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Doctoral Dissertation  
Doctoral Program in Mechanical Engineering

# **Surface Acoustic Waves Propagation and Coupling Fields in Piezomagnetic and Piezoelectric Semiconductor Layered Structures**

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\* \* \* \* \*

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# Abstract

Piezoelectric semiconductors (PSs) are functional materials with both piezoelectric and semiconductor properties. The core property of PSs is the multi-field coupling effect of mechanical-electro-carrier, and its multiple properties are adapting to the needs of the rapid development of electronic devices. In recent years, the research on multi-field coupling behaviors and waves modulation of PSs has become one of the hot topics. By integrating multiferroic materials with PSs, new magnetically regulated composite devices such as magnetically induced piezoelectric potential transistors, magnetically driven nanogenerators, and LED photovoltaic devices can be obtained, which have great potential applications. In this thesis, several fundamental surface acoustic wave (SAW) problems in multiferroic composite semiconductors (MCSs) are systematically studied. The propagation characteristic and regulation mechanism of coupled waves in composite structures are explained, and the electromechanical (ME) coupling responses with multi-physical fields are analyzed, which is of guiding significance for the design of magnetically related SAW devices and piezotronic devices. The main studies include:

(1) Based on the magneto-mechanical-electro-carrier coupling theory, the propagation characteristics of SH-type surface waves in piezomagnetic (PM) and PS layered half-space are studied. Two models of PS/PM layered half-space and PM/PS layered half-space are constructed. In the first model, based on the obtained dispersion equations in the complex domain, the effects of initial carrier concentration, overlay thickness and material properties on the dispersion and attenuation of SH waves are numerically calculated, and the distributions of the real and imaginary parts of electromechanical fields through the thickness are deeply analyzed. The results show that semiconductor properties have a great influence on the mechanical and electrical physical quantities, but a little influence on the magnetic related quantities. In the PM/PS layered half-space, the first three modes of Love waves are obtained for open and short circuits, and for different combinations of materials, respectively, where the wave speed and attenuation in open case are correspondingly smaller than those in short. The analysis shows that the propagation of Love waves depend on the overlay thickness, and the wave speed decreases significantly with the increase of

thickness. PM constant has obvious influence on the first two modes of wave speed, while the carrier concentration and piezoelectric (PE) constant have a small negligible effect.

(2) Considering the PM/PE/semiconductor three-layer composite structure, the effects of the overlay thickness, the initial carrier concentration, PM constant, and the four different material combinations on the Love waves and Rayleigh waves are obtained. The results show that both Love and Rayleigh waves only undergo dispersion without attenuation. Except for the first mode of the Rayleigh waves that starts at the Rayleigh wave velocity at substrate, the remaining modes of the Rayleigh waves and all modes of the Love waves start at the shear wave velocity of substrate. The wave speeds of the first three modes of both Love and Rayleigh waves decrease significantly with the thickness of PM layer, but do not vary with the initial carrier concentration. The influence of material on Rayleigh waves is greater than that on Love waves.

(3) The propagation of SH waves in a PS plate and PM/PS bilayer composite plate under initial stress is investigated, and the influence of initial stress on SH waves under multi-field coupling is analyzed. Numerical results show that the wave velocity of SH waves in PS plate becomes smaller and the attenuation is enhanced with constant initial stress. 100 MPa can be regarded as the inflection point where the phase velocity decreases sharply with increasing initial stress. In addition, both the wave speed and attenuation in short circuit are correspondingly smaller than those in the open case. In the PM/PS composite plate, the effects of carrier concentration, plate thickness, initial stress, and material properties on SH waves are similar to those in pure PS plate, indicating that the PM layer has a limited effect compared to the initial stress. Under the same initial stress, the phase velocity of CdSe/CoFe<sub>2</sub>O<sub>4</sub> is much smaller than that of ZnO/CoFe<sub>2</sub>O<sub>4</sub>, and the attenuation is also much weaker.

(4) The SH waves regulation in PM/PS layered structure by rotation and biasing electric field is studied. The dispersion, attenuation characteristics and frequency drift of SH waves in the layered structure rotating around a fixed axis are obtained in the framework of coupled theory including Coriolis and centrifugal forces. The results show that the wave dispersion and attenuation are also caused by the rotation. With the increase of the angular velocity, the wave speed and attenuation decrease faster, the frequency drift and attenuation coefficient become larger. The rotational sensitivity is much greater at long waves than at short waves. The SH-wave gain can be obtained by applying a horizontal biasing electric field regardless of rotation. There exists a biasing electric field turning point for attenuation and gain, as well as an inflection point where the maximum gain is obtained. With the increase of the biasing electric field, the wave speed will first decrease to the minimum, and then increase rapidly until it levels off.

(5) The bending coupling of a PS bimorph and the electromechanical response of a PS/PM composite beam are studied, respectively. In the PS bimorph, coupled two-dimensional (2D) plate equations are obtained from three-dimensional (3D)



theory. Considering the uniform load in rectangular domain, the distributions of electromechanical fields such as deflection, potential, carriers are obtained by using plate theory series solution and finite element method, respectively. The results of the two methods are similar and the errors are small. Besides, locally self-balancing load induce the potential barriers/wells. The PS/PM composite beam exhibits a complex coupling of bending and extension, and the transverse magnetic field moves the mobile charges to the ends of the beam. All electromechanical fields increase and change significantly with the increase of magnetic field, but the mechanical fields are less sensitive to the change of initial carrier concentration than the electrical physical quantities.

**Keywords:** Piezoelectric semiconductor, Piezomagnetic, Layered structure, Propagation characteristic, Initial stress, Rotation, Coupling behavior

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# Chapter 1

## Introduction

### 1.1 Foreword

Due to the coupling of piezoelectric and semiconductor properties, PSs have functional properties that traditional piezoelectric materials and semiconductor materials do not have. As early as the 1960s, there were many successful attempts on the development of PS microacoustic electronics [1]. In PSs, the interaction between elastic waves and mobile charges is called the acoustoelectric effect [2], and various acoustic devices based on the acoustoelectric effect have been continuously developed in the past few decades [3-8]. However, the piezoelectric properties of early PSs were generally weak, which limits their application in practice. In recent years, researchers have synthesized ZnO nanostructures with large piezoelectric effect such as nanofibers, belts, spirals and thin films [9-11], which have been widely used in functional devices in the form of sensors [12,13], nanogenerators [14,15], field-effect transistors [16,17] and acoustic charge transfer devices [18].

The core property of PSs is mechanical-electro-carrier coupling, and its basic behavior can be described by the macroscopic theory consisting of the traditional piezoelectric theory [19] and the semiconductor drift-diffusion theory [20], which is coupled by doping and moving charges. Based on the coupling theory, various multi-field coupling problems of PSs, including tensile [21,22], bending [23,24], vibration [25,26] and fracture [27-30], have been solved. These studies have laid the foundation for a better understanding of the coupling characteristics of PSs and the developing of novel PS devices.

The wide potential applications of PSs require a better understanding of the elastic waves propagation in PSs. Due to semiconductor properties, elastic waves in PSs are both of dispersion and attenuation [31,32] as compared with in the pure elastic or pure piezoelectric mediums. By applying biasing electric fields in the PS structures, the amplification of the elastic waves can be achieved [33-36]. Therefore, it is necessary to study the effects of boundary conditions, steady-state

carrier concentration, layer thickness, material properties and biasing electric field on the dispersion and attenuation of elastic waves in PSs. Semiconductor properties and elastic waves in PSs can be regulated by generating piezoelectric potentials or polarized electric fields using external loads such as mechanical forces and heat. When PM materials are added or embedded in PS materials, the composite structure will exhibit more complex mageto-electro-semiconductive (MES) coupling. The use of magnetic field to control the semiconductor characteristics of carriers in PSs can realize remote and non-contact operation, and has great application prospects. Based on this, new multifunctional devices such as magnetic induced transistors, magnetic driven nanogenerators and magnetic-optical devices can be made.

However, to our best knowledge, there are still few research on the MES multi-field coupling, and there have been no reported studies on SAWs propagation in MCS structures, which motivate the work.

In the thesis, the propagation characteristics of SAWs in various MCSs are studied. Based on MCS macroscopic theory, the effects of boundary conditions, initial carrier concentration, plate thickness, material properties, initial stress and rotation fields on the dispersion and attenuation of surface waves are analyzed. The coupling of bending and moving charges in bilayer PS plates, and the electromechanical coupling response of PM/PS composite beam under a transverse magnetic field are also obtained.

## **1.2 Multiferroic composite semiconductors**

### **1.2.1 Piezoelectric semiconductors and related applications**

Semiconductor materials have developed from the first generation of elemental semiconductors represented by Si and Ge to the second generation of compound semiconductors such as GaAs and InP, and then to the third generation of semiconductors represented by SiC, GaN, ZnO, diamond and AlN. Compared with the first two generations, the third generation semiconductor have the advantages of large band gap, high breakdown electric field, high thermal conductivity, high electron saturation rate and strong radiation resistance, etc. They are more suitable for the production of high temperature (aerospace engines, underground drilling equipment, etc.), high frequency (GaN fast charging power adapters, 5G/6G base station RF power amplifiers, etc.), radiation resistance (nuclear power plants, satellite power management systems, etc.) and high power devices (photovoltaic inverters, electromagnetic catapult systems, etc.).

The rapid development of electronic science puts forward higher requirements for materials, and traditional piezoelectric materials or semiconductor materials can not fully meet the needs of electronic devices. The development of electronic devices towards miniaturization, functionalization and intelligence has led to an urgent search for functional materials with two or more properties in a single entity. The traditional semiconductors cannot control the electrical signal directly

because they do not have the piezoelectric effect, but the PSs can control the carrier transport through the electric field generated by the mechanical field, so as to manipulate silicon-based electronic devices. The common wurtzite type II-VI and III-V PSs have non-centrosymmetric crystal structures and thus can exhibit remarkable piezoelectric properties.

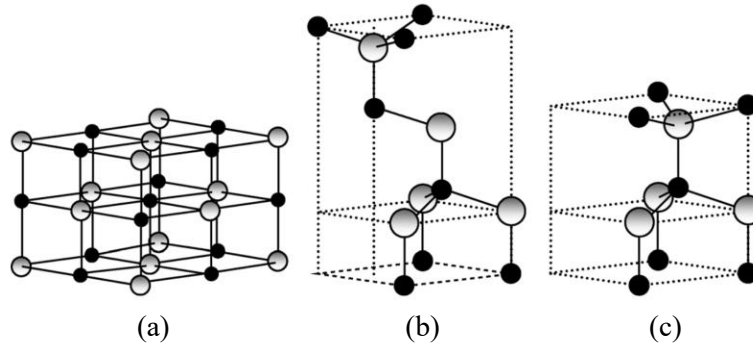


Fig. 1.1 Three different ZnO crystal structures [37] (the gray and black spheres denote Zn and O atoms, respectively): (a) cubic rocksalt, (b) cubic zinc blende, and (c) hexagonal wurtzite

Take ZnO as an example, as shown in Fig. 1.1, there are three zinc oxide crystals with different structures [37]. The wurtzite structure shown in Fig. 1.1(c) is a non-centrosymmetric hexagonal with four “O” ions distributed around each “Zn” ion, which together form a regular tetrahedral structure and are periodically distributed.

According to the presence or absence of doping, semiconductors can be divided into intrinsic semiconductors and impurity semiconductors (n-type and p-type semiconductors), and a small amount of doping can drastically change the conductive properties of semiconductors. In PSs, free electrons and holes act as carriers. The number of more carriers is called the majority of carriers, most of the carriers in n-type semiconductors are electrons, so n-type semiconductors are also called electronic semiconductors, and the corresponding p-type are called hole semiconductors. However, the coupling of polarization charge and semiconductor properties in PSs has long been neglected, and the related functional devices have not been quantified and widely used. The successful fabrication of various nanoscale PSs in the early 21st century [9-11], along with the introduction of the new concepts of Piezotronics [38,39] and Piezo-phototronics [40,41] by Zhonglin Wang's group in 2007 and 2010, respectively, the PSs and the related fields gradually became a research hotspot.

The piezotronic effect uses the piezoelectric potential as a "gate" voltage to modulate the transport of carriers at the contact or junction region [42]. Based on this effect, Zhonglin Wang's team has successfully fabricated a series of novel electronic devices, including various sensors [12,13,43], field-effect transistors [16,17,44], diodes [45] and programmable electromechanical memories [46]. The piezo-phototronic effect controls carrier generation, transport, and recombination through the piezoelectric potential, which is a coupling of polarization and optical



excitation of PSs to improve the performance of optoelectronic devices such as photodetectors [40], light-emitting diodes [47], and solar cells [48]. Longfei Wang [49] outlined the basic principles of piezo-phototronic effects, material systems and the research progress of devices. The three-way coupling of the piezoelectricity, photoexcitation, and semiconductor properties is the basis for the study of piezotronics, piezophotonics, conventional optoelectronics, and piezo-phototronics [42,50,51]. The coupling mechanism and application prospects are shown in Fig. 1.2.

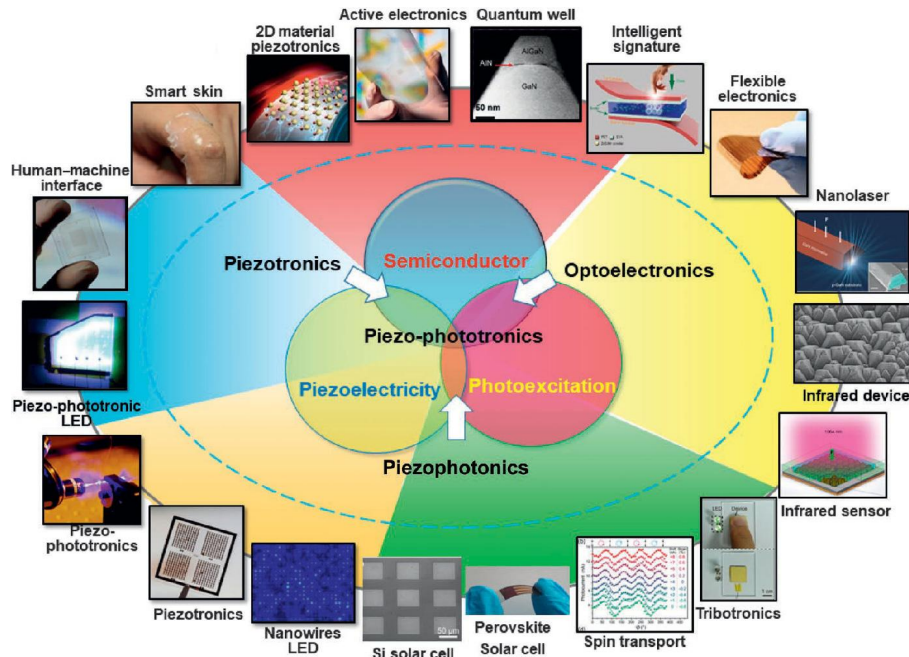


Fig. 1.2 The three-way active coupling among piezoelectricity, photoexcitation and semiconductor & the application prospects of piezotronics and piezo-phototronics [42]

PSs can convert mechanical and electrical signals by using the piezoelectric effect and realize the self-sustainable supply of energy, which will play an important role in the field of energy harvesting [52]. In addition, PSs will occupy an important position in electronic devices and acoustic devices due to the simultaneous acoustoelectric effect and acoustoelectric amplification effect. The theoretical study of the coupling effects and elastic waves in PSs is of great significance for the improvement of PS materials and the development of PS devices and equipment. The macroscopic theory of PSs can be described by theories such as linear piezoelectric equations and charge continuity equations [20,53,54]. In addition, some other scholars have also proposed more comprehensive nonlinear theory [55,56].

### 1.2.2 Multiferroic composite semiconductors and coupling

Electrons have two key properties: charge and spin. The property of charge is widely known, and its directional movement forms the current that conventional computers use to transmit and process data. However, the current also brings

bottlenecks such as “consuming a lot of power and generating a lot of heat”, and the performance of electronic devices is approaching their physical limits. The property of electron spin has not been paid much attention until 1988, when Baibich et al. observed the giant magnetoresistance effect in the superlattice structure composed of Fe/Ce multi-period films [57], and people began to realize the application value of electron spin. The injection of spin-polarized electrons into semiconductor devices allows the development of a new generation of semiconductor spintronic devices [58]. In recent years, spintronics has been studied theoretically and experimentally [59].

However, spin-based electronic devices often require extreme conditions (e.g., low temperatures or strong magnetic fields, etc.), which limits their applications. A flexible method is to design composite devices, such as magnetoelectric and magneto-optical composite devices, etc. Magneto-electro-elastic material is a multifunctional composite material, which can conveniently convert magnetic, electrical and mechanical energies, and macroscopically has magnetoelectric effects (ME). By coupling magnetic, semiconductor and piezoelectric properties, a new type of magnetic control composite device can be obtained, which has a wide application prospect. Liu et al. [60] combined magnetoelectric composites with 2D semiconductors (single-layer  $\text{MoS}_2$ ) to design a novel magnetic-induced-piezopotential transistor, and realized the modulation of the magnetic field to the transistor at room temperature. Hao et al. [61,62] from the Hong Kong Polytechnic University successfully fabricated a magneto-optical composite device by coupling magnetic field, piezoelectric effect and photoelectric effect, and realized the noncontact drive of light by magnetic field. Wang et al. fabricated GaN-based LED optoelectronic devices to achieve magnetic-mechanical-electrical-optical coupling and verified it with experiments [63]. In addition, they reported a new magnetic-driven noncontact nanogenerator (CLNG), which avoids the direct contact between the nanogenerator and the mechanical movement source [64].

In fact, scholars have successfully integrated multiferroic materials with PS materials to explore more multi-functional MCS devices. For example, Li et al. [65] integrated  $\text{BaTiO}_3/\text{CoFe}_2\text{O}_4$  multiferroic heterostructures on GaN substrates to obtain new MCS materials. Thomas et al. [66] reviewed multiferroic thin film deposition technology and multiferroic magnetic integrated semiconductor devices. In addition to magnetic multilayers, research on magnetic semiconductors, non-magnetic semiconductor quantum wells, and semiconductor spin injection has also become very active, greatly enriching the research scope of semiconductor spintronics [67,68].

Semiconductor magnetoelectronics, spintronic devices and magnetic control composite devices have great potential for commercial applications. Compared with traditional stress modulation, magnetically modulated composite devices have the same modulation effect, but with the unique advantages of remote and contactless. For the MCS models, the theory includes piezoelectric effect, carrier transport properties, piezomagnetic effect and magnetoelectric coupling effect.

The corresponding equations are composed of semiconductor current drift-diffusion and the basic equations of multiferroic composite materials. According to existing studies [69-71], the governing equation of MCS models can be expressed as:

$$\begin{aligned} T_{ji,j} &= \rho \ddot{u}_i \\ D_{i,i} &= q(p - n + N_D^+ - N_A^-) \\ B_{i,i} &= 0 \\ J_{i,i}^p &= -q\dot{p}, J_{i,i}^n = q\dot{n} \end{aligned} \quad (1.1)$$

in which Eq. (1.1)<sub>1</sub> stands for the equilibrium equation of motion without body force, Eq. (1.1)<sub>2</sub> and Eq. (1.1)<sub>3</sub> are the equations representing the Gaussian law of quasistatic electromagnetics. The two equations in Eq. (1.1)<sub>4</sub> are the charge conservation equations for holes and electrons. It is worth noting that the above equations ignore the recombination and regeneration of carriers. In the Eq. (1.1),  $T_{ij}$ ,  $D_i$  and  $B_i$  are the stress, electrical displacement and magnetic flux intensity, respectively,  $\rho$  is the density of material.  $q = 1.6 \times 10^{-19}$  C is the elementary charge,  $J_i^p$  and  $J_i^n$  the hole and electron current densities, respectively.

The Cartesian tensor notation and the Einstein summation convention are used. The values of  $i, j, k$  and  $l$  range from 1, 2 to 3. The superscript dot represents the derivative with respect to time, the subscript comma followed by an index  $i$  denotes partial derivative with respect to  $x_i$ .

