accessible to a broad audience, versed in either differential geometry or field-theoretical/components formulations. We gather our closing remarks in Sec. V.

II. THE HOLE AND POINT-COINCIDENCE ARGUMENTS IN A NUTSHELL

Einstein's dual *hole argument* and *point-coincidence argument* were key to his final understanding of the meaning of diffeomorphism covariance leading to the completion of GR, and to his definite views on spacetime—magnificently expressed in the last paragraphs of his appendix to [34]. It is an unfortunate turn of history that subsequently most of the physics community either forgot about them, or misunderstood them as mere distracting philosophical musings diverting Einstein from the straight path to GR. Their fundamental significance was rediscovered and brought to a wider attention by Stachel in 1980 (published in [20]), see [18–21], and [24] for an in-depth modernized account. The literature on the subject is now substantive, even involving parts of the quantum gravity community. Yet it remains misappreciated by most, which has nontrivial consequences.

The core logic of the hole argument goes as follows. By assumption, the field equations $E(\phi) = 0$ of a general-relativistic theory are $\mathrm{Diff}(M)$-covariant, which implies that if ϕ is a solution, so is $\psi^*\phi$ for any $\psi \in \mathrm{Diff}(M)$, since $E(\psi^*\phi) = \psi^* E(\phi) = 0$. Now, let us consider two solutions ϕ, ϕ' belonging to the same $\mathrm{Diff}(M)$-orbit $\mathcal{O}_\phi$, i.e., $\phi' = \psi^*\phi$, for which ψ is a compactly supported diffeomorphism whose support $D_\psi \subset M$ is the "hole"; we have that $\phi = \phi'$ on M/D_ψ, but $\phi \neq \phi'$ on D_ψ. Therefore, the field equations have an ill-defined Cauchy problem, they cannot uniquely determine the evolution of the fields ϕ within D_ψ, so that the theory *prima facie* seems to suffer from indeterminism. The question then arises; how are deterministically evolving physical spatiotemporal d.o.f. represented in GR?

The key conceptual insight came from what Stachel called Einstein's *point-coincidence argument*. It consists in the observation that the physical content of the theory (in particular its possible verifications, and observables) is exhausted by the pointwise coincidental values of fields ϕ, and that the description of such coincidences is $\mathrm{Diff}(M)$-invariant. It means that one has to admit that all solutions within the same $\mathrm{Diff}(M)$-orbit $\mathcal{O}_\phi$ represent the *same* physical state. The deterministically evolving physical d.o.f. are thus encoded in *equivalence classes* $[\phi]$ under $\mathrm{Diff}(M)$; they are not instantiated within the individual mathematical fields ϕ, but by the $\mathrm{Diff}(M)$-invariant *relations* among them.

A further immediate consequence is that the theory, unable to physically distinguish between $\mathrm{Diff}(M)$-related solutions of $E(\phi) = 0$, consequently cannot distinguish $\mathrm{Diff}(M)$-related points of M either. In other words, points of M are not *physical spatiotemporal events*. How, then, is physical spacetime represented in GR? The point-coincidence argument suggested the answer; points of the physical spacetime being *defined*, individuated, as pointwise coincidences of distinct physical field d.o.f., the physical spacetime is thus represented in GR as the $\mathrm{Diff}(M)$-invariant *network of relations* among physical field. The manifold M, like a scaffold to a building, is only there to bootstrap our ability to build a description of the relativistic physics of fields' spatiotemporal d.o.f. And like a scaffold, M is removed from the physical picture by the $\mathrm{Diff}(M)$-covariance of the general-relativistic field equations. As Einstein concluded [34], "*Physical [fields] are not in space, but […] are spatially extended*" so that

"*Space-time does not claim existence on its own, but only as a structural quality of the field*."

The *internal hole argument* has a similar structure, *mutatis mutandis*; the fields ϕ of a GFT have both spatiotemporal and *internal* d.o.f., and the $\mathcal{H}$-covariance of their field equations, $E(\phi^\gamma) = \rho(\gamma)^{-1}E(\phi) = 0$ for any $\gamma \in \mathcal{H}$—with ρ denoting the representations of H to which the fields in the collection ϕ belong to—implies that if ϕ is a solution, so is its gauge transform ϕ^γ. In particular, if γ is a compactly supported element of $\mathcal{H}$ whose support $D_\gamma \subset M$ is the "hole", we have that $\phi = \phi'$ on M/D_γ, while $\phi \neq \phi'$ on D_γ. It would then appear that the Cauchy problem is ill-posed, and the theory undeterministic.

Then enters the *internal point-coincidence argument*. The physical content of a gauge theory is exhausted by the pointwise coincidental values of fields ϕ, whose description is $\mathcal{H}$-invariant. All solutions in the same $\mathcal{H}$-orbit $\mathcal{O}_\phi$ thus represent the *same* physical state. Deterministically evolving physical d.o.f. are not represented by the individual mathematical fields ϕ, but in the $\mathcal{H}$-invariant *relations* among them.

Together, these form a *generalized hole argument* arising from the $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-covariance of the field equation of gRGFT. A *generalized point-coincidence argument* answers it, implying that physical spatiotemporal and internal field d.o.f. are coextensive with the co-defining relations they participate in, and that the network of these relations *define* physical spacetime [24].

It is now clear that the boundary problem, stated for $\mathrm{Diff}(M)$ and/or $\mathcal{H}$, has essentially the same logical structure as a hole argument. It commits the same dual conceptual mistakes of 1) considering mathematical fields ϕ to *directly represent* physical fields, and 2) considering M, and *a fortiori* boundaries ∂U of its bounded sets U, as physical entities existing independently of the fields. It overlooks the relational resolution brought by the generalized point-coincidence argument; the boundary problem evaporates once it is recognized that physical d.o.f. and physical spacetime, and its bounded regions, are *relationally and invariantly defined*.

These misconceptions, together with the various countermeasures put forward to solve the alleged issue (e.g., "edge modes"), would be avoided in a framework where both *relationality* and *strict invariance* are manifest. Such a reformulation of gRGFT would also give ready access to its physical observables. The challenge lies in how to technically implement this idea. This is where the DFM comes into play.

