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A *p*-ary Choquet-based multi-criteria decision-making approach for assessing sustainability indicators in urban development

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ABSTRACT

This research acknowledges the existence of a dichotomy between the weak sustainability (WS) and strong sustainability (SST) paradigms, which can heavily influence decision-making analysis methods. Rather than entering the debate between WS and SST, we take the need for SST as a starting point and concern ourselves with its applicability. To do so, we introduce an original three-tier SST performance range corresponding to as many ways of aggregating and presenting indicators. We adopt the p-ary Choquet integral to encompass these three declinations, and a parsimonious elicitation method introduced by Labreuche and Grabisch to ensure that the problem remains manageable. The advantages of this method in supporting the awareness of Decision Makers regarding their choices are illustrated through the application to a case study concerning the transformation of a historic building located in the city of Turin, Italy.

1. Introduction

The concept of sustainability is nowadays crucial and ubiquitous as shown, for example, by the inception and analysis of Sustainable Development Goals [1]. As shown, among others, by Folke et al. [2], it is widely accepted to distinguish the three dimensions of sustainability: economic, environmental, and social. This, for instance, is reflected in the use of the 3P paradigm — Profit, Planet, People — when it comes to assess the level of sustainability of actions or alternatives. In spite of the agreement on the necessity of considering these three dimensions, currently there is no meeting of minds on *how* to holistically take into account all three dimensions. In particular, there is a dichotomy of paradigms when it comes to solve this problem, namely the Weak Sustainability (WS) and Strong Sustainability (SST) paradigms. According to Gutés [3], the two existing paradigms "represent two opposing ends in the quest to give a workable dimension to sustainability". These can be defined as follows:

- The WS approach allows for compensations between distinct dimensions of the analysis [4]. That is, if the environmental assessment of an alternative decreases, then this can be compensated by an improvement of the performance in another of two dimensions. Note that this approach lends itself naturally to additive methods to aggregate the performances of alternatives with respect to different criteria and dimensions [5].
- The SST approach, instead, considers that a number of environmental resources, once depleted, cannot be restored [6]. For this reason, additive approaches to the aggregation of criteria scores in decision problems may be misleading. On the contrary, this paradigm suggests the use of more conjunctive, and less compensatory, aggregation methods [7]. For example Shmelev and Rodríguez-Labajos [8] suggested the use of the minimum operator and Ziemba [9] proposed to use the PROMETHEE method, which is known for being only partially compensatory [10]. As recalled by Wilson and Wu [11], the most extreme case is perhaps the 'absurd' strong sustainability according to which any indicator should focus exclusively on the environmental dimension.

The natural association of the two declinations of sustainability with different aggregation rules suggests the use of Multi Criteria Decision Analysis (MCDA), that can be seen as a body of knowledge for the representation and resolution of problems involving a multitude of criteria, each representing a different and relevant perspective on the set of decision alternatives [12]. Numerous studies demonstrate the use of these methods in different urban contexts, in relation to sustainability issues and the aggregation of indicators [13–15]. In fact, in MCDA one can find mathematical models that can represent compensatory and non-compensatory behaviors, and they can possibly be viable modeling solutions for either the WS and SST paradigms, respectively. This is due to the fact that the choice of one type of sustainability

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involves a different process of designing and aggregating indicators into a composite indicator [16,17]. Indeed, the way the indicators are aggregated is the element identified by the literature as the one to which the greatest attention should be paid to conduct an assessment in terms of SST [5,11,18] and the mathematical approach used for this aggregation can allow for varying degrees of compensation among indicators, significantly influencing the final outcomes [7,19].

While the scientific debate has traditionally been polarized between WS and SST, we aim to present a new perspective: a declination of SST, directly tied to the mathematical methodologies used for indicator aggregation, to enhance its practical applicability. The paper proposes a dynamic approach to SST, introducing three ways of aggregating indicators that correspond to as many declinations of SST: Strict, Complementary and Integrative Strong Sustainability. More specifically, to encompass the three levels of SST, we employ the p-ary Choquet integral to represent various strategies across distinct regions of the decision space [20]. Following the procedure proposed by Labreuche and Grabisch [20], the present approach employs three distinct Choquet integrals, each corresponding to a specific level of SST. These integrals are associated with different reservation and aspiration thresholds [21,22], which are applied differently depending on the considered level of strong sustainability. Such thresholds provide an alternative to the information on trade-offs and weightings commonly used in other MCDA theories [23]. The adoption of reference thresholds is consistent with recent methodological developments in the literature on sustainability assessment. For instance, Roszkowska et al. [24] employ the Distances to Aspiration Reference Points (DARP) method to analyze the sustainable development of European countries; Ricciolini et al. [22] propose the use of Multiple Reference Point–based multi-criteria methods to evaluate the sustainability progress of European countries; Boggia et al. [21] propose the Multiple Reference Point Weak–Strong Composite Indicators (MRP-WSCI) method to support sustainable choices in agricultural systems; Miao et al. [25] apply the Interval Reference Point Method to the sustainable planning of island microgrid systems; and finally, Karagiannis and Karagiannis [26] develop distance-based methods for constructing indicators applied in the field of sustainable energy. Although the aggregation framework developed in this paper is conceptually general, it is tested within the urban-architectural domain—a data-rich and decision-critical arena in which interactions among natural, social, and manufactured capital are most visible and results can be validated at a tractable scale before broader application. The resulting indicator can facilitate the understanding of the urban context by making phenomena observable [27] and assumes a diagnostic function [28], i.e. it serves to assess the performance of the project prior to its realization, contributing not only to represent it, but also to shape it. To manage the extensive preference information that needs to be gathered, we adopt a parsimonious elicitation method introduced by Labreuche and Grabisch [20], ensuring that the problem remains manageable. The advantages of this method in supporting the awareness of the DMs regarding their choices are illustrated through the application to a case study concerning the transformation of a modern urban monument in the city of Turin, Italy.

It should be noted that this contribution does not intend to promote a rigidly positivist paradigm, according to which sustainability performance can be evaluated solely based on numerical and quantifiable aspects. On the contrary, in line with [29], it is recognized that there is a need to move beyond a vision centered exclusively on technology and the human being, to instead value the interconnections between human and ecological life systems. From this perspective, the proposed framework aims to support decision-makers in understanding project performance and its contribution to sustainable development. Therefore, it is observed that if the selected indicators focus exclusively on anthropocentric and technological aspects, neglecting ecological dimensions, the overall objective of the framework would be compromised. Consequently, careful reflection is required to identify and select the set of indicators, being aware that this process may encounter limits related to their measurability and the definition of appropriate benchmarks.

The paper is organized as follows. In Section 2 we provide a deeper overview of the context and the current dichotomy between WS and SST and propose a new and more flexible definition of strong sustainability. This will be done keeping an eye on its implications in urban planning, which is going to be the focus of our case study. Section 3 introduces the concept of aggregation functions and eventually presents the p-ary Choquet integral, whose customized implementation will be used to model our problem. Section 4 summarized the process of the proposed methodology. Section 5 presents the case study and Section 6 shows the results of the application of the p-ary Choquet integral. Finally, 7 concludes the paper with a discussion.

2. A new definition of strong sustainability

Observing the different definitions of the concepts of WS and SST provided in the literature in the context of the urban environment, a strong homogeneity emerges for the definition of WS (these reflections emerge from a more extensive review of the literature on WS and SST, only a portion of which is cited here): the word 'substitutability' or synonyms is almost always used; rarely is the term 'compensation' made explicit. The two concepts do not coincide: while substitutability focuses on the interchangeability of elements to perform similar functions, compensation focuses on mechanisms that counteract or balance the impacts of changes or losses. Quite often substitutability is assumed to occur between natural capital (e.g., ecosystems and mineral wealth) and manufactured capital (also indicated as "man-made capital", e.g. buildings and urban infrastructure) [4,30,31]. The core element of this paradigm is that total capital, namely the combination of natural and human capital, should remain unchanged [32,33]. This means that the loss of 'nature' can be compensated by a rapid economic growth and urbanization [34,35].