The coupling constitutive relations describing material properties and current density are:

$$\begin{aligned} T_{ij} &= c_{ijkl} S_{kl} - e_{kij} E_k - f_{kij} H_k \\ D_i &= e_{ijk} S_{jk} + \varepsilon_{ij} E_j + g_{ij} H_j \\ B_i &= f_{ijk} S_{jk} + g_{ij} E_j + \alpha_{ij} H_j \\ J_i^p &= qp\mu_{ij}^p E_j - qD_{ij}^p p_{,j} \\ J_i^n &= qn\mu_{ij}^n E_j + qD_{ij}^n n_{,j} \end{aligned} \quad (1.2)$$

in which Eq. (1.2)<sub>1-3</sub> are the constitutive relations describing behaviors of multiferroic materials, and Eq. (1.2)<sub>4-5</sub> are the drift-diffusion model representing the current densities of holes and electrons.  $S_{ij}$  is the strain,  $E_i$  the electric field, and  $H_i$  the magnetic field.  $c_{ijkl}$ ,  $e_{ijk}$ ,  $f_{ijk}$ ,  $\varepsilon_{ij}$ ,  $g_{ij}$  and  $\alpha_{ij}$  are elastic, PE, PM, dielectric, ME coefficients and magnetic permeability, respectively.  $\mu_{ij}^p$  and  $\mu_{ij}^n$  are the mobility constants of holes and electrons,  $D_{ij}^p$  and  $D_{ij}^n$  are the diffusion constants of holes and electrons. The carrier mobility and diffusion constants conform to the Einstein relationship as [20]:

$$\frac{D_{ij}^p}{\mu_{ij}^p} = \frac{D_{ij}^n}{\mu_{ij}^n} = \frac{k_B T_k}{q} \approx 0.0259 \text{ V} \quad (1.3)$$

where  $k_B$  and  $T_k$  are Boltzmann constant and Kelvin temperature, respectively.  $k_B T_k / q$  is the value at room temperature 300K [20]. In the numerical calculation, the small carrier perturbations of the PS layer are usually considered, so the hole and electron concentrations can be represented linearly as  $p = p_0 + \Delta p$  and  $n = n_0 + \Delta n$ . The  $p_0$  and  $n_0$  are the initial hole, electron concentrations under steady state. Considering uniform doping,  $p_0 = N_A^-$  and  $n_0 = N_D^+$ . If the PS in the composite structure is considered as n-type, the internal electron density and donor doping density will be much higher than the hole density and acceptor doping density, i.e.  $n \gg p$ ,  $N_D^+ \gg N_A^-$ . Therefore, in the study, the effect of hole density and acceptor doping density can be ignored, that is, all the equations about hole  $p$  will disappear. The linear governing equation corresponding to PM/n-type PS will be changed from Eq. (1.1) to:

$$\begin{aligned} T_{ji,j} &= \rho \ddot{u}_i \\ D_{i,i} &= -q \Delta n \\ B_{i,i} &= 0 \\ J_{i,i}^n &= q \Delta \dot{n} \end{aligned} \quad (1.4)$$

Accordingly, the constitutive equation (1.2) can also be re-expressed as:

$$\begin{aligned} T_{ij} &= c_{ijkl} S_{kl} - e_{kij} E_k - f_{kij} H_k \\ D_i &= e_{ijk} S_{jk} + \varepsilon_{ij} E_j + g_{ij} H_j \\ B_i &= f_{ijk} S_{jk} + g_{ij} E_j + \alpha_{ij} H_j \\ J_i^n &\cong q n_0 \mu_{ij}^n E_j + q D_{ij}^n (\Delta n)_{,j} \end{aligned} \quad (1.5)$$

It is obvious from Eqs. (1.1) and (1.2) or Eqs. (1.4) and (1.5) that ME effects and semiconductor properties in MCS composite structures are coupled to each other. The above equations lay the theoretical foundation for the modeling of MCSs, as it is obviously not easy to deal with the mageto-electro-semiconductive multifield coupling, especially the nonlinear problems, by mathematical methods. In order to obtain analytical solutions, small perturbations of carriers are usually considered, i.e., the linearized theoretical approach corresponding to Eqs. (1.4) and (1.5).

### 1.3 Elastic waves in piezoelectric semiconductors

The theory of elastic waves in complex media has been widely used in geophysical exploration, non-destructive testing, medical ultrasound imaging, structural health detection and other fields [72]. The propagation characteristics of elastic waves in PSs are also receiving increasing attention. In this Section, the research progress of elastic waves in PSs will be summarized from the following aspects: body, surface and guided waves in pure PSs, elastic waves in layered PSs

and piezoelectric-nonpiezoelectric semiconductor composite structures, as well as elastic waves in PS structures considering small-scale effects (surface effects, flexoelectric effects, etc.) and interface effects.

### 1.3.1 Elastic waves in single piezoelectric semiconductors

Early studies on elastic wave propagation in pure PSs mainly focused on body waves. Huston and White [31] at Bell Telephone Laboratories in the U.S. proposed the linearization theory when studying PS acoustic waves, and found computationally that when piezoelectric crystals were semiconductors at the same time, electric fields would generate current and space charge, resulting in the dispersion and attenuation of waves. In PSs, the interaction between acoustic waves and mobile charges is called the acoustoelectric effect, a special case of a general phenomenon, also called the wave-particle drag effect (WPDE) by Weinreich et al. [73]. Huston et al. [74] obtained experimentally that 18 db and 38 db of wave amplitude gain can be obtained in 7 mm class CdS at 15 Mc/sec and 45 Mc/sec, respectively, and compared the experimental results with theoretical calculations.

In the 1970s and 1980s, researchers utilized the acoustoelectric effect of acoustic waves to produce many acoustoelectric devices. For example, Kino [75] discussed the application of acoustic convolvers in signal processing in 1976, and described optical scanners, acoustic focusing and imaging devices. Heyman [76] in 1978 discussed the working principle of acoustoelectric transducers (AET) based on acoustoelectric effect and gave the optimization steps in use, and found that the data resolution obtained by AET was significantly improved compared with traditional piezoelectric sensors. Busse et al. [77] proposed a CdS based phase-sensitive receiver in 1981 and gave some optimization criteria. Dietz et al. [78] gave experimental results of sensitivity and frequency response of several detectors, and discussed devices such as power meters and spot intensity meters for diagnostic imaging pulses.

Compared with body waves, the amplitude of surface waves and interface waves decreases rapidly with the increase of the distance from the interface or surface, and their energy is also concentrated near the surface or interface [72]. Based on the basic PS half-space, Gu et al. [79] and Cao et al. [80] obtained the propagation characteristics of SH waves and generalized Rayleigh waves under the biasing electric field, respectively. They verified that the semiconductor properties would reduce the wave speed, cause the wave to be dispersed and attenuated, and also verified the gain effect of the biasing electric field. Further, assuming that material parameters increase exponentially along the thickness, Wang et al. [81] studied SH waves in functionally graded piezoelectric semiconductor half-space (FGPS).

For pure PS plates, Tian et al. [82] systematically studied the effects of boundary conditions, carrier concentration, plate thickness and biasing electric field on the wave speed and attenuation of SH and Lamb waves in PS plates

through numerical examples. Zhang et al. [83] studied the propagation of extensional waves in a thin PS rod. Zhu et al. [84] obtained the effect of semiconductor properties on nanogenerators through the study of SH wave propagation in PS plates: mechanically, the semiconductor properties reduce the kinetic energy of the waves; electrically, it was found that energy can be collected during deformation generation and recovery. Liang and Hu [85] obtained the exact solution of the coupled wave in the PS rod, and studied the influence of three time scales on the coupled wave propagation.

Considering the dispersion and attenuation properties of coupled waves in semi-infinite PSs, Jiao et al. [86] studied the reflection of elastic waves at mechanically free, electrically insulation and dielectrically open circuit. Using classical coupled thermoelastic theory, Jahangir et al. [87] studied the influence of free surface boundary on reflected waves in semi-infinite PS medias. For n-type PS half-space and PS plate under initial stress, Chen et al. [88,89] deeply analyzed the effects of boundary conditions, initial carrier concentration and initial stress on SH wave propagation, and the results showed that when the initial stress was large enough, the wave speed in both structures would decrease sharply and the attenuation would be enhanced. Wang et al. [90] investigated the propagation characteristics of SH waves in FGPS when initial stress is applied. Further, Salah et al. [91] proposed a simplified ordinary differential equation (ODE) method for ZnO plates containing prestress, and obtained the effect of residual stress on electron and hole drift.

In addition to half-space, plate, and thin film, researchers have also investigated the properties of axial Lamb waves in PS cylindrical shells using Legendre polynomial method [92]. Based on the PS PN junctions, Wei et al. [93,94] studied the influence of PN junction in different doping types and equivalent imperfect interface models on the coupled waves, and carried out numerical calculations on the reflected and transmitted waves. Kong et al. [95] theoretically studied the influence of elastic longitudinal waves on PN junctions and believed that elastic waves had a tuning effect on the electrical behavior of PN junctions. Recently, Zhang et al. [96] analyzed SH waves in PS nanoplates considering surface effects based on Gurtin-Murdoch model, and illustrated the influence of surface effects on SH waves at the nanoscale. In addition, they revealed the propagation characteristics of terahertz elastic waves in PS rods, finding that both phase and group velocities are 50% to 60% lower in the terahertz range than in the low frequency range [97].

### **1.3.2 Elastic waves in piezoelectric semiconductor composite structures**

The above research focused on single PS structures (pure PS half-space, plate/film and cylindrical shell, etc.). The study of elastic waves in multilayer PSs (PS laminated plate/shell, PS thin film/PS substrate, PS cover/elastic substrate, etc.)

and PE/semiconductor composite structures (semiconductor layer/PE half-space, PE/semiconductor layered structure, etc.) has attracted increasing attention.

By using the series expansion method, Yang et al. obtained the 2D equations of PS laminated plates [35] and PS multilayer shells [98] from the 3D equations, and calculated the extensional waves in the semiconductor/ceramic/semiconductor composite plates and the torsional waves in the PE/semiconductor multilayer shells. In addition, Yang and Zhou also obtained the SH waves in PE plate sandwiched between two semiconductor layers [99]. Using Legendre polynomial expansion, Othmani et al. studied the propagation of Lamb waves in GaAs PS/FGPS/AlAs PS [100], PS laminates composed of AlAs and GaAs [101] and FGPS plates [102] without considering semiconductor properties. Sharma et al. successively studied interfacial waves in three isotropic semiconductor/PE layered half-spaces [103], three PE layer/semiconductor half-space composite structures [104]. In addition, they also obtained dispersion curves, attenuation coefficients, and energy dissipation factors for SAWs in PE/semiconductor/PE sandwich structures [105].

For PS thin film/elastic half-space, Tian et al. investigated the effects of semiconductor properties, film thickness, and biasing electric field on Rayleigh waves [106] and Love waves [107]. At the same time, the analytical solution of SH waves in PS multilayer plate [108] with imperfect interface was obtained, and the effects of stacking order and imperfect interface on SH waves were discussed. Considering the interface effects, Guo et al. [109,110] studied in-plane elastic waves in PS/PE periodic materials and anti-plane elastic waves in 1D PS phononic crystals, which can be used for the optimization of nanoenergy harvesters. Scholars have also made new progress in the reflection and transmission of coupled waves at the interface of laminated PSs. Jiao et al. [111] analyzed wave propagation in AlN PE half-space/ZnO PS plate/AlN PE half-space, and calculated the changes of energy reflection coefficient and transmission coefficient with the incidence angle. Sahu et al. [112] studied the reflection and transmission of quasi-plane waves in AlN/ZnO with initial stress. Based on the electro-thermal-mechanical coupling model, Zhao et al. [113] studied coupled waves in the PS sandwich structure.

Based on the semiconductor/PE layered half-space, Abd-alla et al. [114] numerically analyzed the propagation characteristics of SH waves with rotation and initial stress. Furthermore, Xu et al. [115] obtained Rayleigh waves in layered PSs when the PN junction effects were considered, and the results showed that PN junction had little influence on dispersion and attenuation, but had great influence on electric potential, carrier concentration and electric displacement. By constructing a metal overlay/PS substrate, Xu et al. also studied the influence of Schottky junction effects on SH waves [116] and corrugated boundary on Rayleigh waves [117]. Recently, they analyzed SH waves on n-type PS substrate under thin metal strip gratings [118]. Qu et al. [119] derived infinite power series of 2D differential equations for flexoelectric semiconductors via extended Mindlin method. Jaafar et al. [120] added a titanium/gold guiding layer on the

surface of GaN/Sapphire to achieve a change in the mode of acoustic wave propagation, which can be used for biosensors and wireless communication.

## 1.4 Multifield coupling of piezoelectric semiconductors

The core property of PSs is mechanical-electro-carrier multi-field coupling. It is a common way to control PSs by using external mechanical force to generate piezoelectric potential or polarized electric field to drive carrier transport and redistribution. In addition, non-contact control such as heat and magnetic fields are also effective ways. When the temperature changes, the pyroelectricity and piezoelectricity in the PSs will be coupled to produce a polarized electric field, and then change the carrier distribution. The magnetic field can also control the semiconductor characteristics of the carrier in the PSs through the magnetoelectric coupling effect. This section will discuss the multifield coupling of PSs, focusing on the research progress of stress/thermally/magnetic field regulation.

### 1.4.1 Stress/thermally control of piezoelectric semiconductor coupling behavior

Starting from the 3D equations of the PSs, Zhang et al. [121] first derived the 1D nonlinear equations of a free PS rod, obtained the displacement, electric field and carrier distribution in the self-equilibrium rod, and found that the fields are relatively strong near the ends than in the central part. Further, they also studied the electromechanical fields of ZnO nanofibers under an axial force [21,22], and also reached the conclusion that the electromechanical fields at both ends varied greatly and were more sensitive to axial force. Fan et al. conducted a series of innovative studies on stress-induced PSs. Firstly, for cantilevered circular ZnO nanowire subjected to a concentrated force at the free end, they obtained exact solutions of the electromechanical fields under static bending [122]. Secondly, by studying PN junction subjected to a pair of tensile/compressive stresses at the endpoints, they found that tensile stress would increase the potential barrier of PN junction and make them more difficult to break through, while compressive stress would have the opposite effect [123]. Subsequently, they also investigated the regulation of piecewise stresses (one pair and two pairs of local concentrated forces, and sinusoidally distributed forces) on the barrier and charge of PS nanofibers [124].

For ZnO plates, Qu et al. obtained analytical solutions of rectangular plates under load by using 2D plate theory, and found that local electromechanical fields and barriers would be generated under local loads [125]. In addition, considering the sandwich plate composed of piezoelectric and semiconductor (Si/PZT/Si), they also obtained the coupling between the load and the distribution/motion of the moving charges [126]. By constructing the opposite structure, namely PZT/Si/PZT composite fiber, Cheng et al. [127] used a 1D model to study the piezoelectric effect in stretching, showing that the composite fiber exhibits a



piezoelectric coupling similar to that of a single PS fiber. In the composite fiber discussed by Ju et al. [128], Si is uniform and PZT is piecewise uniform. The results indicate that potential barriers form near the junctions of piecewise piezoelectric layers, demonstrating the effectiveness of the one-dimensional (1D) model in modeling thin piezoelectric fibers.

In addition, He et al. [129] studied the influence of stress on electric currents in PS anti-plane, and obtained current-voltage relationship and current density distribution by using COMSOL software for nonlinear numerical analysis. Yang et al. [130] established a coupling model of PS bipolar junction transistors (PS-BJT) subjected to mechanical loadings. The effect of base region on device performance and the interaction between emitter/base junction and base/collector junction are studied. Recently, in order to quantitatively explain the regulation of stress on carriers, researchers studied the distribution of free carriers under different deformation modes, including extension, bending, thickness-shear and thickness-stretch [131].

When the temperature effects are considered, many PSs exhibit the coupling of thermoelastic, piezoelectricity and pyroelectricity, and the regulation of temperature changes through thermoelastic and pyroelectricity is similar to that of mechanical forces. Using the macroscopic theory of thermoelectric semiconductors, Cheng et al. studied the electrical behavior of PS fibers under uniform temperature [132] and local temperature [133] changes, and the analysis showed that the carriers would be redistributed when the temperature changed. In addition, the research group studied the effect of uniform temperature change on PS PN junction, and found that homogeneous junction has better temperature stability, while heterogeneous junction is more sensitive to temperature change [134]. They also discussed the effect of temperature on moving charges in composite fibers composed of piezoelectric and semiconductors [135].

Based on the 2D equations of thermoelectric semiconductors, Qu et al. studied the effect of temperature on the moving charges in the ZnO PS plate [136] and thin film [137]. The numerical results showed that the temperature field can control the distribution of moving charges in the plates, and the local change of temperature would produce a pair of potential barriers and potential wells in the film. Based on the continuum theory, the interaction between the thermoelectric and mechanical fields in PSs were discussed [138].

Considering the structure of ZnO films sandwiched between  $\text{ZrO}_2$  substrate films, Jin et al. [139] obtained closed form solutions of stress, deformation, electric displacement and electric field under a steady-state thermal gradient. Ju et al. [140] discussed in detail the influence of moving charge on the interfacial thermal stress in PZT/Si/PZT composite rod, obtained the analytical expression of the interfacial distributed shear stress and concentrated shear stress, and found that the shear stress would disappear in the absence of moving charge.

Qiao et al. [141] studied the effect of temperature on the electrical conductivity of GaN by numerical simulation and experimental verification, and established a resistivity prediction model that was in good agreement with the

experimental results. In addition, researchers have also studied the electromechanical coupling behavior of a hollow cylinder composed of thin PS outer/elastic inner layers at uniform temperatures [142].

### 1.4.2 Magnetically controlled piezoelectric semiconductors

In contrast to stress and thermally drive PSs, magnetic fields have high penetration, enabling remote and non-contact operation. In recent years, progress has been made in the theory and application of the magnetic field to control the carrier of the semiconductor by using the magnetoelectric coupling effect in the MCSs.

Cheng et al. [71] constructed a five-layer MCS model consisting of an upper and lower PM layer, an upper and lower piezoelectric layer and a semiconductor center layer, in which the magnetic fields "act" like mechanical forces in the symmetric fiber. Similarly, Wang et al. [143] and Liang et al. [144] constructed a three-layer composite fiber consisting of two PM layers and a PS core, and obtained the carrier transport behavior under axial magnetic field and the electromechanical response under local magnetic field, respectively. In addition, Liang et al. [145] also studied the electromechanical behavior of PN junction in the composite fiber composed of PM/PS/PM, and found that the homogeneous junction was almost unaffected by the magnetic field, while the heterogeneous junction was more sensitive to the magnetic field. Yang et al. [146] studied the coupling behavior of bending and extension in PS/PM asymmetric unimorphs under a transverse magnetic field. Wang et al. [147] proposed an antisymmetric layered MCS consisting of two PM layers and two PS layers and studied the pure bending behavior of cantilever beams under a longitudinal magnetic field.

For the composite cylindrical shell composed of PM and PS layers, the analytical expressions [148] are derived by one-way coupled method and the linear fully-coupled method, respectively, and the effects of the magnetic field and semiconductor layer thickness ratio on the electric potential, electric field and carrier are numerically analyzed. By applying time-dependent magnetic fields to dynamically adjust the semiconductor characteristics of the composite semiconductor fiber, Kong et al. [149] found that the magnetic field has a significant tuning effect on the five-layer symmetric fiber near the natural frequency. Zhang et al. [150] studied the nonlinear dynamic response of PN heterojunctions in MCS composite fibers driven by dynamic magnetic loads. Considering the surface effect, Wang et al [151,152] analyzed the nonlinear magneto-mechanical-thermo coupling characteristics in MCS nanoplates and the vibration of the nanoharvester driven by a time-harmonic magnetic field. Further, Zhao et al. [153] conducted a 2D analysis of the electrical behavior of magnetic field regulation based on the multifield coupling theory. These studies are helpful for the design of piezotronics devices related to magnetic fields, which can be applied in cutting-edge fields such as intelligent sensing, human-computer interaction, biomedicine, nanorobots, and self-powered systems.

In fact, by coupling magnetism, semiconductor and piezoelectricity, some novel magnetically controlled composite devices have been obtained. Using Terfenol-D,  $\text{Al}_2\text{O}_3$  and ferroelectric semiconductor  $\alpha\text{-In}_2\text{Se}_3$ , researchers prepared piezotronics and magnetic dual-gated ferroelectric semiconductor transistor (PM-FEST) [154]. The transistor has a maximum current on/off ratio of up to 1700% at low magnetic fields of less than 200mT, and also has a higher measurement factor than traditional PS devices. Based on the  $\text{AlN/Si/Fe-Co-Si-B}$  structure, Huang et al. [155] proposed a magnetic surface acoustic wave (MSAW) resonator for MEMS. Such resonators have great potential in the realization of wireless magnetic field sensors with miniaturization, low power, and high complementary metal-oxide-semiconductor (CMOS) compatibility.

## 1.5 Outline

As mentioned above, PSs have some advantages that traditional PE materials and semiconductor materials do not have, and their multiple properties are suitable for the rapid development of electronic technology and devices. Elastic wave propagation and multi-field coupling behavior in single PSs and composite PSs have been studied. However, the PE constants of PSs are relatively small, and ferroelectric materials with large PE constants usually have weak semiconductor properties, which limits the effective coupling of piezotronic materials and devices. By adding or embedding PM materials in the PSs, the coupling of piezoelectricity, semiconductor and magnetism can be realized, and the magnetic field is used to drive the carrier to regulate the coupling behavior of the PSs.

Based on the macroscopic theory of magneto-mechanical-electro-carrier multi-field coupling, some typical wave problems and coupling behaviors of MCSs are studied in this paper, which provides a basis for the design of piezotronics devices and acoustic devices based on magnetic field manipulation. This thesis is divided into seven chapters, and the contents of each chapter are as follows:

(1) Chapter 1 is Introduction. The application and coupling mechanism of PSs and MCSs are introduced, the research progress of elastic wave propagation in single PSs and composite PS structures is reviewed, and the coupling behavior of PSs controlled by stress, heat and magnetic field is emphasized. Finally, the whole contents of the thesis are introduced.

(2) In Chapter 2, SH-type waves in PS/PM layered half-space are studied. The effects of steady-state carrier concentration, covering layer thickness and material properties on SH-wave velocity and attenuation are systematically analyzed for the PS layer/PM substrate. The dispersion curves and attenuation curves of Love waves under different boundary conditions and material combinations are obtained by considering the PM/PS layered half-space after the exchange of the cover layer and the substrate.