III. DRESSING FIELD METHOD: BASICS

Via the DFM [27–33] one produces gauge-invariant variables out of the field space $\Phi = \{\phi\}$ of a gRGFT. We here remind, and further expand on, the dressing procedure implemented in the DFM, both in the case of internal gauge symmetries $\mathcal{H}$ and in the case of $\mathrm{Diff}(M)$. We shall also introduce the notion of *dressed spacetime regions*, which will be pivotal in Sec. IV. The following is a field-theoretical presentation, we refer to [27,28] for a formulation in terms of differential bundle geometry.

A. The case of gauge symmetries

Consider a GFT with field content $\phi = \{A, \varphi\}$, where A is the 1-form gauge potential and φ represents the matter fields content, both supporting the action of the gauge group $\mathcal{H}$—e.g., Yang-Mills theory and QED—so that they gauge-transform as

$$A^\gamma \coloneqq \gamma^{-1}A\gamma + \gamma^{-1}d\gamma, \qquad \varphi^\gamma \coloneqq \gamma^{-1}\varphi, \qquad \gamma \in \mathcal{H}. \quad (1)$$

A *dressing field* is, by definition, a smooth map

$$u\colon M \to H, \quad \text{such that } u^\gamma = \gamma^{-1}u, \qquad \gamma \in \mathcal{H}. \quad (2)$$

A key aspects of the DFM it that a dressing field should always be extracted from the field content of the theory. This means that it has to be a *field-dependent dressing field* $u = u[\phi]$, so that $u^\gamma \coloneqq u[\phi^\gamma] = \gamma^{-1}u[\phi]$.

When such a dressing field is found/built, we can define the *dressed fields* $\phi^u = \{A^u, \varphi^u\}$, which are given by

$$A^u \coloneqq u^{-1}Au + u^{-1}du, \qquad \varphi^u \coloneqq u^{-1}\varphi. \quad (3)$$

This illustrates the DFM "rule of thumb"; to dress fields or functional thereof, we compute first their gauge transformations, then formally substitute the gauge parameter γ with the dressing field u. The resulting expressions are $\mathcal{H}$-invariant by construction. We stress, however, that since $u \notin \mathcal{H}$, dressed fields are not gauge-transformed fields, and in particular a dressing via the DFM *is not* a gauge fixing [32,35].

When $u = u[\phi]$ is a field-dependent dressing fields, the DFM has a natural *relational* interpretation [24,28]; the dressed fields $\{A^{u[A,\varphi]}, \varphi^{u[A,\varphi]}\}$ in (3) represent the *gauge-invariant relations* among the physical internal d.o.f. embedded in $\{A, \varphi\}$. They technically implement the internal point-coincidence argument. Still, we remark that these dressed field remain objects defined on the manifold M.

(a) *Dynamics* The above pertain to the kinematics. The dynamics of a GFT is specified by a Lagrangian $L = L(\phi)$, that is typically required to be quasi-invariant under $\mathcal{H}$, i.e., $L(\phi^\gamma) = L(\phi) + db(\gamma; \phi)$, for $\gamma \in \mathcal{H}$. This guarantees the $\mathcal{H}$-covariance of the field equations $E(\phi) = 0$ extracted from L.

Given a dressing u, we define the *dressed Lagrangian*

$$L(\phi^u) \coloneqq L(\phi) + db(u; \phi), \quad (4)$$

which is strictly $\mathcal{H}$-invariant by construction—this is a case of the DFM rule of thumb. The field equations for the dressed fields, $E(\phi^u) = 0$, have the *same functional expression* as those of the "bare" fields; here they still differ by a boundary term, see [28]. The dressed field equations are *deterministic*, they specify uniquely the evolution of the physical internal d.o.f. represented by the dressed fields (3).

B. The case of diffeomorphisms

Consider a general-relativistic theory with field content $\phi = \{A, \varphi, g\}$, where g is a metric field on M, supporting the pullback action of the group of diffeomorphisms,

$$\begin{aligned} &\phi^\psi \coloneqq \psi^*\phi, \qquad \psi \in \mathrm{Diff}(M), \\ &\text{i.e., } \{A^\psi, \varphi^\psi, g^\psi\} \coloneqq \{\psi^*A, \psi^*\varphi, \psi^*g\}. \end{aligned} \tag{5}$$

A dressing field for diffeomorphisms is a smooth map

$$\upsilon\colon N \to M, \quad \text{such that } \upsilon^\psi \coloneqq \psi^{-1} \circ \upsilon, \tag{6}$$

for any $\psi \in \mathrm{Diff}(M)$, N being a model smooth manifold (such that $\dim N = \dim M$, typically $N = \mathbb{R}^n$). As previously, a dressing field should be extracted from the field content of the theory i.e., it should be a field-dependent dressing field $\upsilon = \upsilon[\phi]$, so that $\upsilon^\psi \coloneqq \upsilon(\psi^*\phi) = \psi^{-1} \circ \upsilon[\phi]$.

Given such a dressing field υ, dressed fields defined as

$$\begin{aligned} &\phi^\upsilon \coloneqq \upsilon^*\phi, \\ &\text{i.e., } \{A^\upsilon, \varphi^\upsilon, g^\upsilon\} = \{\upsilon^*A, \upsilon^*\varphi, \upsilon^*g\}, \end{aligned} \tag{7}$$

are $\mathrm{Diff}(M)$-invariant by construction. Remark again at play the DFM rule of thumb, here for diffeomorphisms. For $\upsilon = \upsilon[\phi]$, the dressed fields (7) are manifestly relational variables; they represent $\mathrm{Diff}(M)$-invariant *relations* among the physical spatiotemporal d.o.f. embedded in ϕ. They thus implement the first of the dual insights stemming from the point-coincidence argument. Regarding the second, observe that they *are not* objects defined on the "bare" manifold M.

