Enforcing Dirichlet boundary conditions in physics-informed neural networks and variational physics-informed neural networks

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STRUCTURAL DAMAGE DETECTION IN BRIDGES UNDER OPERATIONAL AND ENVIRONMENTAL VARIABILITY

Leonardo Zunino[1], Joan R. Casas[2], and Marco Domaneschi[1]

[1]Politecnico di Torino - DISEG
Turin, Italy
e-mail: {leonardo.zunino,marco.domaneschi}@polito.it

[2] UPC - BarcelonaTech
Barcelona, Spain
e-mail: joan.ramon.casas@upc.edu

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Abstract. *Ensuring the safety and reliability of bridges, essential elements of civil infrastructure, requires precise assessment methods. Traditional structural health monitoring often associates changes in dynamic response with possible damage. However, for bridges, changes can also derive from operational factors like traffic loads or environmental influences, such as temperature and humidity. These variations, unrelated to structural integrity, complicate damage detection, as they can cause false alarms. To address this, a methodology designed to detect and localize bridge damage while accounting for these external factors is proposed. This approach relies on acceleration data from the Yonghe Bridge, a cable-stayed bridge in China, collected as part of a continuous monitoring effort. The non-stationary nature of these signals limits the effectiveness of the Fast Fourier Transform, prompting the use of Variational Mode Decomposition to separate the data into meaningful Intrinsic Mode Functions. Subsequently, instantaneous frequencies are derived through the Hilbert Huang Transform, identifying damage-sensitive features within the signal. Environmental and operational influences on these features are attenuated via Principal Component Analysis, a dimensionality reduction technique based on variance that enhances interpretability without significant data loss. For the final stage, statistical analysis selects critical features for a clustering process, applying the K-means Machine Learning algorithm to identify damage location. This comprehensive approach has shown a high degree of accuracy in identifying damage under varying traffic and environmental conditions, suggesting its applicability for structural health monitoring systems.*

1 INTRODUCTION

The safety and reliability of long-span bridges significantly depend on effective Structural Health Monitoring (SHM), especially under varying environmental and operational conditions. Conventional SHM typically focuses on detecting structural anomalies through changes in natural frequencies. However, these frequencies are sensitive to external influences such as temperature fluctuations, humidity variations, and dynamic traffic loads, potentially obscuring actual structural damage or causing false alarms.

Recent studies emphasize integrating advanced signal processing with machine learning techniques to address these complexities. Delgadillo et al. [1] validated the use of instantaneous frequencies extracted via Hilbert-Huang Transform (HHT), combined with unsupervised clustering, to detect damage effectively, though environmental variability was not fully addressed. Meixedo et al. [2] analyzed the combined impact of temperature and operational loading on bridge dynamics, highlighting the importance of considering these factors for accurate damage assessment. Similarly, Tenelema et al. [3] demonstrated how Principal Component Analysis (PCA) could separate structural damage indicators from temperature and traffic-induced variations, particularly in numerically simulated scenarios.

Additionally, Diao et al. [4] proposed a comprehensive framework integrating Variational Mode Decomposition (VMD), HHT, and convolutional neural networks (CNN), reinforcing the effectiveness of advanced decomposition methods for extracting reliable damage-sensitive features. While, Santos et al. [5] showed that unsupervised clustering techniques, such as K-means, effectively identify early structural damage under real operational variability, emphasizing their robustness and practical relevance in SHM applications.

The methodology presented in this paper integrates these insights by applying VMD for precise signal decomposition, HHT for instantaneous frequency extraction, PCA to mitigate operational and environmental variability, and K-means clustering for unsupervised and robust damage localization. Inspired by the comprehensive framework of Niyozov et al. [6], initially tested on numerical benchmarks, this research applies this integrated approach to real structural monitoring data from the Yonghe Bridge, recorded during its transition from an undamaged to a damaged condition. Historical visual inspections validate the method's reliability, enhancing its practical applicability.

The paper continues by detailing the proposed methodology, presenting its implementation and outcomes through a case study, and concludes by summarizing the key findings and identifying future research directions.