(3) In Chapter 3, the SAWs in PM/PE/semiconductor composite structures are discussed. In this Chapter, the propagation of Love waves and Rayleigh waves in

three-layer structure is deeply analyzed, and the influence of covering layer thickness, initial carrier concentration, PM constant and different material combination on the waves is studied.

(4) SH waves in MCSs with initial stress are studied in Chapter 4. The effects of boundary conditions, initial carrier concentration, plate thickness and initial stress on SH waves are obtained considering the initial stress. Secondly, the effect of initial stress on SH waves under the coupling of magneto-electro-semiconductor is discussed for the PS/PM bilayer plate and PS/PM layered half-space.

(5) The effects of rotation on SH waves in MCSs are studied in Chapter 5. Based on the coupling theory with Coriolis force and centrifugal force, the propagation characteristics of SH waves in MCSs rotating around a fixed axis are studied, including the dispersion characteristics, attenuation characteristics and frequency drift of SH waves. The influence of semiconductor and biasing electric field on SH waves is also analyzed.

(6) Multi-physics coupling in MCSs is studied. The Chapter 6 focuses on two aspects. Firstly, the coupling of bending and moving charge of PS bilayer plate (bimorph) with opposite  $c$ -axis is studied, and the moving charge distribution and potential barrier/potential well of rectangular plate under uniform load and self-equilibrated load are obtained by using 2D plate theory. Secondly, the electromechanical coupling response of the PS/PM composite beam under the transverse magnetic field is discussed, and the effects of physical and geometric parameters on the electromechanical fields and coupling coefficient are analyzed.

(7) Chapter 7 summarizes and anticipates the full work.

## Chapter 2

# SH-type surface waves propagation in piezoelectric semiconductor and piezomagnetic layered half-space

Due to the advantages of low loss, small size, high reliability and flexible design, SAW devices are widely used in wireless communication, sensing and detection, etc. At the same time, the research interest of surface waves in layered structures has been aroused. As mentioned in Chapter 1, in order to develop new semiconductor acoustic devices, scholars have studied the elastic waves in various PS structures, including body waves and guided waves in PSs, elastic waves in layered PSs and PE/semiconductor composite structures. It also includes the study of the elastic waves of thin layer or thin film covered by semi-infinite substrate such as PS film/elastic half-space, semiconductor layer/PE half-space, p-type PS layer/n-type PS half-space.

SH guided waves in SAWs play an important role in non-destructive testing and structural health monitoring. Miao et al. [156] reviewed the working principle, application and progress of various SH wave transducers. Mandal et al. [157] discussed in detail the detection applications of SH wave sensors in biochemistry, microfluidics and mechanobiology. In recent decades, SH waves propagation has been extensively studied and applied. SH waves in PSs were analyzed taking into account initial stresses [88-91,114], surface effects [96] and Schottky junction effects [116]. In addition to SH waves in PSs, Golub et al. [158,159] studied SH waves in periodic layered structures containing strip cracks and damaged layers, respectively. Nonlinear ultrasonic technology has significant advantages in the detection of material defects. Osika et al. [160] analyzed the nonlinear modes of SH waves in plates based on the analytical and numerical analysis.

As SH-type waves with only one displacement component and high sensitivity in liquids and certain frequency ranges, Love waves are widely used in electronic devices. In 1911, A.E.H. Love [161] first proposed Love waves in

layered structures composed of isotropic elastic layers and substrate. Since then, Love waves in different media such as layered PE materials [162,163] and layered magnetoelectric materials [164-166] has been studied successively.

With the development of better performance and miniaturization of SAW devices, researchers further explored the influence of flexoelectric effects on Love waves [167,168]. Attention has also been paid to the propagation of Love waves in layered half-space with initial stress [169-172], considering the cover layer as a functionally graded material, and embedding a functionally graded buffer layer between the cover layer and the substrate [173-175]. In addition, Kielczynski et al. [176], Billon and Baroudi [177] respectively studied the viscosity of Newton liquid and the influence of finite thickness viscoelastic fluid loaded on the surface of viscoelastic waveguide on Love waves. Love waves are suitable for sensing in liquids to measure the mass density, viscosity and acoustronic properties of liquids [178]. At the same time, Love waves can also be widely used in biochemical fields such as cell monitoring [179] and biosensing [180].

In this Chapter, the propagation characteristics of SH-type waves in layered half-space composed of PSs and PMs are studied. First, for the PS layer/PM half-space composite structure, the basic equations and phase velocity equations are obtained based on the coupling theory, and the dispersion equations in the complex domain are obtained by combining the boundary conditions. The effects of the initial carrier concentration, cover thickness and material properties on the SH-waves speed and attenuation are numerically calculated. Second, considering the PM layer/PS half-space, the open and short circuit boundary conditions, the first three modes of Love waves in the combination of Terfenol-D/ZnO and Terfenol-D/CdSe are obtained, and the effects of initial carrier concentration, PM layer thickness and material properties on the different modes of Love waves are analyzed.

## 2.1 SH waves in a piezoelectric semiconductor thin cover on the piezomagnetic half-space

In this Section, we first consider the PS layer/PM half-space, aiming to reveal the SH waves propagation in MCSs, and obtain the distribution of electromechanical fields through the thickness.

### 2.1.1 Basic equations

Consider a layered composite structure shown in Fig. 2.1, where a thin layer PS with thickness  $h$  is perfectly bonded on a PM half-space substrate. The PS thin cover and PM substrate are both assumed to be transversely isotropic with their polarization directions along the  $x_3$ -axis. The plane strain waves propagate in the positive  $x_1$ -axis.

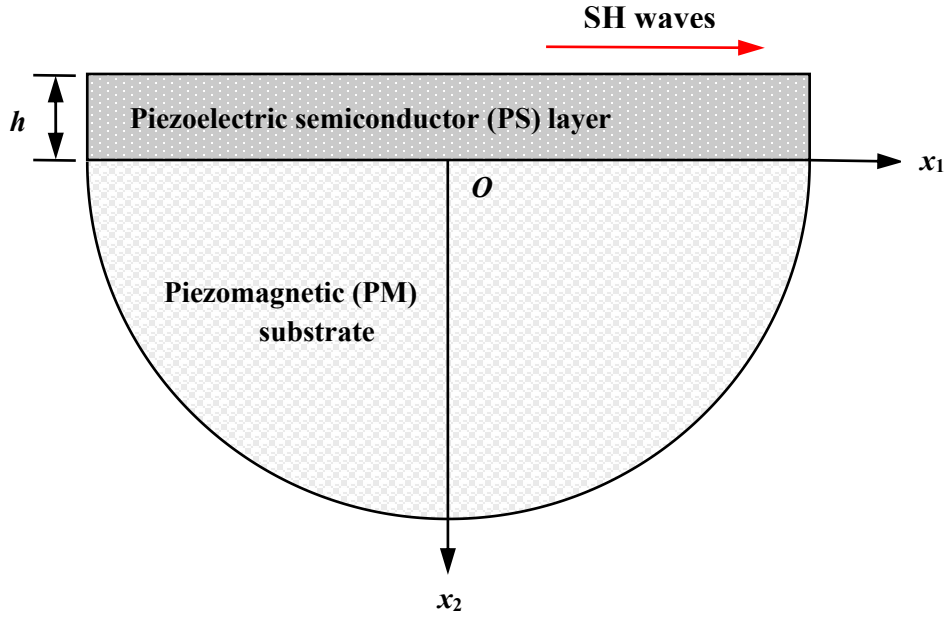


Fig. 2.1 The PS/PM half-space layered composite structure

For the small carrier perturbations in the n-type PS layer, the electron concentration can be represented linearly as  $n = n_0 + \Delta n$ , so it can be solved by using the linearization theory of Eqs. (1.4) and (1.5).

The anti-plane displacement components, electric potential, magnetic potential and perturbed carrier concentrations are assumed as:

$$\begin{aligned} u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2, t) \\ \varphi = \varphi(x_1, x_2, t), \quad \psi = \psi(x_1, x_2, t) \\ \Delta n = \Delta n(x_1, x_2, t) \end{aligned} \quad (2.1)$$

where  $u_1$ ,  $u_2$  and  $u_3$  are the elastic displacement components,  $\varphi$  is the electric potential, and  $\psi$  the magnetic potential.

The strain-displacement relation and field-potential relations are:

$$\begin{aligned} S_{ij} &= (u_{i,j} + u_{j,i})/2 \\ E_i &= -\varphi_{,i}, \quad H_i = -\psi_{,i} \end{aligned} \quad (2.2)$$

For a linear transversely isotropic PS and PM polarized along  $x_3$ -axis, the material constant matrix can be expressed as:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{31} & c_{31} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, [e] = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T, [\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \quad (2.3)$$

where  $c_{66} = (c_{11} - c_{12})/2$ . The material constant matrix structure  $f_{ijk}$  is the same as  $e_{ijk}$ , and  $g_{ij}$ ,  $\alpha_{ij}$ ,  $\mu_{ij}$  and  $D_{ij}$  are the same as  $\varepsilon_{ij}$ .

By substituting Eqs. (2.1) and (2.2) to the linearized coupled constitutive equation (1.5), combined with the material constants, the following non-zero equations can be obtained:

$$\begin{aligned} T_{13} &= c_{44} \frac{\partial u_3}{\partial x_1} + e_{15} \frac{\partial \varphi}{\partial x_1} + f_{15} \frac{\partial \psi}{\partial x_1}, \quad T_{23} = c_{44} \frac{\partial u_3}{\partial x_2} + e_{15} \frac{\partial \varphi}{\partial x_2} + f_{15} \frac{\partial \psi}{\partial x_2} \\ D_1 &= e_{15} \frac{\partial u_3}{\partial x_1} - \varepsilon_{11} \frac{\partial \varphi}{\partial x_1} - g_{11} \frac{\partial \psi}{\partial x_1}, \quad D_2 = e_{15} \frac{\partial u_3}{\partial x_2} - \varepsilon_{11} \frac{\partial \varphi}{\partial x_2} - g_{11} \frac{\partial \psi}{\partial x_2} \\ B_1 &= f_{15} \frac{\partial u_3}{\partial x_1} - g_{11} \frac{\partial \varphi}{\partial x_1} - \alpha_{11} \frac{\partial \psi}{\partial x_1}, \quad B_2 = f_{15} \frac{\partial u_3}{\partial x_2} - g_{11} \frac{\partial \varphi}{\partial x_2} - \alpha_{11} \frac{\partial \psi}{\partial x_2} \\ J_1^n &= -qn_0 \mu_{11}^n \frac{\partial \varphi}{\partial x_1} + qD_{11}^n \frac{\partial \Delta n}{\partial x_1}, \quad J_2^n = -qn_0 \mu_{11}^n \frac{\partial \varphi}{\partial x_2} + qD_{11}^n \frac{\partial \Delta n}{\partial x_2} \end{aligned} \quad (2.4)$$

And then, by substituting the above Eq. (2.4) into the governing equations (1.4), the coupled wave equations of mechanical displacement  $u_3$ , electric potential  $\varphi$ , magnetic potential  $\psi$ , and disturbed carrier concentrations  $\Delta n$  can be obtained:

$$\begin{aligned} c_{44} \nabla^2 u_3 + e_{15} \nabla^2 \varphi + f_{15} \nabla^2 \psi &= \rho \ddot{u}_3 \\ e_{15} \nabla^2 u_3 - \varepsilon_{11} \nabla^2 \varphi - g_{11} \nabla^2 \psi &= -q \Delta n \\ f_{15} \nabla^2 u_3 - g_{11} \nabla^2 \varphi - \alpha_{11} \nabla^2 \psi &= 0 \\ -n_0 \mu_{11}^n \nabla^2 \varphi + D_{11}^n \nabla^2 (\Delta n) &= \Delta \dot{n} \end{aligned} \quad (2.5)$$

where  $\nabla^2 = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2$  is the 2D Laplace operator in Cartesian coordinates.

The dielectric and permeability constants of air are much smaller than those of PS and PM, so air can be treated as vacuum. The electric potential and magnetic potential in vacuum satisfy the Laplace's equations:

$$\nabla^2 \varphi^0 = 0, \quad \nabla^2 \psi^0 = 0 \quad (2.6)$$

The second component of electric displacement  $D_2^0$  and magnetic flux intensity  $B_2^0$  in vacuum meet the following requirements:



$$D_2^0 = -\varepsilon_0 \frac{\partial \varphi^0}{\partial x_2}, \quad B_2^0 = -\mu_0 \frac{\partial \psi^0}{\partial x_2} \quad (2.7)$$

where the superscript 0 represents the physical quantities in vacuum.  $\varepsilon_0 = 8.854 \times 10^{-12}$  F/m is the dielectric constant in vacuum,  $\mu_0 = 4\pi \times 10^{-7}$  H/m is the permeability of vacuum.

The displacement, electric potential and magnetic potential in PM half-space along positive  $x_2$ -axis which satisfy the radiation conditions:

$$x_2 \rightarrow +\infty, \quad u_3^m = 0, \quad \varphi^m = 0, \quad \psi^m = 0 \quad (2.8)$$

where the superscript  $m$  represents the physical quantities in substrate. The radiation conditions in vacuum are:

$$x_2 \rightarrow -\infty, \quad \varphi^0 = 0, \quad \psi^0 = 0 \quad (2.9)$$

The top surface of the PS cover is traction-free and open to the vacuum. For piezoelectric materials with electrical conductivity, as pointed out by Tiersten and Sham [181], a jump condition containing the current term must be added to describe discontinuity arising from the real current. Therefore, the mechanical, electrical and magnetic boundary conditions of the upper surface are:

$$\begin{aligned} T_{23}(x_1, -h) &= 0 \\ \varphi(x_1, -h) &= \varphi^0(x_1, -h), \quad \psi(x_1, -h) = \psi^0(x_1, -h) \\ \dot{D}_2(x_1, -h) + J_2(x_1, -h) &= \dot{D}_2^0(x_1, -h) \end{aligned} \quad (2.10)$$

It is assumed that the perturbed carrier concentrations disappear at the upper and lower surfaces of the PS layer, which is an additional boundary condition compared to the piezoelectric insulation material, namely:

$$\Delta n(x_1, -h) = 0, \quad \Delta n(x_1, 0) = 0 \quad (2.11)$$

At the interface between the PS layer and the PM half-space ( $x_2 = 0$ ), the displacement, stress, potential potential, magnetic potential and current should also satisfy the following continuity conditions:

$$\begin{aligned} u_3(x_1, 0) &= u_3^m(x_1, 0), \quad T_{23}(x_1, 0) = T_{23}^m(x_1, 0) \\ \varphi(x_1, 0) &= \varphi^m(x_1, 0), \quad \psi(x_1, 0) = \psi^m(x_1, 0) \\ \dot{D}_2(x_1, 0) + J_2(x_1, 0) &= \dot{D}_2^m(x_1, 0) \end{aligned} \quad (2.12)$$

### 2.1.2 Solutions of the problem

For the SH waves propagating in the direction  $x_1$ -axis, we can assume that the trial solutions of the Eq. (2.5) are:

$$\begin{aligned} u_3(x_1, x_2, t) &= U_3(x_2) \exp[ik(x_1 - ct)] \\ \varphi(x_1, x_2, t) &= \Phi(x_2) \exp[ik(x_1 - ct)] \\ \psi(x_1, x_2, t) &= \Psi(x_2) \exp[ik(x_1 - ct)] \\ \Delta n(x_1, x_2, t) &= \Delta N(x_2) \exp[ik(x_1 - ct)] \end{aligned} \quad (2.13)$$

where  $i$  is an imaginary number,  $k$  is the wave number and  $c$  the wave speed.  $U_3(x_2)$ ,  $\Phi(x_2)$ ,  $\psi(x_2)$ , and  $\Delta N(x_2)$  are the amplitudes of displacement, electrical potential, magnetic potential and perturbed carrier concentration, respectively. We assume that the solution to the above amplitudes are:

$$\begin{aligned} U_3(x_2) &= A_3 \exp(-k\eta x_2), \quad \Phi(x_2) = A_\Phi \exp(-k\eta x_2) \\ \Psi(x_2) &= A_\Psi \exp(-k\eta x_2), \quad \Delta N(x_2) = A_N \exp(-k\eta x_2) \end{aligned} \quad (2.14)$$

where  $A_3$ ,  $A_\Phi$ ,  $A_\Psi$  and  $A_N$  are undetermined coefficients,  $\eta$  represents the wave attenuation rate at the interface  $x_2 = 0$ . By substituting Eqs. (2.13)-(2.14) into Eq. (2.5) and eliminating the common exponential factor, the homogeneous linear equations with the undetermined coefficients  $A_3$ ,  $A_\Phi$ ,  $A_\Psi$  and  $A_N$  can be obtained as:

$$\begin{aligned} [c_{44}(\eta^2 - 1) + \rho c^2] A_3 + e_{15}(\eta^2 - 1) A_\Phi + f_{15}(\eta^2 - 1) A_\Psi &= 0 \\ e_{15}k^2(\eta^2 - 1) A_3 - \varepsilon_{11}k^2(\eta^2 - 1) A_\Phi - g_{11}k^2(\eta^2 - 1) A_\Psi + qA_N &= 0 \\ f_{15}k^2(\eta^2 - 1) A_3 - g_{11}k^2(\eta^2 - 1) A_\Phi - \alpha_{11}k^2(\eta^2 - 1) A_\Psi &= 0 \\ n_0\mu_{11}^n k(\eta^2 - 1) A_\Phi - [D_{11}^n k(\eta^2 - 1) + ic] A_N &= 0 \end{aligned} \quad (2.15)$$

For the PS cover, the constants  $f_{ijk}$  and  $g_{ij}$  are zero, so the above Eq. (2.15) can be simplified accordingly. It is observed that the simplified third equation  $\alpha_{11}k^2(\eta^2 - 1)A_\Psi = 0$  can be decoupled from the other three equations. Since  $A_\Psi \neq 0$ , we have  $\eta = \pm 1$  from the decoupled equation.

In addition, we take the determinant of the  $3 \times 3$  wave amplitude coefficient matrix. The condition for the existence of non-trivial solutions for  $A_3$ ,  $A_\Phi$  and  $A_N$  is that the determinant of their coefficient matrix equals to zero. For a given wave speed  $c$ , six undetermined coefficients  $\eta$  can be obtained. These six roots include two pairs of complex roots and  $\eta = \pm 1$ . The values of  $\eta_j$  ( $j = 1-6$ ) will depend on wave speed  $c$  and wave number  $k$ .

Therefore, the amplitude equation (2.14) for the PS layer can be rewritten as:

$$\begin{aligned}
U_3(x_2) &= \sum_{j=1}^6 \beta_{1j} A_{\Phi j} \exp(-k\eta_j x_2) \\
\Phi(x_2) &= \sum_{j=1}^6 A_{\Phi j} \exp(-k\eta_j x_2) \\
\Psi(x_2) &= \sum_{j=7}^8 A_{\Phi j} \exp(-k\eta_j x_2) \\
\Delta N(x_2) &= \sum_{j=1}^6 \beta_{2j} A_{\Phi j} \exp(-k\eta_j x_2)
\end{aligned} \tag{2.16}$$

where the subscript  $j$  ranges from 1 to 8,  $A_{\Phi j}$  is the unknown amplitude,  $\beta_{1j}$  and  $\beta_{2j}$  are the amplitude ratios which can be determined by the simplified Eq. (2.15):

$$\begin{aligned}
\beta_{1j} &= -\frac{e_{15}(\eta_j^2 - 1)}{c_{44}(\eta_j^2 - 1) + \rho c^2} \\
\beta_{2j} &= \frac{k^2}{q}(\eta_j^2 - 1)(\varepsilon_{11} - e_{15}\beta_{1j})
\end{aligned} \tag{2.17}$$

Similar to the PS layer, the constants  $e_{ijk}$  and  $g_{ij}$  are zero in the PM substrate. The semiconductor related parameters in the PM half-space will also do not exist, then the fourth equation of semiconductor in Eq. (2.15) will disappear:

$$\begin{aligned}
\left[ c_{44}^m(\eta^{m^2} - 1) + \rho^m c^2 \right] A_3^m + f_{15}(\eta^{m^2} - 1) A_\Psi^m &= 0 \\
\varepsilon_{11}^m k^2(\eta^{m^2} - 1) A_\Phi^m &= 0 \\
f_{15} k^2(\eta^{m^2} - 1) A_3^m - \alpha_{11}^m k^2(\eta^{m^2} - 1) A_\Psi^m &= 0
\end{aligned} \tag{2.18}$$

In the homogeneous linear equation (2.18), the second equation is directly decoupled from the other two equations to yield  $\eta^m = \pm 1$ . We need choose  $\eta^m = 1$  because of the attenuation characteristic at infinity in the positive  $x_2$ -axis.

To obtain non-zero solutions, the determinant of the coefficients of  $A_3^m$  and  $A_\Psi^m$  in Eq. (2.18)<sub>1</sub> and Eq. (2.18)<sub>3</sub> must be zero. For a given wave speed  $c$ , we can get two pairs of complex roots that are opposite to each other. Similarly, the radiation condition must be met when  $x_2 \rightarrow +\infty$ , so only two  $\eta^m$  which are positive in the corresponding real part are retained (in fact, one of the roots is  $\eta^m = 1$ ).