(a) *Dressed regions* The dressed fields (7) live on field-dependent *dressed regions* defined by

$$U^\upsilon = U^{\upsilon[\phi]} \coloneqq \upsilon[\phi]^{-1}(U), \tag{8}$$

with υ^{-1} the inverse map of υ, such that $\upsilon \circ \upsilon^{-1} = \mathrm{id}_M$. Crucially, these are $\mathrm{Diff}(M)$-invariant: Indeed, defining the *right action* of $\mathrm{Diff}(M)$ on subsets $U \subset M$ as $U \mapsto U^\psi \coloneqq \psi^{-1} \circ U$—the action (5) of $\mathrm{Diff}(M)$ on fields ϕ being also a right action—we find

$$\begin{aligned} (U^\upsilon)^\psi &= (\upsilon[\phi]^\psi)^{-1} \circ (U^\psi) \\ &= \upsilon[\phi]^{-1} \circ \psi \circ \psi^{-1} \circ (U) = U^\upsilon. \end{aligned} \tag{9}$$

The reason for the definition (8), and the justification of the claim that dressed fields ϕ^υ live on such regions, simply comes from integration theory: It tells us e.g., that the action is invariant,

$$S \coloneqq \int_U L(\phi) = \int_{\psi^{-1}(U)} \psi^* L(\phi) = \int_{U^\psi} L(\phi^\psi) = S^\psi, \tag{10}$$

where $L(\phi)$ is the Lagrangian providing the dynamics of a general-relativistic theory, required $\mathrm{Diff}(M)$-covariant $L(\phi^\psi) = \psi^* L(\phi)$. By the DFM rule of thumb and (7), this gives

$$S = \int_U L(\phi) = \int_{\upsilon^{-1}(U)} \upsilon^* L(\phi) = \int_{U^\upsilon} L(\phi^\upsilon) \eqqcolon S^\upsilon. \tag{11}$$

The $\mathrm{Diff}(M)$-invariant regions $U^{\upsilon[\phi]}$ thus represent *physical* regions of spatiotemporal events, a physical spatiotemporal event being a field-dependent $\mathrm{Diff}(M)$-invariant point $x^{\upsilon[\phi]} \coloneqq \upsilon[\phi]^{-1}(x) \in U^\upsilon$, realizing the basic intuition behind the point-coincidence argument.

We may call $M^\upsilon \coloneqq \{U^\upsilon |\ \forall\ U \subset M\}$ the *manifold of physical spatiotemporal events*; we remark that it *does not exist* independently from the fields ϕ. The physical *spacetime*, i.e., the physical Lorentzian manifold, is then (M^υ, g^υ). This technically implements the second of the dual insights stemming from the point-coincidence argument, i.e., that physical spacetime is *relationally defined*, in a $\mathrm{Diff}(M)$-invariant way, by its physical field content, and "*does not claim existence on its own, but only as a structural quality of the field*".

(b) *Dynamics* The Lagrangian being $\mathrm{Diff}(M)$-covariant, so are the field equations $E(\phi) = 0$ derived from it. Given a dressing υ, the *dressed Lagrangian*

$$L(\phi^\upsilon) \coloneqq \upsilon^* L(\phi) \tag{12}$$

is strictly $\mathrm{Diff}(M)$-invariant by construction; again a case of the DFM rule of thumb. The field equations for the dressed fields, $E(\phi^u) = 0$, have the *same functional expression* as the "bare" ones [28], but have a well-posed Cauchy problem, uniquely determining the evolution of the physical spatiotemporal d.o.f. represented by the dressed fields (7).

C. Examples of dressing fields

One can find in the gRGFT literature many instances fleshing out the above general framework [28]. Let us mention three basic examples illustrating group-valued dressing fields as defined in Sec. III A.

First, consider scalar electromagnetism, with $\phi = \{A, \phi\}$ and $\mathcal{H} = \mathcal{U}(1)$: the $U(1)$-valued dressing field $u = u[\phi] \coloneqq e^{i\theta}$ is the phase of the $\mathbb{C}$-scalar field $\phi = \rho e^{i\theta}$, supporting indeed the defining $\mathcal{U}(1)$-gauge transformation $u^\gamma = u[\phi^\gamma] = \gamma^{-1}u[\phi]$ for $\gamma \in \mathcal{H} = \mathcal{U}(1)$, so that $\phi^u = \{A^u, \phi^u\} = \{A + id\theta, \rho\}$ are the $\mathcal{U}(1)$-invariant physical fields. This e.g., grounds attempts at providing invariant account of the Aharonov-Bohm effect [36,37], as well as invariant formulation of the Abelian Higgs model bypassing the notion of spontaneous symmetry breaking (SSB). See [38] Sec. 5.3 (also [39] Chapter 6). In spinorial electromagnetism, with $\phi = \{A, \psi\}$ and $\mathcal{H} = \mathcal{U}(1)$, the extraction of a $\mathcal{U}(1)$-dressing field $u = u[A]$ from the (holonomy of the) gauge potential yielding dressed fields $\phi^u = \{A^u, \psi^u\}$ is the formal underpinning of Dirac's proposal [40,41] for an invariant quantization scheme in QED [42].

Secondly, and similarly to the Abelian case, in the electroweak model a $\mathcal{SU}(2)$-dressing field $u = u[\varphi]$ may be extracted from the $\mathbb{C}^2$-scalar field φ; the dressed gauge potential $A^{u[\varphi]} = \{\gamma, W^\pm, Z^0\}$, the dressed fermions $\psi^{u[\varphi]} = \{(\ell_L, \nu_\ell), \ell_R, (q_u, q_d)\}$ and dressed scalar $\varphi^{u[\varphi]} = H$, are the $\mathcal{SU}(2)$-invariant fields existing in *both* the massless and massive phase. See [37,38,43] Sec. 5.4. This grounds invariant approaches avoiding the notion of SSB; from Higgs [44] and Kibble [45,46], to the so-called Fröhlich-Morchio-Strocchi (FMS) approach [48–50], which is in keeping with Elitzur theorem [51] and indeed the basis of concrete lattice implementations of the Standard Model [52,53].