2 METHODOLOGY

2.1 Variational Mode Decomposition and Hilbert Transform

Variational Mode Decomposition (VMD) is a non-recursive signal processing technique aimed at decomposing complex, non-stationary signals into a finite number of narrow-band Intrinsic Mode Functions (IMFs). Unlike empirical approaches such as Empirical Mode Decomposition (EMD), VMD formulates the decomposition process as an optimization problem, minimizing the sum of bandwidths of the extracted modes. Mathematically, the objective function is expressed as:

$$\min_{u_k;\omega_k} \left\{ \sum_{k=1}^{K} \left| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) e^{-j\omega_k t} \right] \right|_2^2 \right\} \quad (1)$$

subjected to the constraint:

$$\sum_{k=1}^{K} u_k(t) = f(t) \tag{2}$$

where $u_k(t)$ are the Intrinsic Mode Functions (IMFs), ω_k their respective center frequencies, $\delta(t)$ the Dirac delta function, $*$ denotes convolution, and j is the imaginary unit.

The Hilbert Transform (HT) is subsequently applied to each IMF to compute instantaneous frequencies, which are damage-sensitive features that also capture structural changes. The analytic signal $z_k(t)$ for each IMF is computed as:

$$z_k(t) = u_k(t) + j\mathcal{H}u_k(t) = a_k(t)e^{j\theta_k(t)} \tag{3}$$

where $\mathcal{H}$ denotes the Hilbert Transform operator, $a_k(t)$ the instantaneous amplitude, and $\theta_k(t)$ the instantaneous phase. The instantaneous frequency $\omega_k(t)$ is derived as the time derivative of the instantaneous phase:

$$\omega_k(t) = \frac{d\theta_k(t)}{dt} \tag{4}$$

The instantaneous frequencies derived through VMD-HHT provide reliable damage-sensitive features, essential for subsequent analysis.

2.2 Principal Component Analysis (PCA)

Structural damage identification is complicated by variations induced by operational factors such as traffic loads and environmental effects (temperature, wind, humidity). Principal Component Analysis (PCA) effectively reduces these variability by transforming the high-dimensional instantaneous frequency data into a lower-dimensional space of orthogonal variables, or Principal Components (PC), ordered by their variance contribution. Mathematically, PCA solves:

$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{5}$$

where $\mathbf{X}$ is the data matrix (features), $\mathbf{U}$ and $\mathbf{V}$ are orthonormal matrices, and $\mathbf{S}$ is a diagonal matrix containing singular values, indicating the variance of each principal component.

Typically, the first principal components capture variations related to temperature or traffic-induced variability. By removing these high-variance principal components, the analysis isolates variations strictly associated with structural damage.

2.3 K-means Clustering and Damage Localization

The unsupervised K-means clustering algorithm partitions the processed data into clusters based on statistical similarity, minimizing the sum of squared distances between data points and their corresponding cluster centroids. This method is especially advantageous in structural health monitoring contexts, where labeled damage data is often unavailable. Mathematically, K-means clustering minimizes the within-cluster variance:

$$\min_{C} \sum_{k=1}^{K} \sum_{\mathbf{x} \in C_k} \|\mathbf{x} - \boldsymbol{\mu}_k\|^2 \tag{6}$$

where $\mathbf{x}$ represents the data points, and $\boldsymbol{\mu}_k$ denotes the centroid of the k-th cluster C_k.

To quantify the degree of separation between clusters, the Difference between Clusters (DC) metric is computed as follows:

$$DC = \frac{1}{K(K-1)} \sum_{k=1}^{K} \sum_{\substack{c=1 \\ c \neq k}}^{K} d_{ck} \tag{7}$$

where d_{ck} represents the Gowda-Diday distance between the centroids of clusters c and k.