The solutions of displacement, electric potential and magnetic potential in the PM substrate are:

$$\begin{aligned}
u_3^m &= \sum_{j=1}^2 \beta_{3j}^m A_{\Psi j}^m e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)} \\
\varphi^m &= A_{\Psi 3}^m e^{-kx_2} e^{ik(x_1 - ct)} \\
\psi^m &= \sum_{j=1}^2 A_{\Psi j}^m e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)}
\end{aligned} \tag{2.19}$$

where  $A_{\Psi_j}^m$  ( $j = 1-3$ ) are the amplitudes to be solved,  $\beta_{3j}^m$  the amplitude ratios which can be obtained from Eq. (2.18):

$$\beta_{3j}^m = \frac{A_{3j}^m}{A_{\Psi_j}^m} = -\frac{f_{15}(\eta_j^{m^2} - 1)}{c_{44}^m(\eta_j^{m^2} - 1) + \rho^m c^2} \quad (2.20)$$

The electric potential and magnetic potential in vacuum satisfy Laplace's equation (2.6), and their solutions are:

$$\begin{aligned} \varphi^0(x_1, x_2, t) &= \sum_{j=1}^6 A_{\Phi_j} e^{kx_2} e^{ik(x_1 - ct)} \\ \psi^0(x_1, x_2, t) &= \sum_{j=7}^8 A_{\Phi_j} e^{kx_2} e^{ik(x_1 - ct)} \end{aligned} \quad (2.21)$$

### 2.1.3 The phase velocity equations

By substituting Eqs. (2.13) and (2.16) into constitutive relations (2.4), the expressions of stress, electric displacement, magnetic flux intensity and current density in the PS cover can be obtained as (the term  $e^{-k\eta_j x_2} \cdot e^{ik(x_1 - ct)}$  is ignored for simplicity) :

$$\begin{aligned} T_{13} &= \sum_{j=1}^6 ik(c_{44}\beta_{1j} + e_{15})A_{\Phi_j}, \quad T_{23} = \sum_{j=1}^6 -k\eta_j(c_{44}\beta_{1j} - e_{15})A_{\Phi_j} \\ D_1 &= \sum_{j=1}^6 ik(e_{15}\beta_{1j} - \varepsilon_{11})A_{\Phi_j}, \quad D_2 = \sum_{j=1}^6 k\eta_j(-e_{15}\beta_{1j} + \varepsilon_{11})A_{\Phi_j} \\ B_1 &= -ik\alpha_{11} \sum_{j=7}^8 A_{\Phi_j}, \quad B_2 = k\alpha_{11} \sum_{j=7}^8 \eta_j A_{\Phi_j} \\ J_1^n &= \sum_{j=1}^6 ikq(-n_0\mu_{11}^n + D_{11}^n\beta_{2j})A_{\Phi_j}, \quad J_2^n = \sum_{j=1}^6 kq\eta_j(n_0\mu_{11}^n - D_{11}^n\beta_{2j})A_{\Phi_j} \end{aligned} \quad (2.22)$$

In the PS materials, electric polarization  $P$  and effective polarization charge density  $\rho^P$  are also important electromechanical fields. They can be gotten as:

$$\begin{aligned} P &= P_2 = D_2 - \varepsilon_0 E_2 = \sum_{j=1}^6 k\eta_j[-e_{15}\beta_{1j} + (\varepsilon_{11} - \varepsilon_0)]A_{\Phi_j} \\ \rho^P &= -P_{2,2} = \sum_{j=1}^6 k^2 \eta_j^2 [-e_{15}\beta_{1j} + (\varepsilon_{11} - \varepsilon_0)]A_{\Phi_j} \end{aligned} \quad (2.23)$$

Similarly, by substituting Eq. (2.19) into Eq. (2.4), the expressions of stress, electric displacement and magnetic flux intensity in the PM substrate can be obtained as follows (the term  $e^{ik(x_1 - ct)}$  is ignored for simplicity) :

$$\begin{aligned}
T_{13}^m &= \sum_{j=1}^2 ik \left( c_{44}^m \beta_{3j}^m + f_{15} \right) e^{-k\eta_j^m x_2} A_{\Psi_j}^m, \quad T_{23}^m = \sum_{j=1}^2 -k\eta_j^m \left( c_{44}^m \beta_{3j}^m + f_{15} \right) e^{-k\eta_j^m x_2} A_{\Psi_j}^m \\
D_1^m &= -ik\epsilon_{11}^m e^{-kx_2} A_{\Psi_3}^m, \quad D_2^m = k\epsilon_{11}^m e^{-kx_2} A_{\Psi_3}^m \\
B_1^m &= \sum_{j=1}^2 ik \left( f_{15} \beta_{3j}^m - \alpha_{11}^m \right) e^{-k\eta_j^m x_2} A_{\Psi_j}^m, \quad B_2^m = \sum_{j=1}^2 k\eta_j^m \left( -f_{15} \beta_{3j}^m + \alpha_{11}^m \right) e^{-k\eta_j^m x_2} A_{\Psi_j}^m
\end{aligned} \tag{2.24}$$

According to Eqs. (2.7) and (2.21), the electric displacement and magnetic flux intensity in vacuum can be obtained as (the term  $e^{kx_2} \cdot e^{ik(x_1-ct)}$  is ignored for simplicity) :

$$D_2^0 = -\epsilon_0 k \sum_{j=1}^6 A_{\Phi_j}, \quad B_2^0 = -\mu_0 k \sum_{j=7}^8 A_{\Phi_j} \tag{2.25}$$

With the solutions in Eqs. (2.13), (2.16) and (2.19), physical quantities expressions (2.22)-(2.25), and then using the boundary conditions and continuous conditions (2.10)-(2.12), we finally obtain homogeneous linear equations with respect to unknown amplitudes  $A_{\Phi_j}$  ( $j = 1-8$ ) and  $A_{\Psi_j}^m$  ( $j = 1-3$ ) as:

$$[M]N = 0 \tag{2.26}$$

where  $N = \{ A_{\Phi_1}, A_{\Phi_2}, A_{\Phi_3}, A_{\Phi_4}, A_{\Phi_5}, A_{\Phi_6}, A_{\Phi_7}, A_{\Phi_8}, A_{\Psi_1}^m, A_{\Psi_2}^m, A_{\Psi_3}^m \}^T$ , and  $[M]$  is a coefficient matrix of  $11 \times 11$ . The nonzero elements of  $[M]$  are:

$$\begin{aligned}
M_{1j} &= (c_{44}\beta_{1j} + e_{15})\eta_j e^{k\eta_j}, \quad M_{2j} = e^{k\eta_j} - e^{-kh} \\
M_{3,7} &= e^{k\eta_7} - e^{-kh}, \quad M_{3,8} = e^{k\eta_8} - e^{-kh} \\
M_{4j} &= (e_{15}\beta_{1j} - \epsilon_{11})ikc\eta_j e^{k\eta_j} + (n_0\mu_{11}^n - D_{11}^n\beta_{2j})q\eta_j e^{k\eta_j} - ikc\epsilon_0 e^{-kh} \\
M_{5j} &= \beta_{2j} e^{k\eta_j}, \quad M_{6j} = \beta_{2j}, \quad M_{7j} = \beta_{1j}, \quad M_{7,9} = -\beta_{31}^m, \quad M_{7,10} = -\beta_{32}^m \\
M_{8j} &= c_{44}\eta_j\beta_{1j} + e_{15}\eta_j, \quad M_{8,9} = -(c_{44}\eta_1^m\beta_{31}^m + f_{15}\eta_1^m) \\
M_{8,10} &= -(c_{44}\eta_2^m\beta_{32}^m + f_{15}\eta_2^m), \quad M_{9j} = 1, \quad M_{9,11} = -1 \\
M_{10,7} &= M_{10,8} = 1, \quad M_{10,9} = M_{10,10} = -1 \\
M_{11j} &= (ikce_{15}\beta_{1j} - ikc\epsilon_{11} + qn_0\mu_{11}^n - qD_{11}^n\beta_{2j})\eta_j, \quad M_{11,11} = ikc\epsilon_{11}^m
\end{aligned} \tag{2.27}$$

where  $j = 1-6$ . To obtain the nonzero solutions of Eq. (2.27), the determinant of matrix must be zero, that is  $\text{Det}[M] = 0$ .

Hence, transcendental equations for phase velocity  $c$  and wave number  $k$  can be obtained. The obtained wave velocity will be complex numbers, with the real part represents the phase velocity which is the real propagation speed. The imaginary part represents the gain or attenuation coefficient. If imaginary part greater than zero, the wave gain occurs, otherwise the attenuation happens.

### 2.1.4 Numerical results and discussions

Based on the dispersion equation, the propagation characteristics of SH waves in the composite structure composed of a ZnO PS thin layer and a CoFe<sub>2</sub>O<sub>4</sub> PM half-space can be obtained by numerical calculation, including the dispersion and attenuation of SH waves and the electromechanical fields through the thickness. The material properties of ZnO [79,80] are shown in Table 2.1, and the material properties of CoFe<sub>2</sub>O<sub>4</sub> [182,183] are shown in Table 2.2.

In the following numerical examples, normalized wave speed  $c/c_{SH}^m$ , and normalized wave number  $kh$  are used, where  $c_{SH}^m$  represents the propagation velocity of shear bulk wave of PM material. Unless otherwise specified in the calculations, it is assumed that  $h = 100 \mu\text{m}$  and  $n_0 = 5 \times 10^{16} \text{ m}^{-3}$ .

Table 2.1 Material properties of the PS layer-ZnO<sup>[79,80]</sup>

| $c_{44}$<br>( $\times 10^9 \text{ N/m}^2$ ) | $e_{15}$<br>( $\text{C/m}^2$ ) | $\varepsilon_{11}$<br>( $\times 10^{-11} \text{ C}^2/\text{Nm}^2$ ) | $\mu_{11}^n$<br>( $\text{m}^2/\text{V}$ ) | $D_{11}^n$<br>( $\text{m}^2/\text{s}$ ) | $\rho$<br>( $\text{kg/m}^3$ ) |
|---------------------------------------------|--------------------------------|---------------------------------------------------------------------|-------------------------------------------|-----------------------------------------|-------------------------------|
| 43                                          | -0.48                          | 7.61                                                                | 1                                         | 0.026                                   | 5700                          |

Table 2.2 Material properties of PM substrate-CoFe<sub>2</sub>O<sub>4</sub><sup>[182,183]</sup>

| $c_{44}^m$<br>( $\times 10^9 \text{ N/m}^2$ ) | $f_{15}$<br>( $\text{N/Am}$ ) | $\varepsilon_{11}^m$<br>( $\times 10^{-11} \text{ C}^2/\text{Nm}^2$ ) | $\alpha_{11}^m$<br>( $\times 10^{-6} \text{ N s}^2/\text{C}^2$ ) | $\rho^m$<br>( $\text{kg/m}^3$ ) |
|-----------------------------------------------|-------------------------------|-----------------------------------------------------------------------|------------------------------------------------------------------|---------------------------------|
| 45.3                                          | 550                           | 8                                                                     | 157                                                              | 5300                            |

#### 2.1.4.1 Propagation characteristics of SH waves in composite structures

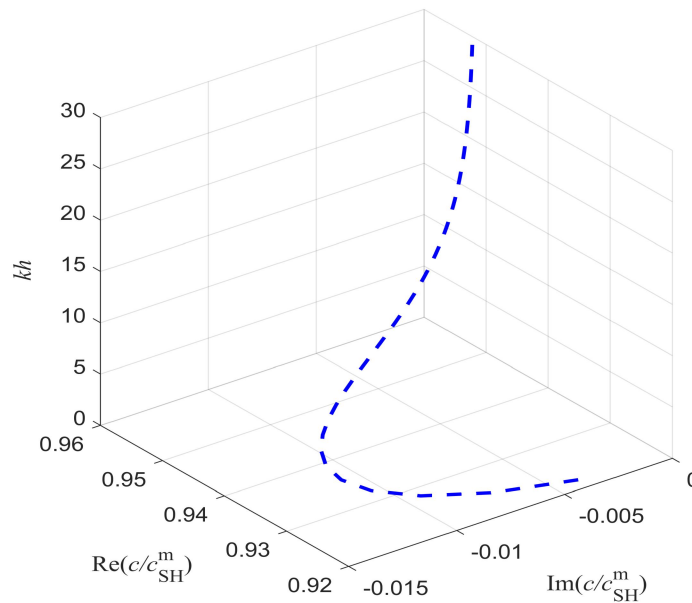


Fig. 2.2 The dispersion and attenuation 3D curves of SH waves in ZnO/CoFe<sub>2</sub>O<sub>4</sub> layered half-space

The wave speed of SH waves in ZnO PS layer/CoFe<sub>2</sub>O<sub>4</sub> PM half-space substrate is complex, so the real and imaginary parts must be discussed separately. In the numerical examples, the 3D curve of SH waves in the composite structure as shown in Fig. 2.2 is obtained firstly. The vertical coordinate  $kh$  is normalized wave number, the  $\text{Re}(c/c_{\text{SH}}^{\text{m}})$  and  $\text{Im}(c/c_{\text{SH}}^{\text{m}})$  on the bottom represent the real part and imaginary part of the wave speed, respectively. As shown in the Fig. 2.2, SH wave attenuates as the imaginary parts are negative.

Further, the 3D curve is placed on the plane to better observe the changes. Figure 2.3 shows the real and imaginary parts of the phase velocity. In this case, the thickness of PS layer  $h = 100 \mu\text{m}$  and the initial carrier concentration  $n_0 = 5 \times 10^{16} \text{ m}^{-3}$  are also fixed.

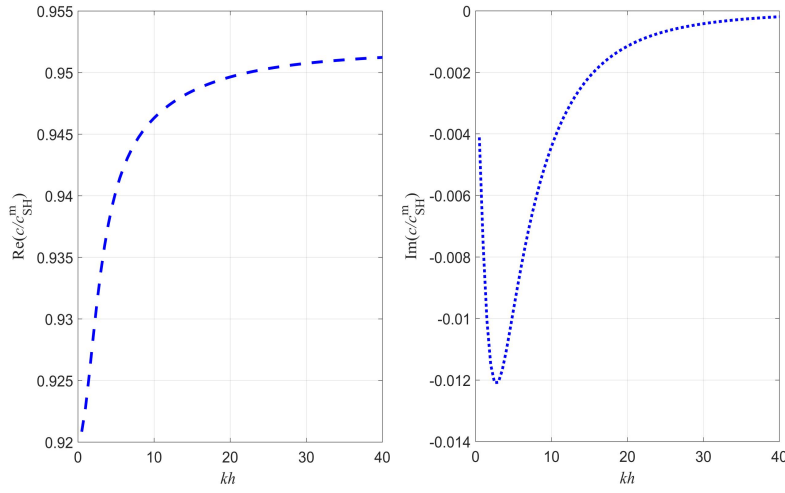


Fig. 2.3 The real part of phase velocity (dispersion curve) and imaginary part (attenuation curve)

Figures 2.2 and 2.3 illustrate that the influence of semiconductor properties cannot be ignored when studying SAWs in PSs, as they cause the waves to be dispersed and attenuated. In fact, the propagation of elastic waves in PSs not only causes the change of mechanical and electric fields, but also causes the carriers to move. The currents and space charges produced by these fields lead to the dispersion and acoustic loss [31]. Furthermore, carriers migration in PSs will reduce the piezoelectric potential, resulting in additional energy loss and wave attenuation [84].

As can be seen from Fig. 2.4(a), phase velocity decreases with the increase of  $n_0$ . This result is consistent with the surface waves in the existing pure PSs [79-81] and PS layered composite structures [106,107], which is easy to understand: the increase of carrier concentration can enhance the damping of surface waves, and the interaction between SAWs and carriers will result in energy dissipation, leading to a decrease in the wave amplitude and thus the decrease of phase velocity; in semiconductors, charge carriers can screen the electric field associated with SAWs, and the screening effect becomes more pronounced as carrier

concentration increases. In addition, there are other effects collectively contribute to the decrease of wave speed with the increase of initial carrier concentration  $n_0$ .

Figure 2.4(b) shows that when the wave in the long wave (i.e. the wave number is small), the attenuation will strengthened to the maximum continuously. As the wave number continues to increase, the attenuation will become weaker, and the lowest point of the attenuation will shift to the right with the increase of  $n_0$ .

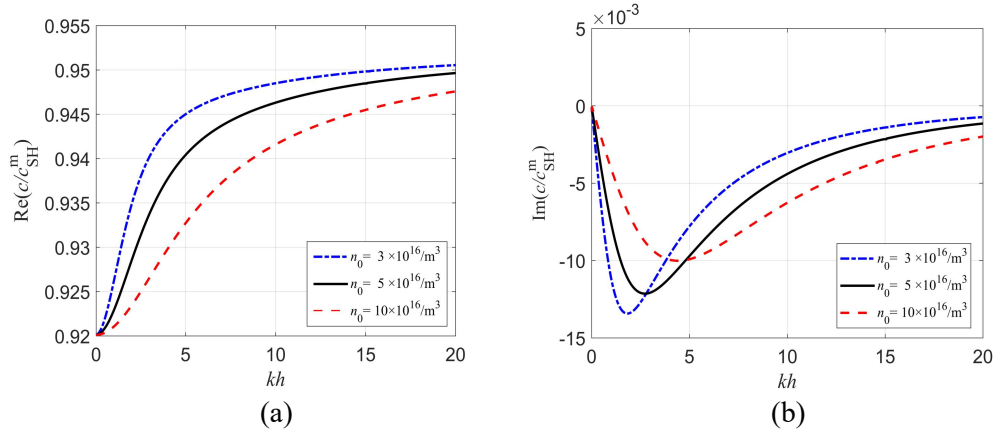


Fig. 2.4 Effect of  $n_0$  on SH wave propagation characteristics: (a) effect of  $n_0$  on phase velocity, and (b) effect of  $n_0$  on attenuation

It is observed that the phase velocity and attenuation depend on the PS plate thickness significantly from Fig. 2.5. Actually, different from SH waves in the corresponding piezoelectric (PE) plate which is independent of plate thickness, the Love waves in PS thin film/elastic half-space [106] and SH waves in PS plate [82] have been proven to be related to the thickness of PS. The difference in the behavior of SH waves in PE and PS materials concerning thickness dependence is primarily due to the additional complexities introduced by the semiconductor properties in the latter. In traditional PE plates, the dominant factor influencing the wave velocity is the material properties (e.g., elastic and PE constants) and the electromechanical coupling, rather than the thickness of the plate. In the case of PSs, the behavior can be more complex.

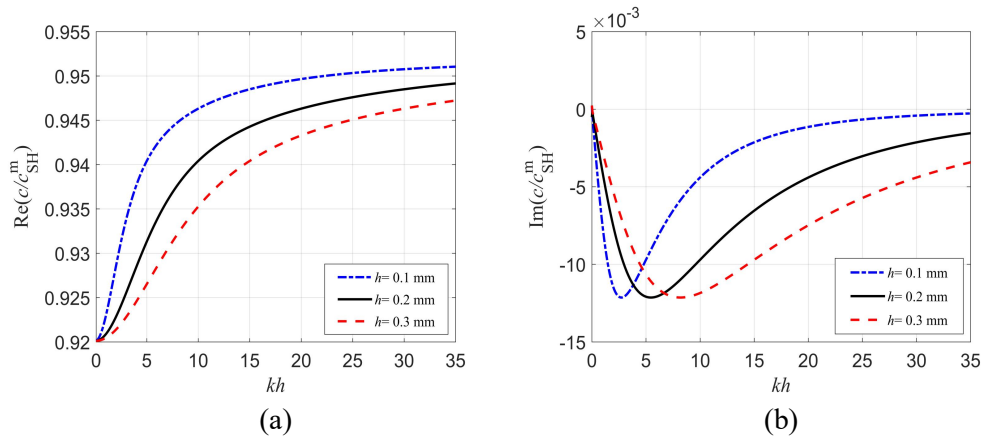


Fig. 2.5 Effect of plate thickness  $h$  on SH wave propagation characteristics: (a) effect of  $h$  on phase velocity, and (b) effect of  $h$  on attenuation



With the increase of  $h$ , the phase velocity of SH wave drops obviously according to Fig. 2.5(a). This is due to the fact that as  $h$  increases, the interaction between the waves and carriers (holes and electrons) becomes more significant, leading to changes in dispersion characteristics, including a reduction in the phase velocity. Besides, from a more understandable perspective, the thicker the PS layer means that there will be relatively more carriers transport behavior, which will cause more energy loss, resulting in a decrease in phase velocity and a shorter wave propagation distance.

The variation of attenuation in Fig. 2.5(b) is not monotonous like in Fig. 2.4(b), it is initially increased to the maximum and subsequently decreased.