Thirdly, consider gauge gravity coupled to an effectively described (i.e., nonspinorial) matter "fluid" φ, with $\mathcal{H} = \mathcal{SO}(1,3)$ and $\phi = \{\tilde{A}, \varphi\} = \{A, e, \varphi\}$ where A is the spin connection and $e = e^a{}_\mu dx^\mu \eqqcolon \bar{e} \cdot dx$ is the tetrad 1-form, so that $\tilde{A} = A + e$ is a Poincaré-valued Cartan connection [54,55]. A natural $\mathcal{SO}$-dressing field is none other than the tetrad field, $u = u[\tilde{A}] \coloneqq \bar{e}$, which satisfies indeed $u^\gamma = u[\tilde{A}^\gamma] = \gamma^{-1}\bar{e}$ for $\gamma \in \mathcal{SO}(1,3)$ [56]. The dressed Cartan connection is $\tilde{A}^u = (A^u, e^u) = (\bar{e}^{-1}A\bar{e} + \bar{e}^{-1}d\bar{e}, \bar{e}^{-1}e) \eqqcolon (\Gamma, dx)$, where Γ is none other than the $\mathcal{SO}$-invariant, $\mathfrak{gl}$-valued affine connection. We remark that the dressed η-transposed tetrad field $e^t = dx \cdot \bar{e}^T\eta$, η the Minkowski metric, just gives the $\mathcal{SO}$-invariant metric on M: $(e^t)^u \coloneqq e^t\bar{e} = dx \cdot \bar{e}^T\eta\bar{e} \eqqcolon dx \cdot \bar{g}$, the metric field being $g = g_{\mu\nu}dx^\mu \otimes dx^\nu \eqqcolon \bar{g}dx \otimes dx$. The bare Lagrangian $L(A, e, \varphi)$ is dressed as $L(\Gamma, g, \varphi)$; the DFM here then accounts for the switch between the tetrad and metric formulations of gravity. This case we revisit in Sec. IV B to illustrate dressings for diffeomorphisms and, thereby, the following core message.

D. No boundary problem in gRGFT

Gathering the results of Secs. III A and III B, we obtain a $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-invariant and manifestly relational reformulation of gRGFT: Defining a *complete* field-dependent dressing field as the pair $(\upsilon, u) = (\upsilon[\phi], u[\phi])$, we define, using (3)–(7), the fully invariant dressed fields

$$\phi^{(\upsilon,u)} \coloneqq \upsilon^*(\phi^u), \tag{13}$$

representing the physical spatiotemporal and internal field d.o.f. and their codefining (coextensive) relations. As above, the fields $\phi^{(\upsilon,u)}$ live on/define the physical manifold of spatiotemporal events $M^{(\upsilon,u)} = M^\upsilon$, and regions U^υ thereof [62]. Their dynamics is given by the invariant dressed Lagrangian $L(\phi^{(\upsilon,u)}) \coloneqq \upsilon^* L(\phi^u)$, from which derive the field equations $E(\phi^{(\upsilon,u)}) = 0$ with a well-posed Cauchy problem. This implements both dual aspects of the generalized point-coincidence argument.

An immediate consequence of all the above is that a physical, relationally-defined, boundary ∂U^υ is by definition $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-invariant. This dissolves the boundary problem, understood as the claim that "$\mathrm{Diff}(M)$ and/or $\mathcal{H}$ symmetries are broken at *spacetime boundaries*". If it is meant literally, it is wrong. If it is meant as "fields ϕ, or functional thereof, defined at ∂U are not $(\mathrm{Diff}(M) \ltimes \mathcal{H})$-invariant" or as "$\partial U$ is not preserved by $\mathrm{Diff}(M)$", it is technically true but trivial and physically inconsequential.

IV. DISSOLVING THE BOUNDARY PROBLEM IN GENERAL RELATIVITY

As should be clear from what precedes, the boundary problem for GFT is easier to tackle/dissolve; it is sufficient to build $\mathcal{H}$-invariant field variables ϕ^u, since the manifold M on which the fields ϕ are defined does not transform under the action of $\mathcal{H}$ [63]. The physical configuration of internal d.o.f. represented by ϕ^u is $\mathcal{H}$-invariant across M, in particular at ∂U for $U \subset M$. For applications of the framework laid in Sec. III A, e.g., to the electroweak model, to electromagnetism coupled to scalar fields, and other GFTs, see [28,38].

The boundary problem in GR physics is more involved since the manifold M itself transforms under $\mathrm{Diff}(M)$. The dressing field $\upsilon[\phi]$ is used to dress both the bare fields ϕ of the theory and (regions U of) M. The framework of Sec. III B encompasses diverse variants of "scalar coordinatization" in GR. For example, in the approach *à la* Kretschmann–Komar-Bergmann [64–67] for vacuum GR, a dressing field is extracted from the bare metric, $\upsilon = \upsilon[g]$ (otherwise seen as a a "g-dependent coordinate system"); the dressed metric $g^\upsilon \coloneqq \upsilon[g]^* g$ is "self-dressed", it represents the invariant structure among the d.o.f. of the physical gravitational field. In approaches à la DeWitt [68], or Brown–Kuchař [69,70], if matter is described effectively as a fluid (gas, particles, dust, etc.), it provides scalars $\varphi = \varphi^a$, $a \in \{0, \ldots, n-1\}$ from which one gets the dressing field $\upsilon[\varphi]$; the dressed metric $g^\upsilon \coloneqq \upsilon[\varphi]^* g$ represents the invariant relational structure between the d.o.f. of the metric and of the effective matter field—otherwise seen as the metric "written" in the physical matter frame [71]. In both cases, even though it is not usually done, other physical matter and/or interaction fields may be described as $\{A^\upsilon, \varphi^\upsilon\}$, while physical regions of events $U^\upsilon \subset M^\upsilon$ are defined either via matter $\upsilon[\varphi]$, or the gravitational field $\upsilon[g]$.

In the following, we illustrate the construction for the case of GR; assuming that a dressing field υ for $\mathrm{Diff}(M)$ has been built from the field content ϕ of the theory, we want to prove the invariance of the dressed metric, i.e., of the physical gravitational field. We shall do so twice over, to showcase both the abstract and computationally concrete versions. For this we need to go over the basics.