Natural capital is that capital which provides raw materials, assimilates waste products, offers amenities, such as the aesthetic beauty of a landscape, and provides the basic functions on which human life depends [36]. It thus represents minerals, air, water, soil, flora, fauna and ecological systems. Manufactured capital refers to the entire capital stock derived from the transformation of raw materials [37]. It includes, on the one hand machines, buildings and infrastructure, which are used and 'consumed' over a period of time longer than a year, and on the other hand plastics, metals and objects, which are instead consumed in less than a year [6].

A new sustainability paradigm, known as SST, emerged to recognize that natural capital available to humanity is finite and vulnerable to degradation. This includes challenges such as the depletion of mineral resources, the accumulation of toxic substances in flora and fauna, soil erosion, deforestation, and the build-up of greenhouse gases in the atmosphere [38].

For the SST, the definitions, while all revolving around the concept of non-substitutability, explore and develop it more extensively: 'cannot be compensated for', 'cannot be accepted', 'is severely limited', is 'inefficient, technically impossible, undesirable from a regulatory point of view', etc. More in details, the primary shared insight emerging from the literature on the urban environment is that this second paradigm asserts that natural and human-made capital cannot be substituted or compensated for [39,40]. In these terms, it is understood that an exchange between environment and economy cannot be accepted [41]; in other words, environmental performance cannot be compensated by economic and social performance [42]. Beckerman [43, p. 191] goes so far as to write that "Strong sustainability, overriding all other considerations, is morally unacceptable as well as impractical". In response to this claim, Daly [37, p. 49] argued that the problem stems from Beckerman's failure to understand that "manmade and natural capital are basically complements", thus introducing the interesting concept of complementarity for SST, later adopted by several authors.

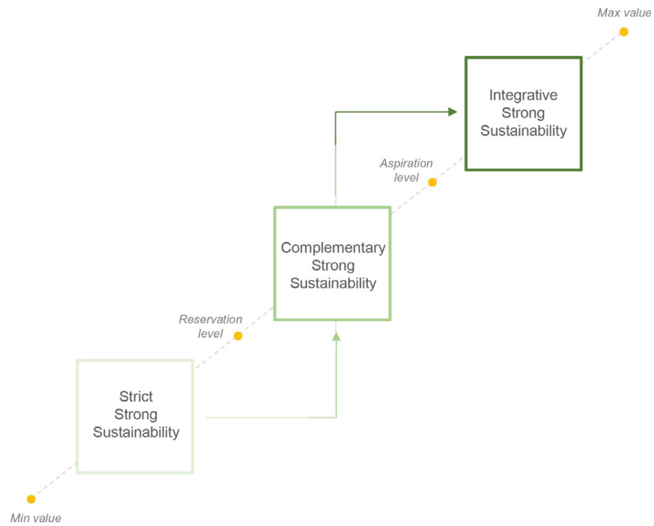


Fig. 1. The three declinations of SST with respect to the fulfillment levels of the criteria: Strict, Complementary and Integrative Strong Sustainability.

Economic activity should, therefore, preserve natural resources and promote social well-being [44]. In this sense, cities and urban regions should develop while maintaining a 'critical' level of biodiversity and ecosystem services necessary for human well-being [45,46]. In other words, cities and their local realities, understood as minimal units in which to develop and materialize sustainability, should shape their development not in quantitative but qualitative terms. The concept of complementarity in the context of SST is enhanced by the idea of integration, shifting toward a more proactive perspective rather than being solely prescriptive: "strong sustainability opts for a strategy in which sustainability and economic development are fully integrated from the beginning" [47, p. 300]. This implies that complementarity is not only about aligning elements to coexist effectively but also actively integrating them to create synergies, fostering innovation and forward-looking solutions.

Adopting the standpoint of United Nations [48, p.6]: "today we see with increasing clarity that economic growth, environmental protection, and social equity are one and the same agenda: the sustainable development agenda", we propose a three-level representation of SST depending on the choice of compensability between the three dimensions and indicators (Fig. 1).

The three declinations are defined according to the achievement of two distinct thresholds [21,22]: the *reservation* level and the *aspiration* level. As the terms themselves suggest, the *reservation* level represents the minimum level necessary for a result to be considered acceptable ("it should be at least"). In other words, it is the value below which the decision maker (DM) should be less inclined to compromise with other criteria. The *aspiration* threshold, on the other hand, indicates what would be ideal ("it would be desirable that…"). It represents the level above which the DM feels satisfied with the result obtained. Based on this, the definition of the three-level representation of the SST is as follows:

- *Strict Strong Sustainability*: it is a choice that ensures that for all criteria considered, values occur as close as possible to the reservation level (to be understood as the threshold of what is at least necessary to occur, 'should be at least'), ensuring that no element is neglected. In this sense, the values are between the minimum value and the reserve level;
- *Complementary Strong Sustainability*: in this case the criteria must reach at least the reservation level, allowing a certain degree of complementarity, meaning that those elements, while retaining their specificities, reinforce each other. Each element brings a unique value that complements that of the other, creating a mutually supportive relationship;
- *Integrative Strong Sustainability*: the criteria must reach at least the aspiration level (i.e. what is desirable to occur, 'it would be desirable for that to happen…'), allowing for integration. Indeed, all the criteria levels are greater than or equal to the aspiration level and, as such, can generate positive effects on the urban environment by working in synergy.

These three declinations of SST correspond to as many ways of aggregating indicators (see Section 3 for the theoretical aspects of the model) and allow to overcome the rigid compensatory/non-compensatory dichotomy.

3. The evaluation model

A widely known problem in decision analysis concerns the definition of methods to aggregate n scores — each score possibly representing a distinct decision criterion — into a unique representative value. Such functions $f : \mathbb{R}^n \to \mathbb{R}$ are called aggregation functions if they satisfy some regularity conditions such as monotonicity: the greater the input values, the greater their aggregate. Aggregation functions whose output value is bounded to be between the smallest and greatest of the inputs are called *averaging functions* [49]. Geometric means, arithmetic means, power means, median — as well as the minimum and maximum representing the two borderline cases — are all averaging functions. Different averaging functions exhibit different behaviors with respect to their disposition to behave similarly to the maximum or to the minimum. Indeed, the similarity of an averaging function to the maximum shall indicate its disjunctive-like behavior (an `or`-like behavior). That is, it is sufficient that there is at least one high input value to make the aggregate value high too. Vice versa, if an averaging function is similar to the minimum, then it displays a conjunctive-like behavior (an `and`-like behavior), as it focuses on the lowest value and therefore a good score can be achieved only if all the scores are good. Recently, non-trivial and general functions have been proposed, so that, by fine tuning their defining parameters, one can find suitable specific averaging functions. A good example is the discrete Choquet integral which, instead of being based on criteria weights, depends on a normalized capacity, that can be interpreted as an assignment of weights to all the subsets of the criteria set. Formally, given a finite non-empty set $N = \{1, \dots, n\}$ and its power set 2^N, a *normalized capacity* is a function $\mu : 2^N \to [0, 1]$ respecting the two properties:

- $\mu(\emptyset) = 0$ and $\mu(N) = 1$ (normalization)
- $A \subseteq B \Rightarrow \mu(A) \leq \mu(B)\ \forall A, B \subseteq N$ (monotonicity)

Example 1. For example, given $N = \{1, 2, 3\}$, the following

$$\mu(\emptyset) = 0$$
$$\mu(\{1\}) = 0.2 \qquad \mu(\{2\}) = 0.5 \qquad \mu(\{3\}) = 0.4$$
$$\mu(\{1,2\}) = 0.8 \qquad \mu(\{1,3\}) = 0.6 \qquad \mu(\{2,3\}) = 0.7$$
$$\mu(\{1,2,3\}) = 1$$

is a normalized capacity.

Given a list $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ and a normalized capacity μ defined over a set $N = \{1, \dots, n\}$, the *(discrete) Choquet integral* of $\mathbf{x}$ with respect to the capacity μ is defined as [49]

$$Ch_\mu(\mathbf{x}) = \sum_{i=1}^{n} \left(x_{\sigma(i)} - x_{\sigma(i-1)}\right) \mu\left(N_{\sigma(i)}\right)$$

where $\sigma : N \to N$ is a permutation such that $\sigma(i)$ is the index of the ith largest element in $\mathbf{x}$ with $\sigma(0) := 0$ and $N_{\sigma(i)} = \{\sigma(i), \dots, \sigma(n)\}$.