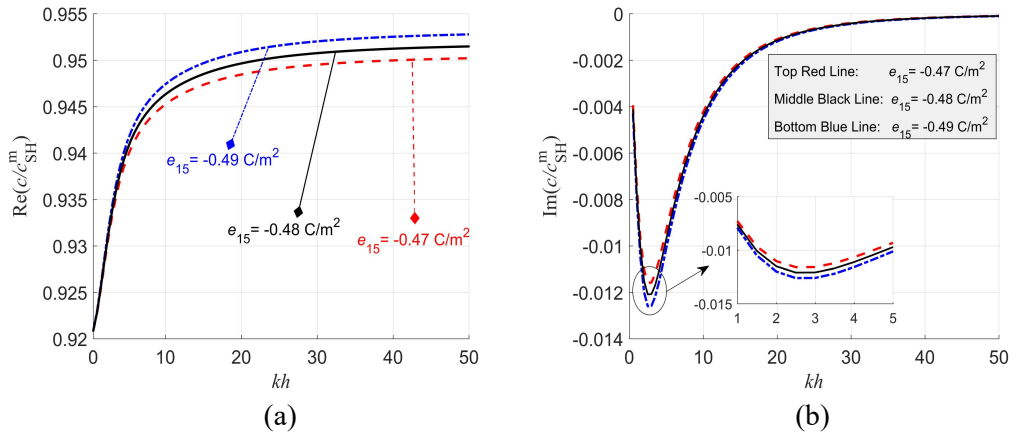


Fig. 2.6 Effect of piezoelectric constant  $e_{15}$  on SH wave propagation characteristics: (a) effect of  $e_{15}$  on phase velocity, and (b) effect of  $e_{15}$  on attenuation

Figure 2.6 show that the effect on the real part is bigger than the effect on the imaginary part. Among the three  $e_{15}$ ,  $-0.48 \text{ C/m}^2$  is the real PE constant of ZnO. In fact, if the material of the PS layer is replaced, other material properties will also change, not just the PE constants. It is important to note that there are various PE constants associated with different types of deformations, and the commonly used PE constants include:  $e_{31}$ ,  $e_{33}$  and  $e_{15}$ . Here, only the PE constant  $e_{15}$ , which characterizes the shear deformation response, is related to SH waves, so only  $e_{15}$  is changed to study its effect on wave propagation.

According to Fig. 2.6(a), we can see that the phase velocity of the SH wave increases with the increase of the absolute value of  $e_{15}$ . This is because a larger absolute value of  $e_{15}$  signifies a strong shear piezoelectric response. Mathematically, the shear bulk wave phase velocity of the PS layer is  $c_{\text{SH}} = \sqrt{(c_{44} + e_{15}^2 / \epsilon_{11}) / \rho}$ . Therefore, with the increase of the absolute value of  $e_{15}$ ,  $c_{\text{SH}}$  will also increase, which can be regarded as one of the important reasons for the increase of phase velocity in this structure.

It is conceivable that a suitable PS material (suitable  $e_{15}$  value) can be selected to achieve the regulation of SH waves.

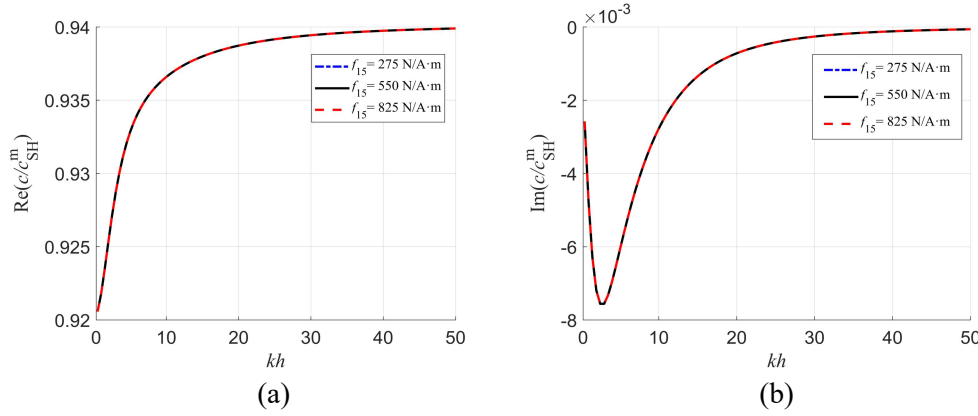


Fig. 2.7 Effect of piezomagnetic constant  $f_{15}$  on SH wave propagation characteristics: (a) effect of  $f_{15}$  on phase velocity, and (b) effect of  $f_{15}$  on attenuation

According to Fig. 2.7, it is found that when the value of PM constant  $f_{15}$  changed in substrate, the three curves coincide completely. There is almost no influence on the real or imaginary part of the SH wave. Among the three  $f_{15}$ , 550 N/A·m is the PM constant of  $\text{CoFe}_2\text{O}_4$ . As with the study of the effect of  $e_{15}$  above, the other two values are assumptions (not  $f_{15}$  values for real PM materials) for comparative study.

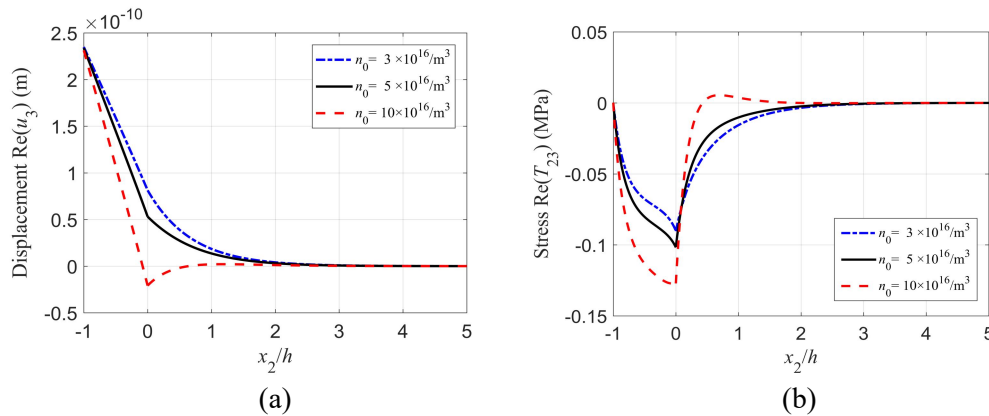
Combined with Figs. 2.4-2.7, it is not difficult to find that both the phase velocity and attenuation of SH wave change more apparent when  $kh$  is small (i.e. long wavelength). This is physically correct. In general, with long waves, the waves involve a larger scale range, so the propagation characteristics of the waves are also more susceptible to the material properties and external factors.

#### 2.1.4.2 Physical quantities distributed through the thickness

##### (1) The electromechanical fields (real part) distribution along thickness

Based on the dispersion relation, the electromechanical fields distribution through the thickness are obtained. Since the wave speeds are complex, all the physical quantities obtained are also complex domains. The distributions of the real and imaginary parts along thickness can be obtained, respectively.

The unknown amplitude  $A_{\Phi 1} = 10^{-3}$  m is assumed and  $kh$  is set to 5 in the calculations. Besides, normalized horizontal axis  $x_2/h$  is used in below figures. In the  $x$ -coordinate region,  $-1 \leq x_2 \leq 0$  represents the PS overlay, while  $x_2 \geq 0$  is the PM substrate.



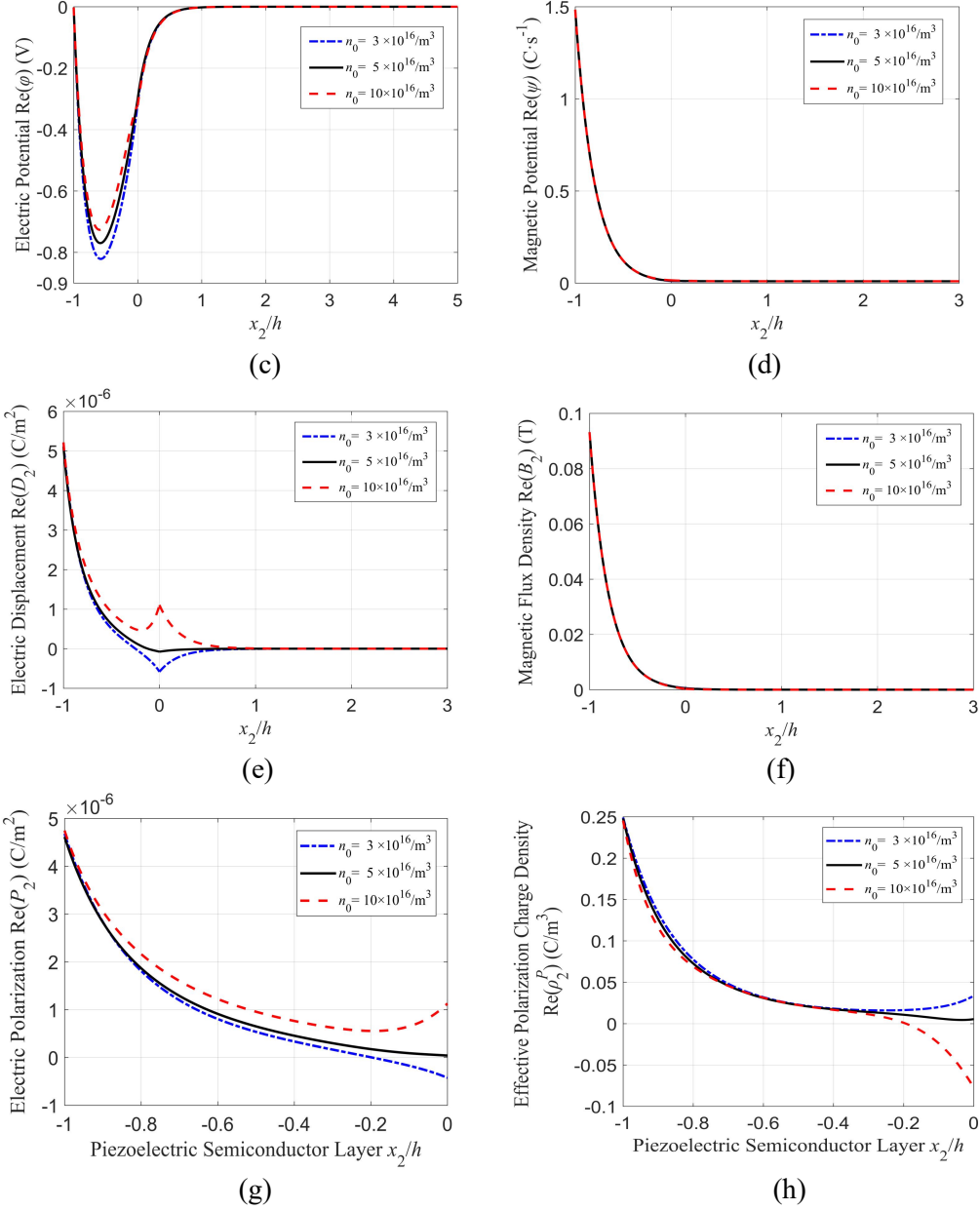


Fig. 2.8 Electromechanical fields (real part) distributed through the thickness at different  $n_0$  ( $kh = 5$ ): (a) displacement, (b) stress, (c) electric potential, (d) magnetic potential, (e) electric displacement, (f) magnetic flux density, (g) electric polarization, and (h) effective polarization charge density

In Figs. 2.8(a)-2.8(f), it can be seen that the displacement, stress, electric potential, magnetic potential, electric displacement and magnetic flux density gradually approach zero through the thickness, which are accord with the radiation conditions in the substrate. As shown in Fig. 2.8(b), the leftmost end of stress is zero, conforming to the traction-free condition of the top surface.

Figures 2.8(g) and 2.8(h) represent the electric polarization and effective polarized charge density distribution, respectively. Since the two physical quantities only exist in the PS layer, the PM substrate is not reflected and only the PS layer is drawn.

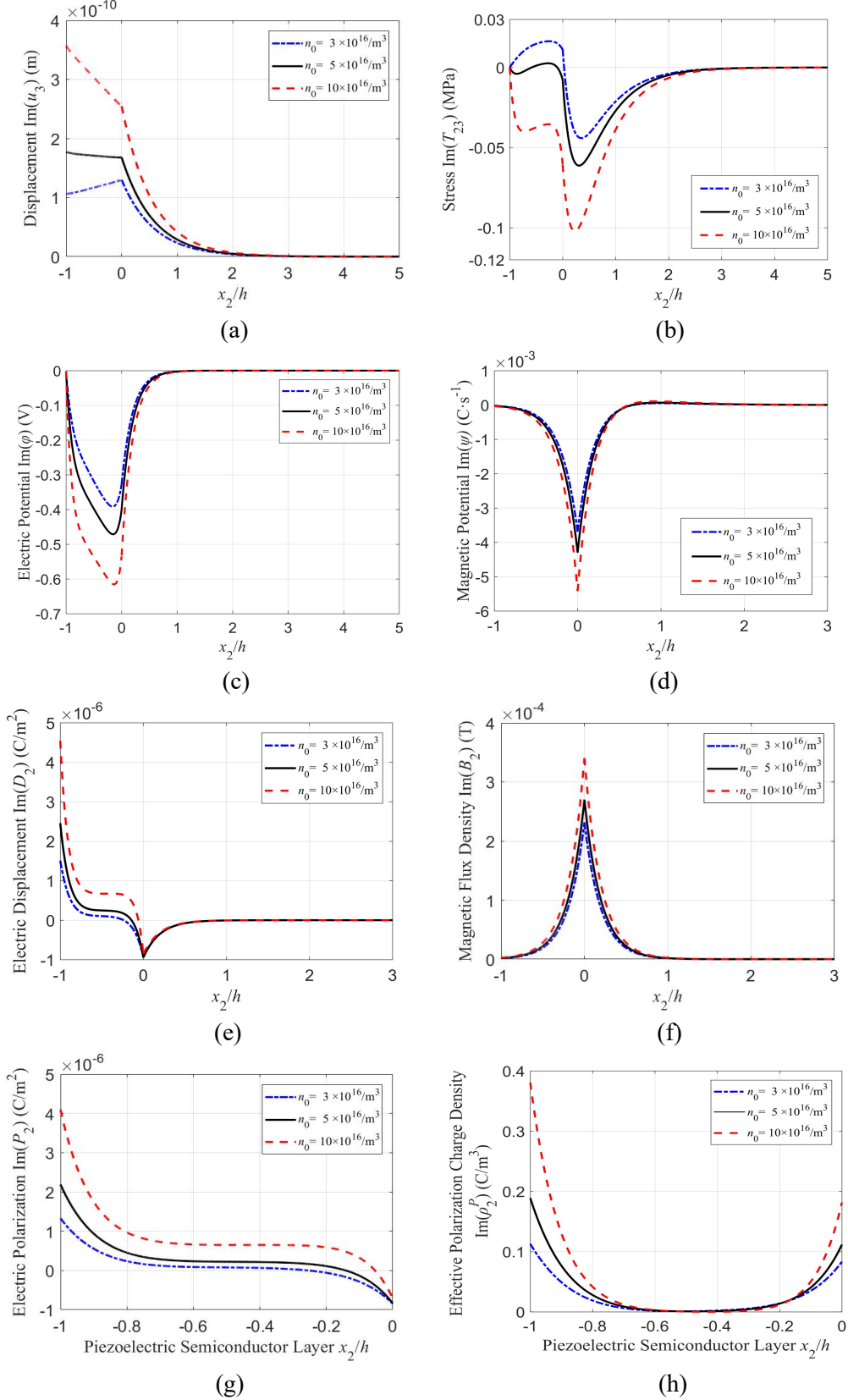


Fig. 2.9 Electromechanical fields (imaginary part) distributed through the thickness at different  $n_0$  ( $kh = 5$ ): (a) displacement, (b) stress, (c) electric potential, (d) magnetic potential, (e) electric displacement, (f) magnetic flux density, (g) electric polarization, and (h) effective polarization charge density

## (2) The electromechanical fields (imaginary part) distribution along thickness

The imaginary parts of displacement, stress, electric potential, magnetic potential, electric displacement and magnetic flux density in Figs. 2.9(a)-2.9(f) also tend to zero along the thickness similar to the real parts in Figs. 2.8(a)-2.8(f), which are also conform to the radiation condition.

Figure 2.9(a) appears yielding points at the contact interface, while stress, electric potential, magnetic potential, electric displacement and magnetic flux density in Figs. 2.9(b)-2.9(f) have their minimum values near the interface.

In Figs. 2.9(g)-2.9(h), the electric polarization and effective polarization charge density will increase with the increase of  $n_0$ . Meanwhile, the carriers in the PS layer are transported and redistributed.

Figures 2.8(d) and 2.9(d) represent the imaginary and real parts of magnetic potential, and Figs. 2.8(f) and 2.9(f) are the imaginary and real parts of magnetic flux intensity. As can be seen from the four figures,  $n_0$  has little influence on magnetic related physical quantities, especially on the real parts. This is because the carrier affects the electric field directly through semiconductor and PE coupling, while its influence on PM is indirect which needs to be realized through ME coupling effect.

## (3) Effects of piezoelectric and piezomagnetic constants on magnetic potential distribution

The influences of PE constant and PM constant on the SH waves are analyzed above. It is necessary to further study their influences on the magnetic potential distribution. Figures 2.10 and 2.11 show the distribution of magnetic potential through the thickness under different  $e_{15}$  and  $f_{15}$ , respectively.

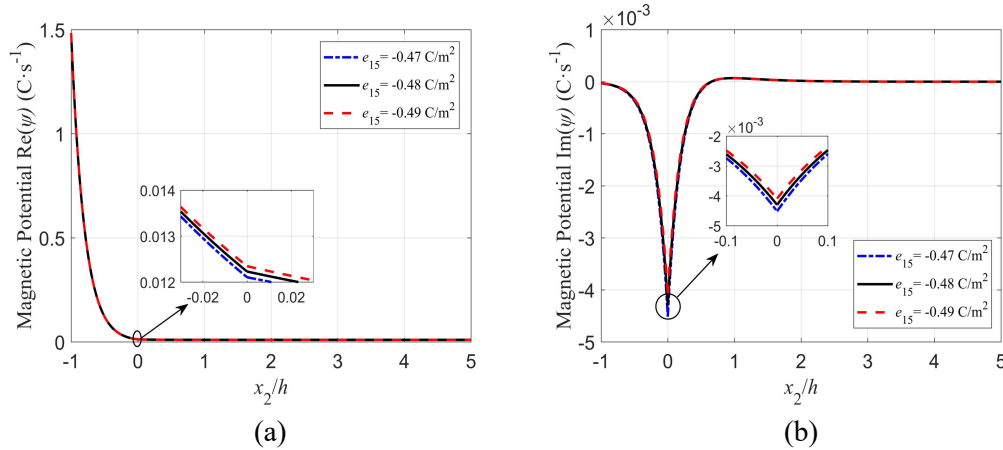


Fig. 2.10 Magnetic potential distributed through the thickness under different  $e_{15}$  ( $kh = 5$ ): (a) real part, and (b) imaginary part

From Fig. 2.10, it can be seen that the effect of  $e_{15}$  on the distribution of the real and imaginary parts of magnetic potential are very small and negligible, and the slight change can be only seen at the very close to the interface. This is because the  $e_{15}$  has a direct effect on the electric potential, while the effect on the magnetic potential is indirectly realized through the ME coupling effect.

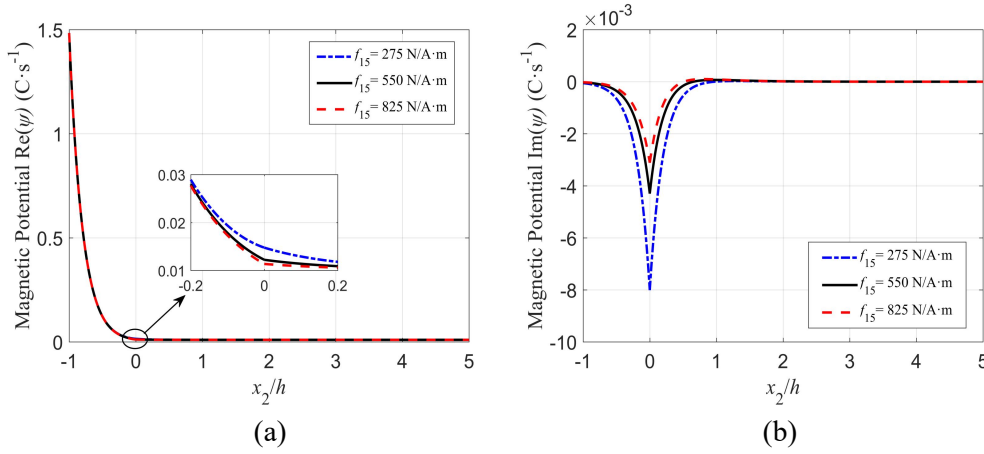


Fig. 2.11 Magnetic potential distributed through the thickness under different  $f_{15}$  ( $kh = 5$ ): (a) real part, and (b) imaginary part

As shown in Fig. 2.11(a), the  $f_{15}$  also has little influence on the real part of the magnetic potential like the effect of  $e_{15}$ . However, its influence on the imaginary part is obvious according to Fig. 2.11(b), which is quite different from Fig. 2.10(b). No matter how big  $f_{15}$  is, the real and imaginary parts tend to zero with depth and the imaginary part of magnetic potential on the upper surface is zero.

#### (4) The distribution of displacement at different times

The above electromechanical fields are all at the time of  $t = 0$ . Displacement is selected as an example to analyze the distribution at different times. By selecting time  $\omega t = 0$ ,  $\omega t = \pi/2$ ,  $\omega t = \pi$  and  $\omega t = 3\pi/2$ , the distribution of  $u_3$  at four different times can be obtained, respectively.