A. Physical boundaries in relational GR

Recall that diffeomorphisms $\psi \in \mathrm{Diff}(M)$ are smooth maps $\psi\colon M \to M$, $x' \mapsto \psi(x')$, with smooth inverse $x \mapsto \psi^{-1}(x)$. Their linearization yields vector fields $X \coloneqq \frac{d}{d\tau}\psi_\tau|_{\tau=0}$, with $\psi_{\tau=0} = \mathrm{id}_M$, which constitute the Lie algebra $\mathfrak{diff}(M) \simeq \Gamma(TM)$. As previously mentioned, the right action of $\mathrm{Diff}(M)$ on regions $U \subseteq M$ is defined as is $U \mapsto \psi^{-1}(U)$. The group $\mathrm{Diff}(M)$ acts on vector fields by *pushforward*, $\psi_*\colon TM \to TM$, $\mathfrak{X}_{|x} \mapsto (\psi_*\mathfrak{X})_{|\psi(x)}$. Its action on differential forms, and covariant tensors, defines the *pullback*, $\psi^*\colon T^*M \to T^*M$, $\alpha_{|x} \mapsto (\psi^*\alpha)_{|\psi^{-1}(x)}$. The latter is a also a right action. The linearization of these actions defines the Lie derivative on vector fields, $\mathcal{L}_X\mathfrak{X} \coloneqq \frac{d}{d\tau}\psi_{\tau *}\mathfrak{X}|_{\tau=0} = [X, \mathfrak{X}]$. On differential forms and covariant tensors, $\mathcal{L}_X\alpha \coloneqq \frac{d}{d\tau}\psi^*_\tau\alpha|_{\tau=0}$.

The $\mathrm{Diff}(M)$-transformation of the metric, a symmetric covariant 2-tensor, is thus $g_{|x} \mapsto (\psi^*g)_{|\psi^*(x)}$ or $g_{|\psi^{-1}(x)} \mapsto (\psi^*g)_{|x}$. And linearly, $\mathcal{L}_X g = \frac{d}{d\tau}\psi^*_\tau g|_{\tau=0}$. We introduce the following compact matrix notation:

$$g = \bar{g}dx \otimes dx, \qquad \mathfrak{X} = \bar{\mathfrak{X}} \cdot \partial_x, \tag{14}$$

where $\bar{g}$ and $\bar{\mathfrak{X}}$ are matrix-valued 0-forms on M, i.e., the 'coordinates representatives' of g and $\mathfrak{X}$, and $dx(\partial_x) = 1$. Explicitly, in components this is $g = g_{\mu\nu}dx^\mu \otimes dx^\nu$ and $\mathfrak{X} = \mathfrak{X}^\mu\partial_\mu$, i.e., $\bar{g} = g_{\mu\nu}$ and $\bar{\mathfrak{X}} = \mathfrak{X}^\mu$, and $dx^\mu(\partial_\nu) = \delta^\mu_\nu$, with $\mu, \nu \in \{0, 1, \ldots, n-1\}$. We have then by definition of the metric that $g_{|x}(\mathfrak{X}_{|x}, \mathfrak{Y}_{|x}) = \bar{\mathfrak{X}}^T_{|x}\bar{g}_{|x}\bar{\mathfrak{Y}}_{|x}$ is a number, with $\bar{\mathfrak{X}}^T$ the matrix transpose of the column matrix $\bar{\mathfrak{X}}$.

We have the expression for the pushforward,

$$\psi_*\mathfrak{X} = (\overline{\psi_*\mathfrak{X}})_{|x} \cdot \partial_x = G(\psi)_{|x}\bar{\mathfrak{X}}_{|x} \cdot \partial_x, \tag{15}$$

where $G(\psi) \coloneqq \frac{\partial\psi}{\partial x}$ denotes the Jacobian matrix of ψ. The fundamental duality relation between pullback and pushforward is, applied to g,

$$(\psi^*g)_{|x}(\mathfrak{X}_{|x}, \mathfrak{Y}_{|x}) = g_{|\psi(x)}(\psi_*\mathfrak{X}_{|\psi(x)}, \psi_*\mathfrak{Y}_{|\psi(x)}), \tag{16}$$

which yields the $\mathrm{Diff}(M)$-transformation of $\bar{g}$,

$$(\overline{\psi^*g})_{|x} = G(\psi)^T_{|x}[\bar{g} \circ \psi(x)]G(\psi)_{|x}, \tag{17}$$

from which we can find its Lie derivative,

$$\begin{aligned}
\mathcal{L}_X\bar{g}_{|x} \coloneqq{}& \frac{d}{d\tau}(\overline{\psi^*_\tau g})_{|x}|_{\tau=0} \\
={}& \left(\frac{d}{d\tau}G(\psi_\tau)^T_{|x}|_{\tau=0}\right)[\bar{g} \circ \psi_{\tau=0}(x)]G(\psi_{\tau=0})_{|x} \\
&+ G(\psi_{\tau=0})^T_{|x}[\bar{g} \circ \psi_{\tau=0}(x)]\left(\frac{d}{d\tau}G(\psi_\tau)_{|x}|_{\tau=0}\right) \\
&+ G(\psi_{\tau=0})^T_{|x}\underbrace{\frac{d}{d\tau}[\bar{g} \circ \psi_\tau(x)]|_{\tau=0}}_{=:X(\bar{g}) = \iota_X d\bar{g}}G(\psi_{\tau=0})_{|x},
\end{aligned} \tag{18}$$

where ι_X denotes the contraction operator on forms. Introducing the linearized Jacobian

$$J(X)_{|x} \coloneqq \frac{d}{d\tau}G(\psi_\tau)_{|x}|_{\tau=0}, \tag{19}$$

we finally obtain the known expression,

$$\mathcal{L}_X\bar{g} = \iota_X d\bar{g} + J(X)^T\bar{g} + \bar{g}J(X). \tag{20}$$

This recap was necessary to fully understand how the DFM ought to apply in concrete computations.

Given a dressing field $\upsilon = \upsilon[\phi]$ for diffeomorphisms, the proof of the invariance of dressed points $x^\upsilon \coloneqq \upsilon^{-1}(x)$ and bounded regions $U^\upsilon \coloneqq \upsilon^{-1}(U)$ with boundaries ∂U^υ of the physical manifold of events is already given by (9). As already mentioned, the *physical spacetime* is (M^υ, g^υ), with $g^\upsilon \coloneqq \upsilon^*g$ the dressed metric field. We need only to prove its $\mathrm{Diff}(M)$-invariance to definitely establish that there is no boundary problem.