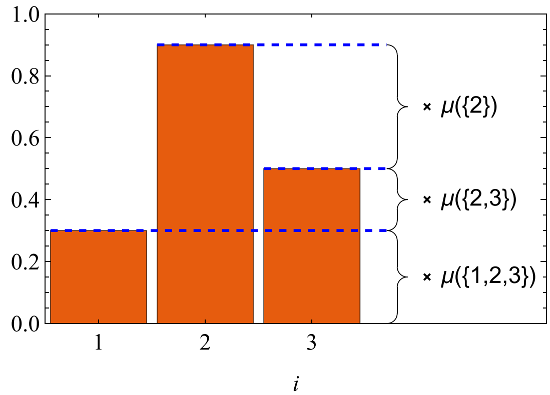


Fig. 2. Illustration for the calculation of the discrete Choquet integral on $\mathbf{x} = (0.3, 0.9, 0.5)$.

Example 2. Consider $\mathbf{x} = (0.3, 0.9, 0.5)$, then the Choquet integral collapses into

$$Ch_\mu(0.3, 0.9, 0.5) = (0.3 - 0) \cdot \mu(\{1, 2, 3\}) + (0.5 - 0.3) \cdot \mu(\{2, 3\}) + (0.9 - 0.5) \cdot \mu(\{2\}),$$

and can be interpreted as in Fig. 2.

When used in multi-criteria decision analysis, the relative importance of criteria can be extracted from the normalized capacity μ thanks to the *Shapley value* [50, p. 105], i.e., the relative importance of the ith criterion is:

$$\phi(i) = \sum_{A \subseteq N \setminus \{i\}} \frac{(n - |A| - 1)!}{n!} [\mu(A \cup \{i\}) - \mu(A)]. \tag{1}$$

Note that the Shapley value can be interpreted as a weighed average of the contribution of the ith criterion when it is added to the subsets $A \subseteq N$ that do not contain it yet.

Given two indices $\{i, j\}$, their *interaction index* was defined [50, p. 105] as

$$I_{i,j} = \sum_{A \subseteq N \setminus \{i,j\}} \frac{(n - |A| - 2)!|A|!}{(n - 1)!} [\mu(A \cup \{i, j\}) - \mu(A \cup \{i\}) - \mu(A \cup \{j\}) + \mu(A)]. \tag{2}$$

Similarly to the Shapley value, it can be interpreted as a weighted sum of the degrees to which the joint addition of both elements $\{i, j\}$ increases the measure more (or less) than the sum of the two separate additions of $\{i\}$ and $\{j\}$. Index (2) can be extended to quantify the interaction levels of sets with more than two elements. Furthermore, the closeness of the Choquet-based aggregation to the max-operator can be characterized by the following measure of *orness* [50, p. 108],

$$orness(\mu) = \frac{1}{n - 1} \sum_{A \subset N} \frac{(n - |A|)!|A|!}{n!} \mu(A) \in [0, 1] \tag{3}$$

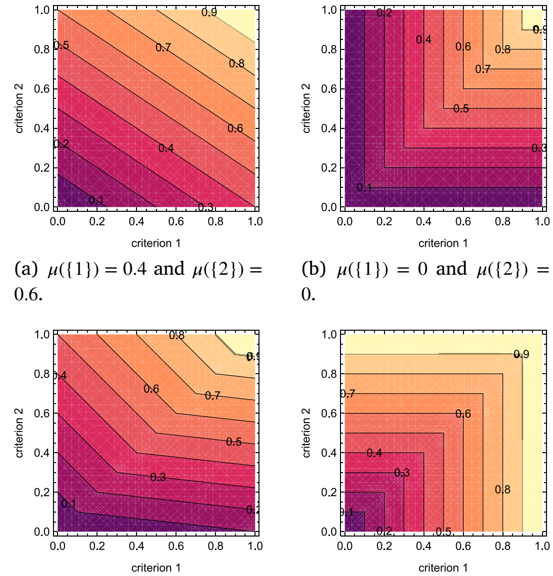
Note that the two extremes describe the maximum and the minimum operators, respectively. That is,

$$orness(\mu) = 1 \quad \Leftrightarrow \quad Ch_\mu(\mathbf{x}) = \max(\mathbf{x})$$
$$orness(\mu) = 0 \quad \Leftrightarrow \quad Ch_\mu(\mathbf{x}) = \min(\mathbf{x})$$

Indices (1), (2), and (3) are not only used to describe the Choquet integral, but, as shown by Grabisch et al. [51], they can also be used, the other way round, as constraints to identify normalized capacities complying with some *a priori* specified desiderata.

3.1. The p-ary Choquet integral for strong sustainability

In the case of two arguments, i.e. $\mathbf{x} = (x_1, x_2)$, it is possible to inspect the contour plots of the Choquet integral. Fig. 3 shows some special cases with their associated capacities, keeping in mind that the conclusions can be extended to the case with more than 2 attributes.



(a) $\mu(\{1\}) = 0.4$ and $\mu(\{2\}) = 0.6$.
(b) $\mu(\{1\}) = 0$ and $\mu(\{2\}) = 0$.
(c) $\mu(\{1\}) = 0.1$ and $\mu(\{2\}) = 0.5$.
(d) $\mu(\{1\}) = 1$ and $\mu(\{2\}) = 1$.

Fig. 3. Different behaviors of the Choquet integral induced by the underlying normalized capacity.

The Choquet integral generalizes various averaging functions, since it can weigh criteria *and* mimic different attitudes of the averaging process. In the situation depicted in Fig. 3(a) the Choquet integral collapses into an additive value function with a constant substitution rate between the two criteria. In this specific case, the decrease of one attribute can be compensated by a proportionate increase in the other, regardless of the position of the starting point. This seems to be compatible with the WS paradigm. If we, instead, consider the extreme behavior shown in Fig. 3(b) we note that it does *not* allow compensation. An increase in the value function can be obtained only by increasing the lowest value, which acts as an anchor for the function. Similarly, this seems to be coherent with the notion of SST. Note that other authors, e.g., Boggia et al. [21], have already pointed out the suitability of the weighted average and the minimum functions (cf. Figs. 3(a) and 3(b)), to model WS and SST, respectively. Nevertheless, the Choquet integral is a more general approach, spanning both extremes and encompassing compromise solutions too, like the one shown in Fig. 3(c)—such a function resembles the minimum and yet it retains some features of the additive approach. That is, it combines the tendency to favor increases of the lowest values with the possibility to still account for distinct importance of the two criteria. Therefore, under some circumstances it may be preferable over the too radical min operator. Finally, Fig. 3(d) shows the contour plot of the maximum operator. Indeed, the Choquet integral can also be used to model midway behaviors between the additive model and the maximum operator. The Choquet integral has already been used to model decisions related to sustainability.

In our problem, we are less willing to adopt a compensatory approach if, for example, an alternative has an extremely low score in (at least) one criterion. On the contrary, when all criteria levels are satisfactory we are more willing to accept compromises and tradeoffs between them. Hence, we need a more dynamic method and we chose to adopt a customized implementation of the p-ary Choquet integral originally introduced by Labreuche and Grabisch [20] to increase the flexibility of the basic Choquet integral.

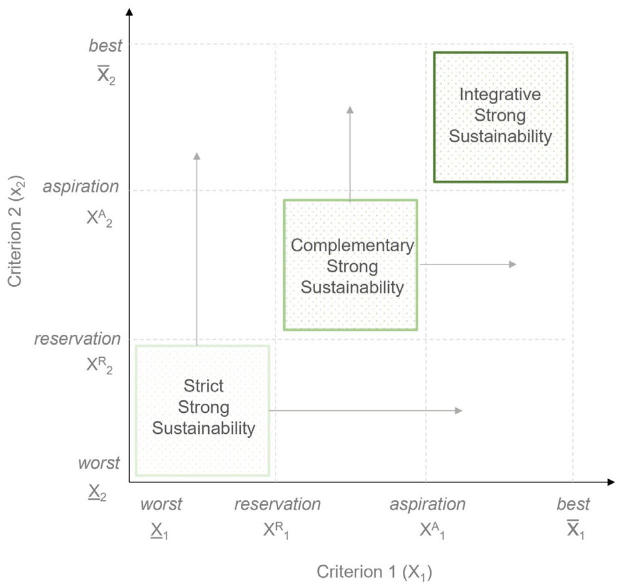


Fig. 4. Desirable attitudes of the aggregation function V in regions of decision space in the case of two attributes. Arrows indicate when the properties of a given region should be propagated to other regions.