A Damage Index (DI) is subsequently calculated to assess the potential presence of structural damage:

$$DI = \frac{\max_i(DC_i - CB)}{\text{median}(DC)}, \quad CB = \text{median}(DC_i) + t_\alpha \frac{\text{MAD}(DC)}{\sqrt{S}} \tag{8}$$

Here, t_α is the critical value from the Student's t-distribution corresponding to the chosen confidence level (typically 99.9%), MAD indicates the Median Absolute Deviation, and S is the sample size used in calculating the confidence boundary (CB). Positive DI values indicate potential damage occurrence, thereby enabling reliable structural damage detection and localization.

3 CASE STUDY: YONGHE BRIDGE

The selected case study is the cable-stayed Yonghe Bridge, located in China and characterized by a main span of 260 m. The bridge is equipped with a continuous structural health monitoring system comprising 14 deck-mounted uniaxial accelerometers and 2 biaxial accelerometers located on the towers, recording vibration responses induced by traffic loads at a sampling rate of 100 Hz.

For this study, acceleration signals were selected from three distinct structural conditions: an initial undamaged condition recorded shortly after rehabilitation (January), a transitional undamaged condition (June), and a subsequent damaged condition following documented cracking and cable deterioration (August). For each of these three conditions, 16 different acceleration signals of 60 seconds each were analyzed. These signals were specifically chosen from four separate days within each month, capturing four distinct time intervals per day to ensure representativeness across operational variations.

Table 1 summarizes the selection criteria and the number of signals analyzed:

Condition	Month	Number of signals
Undamaged	January	16
Undamaged (transition)	June	16
Damaged	August	16

Table 1: Summary of analyzed vibration signals from Yonghe Bridge.

These selected datasets span significant temperature fluctuations from approximately $-2°$C (January) to $+25°$C (August), further contributing to the complexity and realism of the data and reflecting the operational and environmental variability faced in practical SHM applications.

4 RESULTS AND DISCUSSION

4.1 Feature Extraction

Following VMD, the acceleration signals were optimally decomposed into K IMFs (Figure 1 shows a decomposition example with 6 IMFs). Subsequently, HHT was applied to each IMF to extract instantaneous frequencies (Figure 2). These instantaneous frequencies were then processed through PCA, removing the first principal component which exhibited the highest variance and thus primarily captured variability due to traffic and temperature fluctuations. The cleaned instantaneous frequencies obtained after PCA formed the basis for damage detection (Figure 3). Specifically, the K-th instantaneous frequency (corresponding to the lowest frequency band) was selected as it better represented the structural behavior of the bridge, given that the fundamental vibration mode of a long-span cable-stayed bridge typically lies in the range of 0.2–0.4 Hz. This is demonstrated by the frequency domain analysis shown in the zoomed view of Figure 1, where it can be observed that the last instantaneous frequency closely matches the structural frequencies.