(a) *Abstract proof* The intrinsic, abstract, proof of the $\mathrm{Diff}(M)$-invariance of the dressed metric g^υ is easy,

$$\begin{aligned}
(g^\upsilon)^\psi &= (\upsilon^\psi)^*g^\psi = (\psi^{-1} \circ \upsilon)^*(\psi^*g) \\
&= \upsilon^*\psi^{-1*}\psi^*g = \upsilon^*g \eqqcolon g^\upsilon.
\end{aligned} \tag{21}$$

It is correspondingly trivial to prove the vanishing of its Lie derivative,

$$\mathcal{L}_X g^\upsilon \coloneqq \frac{d}{d\tau}(g^\upsilon)^{\psi_\tau}\Big|_{\tau=0} = \frac{d}{d\tau}(g^\upsilon)\Big|_{\tau=0} \equiv 0. \tag{22}$$

Remark that the spirit of the proof applies to any tensor or pseudotensor.

(b) *Computational proof* The previous results are not surprising, conceptually; as we have stressed, the dressed metric g^υ does not "live" on M, but on the physical manifold M^υ. The dressing field being a smooth map, $\upsilon\colon M^\upsilon \to M$, $y \coloneqq x^\upsilon \mapsto \upsilon(y) = x$, its pushforward is $\upsilon_*\colon TM^\upsilon \to TM$,

$$\mathcal{X} \mapsto \upsilon_*\mathcal{X} = (\overline{\upsilon_*\mathcal{X}})_{|y} \cdot \partial_y = G(\upsilon)_{|y}\bar{\mathcal{X}}_{|y} \cdot \partial_y, \tag{23}$$

where $G(\upsilon) \coloneqq \frac{\partial \upsilon}{\partial y}$ is the Jacobian matrix of the dressing field. We have then the pullback/pushforward duality relation between the bare and dressed metrics,

$$(\upsilon^* g)_{|y}(\mathcal{X}_{|y}, \mathcal{Y}_{|y}) = g_{|\upsilon(y)}(\upsilon_*\mathcal{X}_{|\upsilon(y)}, \upsilon_*\mathcal{Y}_{|\upsilon(y)}), \tag{24}$$

which yields the component expression of the dressed metric in terms of that of the bare metric,

$$\overline{g^\upsilon}_{|y} = \overline{\upsilon^* g}_{|y} = G(\upsilon)^T_{|y}[\bar{g} \circ \upsilon(y)]G(\upsilon)_{|y}. \tag{25}$$

Again, this can be understood as the components of the physical metric/gravitational field as determined by the reference frame $\upsilon[\phi]$ provided by the other fields ϕ.

Now, the $\mathrm{Diff}(M)$-transformation of the Jacobian of υ is

$$\begin{aligned} G(\upsilon)^\psi &\coloneqq G(\upsilon^\psi) = G(\psi^{-1} \circ \upsilon) = G(\psi^{-1})G(\upsilon) \\ &= G(\psi)^{-1}G(\upsilon), \end{aligned} \tag{26}$$

where in the last step we use the well-known property that the Jacobian of the inverse of a map is the inverse of the Jacobian of the map [72]. Using (17) and (26), we can check explicitly the invariance of $\overline{g^\upsilon}$:

$$\begin{aligned} \overline{g^\upsilon}^\psi &= G(\upsilon)^{\psi T}[\bar{g}^\psi \circ \upsilon^\psi]G(\upsilon)^\psi \\ &= G(\upsilon)^T G(\psi)^{-1^T}[G(\psi)^T(\bar{g} \circ \psi \circ \psi^{-1} \circ \upsilon)G(\psi)] \\ &\quad \cdot G(\psi)^{-1}G(\upsilon) \\ &= G(\upsilon)^T[\bar{g} \circ \upsilon]G(\upsilon) \eqqcolon \overline{g^\upsilon}. \end{aligned} \tag{27}$$

Correspondingly, using the linear version of (26),

$$\mathcal{L}_X G(\upsilon) = \frac{d}{d\tau} G(\upsilon)^{\psi_\tau}\Big|_{\tau=0} = -J(X)G(\upsilon), \tag{28}$$

we may explicitly compute,

$$\begin{aligned} \mathcal{L}_X\overline{g^\upsilon} &= \mathcal{L}_X(G(\upsilon)^T[\bar{g} \circ \upsilon]G(\upsilon)) \\ &= (\mathcal{L}_X G(\upsilon)^T)[\bar{g} \circ \upsilon]G(\upsilon) + G(\upsilon)^T[\bar{g} \circ \upsilon](\mathcal{L}_X G(\upsilon)) + G(\upsilon)^T[\mathcal{L}_X \bar{g}_{|x}]G(\upsilon) + G(\upsilon)^T \frac{d}{d\tau}[\bar{g} \circ \psi_\tau^{-1}(x)|_{\tau=0}]G(\upsilon) \\ &= -G(\upsilon)^T J(X)^T[\bar{g} \circ \upsilon]G(\upsilon) + G(\upsilon)^T[\bar{g} \circ \upsilon](-J(\upsilon)G(\upsilon)) + G(\upsilon)^T\{\iota_X d\bar{g} + J(X)^T\bar{g} + \bar{g}J(X)\}G(\upsilon) \\ &\quad + G(\upsilon)^T[\underbrace{-X(\bar{g})}_{-\iota_X d\bar{g}}]G(\upsilon) \equiv 0. \end{aligned} \tag{29}$$

Again, the principle of the proof applies to any tensor or pseudotensor.