We consider n criteria and assume that the ith criterion is associated to a set of criteria levels X_i, within which we identify the best and worst levels $\overline{x}_i$ and $\underline{x}_i$, respectively. We assume the existence of measurable attribute value functions that are normalized with respect to the worst and best levels, $v_i(\underline{x}_i) = 0$ and $v_i(\overline{x}_i) = 1$. These attribute value functions can be elicited using well-known methods like the mid-value splitting technique [52]. Furthermore, we ask the experts to identify the *reservation* (x_i^R) and *aspiration* levels (x_i^A) and we associate them with some values in the interval $[0,1]$, such that

$$\underbrace{v(\underline{x}_i)}_{=0} \leq v(x_i^R) \leq v(x_i^A) \leq \underbrace{v(\overline{x}_i)}_{=1} .$$

For illustrative purposes, hereafter we shall consider the case with two criteria, bearing in mind that our conclusions can readily be extended to the case with more criteria. The Choquet integral, as shown in Figs. 3(a)–3(d) can mimic different behaviors, but it is static as its behavior does not change over its domain (the decision space). Conversely, as argued before, we may prefer a dynamic approach and wish that the aggregation function exhibits different attitudes over the domain. An example is represented in Fig. 4. When both attribute levels are between reservation and aspiration levels we are willing to adopt a compensatory approach and allow for trade-offs between attributes (cf. Complementary Sustainability in Fig. 4). Conversely, when both attributes are below the reservation levels, we desire that the lowest one may impede the value function to grow excessively, thus leaning toward the strong sustainability sometimes modeled with the min operator (cf. Strict Strong Sustainability in Fig. 4). Whereas we do not exclude the use of the min function, we prefer to keep a more open view on the aggregation procedure and allow for situations as the one in Fig. 4. Moreover, let us note that the use of this latter approach allows us to consider, even if not fully compensatorily, importance levels of criteria which are, instead, neglected by the min operator. The mirror image of this behavior can be found where both attributes exceed the aspiration. In this situation we are more willing to focus on the highest scoring attribute (cf. Integrative Sustainability in Fig. 4): the so-called cherry on top of the cake.

The most widely used MCDA methods, e.g., the analytic hierarchy process [53], simple additive weighting [54], and additive value models [55], are fully compensatory and they could, in the best case, represent the desired behavior of the value function V only in the region between reservation and aspiration levels. We have also seen that a more general aggregation strategy like the Choquet integral can mimic different behaviors, but a single one is not flexible enough to represent all the expected behaviors of V in Fig. 4. To overcome these limitations, Labreuche and Grabisch [20] suggested to use distinct Choquet integrals for the three areas indicated in Fig. 4, and then define the behavior of the value function in the other areas by interpolation such that (i) they inherit the behavior of the diagonal quadrants and (ii) the entire value function satisfies some basic properties such as continuity. In our customized implementation of the p-ary Choquet integral, we are inspired by the steps proposed by Labreuche and Grabisch [20]. We shall here sketch the procedure, bearing in mind that more specific details will be given later in the case study.

Step 1: We define the values of the attribute levels x_i^R and a_i^A in the range $[0,1]$. As the perception of reservation and aspiration is attribute independent, we know that

$$v_i(x_1^R) = \cdots = v_i(x_n^R)$$
$$v_i(x_1^A) = \cdots = v_i(x_n^A)$$

must hold. Since we are using averaging functions, which are idempotent, we can deduce that:

$$V(\underline{x}_1, \ldots, \underline{x}_n) = 0$$
$$V(x_1^R, \ldots, x_n^R) = v_i(x_i^R)$$
$$V(x_1^A, \ldots, x_n^A) = v_i(x_i^A)$$
$$V(\overline{x}_1, \ldots, \overline{x}_n) = 1$$

In this way, we could determine the antidiagonal values on the line intersections.

Step 2: Shapley values $\phi(i)$ are used to find values on the line intersections right above and below the diagonal. We consider the compensatory region in Fig. 4. In this case, as we know that the behavior must be additive, we can obtain such values by solving

$$\frac{V(x_1^R, \ldots, x_{i-1}^R, x_i^A, x_{i+1}^R, \ldots, x_n^R) - V(x_1^R, \ldots, x_n^R)}{V(x_1^A, \ldots, x_n^A) - V(x_1^R, \ldots, x_n^R)} = \phi(i)$$

for all i. A similar procedure is followed to find the values of the other intersections above and below the antidiagonal, this time keeping in mind that different levels or orness are required (i.e. additivity does not hold within those quadrants) to emulate more disjunctive and conjunctive behaviors. The points obtained in Steps 1 and 2 are represented in Fig. 7.

Step 3: The remaining values at the intersection points on the grid are obtained by interpolation using the method proposed by Labreuche and Grabisch [20].

Step 4: The p-ary Choquet integral is defined using different Choquet integrals for different quadrants of the objective space. The interpolation procedure used at Step 3 ensures that the resulting value function V respects desirable properties such as continuity, on top of the behavior shown in Fig. 4.

4. A flow-chart of the applied methodology

In this section, we provide a brief overview of the methodology proposed and applied to the case study, summarized through the flowchart in Fig. 5.

Phase 1. The first phase consists in the problem definition, which involves identifying the evaluation dimensions and their indicators, selecting (and possibly creating) the alternatives to be assessed, and constructing the performance matrix. The latter, also referred to as the decision matrix, represents the alternatives in relation to each specific

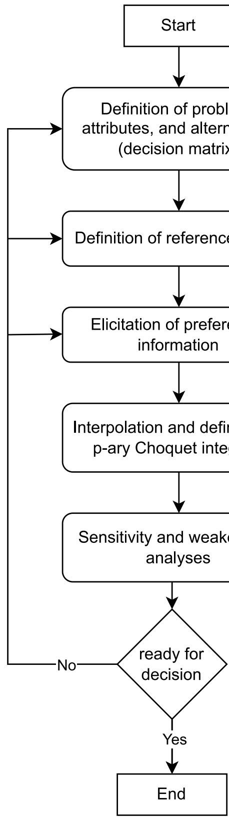


Fig. 5. Flow diagram for the proposed approach.

indicator. This phase is carried out by the analyst in collaboration with the DM/s.

Phase 2. Once the indicators have been defined, it is necessary to establish the corresponding thresholds—namely, the minimum value, the reservation and aspiration levels, and the maximum value. These thresholds must be specified for each indicator and can be determined based on existing regulations, institutional and/or scientific literature, or expert judgment. In this sense, the phase can be conducted independently by the analyst through a review of the relevant and available documentation, or it may involve interaction with experts to gather additional insights.

Phase 3. Next, the DM/s is again consulted to understand his/her preferences and tradeoffs. This is done using a parsimonious approach that leaves some values undetermined.

Phase 4. This part, automatized, uses interpolation to estimate the missing information (in the form of further reference points) as coherently as possible with the elicited one. The final output consists of as many p-ary Choquet integral as the dimensions of the analysis. Each one can be used to score alternatives with respect to a dimension.

Phase 5. In the last phase the results are analyzed by means of a sensitivity analysis with respect to the weights associated to the dimensions and a weakest link analysis where the weakest aspects of each alternative are compared.

Phase 6. The final phase of the process involves presenting the results to the DM/s through graphical representations that facilitate a visual understanding of the relative positioning of the alternatives, the reservation or aspiration thresholds met, and the degree to which each option is able to implement a strong sustainability model. This phase enables the DM/s to assess the level of sustainability that the urban interventions can realistically achieve and, consequently, to pursue one of the following two courses of action: (i) Confirm the obtained results by selecting and implementing the alternative evaluated as most satisfactory with respect to the defined thresholds and the adopted sustainability model; (ii) Return to the initial phase of the process to redesign those alternatives that are potentially preferred by the DM/s but which are insufficient to meet the required thresholds, with the aim of enhancing their performance and aligning them more closely with a model of Strict Strong Sustainability. From this perspective, the process is configured as a circular and iterative pathway, which not only supports the identification of the most suitable alternative and the understanding of its sustainability profile, but also allows for the detection of weaknesses in the DM/s preferred alternatives, thus enabling their improvement in line with more rigorous sustainability criteria.