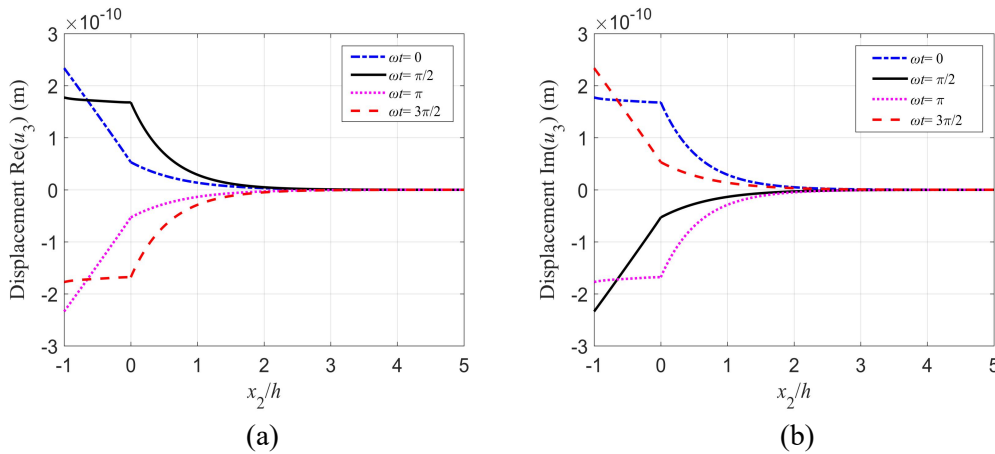


Fig. 2.12 Displacement distributed along the thickness for a different time  $\omega t$  ( $kh = 5$ ): (a) real part, and (b) imaginary part

It can be observed from Fig. 2.12 that both the real and imaginary parts of displacement are symmetric about the horizontal axis of  $\text{Re}(u_3) = 0$  at  $\omega t = 0$  and  $\omega t = \pi$ . The displacement distribution at  $\omega t = \pi/2$  and  $\omega t = 3\pi/2$  are similarly symmetric. In addition, it is found that the distribution of real and imaginary parts at  $\omega t = 2\pi$  coincide with  $\omega t = 0$ , indicating that the displacement is distributed periodically at  $2\pi$ . It should be pointed out that the distribution of other physical

quantities are also distributed with the period of  $2\pi$  which are not repeated for simplification.

## 2.2 Love waves in a layered piezomagnetic/piezoelectric semiconductor half-space

In this Section, the propagation characteristic of Love waves in a layered PM/PS half-space with different boundary conditions and material combinations are discussed. Combined with the numerical results, the effects of initial carrier concentration, PM layer thickness and material properties on the wave speed and attenuation are systematically analyzed.

### 2.2.1 Statement of the problem

Consider a n-type PS half-space covered with a PM layer of thickness  $h$  shown in Fig. 2.13. Both materials possess the transversely isotropic behavior and are perfectly bonded along the interface. The poling directions are both along the  $x_3$ -axis.

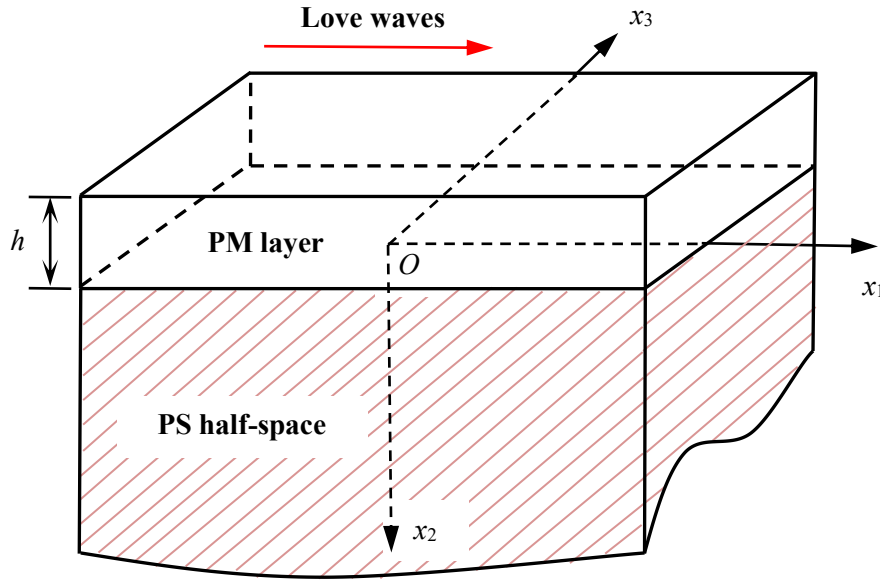


Fig. 2.13 A piezomagnetic (PM)/piezoelectric semiconductor (PS) layered half-space

We consider the Love waves propagating along the  $x_1$ -axis and small perturbations in the n-type PS substrate. Therefore, the basic equations in this Section (governing equations, constitutive relations and geometric relations, etc.) are the same as in the previous Section and will not be repeated. The coupled wave governing equation which is consistent with Eq. (2.5) can also be obtained.

The electric and magnetic potential, electric displacement and magnetic induction in vacuum satisfy the same conditions as Eqs. (2.6)-(2.7). In addition, due to the exchange between the cover layer and the substrate compared with Fig. 2.1, the radiation conditions are slightly changed:

$$\begin{aligned} x_2 \rightarrow +\infty, u_3^b = 0, \varphi^b = 0, \psi^b = 0, \Delta n^b = 0 \\ x_2 \rightarrow -\infty, \varphi^0 = 0, \psi^0 = 0 \end{aligned} \quad (2.28)$$

where the superscript  $b$  and  $0$  denote the physical quantities in base and vacuum, respectively. The top surface of the PM layer is traction free, that is:

$$T_{23}(x_1, -h) = 0 \quad (2.29)$$

Two kinds of the electric-magnetic boundary conditions at the free surface ( $x_2 = -h$ ) will be considered.

(i) the electrically and magnetically open case (denoted by BC1)

$$D_2(x_1, -h) = 0, \quad \psi(x_1, -h) = 0 \quad (2.30)$$

(ii) the electrically and magnetically short case (denoted by BC2)

$$B_2(x_1, -h) = 0, \quad \varphi(x_1, -h) = 0 \quad (2.31)$$

The continuous conditions at the interface ( $x_2 = 0$ ) can be written as:

$$\begin{aligned} u_3(x_1, 0) &= u_3^b(x_1, 0), \quad T_{23}(x_1, 0) = T_{23}^b(x_1, 0) \\ \varphi(x_1, 0) &= \varphi^b(x_1, 0), \quad D_2(x_1, 0) = D_2^b(x_1, 0) \\ \psi(x_1, 0) &= \psi^b(x_1, 0), \quad B_2(x_1, 0) = B_2^b(x_1, 0) \\ \dot{D}_2(x_1, 0) &= \dot{D}_2^b(x_1, 0) + J_2^b(x_1, 0) \end{aligned} \quad (2.32)$$

### 2.2.2 Solutions of the problem

Since the basic equations in this Section are the same as those in the previous Section, the homogeneous linear equations for the undetermined coefficients  $A_3$ ,  $A_\Phi$ ,  $A_\Psi$  and  $A_N$  can be obtained exactly as in Eq. (2.15). Considering the interchanges between overlay and substrate, it is necessary to reformulate their complete solutions.

The solutions for displacement, potential and magnetic potential in the PM cover can be written as:

$$\begin{aligned} u_3 &= \sum_{m=1}^4 \beta_{1m} A_{\Psi m} e^{-k\eta_m x_2} e^{ik(x_1 - ct)} \\ \varphi &= \begin{cases} A_{\Psi 5} e^{-kx_2} e^{ik(x_1 - ct)} \\ A_{\Psi 6} e^{kx_2} e^{ik(x_1 - ct)} \end{cases} \\ \psi &= \sum_{m=1}^4 A_{\Psi m} e^{-k\eta_m x_2} e^{ik(x_1 - ct)} \end{aligned} \quad (2.33)$$



where  $\beta_{1m}$  ( $m = 1-4$ ) represent the amplitude ratio.  $A_{\Psi m}$ ,  $A_{\Psi 5}$ , and  $A_{\Psi 6}$  are unknown amplitudes. For given wave speed, the Eq. (2.18) gives six roots of  $\eta_m$ .

In the same way, the complete solution of the PS substrate can be obtained:

$$\begin{aligned} u_3^b &= \sum_{s=7}^9 \beta_{2s}^b A_{\Phi s}^b e^{-k\eta_s^b x_2} e^{ik(x_1-ct)}, \quad \varphi^b = \sum_{s=7}^9 A_{\Phi s}^b e^{-k\eta_s^b x_2} e^{ik(x_1-ct)} \\ \psi^b &= A_{\Phi 10}^b e^{-kx_2} e^{ik(x_1-ct)}, \quad \Delta n^b = \sum_{s=7}^9 \beta_{3s}^b A_{\Phi s}^b e^{-k\eta_s^b x_2} e^{ik(x_1-ct)} \end{aligned} \quad (2.34)$$

where  $\beta_{2s}^b$  and  $\beta_{3s}^b$  ( $s = 7-9$ ) represent the amplitude ratio.  $A_{\Phi s}^b$  and  $A_{\Phi 10}^b$  are unknown amplitudes. By setting  $f_{ijk}$  and  $g_{ij}$  to zero from Eq. (2.15), eight  $\eta^b$  are obtained, which are two positive and negative complex roots and two  $\eta^b = \pm 1$ . Due to the radiation conditions, only four  $\eta^b$  with positive real parts are retained.

The electric and magnetic potential in vacuum are:

$$\begin{aligned} \varphi^0(x_1, x_2, t) &= \sum_{m=5}^6 A_{\Psi m} e^{kx_2} e^{ik(x_1-ct)} \\ \psi^0(x_1, x_2, t) &= \sum_{m=1}^4 A_{\Psi m} e^{kx_2} e^{ik(x_1-ct)} \end{aligned} \quad (2.35)$$

## 2.2.3 Dispersion equations

### (1) Phase velocity equations of each layer

By substituting Eq. (2.33) to the constitutive equation (2.4), the expressions of stress, electric displacement and magnetic induction intensity in the PM layer can be obtained (the  $e^{ik(x_1-ct)}$  term is uniformly omitted):

$$\begin{aligned} T_{13} &= \sum_{m=1}^4 ik(c_{44}\beta_{1m} + f_{15})e^{-k\eta_m x_2} A_{\Psi m}, \quad T_{23} = \sum_{m=1}^4 -k\eta_m(c_{44}\beta_{1m} + f_{15})e^{-k\eta_m x_2} A_{\Psi m} \\ D_1 &= -ik\varepsilon_{11}(e^{-kx_2} A_{\Psi 5} + e^{kx_2} A_{\Psi 6}), \quad D_2 = k\varepsilon_{11}(e^{-kx_2} A_{\Psi 5} - e^{kx_2} A_{\Psi 6}) \\ B_1 &= \sum_{m=1}^4 ik(f_{15}\beta_{1m} - \alpha_{11})e^{-k\eta_m x_2} A_{\Psi m}, \quad B_2 = \sum_{m=1}^4 -k\eta_m(f_{15}\beta_{1m} - \alpha_{11})e^{-k\eta_m x_2} A_{\Psi m} \end{aligned} \quad (2.36)$$

Similarly, by substituting Eq. (2.34) into Eq. (2.4), expressions in the PS substrate (also omitting the  $e^{ik(x_1-ct)}$  term) can be obtained:

$$\begin{aligned} T_{13}^b &= \sum_{s=7}^9 ik(c_{44}^b\beta_{2s}^b + e_{15})e^{(-k\eta_s^b x_2)} A_{\Phi s}^b, \quad T_{23}^b = \sum_{s=7}^9 -k\eta_s^b(c_{44}^b\beta_{2s}^b + e_{15})e^{(-k\eta_s^b x_2)} A_{\Phi s}^b \\ D_1^b &= \sum_{s=7}^9 ik(e_{15}\beta_{2s}^b - \varepsilon_{11}^b)e^{(-k\eta_s^b x_2)} A_{\Phi s}^b, \quad D_2^b = \sum_{s=7}^9 -k\eta_s^b(e_{15}\beta_{2s}^b - \varepsilon_{11}^b)e^{(-k\eta_s^b x_2)} A_{\Phi s}^b \\ B_1^b &= -ik\alpha_{11}^b e^{-kx_2} A_{\Phi 10}^b, \quad B_2^b = k\alpha_{11}^b e^{-kx_2} A_{\Phi 10}^b \\ J_1^b &= \sum_{s=7}^9 -ikq(n_0\mu_{11} - D_{11}\beta_{3s}^b)e^{(-k\eta_s^b x_2)} A_{\Phi s}^b, \quad J_2^b = \sum_{s=7}^9 kq\eta_s^b(n_0\mu_{11} - D_{11}\beta_{3s}^b)e^{(-k\eta_s^b x_2)} A_{\Phi s}^b \end{aligned} \quad (2.37)$$

In addition, according to Eqs. (2.7) and (2.35), the electric displacement and magnetic induction in vacuum are ( $e^{kx_2} \cdot e^{ik(x_1-ct)}$  term omitted):

$$D_2^0 = -\varepsilon_0 k \sum_{m=5}^6 A_{\Psi_m}, \quad B_2^0 = -\mu_0 k \sum_{m=1}^4 A_{\Psi_m} \quad (2.38)$$

## (2) Dispersion equations

Therefore, by substituting Eqs. (2.36)-(2.38) to boundary conditions and continuous conditions (2.29)-(2.32), the dispersion equation of this model can be obtained:

$$[N]A = 0 \quad (2.39)$$

where  $A = \{ A_{\Psi_1}, A_{\Psi_2}, A_{\Psi_3}, A_{\Psi_4}, A_{\Psi_5}, A_{\Psi_6}, A_{\Phi_7}^b, A_{\Phi_8}^b, A_{\Phi_9}^b, A_{\Phi_{10}}^b \}^T$ ,  $[N]$  is a  $10 \times 10$  coefficient matrix. For the boundary condition BC1, the non-zero elements of its coefficient matrix  $[N]$  are:

$$\begin{aligned} N_{1m} &= (c_{44}\beta_{1m} + f_{15})\eta_m \exp(kh\eta_m), \quad N_{2m} = \beta_{1m}, \quad N_{2s} = -\beta_{2s}^b \\ N_{3m} &= (c_{44}\beta_{1m} + f_{15})\eta_m, \quad N_{3s} = -(c_{44}^b\beta_{2s}^b + e_{15})\eta_s^b \\ N_{4,5} &= N_{4,6} = 1, \quad N_{4s} = -1, \quad N_{5m} = 1, \quad N_{5,10} = -1 \\ N_{6,5} &= \varepsilon_{11}, \quad N_{6,6} = -\varepsilon_{11}, \quad N_{6s} = (e_{15}\beta_{2s}^b - \varepsilon_{11}^b)\eta_s^b \\ N_{7m} &= (f_{15}\beta_{1m} - \alpha_{11})\eta_m, \quad N_{7,10} = \alpha_{11}^b \\ N_{8,5} &= ikc\varepsilon_{11}, \quad N_{8,6} = -ikc\varepsilon_{11}, \quad N_{8s} = (ikce_{15}\beta_{2s}^b - ikc\varepsilon_{11}^b + qn_0\mu_{11} - qD_{11}\beta_{3s}^b)\eta_s^b \\ N_{9,5} &= \exp(kh), \quad N_{9,6} = -\exp(-kh), \quad N_{10m} = \exp(kh\eta_m) \end{aligned} \quad (2.40)$$

where  $m = 1-4$  and  $s = 7-9$ . In order to obtain non-zero solutions to the Eq. (2.39), the determinant of its coefficients must be zero, that is,  $\text{Det}[N] = 0$ .

In addition, for the BC2 case, row nine and row ten in matrix  $[N]$  are  $N_{9,6} = \exp(-kh)$  ( $N_{9,5}$  in the row nine is unchanged),  $N_{10m} = (f_{15}\beta_{1m} - \alpha_{11})\eta_m \exp(kh\eta_m)$ . The remaining rows are left unchanged simultaneously, then the dispersion equation at BC2 can be obtained.

## 2.2.4 Numerical results

For the BC1 and BC2, Terfenol-D and ZnO are considered as the PM layer and PS substrate in numerical examples, and the Terfenol-D/CdSe is selected for comparison. Their material properties are listed in Table 2.3. The electrical conductivity of Terfenol-D is not taken into account. Unless particularly stated, the Terfenol-D/ZnO with BC1 is considered. In addition, the PM layer thickness is  $h = 0.1\text{mm}$ , and the initial carrier concentration  $n_0 = 5 \times 10^{16}/\text{m}^3$ .

Table 2.3 The properties of Terfenol-D<sup>[165]</sup> and CdSe<sup>[103]</sup>

| Material constants                                                      | Terfenol-D <sup>[165]</sup> | CdSe <sup>[103]</sup> |
|-------------------------------------------------------------------------|-----------------------------|-----------------------|
| $\rho$ (kg/m <sup>3</sup> )                                             | 9230                        | 5504                  |
| $c_{44}$ (10 <sup>9</sup> N/m <sup>2</sup> )                            | 5.99                        | 13.2                  |
| $\varepsilon_{11}$ (10 <sup>-11</sup> C <sup>2</sup> /Nm <sup>2</sup> ) | 5                           | 8.26                  |
| $f_{15}$ (N/Am)                                                         | 167.66                      | /                     |
| $\alpha_{11}$ (10 <sup>-6</sup> Ns <sup>2</sup> / C <sup>2</sup> )      | 3.97                        | /                     |
| $e_{15}$ (C/m <sup>2</sup> )                                            | /                           | -0.138                |
| $\mu_{11}$ (m <sup>2</sup> /V)                                          | /                           | 4                     |
| $D_{11}$ (m <sup>2</sup> /s)                                            | /                           | 0.104                 |

In the calculations, phase velocity and wave number are normalized as  $c / c_{sh}^b$  and  $kh$ .  $c_{sh} \approx 1190.00$  m/s is the shear wave velocity of Terfenol-D, and  $c_{sh}^b \approx 2841.66$  m/s is the shear wave velocity of ZnO half-space (the  $c_{sh}^b$  of CdSe is 1562.10 m/s).

The magneto-electromechanical coupling factor for surface waves can be defined as [164]:

$$k^2 = 2 \left( c_{(\text{open})} - c_{(\text{short})} \right) / c_{(\text{open})} \quad (2.41)$$

where  $c_{(\text{open})}$  and  $c_{(\text{short})}$  are the wave speed of the  $m$ th mode at magneto-electrically open and shorted cases, respectively.

#### 2.2.4.1 Propagation characteristics of Love waves in a PM/PS layered half-space

##### (1) Propagation characteristics of Love waves under different boundary

The propagation of Love waves in PM/PS layered half-space under the electrically and magnetically open case (BC1) is first studied. It should be noted that in addition to considering the influence of boundary conditions on the Love waves, the open case is considered in the rest parts.

Figure 2.14 shows the first five modes of Love wave propagation in Terfenol-D/ZnO layered half-space under BC1. The trend of dispersion curves of the first five modes in Fig. 2.14(a) is similar to that in [162,163,165,166], which confirms the correctness and reliability of the numerical calculation. Each mode has a lower cutoff value of  $kh$ . The wave velocities start from the bulk shear wave velocity in ZnO and tend to the bulk shear wave velocity in Terfenol-D. In addition, it is found from Fig. 2.14(a) that the wave speed for each mode monotonically decreases with the increase of  $kh$ , and the wave speed decreases with increasing mode order.

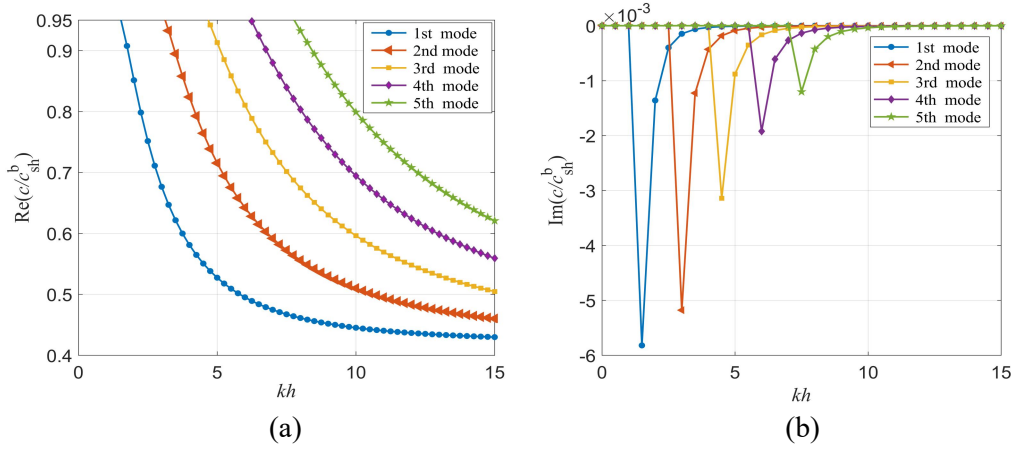


Fig. 2.14 The first five modes of Love waves in the Terfenol-D/ZnO layered half-space with open condition: (a) dispersion curves with open condition, and (b) attenuation curves with open condition

Due to the semiconductor properties in the PS substrate, the attenuation characteristics in Fig. 2.14(b) are not present in the Love waves of PE or ME layered structures [162-166]. Besides, due to the coupling of PM characteristic in the MCS structure, the attenuation of Fig. 2.14(b) are not completely consistent with the Love waves in the PS film/elastic dielectric half-space without PM [106]. It can also be seen from Fig. 2.14(b) that the attenuation decreases continuously from the lower modes to the higher modes.