B. Scalar coordinatization of GR via the DFM

We now illustrate the above to the case of $n = 4$ GR, with cosmological constant Λ, and matter described phenomenologically as a (perfect) fluid [73]; it supplies a set of scalar fields $\varphi = \varphi^a\colon U \subseteq M \to N = \mathbb{R}^4$, $a = \{1, \ldots, 4\}$, characterizing the matter/fluid distribution and entering the expression of its covariantly conserved stress-energy tensor $\mathsf{T} = \mathsf{T}(g, \varphi)$, $\nabla^g\mathsf{T} = 0$, itself derivable as the Hilbert stress-energy tensor of an effective Lagrangian $L_{\text{matter}}(g, \varphi)$—whose precise form is not needed here [74] The fields considered $\phi = \{g, \varphi\}$ $\mathrm{Diff}(M)$-transform as

$$\phi^\psi = \{\psi^* g, \psi^*\varphi\} = \{\psi^* g, \varphi \circ \psi\}, \tag{30}$$

and the Lagrangian of the theory is

$$\begin{aligned} L_{\text{GR}}(\phi) &= L_{\text{GR}}(g, \varphi) \\ &= \frac{1}{2\kappa}\mathrm{vol}_g(\mathsf{R}(g) - 2\Lambda) + L_{\text{matter}}(g, \varphi), \end{aligned} \tag{31}$$

where $\kappa = \frac{8\pi G}{c^4}$ is the gravitational coupling constant, vol_g is the volume 4-form induced by g, and $\mathsf{R}(g)$ is the Ricci scalar. The Lagrangian $L_{\text{GR}}(\phi)$ is $\mathrm{Diff}(M)$-covariant, and so are the Einstein equations derived from it: $E(\phi) = \mathsf{G}(g) + \Lambda g - \kappa\mathsf{T}(g, \varphi) = 0$, where $\mathsf{G}(g)$ is the Einstein tensor. Remark that $\nabla^g\mathsf{T} = 0$ implies that the fluid particles are in geodesic motion either if the pressure gradient vanishes, or if pressure itself does, in which case the fluid is a dust field (and, as said earlier, we then make contact with [69,70]).

We can identify a $\mathrm{Diff}(M)$-dressing field as

$$\upsilon = \upsilon[\varphi] \coloneqq \varphi^{-1}\colon \mathbb{R}^4 \to M. \tag{32}$$

As indeed, $\upsilon^\psi = \upsilon[\varphi^\psi] = \psi^{-1} \circ \upsilon[\varphi]$. It allows to define dressed regions

$$U^\upsilon \coloneqq \upsilon^{-1}(U), \quad \text{such that } (U^\upsilon)^\psi = U^\upsilon. \tag{33}$$

This gives a $\mathrm{Diff}(M)$-invariant relational definition of physical regions of events via (the scalar distribution of) matter as a *physical* reference system, as is expected from the point-coincidence argument. The dressed fields are

$$\phi^{\upsilon} = \{g^{\upsilon}, \varphi^{\upsilon}\} = \{\upsilon^* g, \mathrm{id}_{U^{\upsilon}}\}. \tag{34}$$

Here $\varphi^{\upsilon} \coloneqq \upsilon^*\varphi = \varphi \circ \upsilon = \mathrm{id}_{U^{\upsilon}}$, the matter distribution being "self-dressed", just expresses the fact that the (values of) the scalars now *are* the coordinates—also known as *Lagrangian* or *comoving* coordinates (see footnote [68]). The $\mathrm{Diff}(M)$-invariant dressed metric g^{υ}, encoding the geometric properties of M^{υ}, can then be understood as the physical gravitational field as measured in the coordinate system supplied by the matter distribution φ. In abstract index notation, (25) gives

$$g^{\upsilon}_{ab} = G(\upsilon)_a{}^{\mu} g_{\mu\nu} G(\upsilon)^{\nu}{}_b, \tag{35}$$

with the Jacobian $G(\upsilon) = G(\varphi^{-1}) = G(\varphi)^{-1} = (\frac{\partial \varphi^a}{\partial x^{\mu}})^{-1}$.

The Lagrangian of the dressed, relational, theory is

$$\begin{aligned} L_{\mathrm{GR}}(g^{\upsilon}, \varphi^{\upsilon}) &\coloneqq \upsilon^* L_{\mathrm{GR}}(g, \varphi) \\ &= \frac{1}{2\kappa}\mathrm{vol}_{g^{\upsilon}}(\mathsf{R}(g^{\upsilon}) - 2\Lambda) + L_{\mathrm{matter}}(g^{\upsilon}, \varphi^{\upsilon}). \end{aligned} \tag{36}$$

From it we derive the *relational Einstein equations,*

$$E(\phi^{\upsilon}) = \mathsf{G}(g^{\upsilon}) + \Lambda g^{\upsilon} - \kappa \mathsf{T}(g^{\upsilon}, \varphi^{\upsilon}) = 0, \tag{37}$$

with $\mathsf{T}(g^{\upsilon}, \varphi^{\upsilon}) \eqqcolon \mathsf{T}^{\upsilon}$ the conserved dressed stress-energy tensor, $\nabla^{g^{\upsilon}} T^{\upsilon} = 0$ (controlling the dynamics of φ^{υ}). These are strictly $\mathrm{Diff}(M)$-invariant, and have a well-posed Cauchy problem [84]. Notice that, contrary to the bare equations, there is no more "pure metric side" in the dressed field equations (37); all its terms involve *both* the physical metric and matter d.o.f. These are the equations that are confronted to experimental tests [28] (see e.g., [85] for an application to galaxy rotation curves analysis).

V. CONCLUSIONS

We have addressed the claim, often encountered in general-relativistic and gauge field theoretic physics, that "diffeomorphism symmetry, $\mathrm{Diff}(M)$, and/or gauge symmetries, $\mathcal{H}$, are broken at spacetime boundaries", which we refer to as the boundary problem. We have demonstrated this claim to have the same conceptual structure as a hole-type argument, and is thus defused by the point-coincidence argument. Upon recognizing physical (spacetime) regions and boundaries as invariant structures, relationally defined by the physical fields, the boundary problem conceptually dissolves.

This resolution was technically realized through a manifestly invariant and relational reformulation of general-relativistic gauge field theories via the dressing field method, whereby the point-coincidence argument is implemented automatically by defining dressed fields and regions. We have illustrated our framework with a concrete application to the general-relativistic case, where the $\mathrm{Diff}(M)$-invariance of the dressed metric and spacetime regions is proven twice over, abstractly and in a concrete computational way, showing that no boundary problem can arise technically.