Fig. 6. Turin Exposition complex.[1]

5. Description of the case study

The case study concerns the urban renewal of the Turin Exposition complex, located near Valentino Park in Turin, Italy, as part of the Politecnico di Torino's expansion strategy. This initiative focuses on the reuse and revitalization of a significant modern urban monument — the Exposition complex designed in 1938 by renowned Italian architect and engineer Ettore Sottsass and Pier Luigi Nervi — positioned at the edge of the historic park (Fig. 6).

The project, promoted by the City of Turin (owner) and Politecnico di Torino (user), envisions transforming three pavilions into a new university pole. The selection of the case study is based on two main motivations. The first relates to the project's affiliation with the Politecnico di Torino, the institution to which the first two authors belong, which has ensured easier access to relevant data and information. The second motivation lies in the urban significance of the project itself: it represents a major regeneration initiative aimed both at reclaiming an abandoned and degraded area and at converting it into a space dedicated to cultural and social activities. This reappropriation of urban space serves as a point of interaction between the university, research, laboratories, and civil society, fostering new forms of exchange and dialogue. The transformation of the area into a university campus makes this case particularly suitable for analyzing the trade-offs inherent in sustainability. University campuses, indeed, are complex environments that have to face environmental, economic, and social challenges [56], and as such, should serve as active laboratories for sustainable development strategies. Four hypothetical design alternatives are considered and assessed using seven indicators and relative thresholds. Detailed space organization is omitted to focus on demonstrating the proposed method's applicability.

Therefore the set of indicators has been defined on a scientific basis [57–60], instead the reference thresholds has been fixed on the

[1] available at https://iltorinese.it/2024/06/10/allurban-lab-uno-sguardo-sulla-torino-di-domani-rivelare-il-futuro/

basis of the literature or through interfacing with experts (specific details are outlined in the following subsections). As for the design performance of the compared scenarios, these have been defined to demonstrate the potential applicability of the proposed method.

The seven indicators, divided into three sustainability dimensions, were defined based on literature and expert input. Best, worst, reservation, and aspiration levels were set using: (i) literature and standards, and (ii) expert questionnaires. Social interaction thresholds were reviewed by a social issues expert, safety thresholds by a safety expert, and daylight illumination thresholds by a technical physics expert. A detailed description of each indicator and its thresholds follows.

5.1. Economic dimension

Construction cost. The indicator measures the construction cost, i.e. the amount of expenditure incurred for the realization of the property in material terms [58]. In this case, it refers to the costs for the renovation of the property quantified in monetary terms [€/m^2]. Concerning the thresholds, there are no fixed and agreed parameters as the acceptability of criterion levels depends on the availability of the investors. Therefore, in order to define the thresholds in this specific case, reference is made to similar projects that are part of the same University Masterplan project, and to the total cost value for the renovation work estimated by the financial framework of the executive technical plan of the complex under analysis.[2]

On the basis of this data, the aspiration level is set equal to the budgeted value of 1773 €/m^2, while the reservation level is set equal to 2203 €/m^2. The minimum value is set equal to 0 €/m^2, corresponding to the no-intervention situation, and the maximum value equal to the expenditure for the new construction of a classroom block, 2 278 €/m^2.

Reuse of existing structures. With the aim of encouraging the reuse of as much of the existing building space as possible, disincentivising demolition and reconstruction [61], the indicator proposed in [57], measures the percentage of reused area in relation to the total area:

$$\frac{\text{horizontal and/or inclined surface reused}}{\text{total horizontal and/or inclined surface}} \times 100$$

Considering what is stated in the GBC Neighborhoods Sustainability Protocol,[3] the reservation level is set at 50% of the building's reuse. Indeed, it is reported that "where the intervention involves only one building, re-use at least 50% of that building on the basis of the visible surface area. The calculation must include all structural elements (internal and external, including intermediate floors and roofs) and the building envelope (excluding external vertical and horizontal frames and non-structural roof elements) of the existing building".[4] The aspiration level is set at 92% considering that one of the pavilions, lacking historical value and in critical structural condition, can be demolished and rebuilt (5% of the surface area), as well as a post-Olympics 2006 outpost that alters the original façade and reduces pedestrian space (3% of the surface area).[5]

5.2. Social dimension

Internal spaces for social interaction. This indicator acknowledges that architectural design and spatial configurations can shape social interaction [59,62]. Accordingly, it aims to measure spatial quality in terms of its capacity to foster such interactions. [63] confirm that narrow corridors discourage social interaction, while large, circular spaces with features like fountains and activities promote aggregation. Additionally, Mamaghani et al. [62] emphasized that the presence of seating is essential to enable individuals to spend time and interact with others. Since the indicator is intended to measure intangible and non-standardized aspects of social life, there are currently no shared and accepted reference thresholds to comply with. Therefore, reservation and aspiration levels were set according to the opinion of an expert in social issues in the architectural context. These were set respectively on level 3 and 4, on a scale from 1 to 5.

Outdoor spaces for social interaction. This indicator is based on the same conceptual principles as the previous one, but applies specifically to outdoor spaces. Accordingly, it is assessed using a qualitative scale ranging from 1 to 5. As with the previous indicator, it measures intangible, non-standardized aspects of social life for which no commonly accepted reference thresholds exist. Consequently, reservation and aspiration levels were determined through consultation with an expert in social aspects of architectural design, and established at levels 3 and 4, respectively.

Degree of security. This indicator evaluates the security measures considered by the project, drawing on literature that identifies security as a social factor enhanced by self-help neighborhood systems, lighting, access control, and specialized personnel [64]. It is measured on a qualitative scale from 1 to 5. Since it addresses intangible and non-standardized aspects of social life, no established reference thresholds exist. Therefore, reservation and aspiration levels (set at 2 and 4, respectively) were defined in consultation with experts in social and safety issues within the architectural context.

5.3. Environmental dimension

Useful daylight illumination (UDI). The indicator observes the daylight level in a 'useful' range, i.e. set between 100 and 3000 lux [60]. According to Palarino and Piderit [65], illuminance between 100–300 lux suffices for tasks with low visual demands, 300–3000 lux is ideal for visual tasks, and over 3000 lux may cause discomfort. Since the case study includes lecture rooms, study rooms, and offices requiring visual tasks, it is essential to evaluate visual quality without artificial lighting. The indicator measures the percentage of annual hours achieving useful natural illuminance [66]. With regard to thresholds, it is noted that the minimum value set for learning spaces is 80% [67]. Thus, the reservation level is set at 80%, while the aspiration level is set according to the opinion of a technical physics expert at 90%.

Speech transmission index (STI). The indicator measures the quality of speech transmission (STI), relevant for lecture halls on a university campus. STI values range from 0 to 1, with the 2010 UNI 11367 standard setting STI ≥ 0.6 dB for speech environments and STI ≥ 0.5 dB for sports spaces. The 2011 BS EN 60268-16 standard categorizes STI as follows: 0.3 dB bad-poor quality; 0.45 dB poor-fair quality; 0.6 dB fair-good quality; and 0.75 dB good-excellent quality. Based on these standards, the aspiration level is set at 0.75 dB for good-excellent speech transmission quality, while the reservation level is 0.6 dB as the minimum acceptable standard.

6. Results of the application to the case study

In the real-world case study, we considered economic, social and environmental dimensions of the projects, each articulated into sub-

[2] Comune di Torino, Complesso di Torino esposizioni. piano tecnico esecutivo. relazione illustrativa, http://geoportale.comune.torino.it/web/sites/default/files/mediafiles/allegato_n_001-piano_tecnico_esecutivo_rev2022_07_12.pdf, 2022. Accessed: 13/09/2024.

[3] Green Building Council Italia, Urban sustainability: how can cities become sustainable?, https://gbcitalia.org/wp-content/uploads/2021/08/Manuale-GBC-QUARTIERI-2015-def.pdf, 2015. Accessed: 14/09/2024.