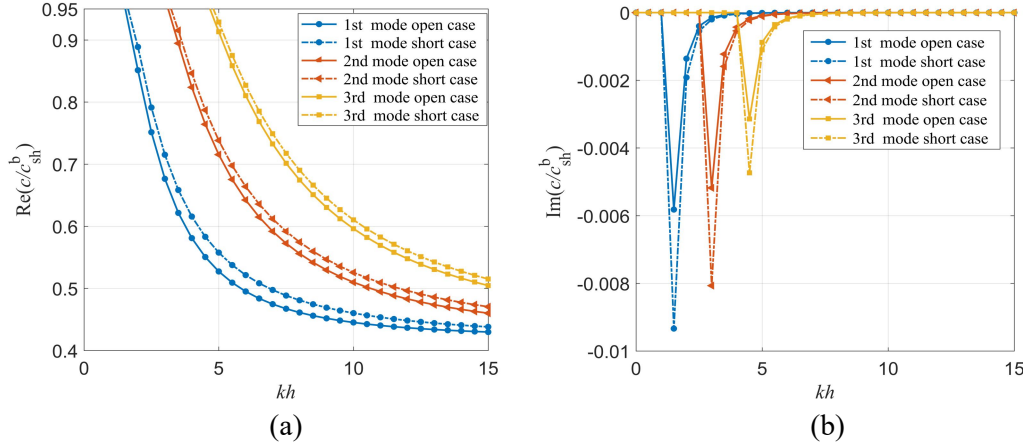


Fig. 2.15 The first three modes of Love waves in the Terfenol-D/ZnO layered half-space with open and short conditions: (a) dispersion curves with different boundary conditions, and (b) attenuation curves with different boundary conditions

To check the influence of boundary conditions on Love waves, the dispersion curves and attenuation curves in the Terfenol-D/ZnO layered half-space under BC1 and BC2 are presented in Fig. 2.15. The three solid lines represent the results of the open boundary, while the three dots are marked for the short case. It is worth noting that since the curves of first three modes already illustrate the situation, it is not necessary to plot the first five modes as shown in Fig. 2.14.

It can be observed that the trend of dispersion and attenuation curves in Fig. 2.15 are the same under BC1 and BC2, but the wave speed and attenuation with open case are correspondingly smaller than those with short case.

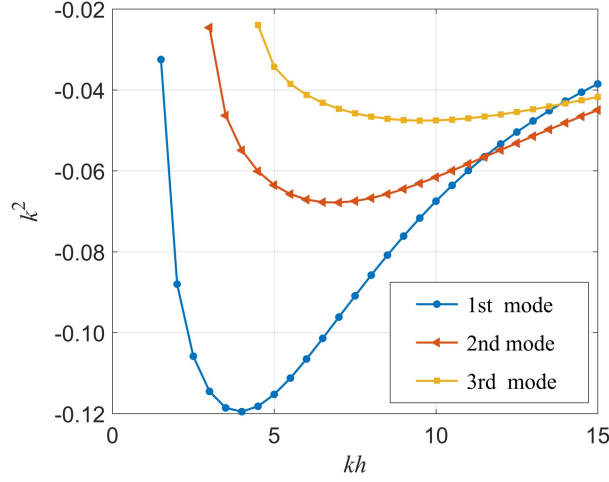


Fig. 2.16 Magneto-electromechanical coupling factors of the first three modes

Based on the wave speeds of open and short case in Fig. 2.15, the magneto-electromechanical coupling factors  $k^2$  of the first three modes shown in Fig. 2.16 can be further obtained according to Eq. (2.41). Under the same wave number, the  $k^2$  of the second and third modes are obviously larger than that of the first mode. In addition,  $k^2$  first decrease to the minimum and then increase with  $kh$  increasing in all three modes.

## (2) Propagation characteristics of Love waves in different material combinations

Considering two different PS substrate, we obtained the propagation characteristics of the first three modes of Love waves in Terfenol-D/ZnO and Terfenol-D/CdSe shown in Fig. 2.17. From the lower modes to the higher modes, the wave speed and attenuation will decrease.

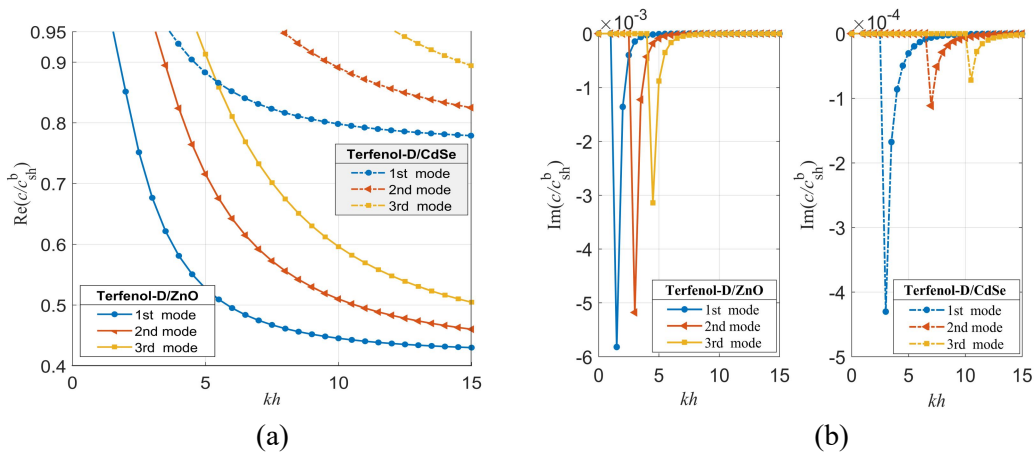


Fig. 2.17 The first three modes of Love waves in the Terfenol-D/ZnO & Terfenol-D/CdSe layered half-space: (a) dispersion curves comparison of two PM/PS layered half-spaces, and (b) attenuation curves comparison of two PM/PS layered half-spaces

Figure 2.17(a) shows that the wave speeds of Love waves in Terfenol-D/ZnO composite structure are relatively smaller. In Fig. 2.17(b), the imaginary parts of each mode in Terfenol-D/ZnO are correspondingly smaller than those in Terfenol-D/CdSe, indicating that Love waves attenuate more obviously in the former. The Terfenol-D/ZnO combination will be considered in the following calculations.

#### 2.2.4.2 Effect of initial carrier concentration on Love waves propagation

Steady-state carrier concentration is one of the important parameters of semiconductor materials. It has been proved that the steady-state carrier concentration has a great influence on the propagation of SAWs. Semiconducting properties reduce the propagation speed and cause attenuation.

Different from the above structures, the PS material is used as the substrate in this Section, and the Love waves mainly propagate in the PM layer. Therefore, the influence of ZnO semiconducting properties on Love waves is indirect rather than direct. Figure 2.18 and Fig. 2.19 show the influence of the initial carrier concentration  $n_0$  on the first and second modes of Love waves. According to Figs. 2.18(a) and 2.19(a),  $n_0$  has little effect on the wave speed of the first and second modes. After local amplification, it can be seen that the wave speed decreases with the increase of  $n_0$  (the change is too small to be ignored), which is similar to the results in the mentioned PS structures.

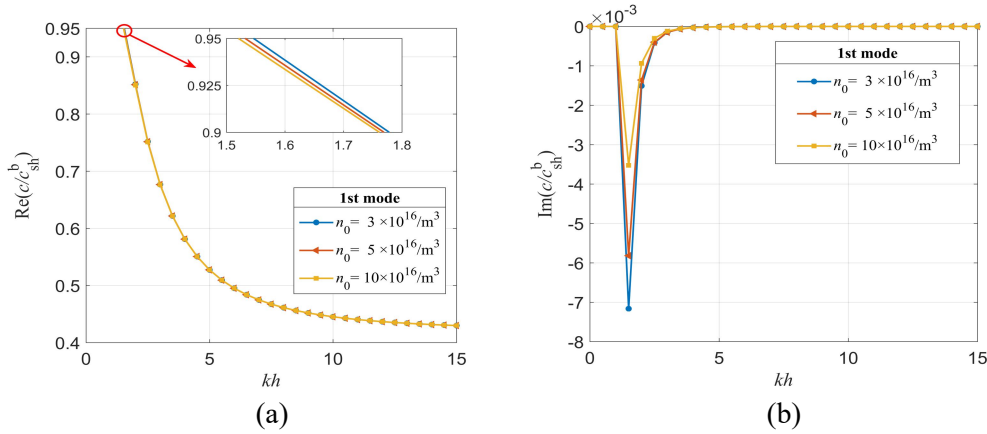


Fig. 2.18 Effect of  $n_0$  on first mode of Love waves in Terfenol-D/ZnO layered half-space: (a) effect of  $n_0$  on wave speed, and (b) effect of  $n_0$  on attenuation

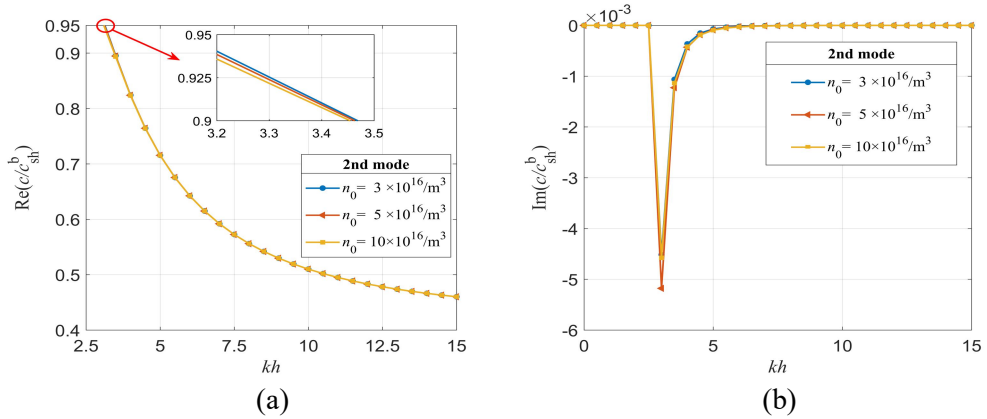


Fig. 2.19 Effect of  $n_0$  on second mode of Love waves in Terfenol-D/ZnO layered half-space: (a) effect of  $n_0$  on wave speed, and (b) effect of  $n_0$  on attenuation

In Fig. 2.18(b), the change of  $n_0$  has a certain effect on the attenuation of the first mode of Love waves. However, Fig. 2.19(b) indicates that  $n_0$  has almost no influence on the attenuation of second modes.

#### 2.2.4.3 Effect of piezomagnetic layer thickness on Love waves propagation

Like the Love waves in the PS thin film on an elastic dielectric half-space [106], Figure 2.20 shows that the wave speed depend significantly on the PM layer thickness  $h$ . With the increase of  $h$ , the wave speed will decrease, especially when the thickness changes from  $h = 0.1\text{mm}$  to  $h = 0.2\text{mm}$  (blue and red lines in Figs. 2.20(a) and 2.20(b)). It can be observed that the wave speed changes for the second mode in Fig. 2.20(b) are more obvious.

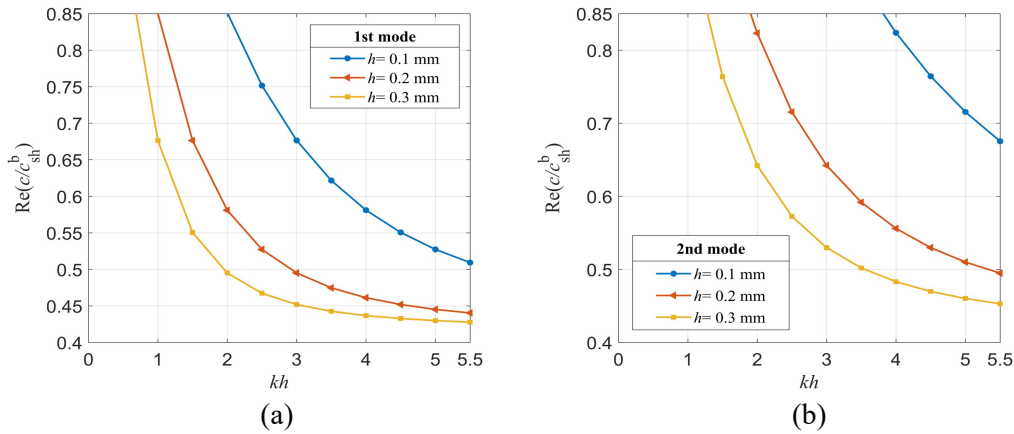


Fig. 2.20 Effect of  $h$  on the first mode and second mode wave speeds of Love waves: (a) effect of  $h$  on wave speed of the first mode, and (b) effect of  $h$  on wave speed of the second mode

#### 2.2.4.4 Effects of piezoelectric constant and piezomagnetic constant on Love waves propagation

Figure 2.21 and Fig. 2.22 show the influence of piezoelectric (PE) constant  $e_{15}$  in ZnO substrate on the first two modes in the layered half-space. It can be seen that the trends of dispersion and attenuation curves in Fig. 2.21 and Fig. 2.22 are similar, only the values change.

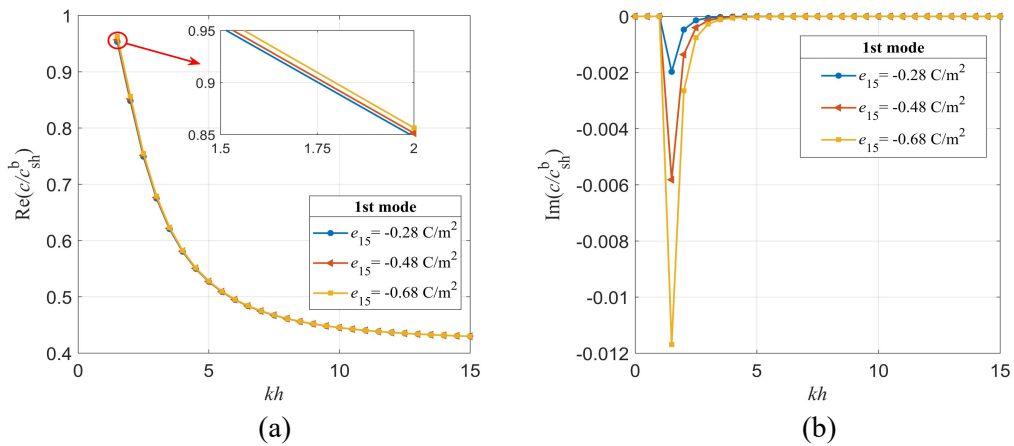


Fig. 2.21 Effect of  $e_{15}$  on the first mode of Love waves: (a) effect of  $e_{15}$  on the wave speed of first mode, and (b) effect of  $e_{15}$  on the attenuation of first mode

In the numerical examples,  $e_{15}$  is selected as  $-0.28\text{C/m}^2$ ,  $-0.48\text{C/m}^2$  and  $-0.68\text{C/m}^2$ , respectively (where  $e_{15} = -0.48\text{C/m}^2$  is the real PE constant of ZnO). According to Fig. 2.21(a),  $e_{15}$  has little effect on the wave speed, and only slight changes can be seen through local amplification. It is also found from Fig. 2.21 that the wave speed and attenuation decrease with the increase of  $e_{15}$ . In Fig. 2.22, the second mode of the Love waves has the same trend as the first mode, so it will not be discussed again.

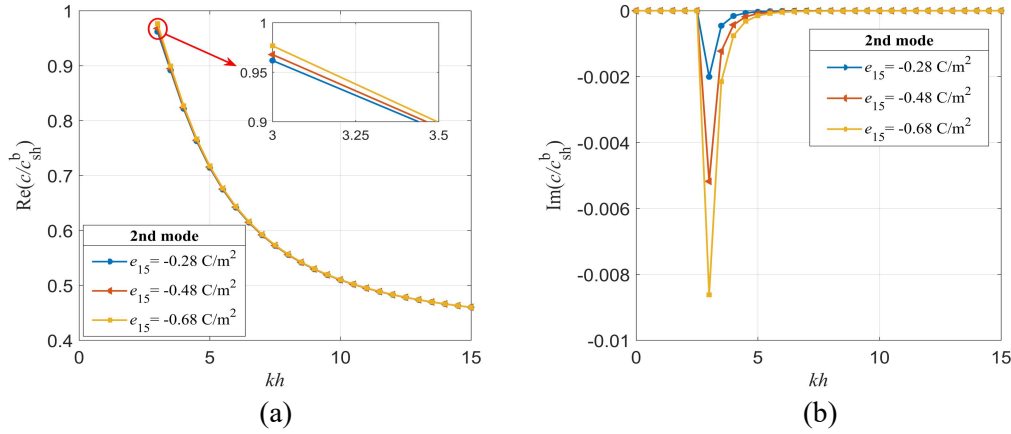


Fig. 2.22 Effect of  $e_{15}$  on the second mode of Love waves: (a) effect of  $e_{15}$  on the wave speed of second mode, and (b) effect of  $e_{15}$  on the attenuation of second mode

Figure 2.23 shows the dispersion curves of the first two modes with three different piezomagnetic (PM) constant  $f_{15}$ . Unlike the PE constant  $e_{15}$  in Figs. 2.21(a) and 2.22(a), we can find that PM constant  $f_{15}$  has an obvious effect on wave speed. In addition, the trends of the first mode in Fig. 2.23(a) and the second mode in Fig. 2.23(b) are also similar. In Fig. 2.23,  $f_{15}$  is  $100\text{N/Am}$ ,  $167.66\text{N/Am}$ , and  $200\text{N/Am}$ , respectively (where  $f_{15} = 167.66\text{N/Am}$  is the PM constant of Terfenol-D). It can be concluded that the wave speed will decrease with the increase of  $f_{15}$  at the same  $kh$ . At the same time, the decrease is more obvious when  $kh$  is small (long wavelengths) than  $kh$  is large (short wavelengths).

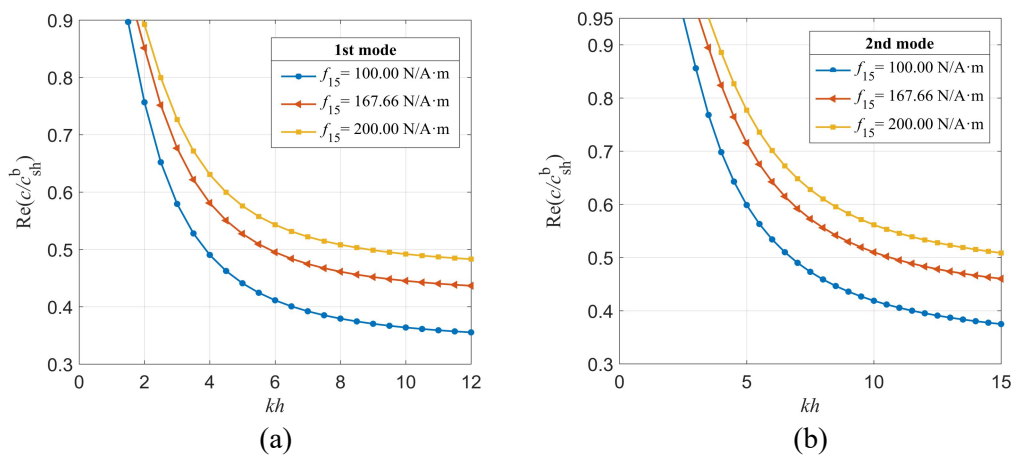


Fig. 2.23 Effect of  $f_{15}$  on the first mode and second mode wave speeds of Love waves: (a) effect of  $f_{15}$  on the wave speed of first mode, and (b) effect of  $f_{15}$  on the wave speed of second mode



In this study, the cover layer of MCS is PM material, and its PM constant  $f_{15}$  has a direct effect on the Love waves. As the semi-infinite substrate, the PE constant  $e_{15}$  has an indirect effect on the Love waves, which needs to be influenced by magnetoelectric coupling effect. Therefore, the influence of magnetically related  $f_{15}$  in the cover on Love waves is significantly greater than that of piezoelectricity related  $e_{15}$  and semiconductor related  $n_0$  in the substrate.

## 2.3 Summary

In this Chapter, the dispersion equations of SH-type surface waves in PS/PM layered half-space are derived. The effects of semiconductor properties, cover thickness and material properties on the dispersion and attenuation of surface waves are discussed. Two models are studied in this Chapter: SH waves propagation in PS/PM half-space and Love waves propagation in PM/PS half-space. For the first study, the conclusions are as follows:

(1) The increase of initial carrier concentration, cover thickness and PE constant will make the wave speed smaller, while the change of PM constant has little effect on the wave speed and attenuation.

(2) The initial carrier concentration has a great influence on the distribution of mechanical field and electrical related physical quantities, while the influence on magnetic related physical quantities is indirect and small. The effect of the PE constant on the magnetic potential is negligible, and only a slight change can be seen near the interface.

For the study of Love waves in PM/PS half-space, the following conclusions can be drawn:

(1) The wave speed and attenuation of the first three modes of the Love waves in the open circuit are smaller than that in the short case of the corresponding mode. The first three mode wave speeds of Terfenol-D/ZnO are correspondingly smaller than those of Terfenol-D/CdSe, but the attenuation of Love waves in the former is more obvious than that in the latter.

(2) Under the same wave number, the wave speed decreases significantly with the increase of the PM layer thickness, especially in the second mode.