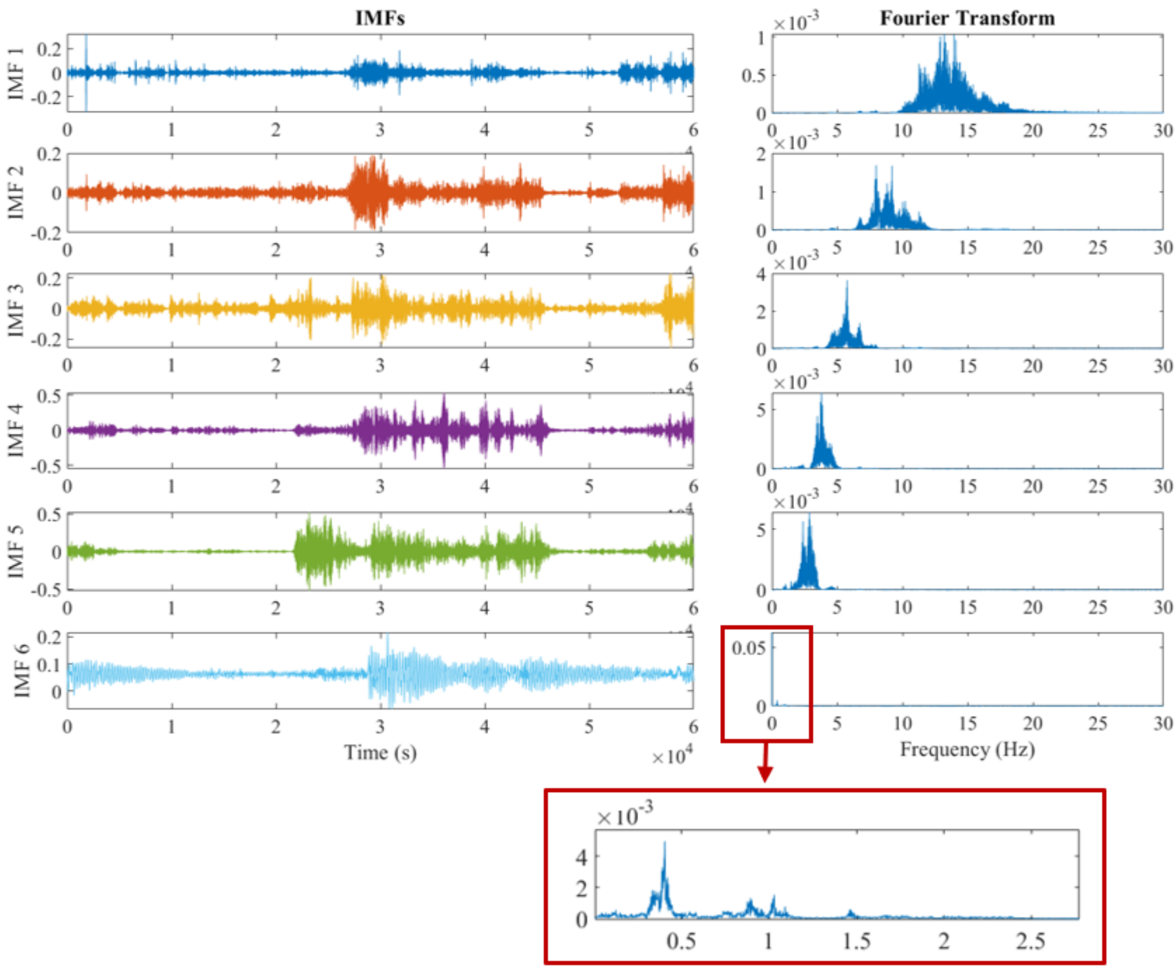


Figure 1: Signal decomposition using VMD (with a focus on the frequency domain of the last IMF).

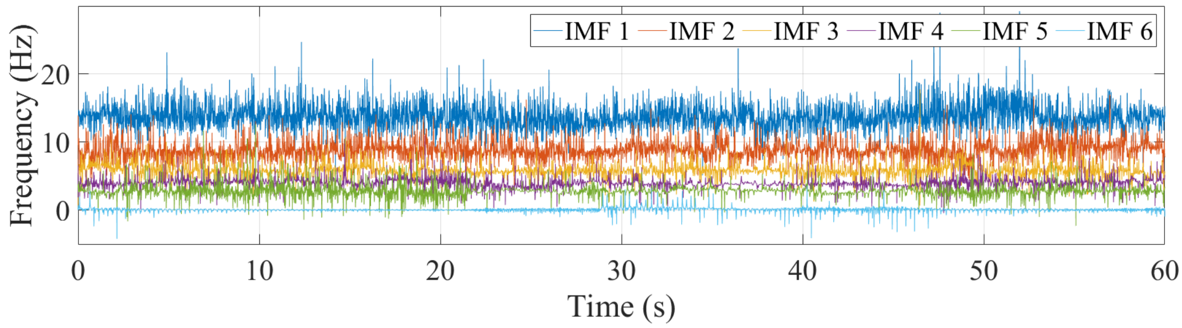


Figure 2: Instantaneous frequencies extraction from IMFs using HHT.