[4] our translation p. 345, from Green Building Council Italia, Urban sustainability: how can cities become sustainable?, https://gbcitalia.org/wp-content/uploads/2021/08/Manuale-GBC-QUARTIERI-2015-def.pdf, 2015. Accessed: 14/09/2024.

[5] Comune di Torino, Complesso di Torino esposizioni. piano tecnico esecutivo. relazione illustrativa, http://geoportale.comune.torino.it/web/sites/default/files/mediafiles/allegato_n_001piano_tecnico_esecutivo_rev2022_07_12.pdf, 2022. Accessed: 13/09/2024.

Table 1
Set of indicators, with their units of measurement, thresholds and Shapley values.

ID	Indicator name	Polarity	Unit of measure	$\underline{x}_i$	x_i^R	x_i^A	$\overline{x}_i$	$\phi(i)$
eco1	Construction cost	↓	€/m²	2 278	2 203	1 773	0	0.7
eco2	Reuse of existing structures	↑	%	0	50	92	100	0.3
soc1	Internal spaces for social interaction	↑	Qualitative [1-5]	1	3	4	5	0.3
soc2	Outdoor spaces for social interaction	↑	Qualitative [1-5]	1	3	4	5	0.3
soc3	Degree of security	↑	Qualitative [1-5]	1	2	4	5	0.4
env1	Useful daylight illumination	↑	%	0	80	90	100	0.5
env2	Speech transmission index	↑	dB	0	0.6	0.75	1	0.5

Table 2
Decision table containing the description of the alternatives (projects) with respect to the indicators.

ID	Indicator name	Project 1	Project 2	Project 3	Project 4
eco1	Construction cost	1 759 €/m²	2 100 €/m²	2 215 €/m²	1 685 €/m²
eco2	Reuse of existing structures	80%	100%	100%	90%
soc1	Internal spaces for social interaction	5	5	3	4
soc2	Outdoor spaces for social interaction	3	4	5	4
soc3	Degree of security	4	3	3	1
env1	Useful daylight illumination	0.55 dB	0.68 dB	0.72 dB	0.6 dB
env2	Speech transmission index	79%	80.5%	60%	77%

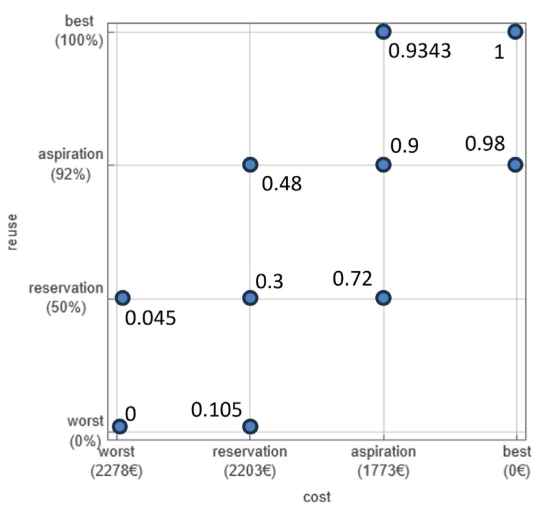


Fig. 7. Reference values for the economic dimension.

indicators. Experts were asked to assess bounds for attribute levels as well as reservation and aspiration levels. This information, together with desired local Shapley values of the indicators, is reported in Table 1.

The decision table containing the attribute levels of the four alternative projects is reported in Table 2. Note that no project is Pareto dominated by others. This means that they are all potentially optimal and that, a priori, none of them can be excluded from the analysis.

At this point, three distinct p-ary Choquet integrals can be built to aggregate scores locally, for each dimension. For illustrative purposes, we show the full operationalization only for the economic dimension, with the two criteria eco1 (cost) and eco2 (reuse). In particular we discuss how the values in Fig. 7 were obtained. We do so by following and detailing the same steps recalled in Section 3.1.

Step 1: The extreme antidiagonal values were first identified: due to normalization, the values of the "bottom left" and the "upper right" extreme points in Fig. 7 are

$$V(\underline{x}_{\text{cost}}, \underline{x}_{\text{reuse}}) = 0 \qquad V(\overline{x}_{\text{cost}}, \overline{x}_{\text{reuse}}) = 1$$

Thanks to the experts we identified the values of reservation and aspiration levels as equal to 0.3 and 0.9. Therefore,

$$V(x^R_{\text{cost}}, x^R_{\text{reuse}}) = 0.3 \qquad V(x^A_{\text{cost}}, x^A_{\text{reuse}}) = 0.9$$

Step 2: The Shapley values of the two criteria were defined, by the expert, as 0.7 and 0.3. Then, we can find two more values in the objective space

$$V(x^A_{\text{cost}}, x^R_{\text{reuse}}) = 0.72 \qquad V(x^R_{\text{cost}}, x^A_{\text{reuse}}) = 0.48$$

This was done bearing this in mind that, in the "middlemost" part, the value function V is compensatory, and thus solving

$$\frac{V(x^A_{\text{cost}}, x^R_{\text{reuse}}) - \overbrace{V(x^R_{\text{cost}}, x^R_{\text{reuse}})}^{0.3}}{\underbrace{V(x^A_{\text{cost}}, x^A_{\text{reuse}})}_{0.9} - \underbrace{V(x^R_{\text{cost}}, x^R_{\text{reuse}})}_{0.3}} = \overbrace{\phi(\text{cost})}^{0.7}$$

$$\frac{V(x^R_{\text{cost}}, x^A_{\text{reuse}}) - \overbrace{V(x^R_{\text{cost}}, x^R_{\text{reuse}})}^{0.3}}{\underbrace{V(x^A_{\text{cost}}, x^A_{\text{reuse}})}_{0.9} - \underbrace{V(x^R_{\text{cost}}, x^R_{\text{reuse}})}_{0.3}} = \overbrace{\phi(\text{reuse})}^{0.3}$$

Consider now the lower bottom square in Fig. 7. In this case, we may desire a lower orness to induce a more conjunctive behavior. It was decided that 0.25 was a desirable level of orness as it represents a midway compromise between the weighted average and the min operator. This implies that

$$V(x^A_{\text{cost}}, \underline{x}_{\text{reuse}}) + V(\underline{x}_{\text{cost}}, x^A_{\text{reuse}}) = 0.15$$

Note that 0.15 is half of 0.3. The values $V(x^A_{\text{cost}}, \underline{x}_{\text{reuse}}) = 0.105$ and $V(\underline{x}_{\text{cost}}, x^A_{\text{reuse}}) = 0.045$ were found by splitting 0.15 using the Shapley values of the two criteria. Formally, we solved the equations

$$\frac{V(x^A_{\text{cost}}, \underline{x}_{\text{reuse}}) - 0}{0.15 - 0} = \overbrace{\phi(\text{cost})}^{0.7}$$

$$\frac{V(\underline{x}_{\text{cost}}, x^A_{\text{reuse}}) - 0}{0.15 - 0} = \overbrace{\phi(\text{reuse})}^{0.3}$$

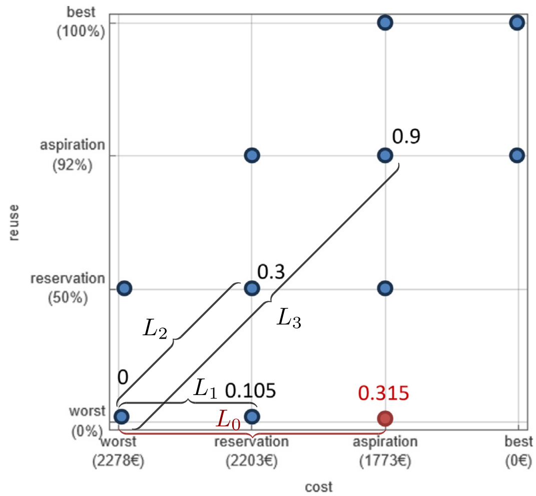


Fig. 8. Geometric interpretation of the interpolation procedure. Some known values (in blue) are used to estimate a missing value (in red) by means of interpolation.