A related consequence of our framework is that it dissolves one aspect of the multifaceted "problem of time", i.e., the worry that, since in classical GR proper time evolution on M can be seen as a special case of diffeomorphisms, "time evolution is 'pure gauge' in general-relativistic physics", so that physical observables have no dynamics. This is otherwise known as the "frozen formalism problem" [86–88]. The issue arises only if one overlooks that M *is not* the true physical spacetime. This aspect of the problem of time dissolves in a way analogous to the boundary problem, on account of the relational core of general-relativistic physics. The same "problem" arises in parametrized Mechanics (see e.g., [89,90]) and is likewise resolved relationally; we give the detailed treatment via DFM in [91], leading to what we call *relational quantization* (see also [31]).

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DATA AVAILABILITY

No data were created or analyzed in this study.

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[63] The origin of the latter being indeed the group $\mathrm{Aut}_v(P)$ of vertical automorphisms of the principal fiber bundle P over M, whose geometry underlies the kinematics of GFT. These automorphisms of P induce, by definition, the identity transformation id_M of M. They form a (normal) subgroup of the full group $\mathrm{Aut}(P)$ of automorphisms of P, which induces diffeomorphisms $\mathrm{Diff}(M)$ of M. The geometry of principal bundles is the unifying geometric framework for gRGFT. See [24,28].
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[71] DeWitt [68] would say that the dressing field $\upsilon[\phi]$ provides what he calls "intrinsic coordinates", whereby one may build invariants: in our language, the dressed fields $\phi^\upsilon := \upsilon^*\phi$. To quote him more extensively, he argues that "what appears to be needed, if one insists on maintaining manifest covariance [we would rather say, $\mathrm{Diff}(M)$-invariance], is a mean of constructing *local invariants*. A possible procedure is to introduce four independent scalars ζ^A, $A = 0, 1, 2, 3$ [yielding a dressing field for $\mathrm{Diff}(M)$ as defined in Eq. (6)], formed out of the metric and its derivative [i.e., a dressing field $\upsilon[g]$] and then to use these to define an *intrinsic coordinate system*." His emphasis. DeWitt cites Komar and Bergmann [64,65] as doing just that, but noting possible difficulties with using the metric; to avoid them he continues "[...] we shall introduce directly into the discussion an additional physical system. This system will serve to furnish us with a reasonably full-proof set of intrinsic coordinates [i.e., a dressing field $\upsilon[\phi]$] while at the same time forming a *combined physical system* with the gravitational field." The emphasis here is ours, and highlights the *relational* character of the physical system to be analyzed. He continues "In principle, any additional system which provides a 'useful' set of four scalars will do."—see indeed Section IV B for an illustration—adding "It might be supposed that [the additional system] has merely a technical utility, constituting an otherwise foreign element in the discussion. Such is by no means the case. The role played by the medium in providing a *physical coordinate system* proves to be a fundamental one [...]" (our emphasis again), with which we fully agree. Echoing (perhaps unwittingly) Einstein's point-coincidence argument described earlier, DeWitt indeed insists that "A [...] medium of some kind is needed [...] in order to give an operational meaning to the concept of 'space-time geometry' in the first place." We thank a referee for pointing out this reference by DeWitt.

[72] Remark that, seeing $G(\upsilon): M^\upsilon \to GL(n)$, this transformation closely resembles that of an 'internal' dressing (2).

[73] We may e.g., quote DeWitt [68] again; if "[i]n principle, any additional system which provides a 'useful' set of four scalars [dressing $\upsilon[\phi]$] will do. Actually, we shall choose the most intuitively obvious system possible, namely, a stiff elastic medium carrying a framework of clocks." whose "physical constitution" is considered "only phenomenologically".

[74] The scalars could be taken to be, if not as the *components* of the 4-velocity of the fluid particles $\varphi^a = u^a$ (as determined by any arbitrary *frame field*, i.e., a section of the frame bundle LM of M), at least as a (sub)set of scalars in terms of which the fluid 4-velocity is expressed, $u^a = u^a(\varphi)$, as is done in the so-called "velocity-potential" representations. A typical choice of effective Lagrangian is $L_{\text{matter}}(g,\varphi) = \rho \mathrm{vol}_g$, with vol_g the volume form induced by the metric field g and $\rho = \rho(\varphi, \ldots)$ the rest energy density of the fluid expressed as a function of the scalars φ and possibly of other thermodynamical parameters (entropy per baryon, chemical potential, etc.). The field equations for the scalars are equivalent to particle (baryon) number conservation and covariant conservation of the stress-energy tensor, $\nabla \mathsf{T}(g.\varphi) = 0$. Such Lagrangian description was pioneered notably by Taub [75,76], Schutz [77], as well as Carter [78], Kijowski and Tulczyjew [79]—see also Brown [80,81]—and is an integral part of the field of relativistic fluid dynamics and numerical relativity [82,83]. In this field, it is also often the case that the fluid distribution is described by a set of scalars fields labelling fluid particles and called "Lagrangian coordinate fields", or yet "comoving coordinates" (standard, unphysical, coordinates on M being called "Eulerian coordinates"); these are clearly fit for our purpose (e.g., DeWitt [68] uses just this viewpoint). Velocity-potentials, or a subset thereof, are sometimes used as Lagrangian coordinates. Keen readers may detect in this literature many instances of the DFM philosophy; e.g., the reference manifold N (that is R^4 in the case at hand), the source space of the dressing field υ, generalizes what is variously called "material space" [79], "fluid space" [80], and "matter space" [81,82]—or yet "fleet" (of fluid particles) [81]. Also, dressed fields ϕ^υ extend what Carter calls "material tensors" [78], while g^υ relates e.g., to the "matter space/fleet metric" of [81].

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[84] As stressed already above, the dressed field equations have the same functional expression as the bare ones. In components, they are related by

$$\begin{aligned} &\mathsf{G}^\upsilon_{ab} + \Lambda g^\upsilon_{ab} - \kappa \mathsf{T}^\upsilon_{ab} \\ &\quad = G(\upsilon)_a{}^\mu (\mathsf{G}_{\mu\nu} + \Lambda g_{\mu\nu} - \kappa \mathsf{T}_{\mu\nu}) G(\upsilon)^\nu{}_b = 0. \end{aligned}$$

Which superficially resembles the general-covariance of Einstein's equation, yet is conceptually distinct.

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