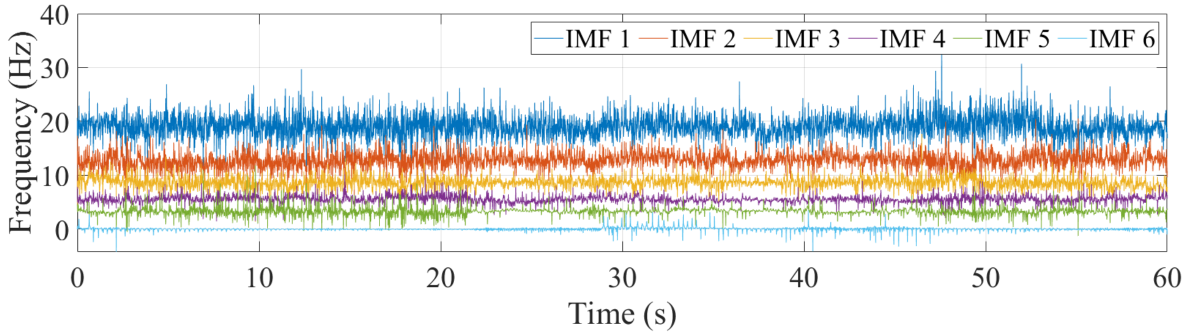


Figure 3: Rescaled Instantaneous frequencies after PCA.

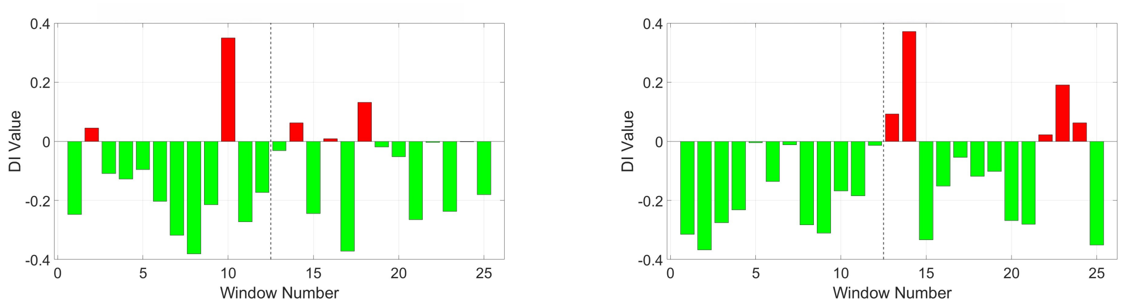
4.2 Unsupervised Clustering

The K-means clustering algorithm was employed to detect structural damage through the analysis of the processed instantaneous frequencies. For each sensor, the last instantaneous frequency, obtained after PCA reconstruction, was selected due to its enhanced sensitivity to structural variations. A continuous time series was created by concatenating the instantaneous frequencies from both undamaged and damaged test cases, thus clearly separating undamaged conditions (left side of the time series) from damaged conditions (right side).

These concatenated data were segmented into symbolic sequences characterized by their median and interquartile range (IQR), and analyzed using a sliding window approach (window size equal to 5 segments, overlapping). For each sliding window, silhouette analysis determined the optimal number of clusters (between 2 and 5), maximizing intra-cluster homogeneity and inter-cluster separation. The chosen clusters were then used to compute the Difference between Clusters (DC), quantifying the degree of separation.

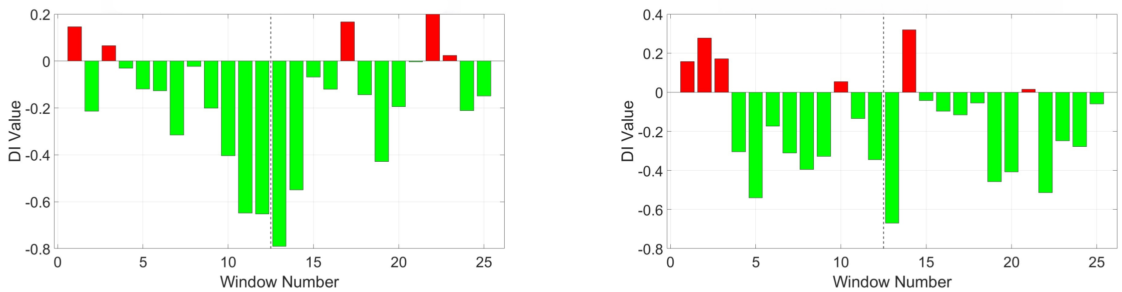
A Detection Index (DI) was derived from the DC values to objectively assess structural changes. A confidence boundary (CB) was established based on the statistical properties of DC (mean, standard deviation, and a t-distribution at a 99.9% confidence level). Positive DI values exceeding this boundary indicated significant structural changes potentially associated with damage events.