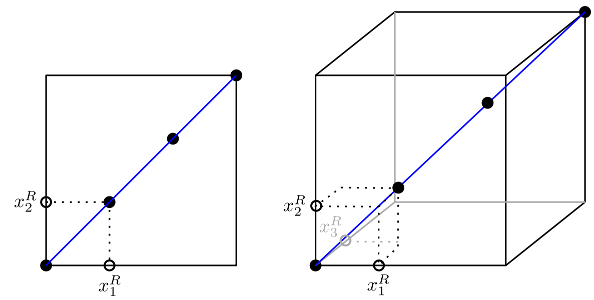


Fig. 9. Interpretation of threshold levels along the diagonal for 2 and 3 criteria.

A similar procedure was followed to find the values in the upper right part of the graph, this time using a value of orness greater than 0.5 to obtain a more disjunctive attitude, with the values $V(\overline{x}_{\text{cost}}, x^A_{\text{reuse}}) = 0.98$ and $V(x^A_{\text{cost}}, \overline{x}_{\text{reuse}}) \approx 0.9343$. The resulting values for the grid intersection points are represented in Fig. 7.

Step 3: We used the procedure proposed by Labreuche and Grabisch [20] to find the missing values at the other intersection points. This procedure is sketched in Fig. 8 where the values of the letters indicate the difference of values between different points. In this case, we are interested in the interpolation of the value $V(x^A_{\text{cost}}, \underline{x}_{\text{reuse}})$, which we denote as L_0. As shown, it can be found as the solution of $L_0/L_1 = L_3/L_2$, where L_1, L_2, L_3 are known. In this precise case,

$$V(x^A_{\text{cost}}, \underline{x}_{\text{reuse}}) = L_0 = \frac{L_3}{L_2} L_1 = \frac{0.9}{0.3} 0.105 = 0.315.$$

The extension to the 3 dimensional case, representative of a problem with 3 criteria, requires more information and more points need to be interpolated: at the end of the process 4^3 points are needed. Nonetheless, the working principles remain the same, as shown in Fig. 9.

Step 4: In the last step a distinct Choquet integral is placed in each subregion of the decision space: the resulting value function V, for the economics dimension, is shown in Fig. 10. By inspection, we can conclude that the p-ary Choquet integral represented in Fig. 10 fulfills the desiderata expressed in Fig. 4. More precisely, in the central square of Fig. 10 the value function is additive and in the bottom-left square it is more conjunctive, with a behavior similar to the one shown in 3(c). Similarly, in the top-right square, it is a mix between the additive model and the maximum.

From Fig. 10, we can see that low values of a single attribute are sufficient to drag down the aggregate value. This can be interpreted as a continuous barrier that tends to discourage the adoption of alternatives in which at least one attribute does not meet its reservation level. Furthermore, although they cannot be visualized, p-ary Choquet integrals for more than 2 attributes are similar to the one shown in Fig. 10, in that they also penalize low values of attributes.

Once the three value function V_{eco}, V_{soc} and V_{env} are obtained, they can be used to evaluate the attribute levels of each alternative with

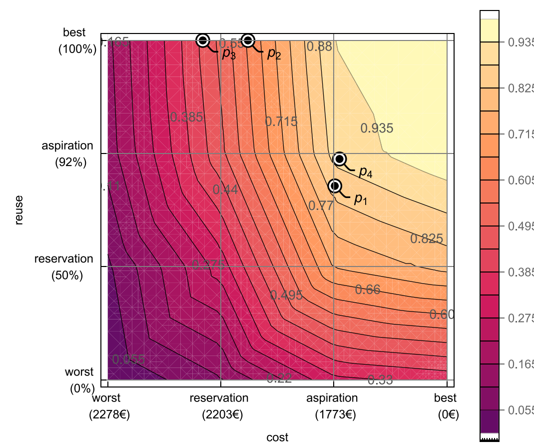


Fig. 10. Contour plot of the p-ary Choquet integral for the economic dimension obtained by interpolating the values in Fig. 8. The black dots represent the four projects in the decision space. Note that, for reasons of scalability, the four reference values on each axes have been mapped, as suggested by Labreuche and Grabisch [20, Fig. 3] into the values 0, 1, 2, and 3, respectively.

respect to the different dimensions. If we call

$$\mathbf{v}_k = \begin{pmatrix} v_1^{(k)} \\ v_2^{(k)} \\ v_3^{(k)} \\ v_4^{(k)} \end{pmatrix}$$

the vector collecting the scores of the four alternatives with respect to the kth dimension, we can collect the results into

$$\mathbf{v}_{\text{eco}} = \begin{pmatrix} 0.8492 \\ 0.6120 \\ 0.4524 \\ 0.8954 \end{pmatrix} \quad \mathbf{v}_{\text{soc}} = \begin{pmatrix} 0.7331 \\ 0.8345 \\ 0.4621 \\ 0.3927 \end{pmatrix} \quad \mathbf{v}_{\text{env}} = \begin{pmatrix} 0.4908 \\ 0.5547 \\ 0.2775 \\ 0.5103 \end{pmatrix} \tag{4}$$

For example, $\mathbf{v}_{\text{eco}}$ collects the scores of the four projects with respect to the economics dimension. Namely, the values in $\mathbf{v}_{\text{eco}}$ are the values associated to the four black points in Fig. 10. From the three vectors, we can already see that Project 4 is the best from the economic point of view whereas Project 2 is the best from the social and environmental points of view. A common way to proceed would be that of aggregating the different ratings, possibly be means of a weighted average, to obtain an overall rating of the alternatives $\mathbf{v}^*$. Namely,

$$\begin{aligned}\mathbf{v}^* &= \omega_{\text{eco}} \mathbf{v}_{\text{eco}} + \omega_{\text{soc}} \mathbf{v}_{\text{soc}} + \omega_{\text{env}} \mathbf{v}_{\text{env}} \\ &= \omega_{\text{eco}} \begin{pmatrix} 0.8492 \\ 0.6120 \\ 0.4524 \\ 0.8954 \end{pmatrix} + \omega_{\text{soc}} \begin{pmatrix} 0.7331 \\ 0.8345 \\ 0.4621 \\ 0.3927 \end{pmatrix} + \omega_{\text{env}} \begin{pmatrix} 0.4908 \\ 0.5547 \\ 0.2775 \\ 0.5103 \end{pmatrix}\end{aligned}$$

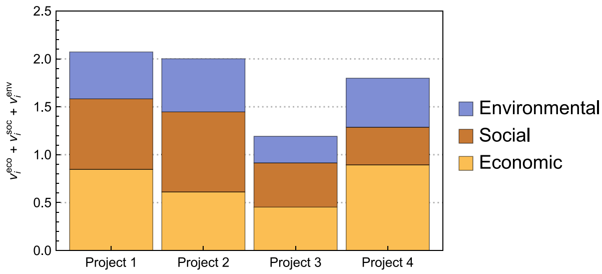


Fig. 11. Sums of the scores achieved by each project with respect to the dimensions of the analysis.

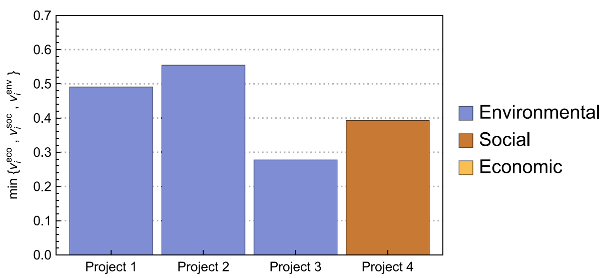


Fig. 12. Weakest aspect analysis of the four alternative projects.

where $\omega_{\text{eco}}, \omega_{\text{soc}}, \omega_{\text{env}} \geq 0$ are scaling constants such that $\omega_{\text{eco}} + \omega_{\text{soc}} + \omega_{\text{env}} = 1$. Let us note that Project 3 appears to be dominated. Nevertheless, for completeness we will still consider it in the analysis.

6.1. Equal weights and weakest links

In the absence of explicit information on the weights, a pragmatic solution to the weighting problem would be adopting Laplace principle of insufficient reason and assign equal weights to the three dimensions, i.e., $\omega_{\text{eco}} = \omega_{\text{soc}} = \omega_{\text{env}} = 1/3$, thus making no discrimination among them. This approach would be coherent with a WS vision of the problem, and it will be discussed later on. Conversely, if we want to adopt the SST paradigm, we take inspiration from [68] and consider the lowest scores achieved by each alternative with respect to the dimensions, interpreting them as the weakest links of a chain in a framework where the chain is only as strong as its weakest link. This calculation can be easily done on the vectors $\mathbf{v}_{\text{eco}}$, $\mathbf{v}_{\text{soc}}$, $\mathbf{v}_{\text{env}}$ by solving

$$i^* = \underset{i=1,\ldots,4}{\arg\max} \min \left\{ v_i^{\text{eco}}, v_i^{\text{soc}}, v_i^{\text{env}} \right\}$$

and discovering that Project 2 has the best weakest link and therefore may be preferable according to the strong sustainability paradigm. In addition, such a procedure has a simple graphical interpretation. Each segmented bar in the barchart in Fig. 11 represents the sums of the scores of the corresponding project with respect to the three dimensions. For instance, the height of the bar associated with Project 1 is obtained as

$$v_1^{\text{eco}} + v_1^{\text{soc}} + v_1^{\text{env}} = 0.8492 + 0.7331 + 0.4908 = 2.0731$$

Then, according to this principle, the best alternative is the one associated to the bar whose smallest subrectangle is the largest. For sake of clarity, the smallest rectangles, representing the weak aspects of each alternative, can be isolated, as shown in Fig. 12, and then compared, clarifying in this way that Project 2 shall be, even if by a small amount, preferred.

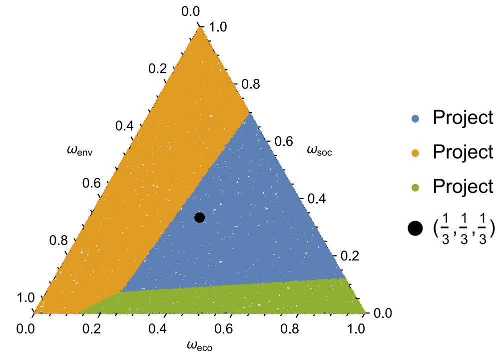


Fig. 13. Ternary plot indicating the best alternative with respect to the values ω_{eco}, ω_{soc}, ω_{env}.

6.2. An explicit sensitivity analysis

Most likely, any DM may encounter troubles when he/she tries to express any information on the weights ω_{eco}, ω_{soc}, ω_{env}: these three dimensions do not satisfy the requirements that are usually required for decision attributes [69], they already act as composite indicators, and their global meaning is harder to grasp than the meaning attached to local indicators like cost and reuse of structure. Secondly, any preferential information on weights — regardless of the aggregation method that is used — could be interpreted as a political statement on which one of the three dimensions should be favored and even the more agnostic choice of assigning equal weights to each dimension is a statement on the relative importance of the three dimensions. For these reasons we proceed with a graphical analysis of different scenarios obtained by randomly sampling vectors $\boldsymbol{\omega} = (\omega_{\text{eco}}, \omega_{\text{soc}}, \omega_{\text{env}})$ of scaling constants from the 2-dimensional simplex

$$\Delta = \{\boldsymbol{\omega} \,|\, \omega_{\text{eco}}, \omega_{\text{soc}}, \omega_{\text{env}} \geq 0, \;\; \omega_{\text{eco}} + \omega_{\text{soc}} + \omega_{\text{env}} = 1\}$$

representing the set of all possible combinations of scaling constants that can be assigned to the three dimensions. Results are shown in the ternary plot in Fig. 13.

The absence of Project 3 is coherent with our previous observation that it was dominated, as was seen in (4). Furthermore, Fig. 13 supports the idea that Projects 2 and 4 excel in certain specific contexts, while also investigating whether their advantages persist even when one viewpoint is combined with the other two. Additionally, it highlights the relevance of Project 1 in situations where a compromise solution, such as equally weighing all three dimensions, is desirable.

Further analysis can be conducted. For instance, if we begin from a scenario where the economic criterion is the dominant factor and gradually decrease its significance (moving up-left from the bottom right corner) to enhance the social aspects, we find that Project 4 is replaced by Project 1. Conversely, if we start from the rightmost point and reduce the emphasis on economic factors to prioritize environmental aspects (moving left from the bottom right corner), Project 4 takes longer to lose its appeal.

Last but not least, a quantitative analysis of the ternary plot in Fig. 13 reveals that the 36.4%, 48.9%, 14.7% of its total area is covered by Projects 1, 2 and 4, respectively. In the total absence of preferential information, and the rejection of Laplace's principle of insufficient reason, such values can be used for the selection of the best project.

7. Discussion and conclusions

In public and political debate, the relationship between measurement and policy is often seen as linear, assuming that numbers automatically lead to effective decisions. However, this view overlooks the fact that numerical data require interpretation: their meaning depends on context and goal-oriented judgment [70]. Indicators, in fact, are not neutral—they selectively highlight certain aspects of urban reality while leaving others in the background [27,28].

This ambiguity becomes particularly relevant in contexts where indicators are used not only to evaluate outcomes but also to justify interventions. Measurement, in such cases, precedes policy—it becomes a concurrent cause, or at least a discursive justification for action. From this perspective, indicators acquire both diagnostic and persuasive functions: they help to assess and to legitimize, to describe and to prescribe. At the same time, it is important to recognize that policy decisions are not based solely on rational interpretation of data. As behavioral research has consistently shown [71], decision-making processes are shaped by cognitive heuristics, institutional routines, and resistance to change. The assumption that data, once available, will be acted upon in a rational and timely manner, does not hold up empirically (see also [72]). The barrier is not only informational, but political and behavioral. The model proposed in this paper should therefore be understood in these terms. Like any model, it has limitations. Its contribution does not lie solely in technical precision, but also in how it is interpreted, communicated, and situated within broader policy narratives. It aims to support decision processes by structuring the interpretation of complex sustainability challenges, but it does not presume to overcome the deeper governance and implementation gaps that often prevent action, even in the face of overwhelming evidence.

Rather than entering the scientific debate on the dichotomy between WS and SST, this research takes the need for strong sustainability as a starting point and focuses with its applicability. In the model presented in this research — proposed at a general conceptual level and tested in the field of urban architecture — both the compensatory and non-compensatory approaches are considered acceptable under certain conditions. Specifically, we adopted a customized implementation of the p-ary Choquet integral to link the DM's willingness to accept (or not accept) compromises to the characteristics of the alternative, thus still allowing for SST. In addition to the use of the p-ary Choquet integral to aggregate indicators within the different dimensions of the analysis, we kept the rest of the analysis transparent, trying, as much as we could to present and interpret the output in a graphical form without resorting to the identification of a unique final ranking. The case study shows that conservation and economic development can be integrated into a single strategy. The approach proposed in this article can help decision-makers collaborate with analysts to design sustainable urban interventions based on specific assumptions.

The research has some limitations. First, the application shows the visualization of the relationship between only two indicators, which is particularly effective, whereas we are aware that when adding other criteria, this visualization would become decidedly less user-friendly. Secondly, we simulated a decision-making process related to an ongoing urban transformation, incorporating certain simplifications in an illustrative example—made possible specifically because real DMs were not involved. Thirdly, the research is limited by the application of the methodology to a single case study, which provides an initial validation but does not fully explore its potential adaptability and scalability. To overcome these simplifications, future research could apply the methodology in real-world decision-making settings involving actual stakeholders. This would provide insights into how the proposed approach performs under real constraints, with varying levels of complexity, stakeholder compositions, decision-making frameworks, and would help to better assess its methodological versatility and robustness in addressing architectural and urban planning challenges. We are aware that decisions on city development are highly discretionary decisions, constituting geopolitical assessments but we believe that, to trigger real operations of sustainable policies, both a reflection on the ways in which sustainability is actually measured and a broader analytical and critical approach that questions its appropriateness, validity and use are needed.

Finally, we note that the choice of the p-ary Choquet integral was motivated not only by its flexibility but also by its appealing geometric interpretation. Nonetheless, other MCDA methods could also be suitable. For example, the level-dependent Choquet integral [73] bears strong similarities to the p-ary Choquet integral proposed in this paper; the weighted OWA operator [74] offers the ability to model varying degrees of compensation while assigning weights to different attributes; and the ELECTRE methods perform well when dealing with a small set of alternatives and allow for the definition of veto thresholds [75]. These approaches open up promising directions for future research on the applicability of alternative MCDA methods and on comparing their effectiveness.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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