Figure 4 presents the clustering results for Sensor 1, located near the documented damaged areas. The Detection Index (DI) distinctly shows positive values exclusively in the damaged condition (right side of the plot), accurately reflecting the actual structural degradation. In contrast, the clustering results for Sensor 3 (Figure 5), positioned away from damaged regions, reveal several false positives distributed across both the undamaged and damaged conditions. The absence of a clear separation between left (undamaged) and right (damaged) portions in the DI indicates that the structural response at Sensor 3 is similar in both scenarios. The use of a global Confidence Boundary (CB) in such cases can lead to misleading interpretations, highlighting the necessity of sensor-specific analyses or adaptive thresholding to enhance damage detection accuracy.



(a) (b)

Figure 4: Detection Index for Sensor 1: (a) Undamaged condition (January, left) vs undamaged condition (June, right); (b) Undamaged condition (January, left) vs damaged condition (August, right).



(a) (b)

Figure 5: Detection Index for Sensor 3: (a) Undamaged condition (January, left) vs undamaged condition (June, right); (b) Undamaged condition (January, left) vs damaged condition (August, right).

5 CONCLUSIONS

The effectiveness of a multi-stage structural damage detection methodology incorporating VMD, Hilbert Transform, PCA, and unsupervised K-means clustering was demonstrated on the Yonghe Bridge under realistic operational and environmental conditions. The approach successfully isolated structural damage-sensitive features by effectively mitigating influences from traffic loads and temperature variations through PCA. The clustering results highlighted clear distinctions between undamaged and damaged conditions, particularly at sensors near documented damage locations. The robustness of the proposed method was confirmed by consistent outcomes despite variations in clustering parameters. Future research will focus on implementing adaptive filtering techniques to enable faster and more precise online damage detection, ultimately reducing the latency between damage occurrence and automated structural health warnings.

REFERENCES

[1] R. M. Delgadillo, J. R. Casas, Bridge damage detection via improved completed ensemble empirical mode decomposition with adaptive noise and machine learning algorithms. *Structural Control and Health Monitoring*, 2022, doi:10.1002/stc.2966.

[2] A. Meixedo, J. Santos, D. Ribeiro, R. Calçada, M. Todd, Damage detection in railway bridges using traffic-induced dynamic responses. *Engineering Structures*, **238**, 112189, 2021, doi:10.1016/j.engstruct.2021.112189.

[3] F. J. Tenelema, R. M. Delgadillo, J. R. Casas, Bridge damage detection and quantification under environmental effects by Principal Component Analysis. *Proceedings of the 1st Conference of the European Association on Quality Control of Bridges and Structures*, Springer, 2022, doi:10.1007/978-3-030-91877-4_22.

[4] Y. Diao, J. Lv, Q. Wang, X. Li, J. Xu, Structural damage identification based on variational mode decomposition–Hilbert transform and CNN. *Journal of Civil Structural Health Monitoring*, **13**, 1415–1429, 2023, doi:10.1007/s13349-023-00715-3.

[5] J. P. Santos, C. Crémona, L. Calado, P. Silveira, A. D. Orcesi, On-line unsupervised detection of early damage. *Structural Control and Health Monitoring*, **23**, 1047–1069, 2016, doi:10.1002/stc.1825.

[6] S. A. Niyozov, M. Domaneschi, J. R. Casas, R. M. Delgadillo, Temperature effects removal from non-stationary bridge–vehicle interaction signals for ML damage detection. *Sensors*, **23**(5187), 2023, doi:10.3390/s23115187.