(3) The wave speed decreases with the increase of initial carrier concentration, PE constant and PM constant. The effect of PM constant on the wave speed of the first and second modes is obvious.

## Chapter 3

# Love waves and Rayleigh waves in a piezomagnetic/piezoelectric/semiconductor composite structure

In recent years, tube, belt, spiral, fiber and thin film [9-11] ZnO nanostructures with large PE effect have been synthesized. However, the PSs have a low electro-mechanical coupling coefficient compared with the PE medium without semiconductor effect and low carrier mobility compared with some semiconductor materials without PE characteristics, making it unable to meet the development needs of functional devices. As mentioned in the Introduction, composite structures composed of PE and non-piezoelectric semiconductors can also achieve mechanical-electric-carrier coupling, thus exhibiting similar or even better performance than single PSs [103-105,127,128].

On the other hand, in the field of ferromagnetic materials, two-phase composites with magnetoelectric effects can usually be divided into 0-3 type, 2-2 type and 1-3 type. Specifically, the 0-3 structures are composed of PE and magnetic oxide particles, 2-2 are layered magnetoelectric structure composed of PE and magnetic oxide particles, and 1-3 are embedment of one phase fiber into another phase matrix [184]. Generally, the 2-2 layer composite structures are the most convenient to prepare and easy to obtain greater magnetoelectric effect, so this structure has been widely studied.

Silicon as the main semiconductor material of electronic devices, the reason is that people have long mastered the low-cost, high-volume manufacturing, low-defect and high-purity monocrystalline silicon technology and various process technologies for its semiconductor processing. Silicon is usually used as substrates in the product. In this Chapter, Si is selected as the semiconductor substrate, Terfenol-D and  $\text{CoFe}_2\text{O}_4$  as the PM layer, and  $\text{BaTiO}_3$ , PZT-4 and PZT-5H as the PE layer, so as to realize different material combinations. The influenc-

es of PM and PE materials on Love waves and Rayleigh waves in layered structures were investigated.

### 3.1 Statement of the problem and basic equations

Consider the PM/PE/semiconductor three-layer composite structure as shown in Fig. 3.1, where the covering layer is a PE medium with thickness  $h_1$  and a PM material with thickness  $h_2$ , and the substrate is n-type semiconductor. The Interface 1 and Interface 2 are perfectly connected.

Since the thickness of the substrate is much greater than that of the two covering layers, the substrate can be regarded as a semi-infinite space. The region with  $-h \leq x_2 \leq 0$  is the covering layer, while the region with  $x_2 > 0$  is the substrate.

Both Love and Rayleigh waves propagate along the  $x_1$ -axis, and the polarization direction of all the materials are along the  $x_3$ -axis.

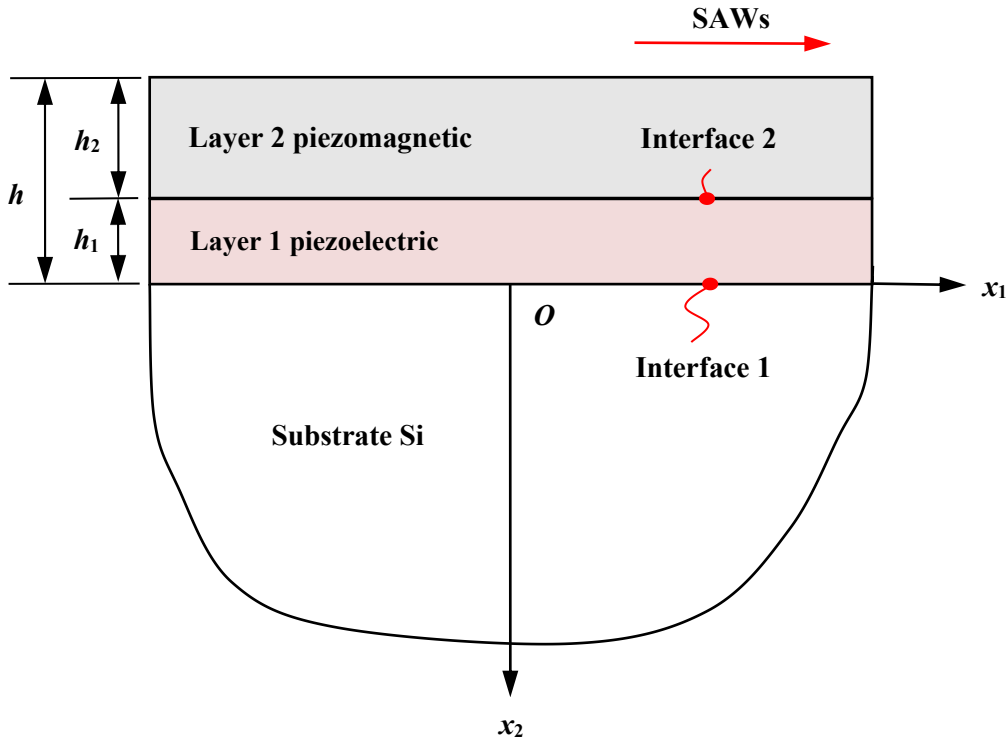


Fig. 3.1 The piezomagnetic/piezoelectric/semiconductor composite structure

As in Chapter 2, smaller carrier perturbations are considered, so the linearized macroscopic theory can be used. It is assumed that the displacement components, electric potential, magnetic potential and carrier concentration perturbation of surface waves are:

$$\begin{aligned} u_1(x_1, x_2) &= u_1(x_1, x_2, t), \quad u_2(x_1, x_2) = u_2(x_1, x_2, t), \quad u_3 = u_3(x_1, x_2, t) \\ \varphi &= \varphi(x_1, x_2, t), \quad \psi = \psi(x_1, x_2, t), \quad \Delta n = \Delta n(x_1, x_2, t) \end{aligned} \quad (3.1)$$

where  $u_1$ ,  $u_2$  and  $u_3$  are displacement components. Love waves have only one dis-

placement component, that is  $u_3$ , and the other two displacement components should be considered when studying Rayleigh waves.  $\varphi$  and  $\psi$  denote the electric potential and magnetic potential.  $\Delta n$  is the small carrier concentration perturbation.

### 3.1.1 Fundamental equations of piezomagnetic layer

For PM materials, the equilibrium equations regardless of body force and initial stress are:

$$\begin{aligned} T_{ji,j}^m &= \rho^m \ddot{u}_i^m \\ D_{i,i}^m &= 0 \\ B_{i,i}^m &= 0 \end{aligned} \quad (3.2)$$

where the superscript  $m$  represents the material properties and field quantity in the PM layer. The constitutive equations of PM layer are as follows:

$$\begin{aligned} T_{ij}^m &= c_{ijkl}^m S_{kl}^m - f_{kij}^m H_k^m \\ D_i^m &= \varepsilon_{ij}^m E_j^m \\ B_i^m &= f_{ijk}^m S_{jk}^m + \alpha_{ij}^m H_j^m \end{aligned} \quad (3.3)$$

where  $c_{ijkl}^m$ ,  $f_{ijk}^m$ ,  $\varepsilon_{ij}^m$  and  $\alpha_{ij}^m$  are the elastic constant, PM constant, dielectric constant and permeability of the PM material, respectively.

The generalized geometric relations are:

$$\begin{aligned} S_{ij}^m &= (u_{i,j}^m + u_{j,i}^m)/2 \\ E_i^m &= -\varphi_{,i}^m, H_i^m = -\psi_{,i}^m \end{aligned} \quad (3.4)$$

By substituting Eqs. (3.1), (3.3) and (3.4) into the governing equation (3.2), the coupled wave equations of the PM covering layer can be obtained:

$$\begin{aligned} c_{11}^m \frac{\partial^2 u_1^m}{\partial x_1^2} + c_{66}^m \frac{\partial^2 u_1^m}{\partial x_2^2} + (c_{12}^m + c_{66}^m) \frac{\partial^2 u_2^m}{\partial x_1 \partial x_2} &= \rho^m \ddot{u}_1^m \\ c_{66}^m \frac{\partial^2 u_2^m}{\partial x_1^2} + c_{11}^m \frac{\partial^2 u_2^m}{\partial x_2^2} + (c_{12}^m + c_{66}^m) \frac{\partial^2 u_1^m}{\partial x_1 \partial x_2} &= \rho^m \ddot{u}_2^m \\ c_{44}^m \nabla^2 u_3 + f_{15}^m \nabla^2 \psi &= \rho^m \ddot{u}_3 \\ \nabla^2 \varphi^m &= 0 \\ f_{15}^m \nabla^2 u_3 - \alpha_{11}^m \nabla^2 \psi &= 0 \end{aligned} \quad (3.5)$$

where  $\nabla^2 = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2$  is the Laplacian operator in Cartesian coordinates.

### 3.1.2 Equations of piezoelectric semiconductor layer

The constitutive equations of PE overlay are:

$$\begin{aligned} T_{ij}^e &= c_{ijkl}^e S_{kl}^m - e_{kij}^e E_k^e \\ D_i^e &= e_{ijk}^e S_{jk}^e + \varepsilon_{ij}^e E_j^e \\ B_i^e &= \alpha_{ij}^e H_j^e \end{aligned} \quad (3.6)$$

where the superscript  $e$  denotes the material properties and field quantity in the PE layer.

The equilibrium equations and geometric relations of PE layer are the same as those of PM layer. Combined with the constitutive equation (3.6), we can obtain the coupled wave equations of the PE layer:

$$\begin{aligned} c_{11}^e \frac{\partial^2 u_1^e}{\partial x_1^2} + c_{66}^e \frac{\partial^2 u_1^e}{\partial x_2^2} + (c_{12}^e + c_{66}^e) \frac{\partial^2 u_2^e}{\partial x_1 \partial x_2} &= \rho \ddot{u}_1^e \\ c_{66}^e \frac{\partial^2 u_2^e}{\partial x_1^2} + c_{11}^e \frac{\partial^2 u_2^e}{\partial x_2^2} + (c_{12}^e + c_{66}^e) \frac{\partial^2 u_1^e}{\partial x_1 \partial x_2} &= \rho \ddot{u}_2^e \\ c_{44}^e \nabla^2 u_3 + e_{15}^e \nabla^2 \varphi &= \rho \ddot{u}_3 \\ e_{15}^e \nabla^2 u_3 - \varepsilon_{11}^e \nabla^2 \varphi &= 0 \\ \nabla^2 \psi^e &= 0 \end{aligned} \quad (3.7)$$

### 3.1.3 Equations of semiconductor substrate

The equilibrium equations in the semiconductor substrate, excluding body force and initial stress, can be expressed as:

$$\begin{aligned} T_{ji,j}^s &= \rho^s \ddot{u}_i^s \\ D_{i,i}^s &= -q \Delta n \\ J_{i,i} &= q \Delta \dot{n} \end{aligned} \quad (3.8)$$

where the superscript  $s$  denotes the material properties and field quantity in the semiconductor substrate.

The constitutive equations of semiconductor material are as follows:

$$\begin{aligned} T_{ij}^s &= c_{ijkl}^s S_{kl}^s \\ D_i^s &= \varepsilon_{ij}^s E_j^s \\ J_i &\cong q n_0 \mu_{ij} E_j + q D_{ij} (\Delta n)_{,j} \end{aligned} \quad (3.9)$$

Eq. (3.9)<sub>3</sub> is the current equation of the semiconductor.  $\mu_{ij}$  and  $D_{ij}$  are the m-

obility and diffusion coefficient of the carrier, respectively. They satisfy the Einstein relation that  $D_{ij}/\mu_{ij} = k_B/T_k \approx 0.0259$  V, where  $k_B$  and  $T_k$  are Boltzmann constant and absolute temperature, respectively.

Similarly, the coupled wave equations for the substrate can be obtained:

$$\begin{aligned}
 c_{11}^s \frac{\partial^2 u_1^s}{\partial x_1^2} + c_{66}^s \frac{\partial^2 u_1^s}{\partial x_2^2} + (c_{12}^s + c_{66}^s) \frac{\partial^2 u_2^s}{\partial x_1 \partial x_2} &= \rho \ddot{u}_1^s \\
 c_{66}^s \frac{\partial^2 u_2^s}{\partial x_1^2} + c_{11}^s \frac{\partial^2 u_2^s}{\partial x_2^2} + (c_{12}^s + c_{66}^s) \frac{\partial^2 u_1^s}{\partial x_1 \partial x_2} &= \rho \ddot{u}_2^s \\
 c_{44}^s \nabla^2 u_3 &= \rho \ddot{u}_3 \\
 \varepsilon_{11}^s \nabla^2 \varphi^s &= q \Delta n \\
 -n_0 \mu_{11} \nabla^2 \varphi^s + D_{11} \nabla^2 (\Delta n) &= \Delta n
 \end{aligned} \tag{3.10}$$

### 3.1.4 Boundary conditions and continuity conditions

When  $x_2 \rightarrow +\infty$ , the displacement, potential and perturbation carrier concentration of substrate tend to zero. Similarly, as  $x_2 \rightarrow -\infty$ , the electric and magnetic potential in the vacuum also tend to zero as the radiation condition:

$$\begin{aligned}
 x_2 \rightarrow +\infty, u_3^s &= 0, \varphi^s = 0, \Delta n = 0 \\
 x_2 \rightarrow -\infty, \varphi^0 &= 0, \psi^0 = 0
 \end{aligned} \tag{3.11}$$

where the superscript 0 represents the physical quantity in vacuum.

On the free surface of the PM covering layer, there are:

$$\begin{aligned}
 T_{2i}^m(x_1, -h) &= 0, \quad i = 1-3 \\
 D_2^m(x_1, -h) &= 0 \\
 \psi^m(x_1, -h) &= 0
 \end{aligned} \tag{3.12}$$

Assuming that the disturbed carrier concentration disappears on the upper surface of the semiconductor and the two interfaces are perfectly connected, the disturbed carrier concentration and continuity conditions at  $x_2 = 0$  (Interface 1) are:

$$\begin{aligned}
 \Delta n(x_1, 0) &= 0 \\
 u_i^e(x_1, 0) &= u_i^s(x_1, 0), \quad T_{2i}^e(x_1, 0) = T_{2i}^s(x_1, 0) \\
 \varphi^e(x_1, 0) &= \varphi^s(x_1, 0), \quad D_2^e(x_1, 0) = D_2^s(x_1, 0) \\
 \dot{D}_2^e(x_1, 0) &= \dot{D}_2^s(x_1, 0) + J_2(x_1, 0)
 \end{aligned} \tag{3.13}$$

In addition, the continuity condition at  $x_2 = -h_1$  (Interface 2) is:

$$\begin{aligned}
 u_i^m(x_1, -h_1) &= u_i^e(x_1, -h_1), \quad T_{2i}^m(x_1, -h_1) = T_{2i}^e(x_1, -h_1) \\
 \varphi^m(x_1, -h_1) &= \varphi^e(x_1, -h_1), \quad D_2^m(x_1, -h_1) = D_2^e(x_1, -h_1) \\
 \psi^m(x_1, -h_1) &= \psi^e(x_1, -h_1), \quad B_2^m(x_1, -h_1) = B_2^e(x_1, -h_1)
 \end{aligned} \tag{3.14}$$

## 3.2 Solutions of the problem and dispersion equations

### 3.2.1 Solutions of each layer

#### (1) Solution of the PM layer

We assume that the solution of the PM coupled wave equation (3.5) is:

$$\begin{Bmatrix} u_1^m \\ u_2^m \\ u_3^m \\ \varphi^m \\ \psi^m \end{Bmatrix} = \begin{Bmatrix} A_1^m \\ A_2^m \\ A_3^m \\ A_\Phi^m \\ A_\Psi^m \end{Bmatrix} e^{-k\eta^m x_2} e^{ik(x_1 - ct)} \quad (3.15)$$

where  $A_1^m$ ,  $A_2^m$ ,  $A_3^m$ ,  $A_\Phi^m$ , and  $A_\Psi^m$  are unknown amplitudes.  $\eta^m$  is the undetermined parameter,  $k$  is the wave number,  $c$  is the wave speed, and  $i$  is the imaginary unit.

Substituting Eq. (3.15) into Eq. (3.5), a homogeneous linear system of undetermined coefficients  $A_1^m$ ,  $A_2^m$ ,  $A_3^m$ ,  $A_\Phi^m$  and  $A_\Psi^m$  can be obtained:

$$\begin{aligned} (c_{11}^m - c_{66}^m \eta^{m^2} - \rho^m c^2) A_1^m + i\eta^m (c_{12}^m + c_{66}^m) A_2^m &= 0 \\ i\eta^m (c_{12}^m + c_{66}^m) A_1^m + (c_{66}^m - c_{11}^m \eta^{m^2} - \rho^m c^2) A_2^m &= 0 \\ [c_{44}^m (\eta^{m^2} - 1) + \rho^m c^2] A_3^m + f_{15}^m (\eta^{m^2} - 1) A_\Psi^m &= 0 \\ (\eta^{m^2} - 1) A_\Phi^m &= 0 \\ f_{15}^m (\eta^{m^2} - 1) A_3^m - \alpha_{11}^m (\eta^{m^2} - 1) A_\Psi^m &= 0 \end{aligned} \quad (3.16)$$

Eq. (3.16)<sub>4</sub> is decoupled from the other four equations to get  $\eta^m = \pm 1$ . In addition, Eq. (3.16)<sub>1</sub> is coupled to Eq. (3.16)<sub>2</sub>. Eq. (3.16)<sub>3</sub> and Eq. (3.16)<sub>5</sub> are also coupled, but the two sets of equations are not coupled to each other. In order to obtain nontrivial solutions for  $A_1^m$ ,  $A_2^m$ ,  $A_3^m$  and  $A_\Psi^m$ , the determinants of the coefficients of the two sets of equations are zero, respectively. Thus, the complete solution of the PM layer can be written as:

$$\begin{Bmatrix} u_1^m \\ u_2^m \\ u_3^m \\ \varphi^m \\ \psi^m \end{Bmatrix} = \begin{Bmatrix} \sum_{j=1}^4 \beta_{1j}^m A_{2j}^m \\ \sum_{j=1}^4 A_{2j}^m \\ \sum_{j=5}^8 \beta_{2j}^m A_{\Psi j}^m \\ \sum_{j=9}^{10} A_{\Phi j}^m \\ \sum_{j=5}^8 A_{\Psi j}^m \end{Bmatrix} e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)} \quad (3.17)$$

where  $A_{2j}^m$  ( $j = 1-4$ ) and  $A_{\Psi j}^m$  ( $j = 5-8$ ) are the amplitude (it is worth noting that the range of  $j$  will be adjusted according to the dispersion equation).  $\beta_{1j}^m$  and  $\beta_{2j}^m$  are amplitude ratios, which can be obtained from the simplified Eq. (3.16) :

$$\begin{aligned}\beta_{1j}^m &= \frac{-i\eta^m (c_{12}^m + c_{66}^m)}{(c_{11}^m - c_{66}^m \eta^{m^2} - \rho^m c^2)}, j = 1-4 \\ \beta_{2j}^m &= \frac{-f_{15}^m (\eta^{m^2} - 1)}{c_{44}^m (\eta^{m^2} - 1) + \rho^m c^2}, j = 5-8\end{aligned}\quad (3.18)$$

## (2) Solution of the PE layer

Similarly, we can obtain homogeneous linear equations for the undetermined coefficients  $A_1^e$ ,  $A_2^e$ ,  $A_3^e$ ,  $A_\Phi^e$  and  $A_\Psi^e$  in the PE layer:

$$\begin{aligned}(c_{11}^e - c_{66}^e \eta^{e^2} - \rho^e c^2) A_1^e + i\eta^e (c_{12}^e + c_{66}^e) A_2^e &= 0 \\ i\eta^e (c_{12}^e + c_{66}^e) A_1^e + (c_{66}^e - c_{11}^e \eta^{e^2} - \rho^e c^2) A_2^e &= 0 \\ [c_{44}^e (\eta^{e^2} - 1) + \rho^e c^2] A_3^e + e_{15}^e (\eta^{e^2} - 1) A_\Phi^e &= 0 \\ e_{15}^e (\eta^{e^2} - 1) A_3^e - \varepsilon_{11}^e (\eta^{e^2} - 1) A_\Phi^e &= 0 \\ (\eta^{e^2} - 1) A_\Psi^e &= 0\end{aligned}\quad (3.19)$$

By decoupling Eq. (3.19)<sub>5</sub> from the other four equations,  $\eta^e = \pm 1$  is obtained. In addition, in order to obtain non-trivial solutions for  $A_1^e$ ,  $A_2^e$ ,  $A_3^e$  and  $A_\Phi^e$ , the coefficients of the two sets of decoupling equations (3.19)<sub>1</sub>, (3.19)<sub>2</sub> and (3.19)<sub>3</sub>, (3.19)<sub>4</sub> are zero, respectively. Thus, the complete solution of the PE layer can be expressed as:

$$\begin{Bmatrix} u_1^e \\ u_2^e \\ u_3^e \\ \varphi^e \\ \psi^e \end{Bmatrix} = \begin{Bmatrix} \sum_{j=1}^4 \beta_{1j}^e A_{2j}^e \\ \sum_{j=1}^4 A_{2j}^e \\ \sum_{j=5}^8 \beta_{2j}^e A_{\Phi j}^e \\ \sum_{j=5}^8 A_{\Phi j}^e \\ \sum_{j=9}^{10} A_{\Psi j}^e \end{Bmatrix} e^{-k\eta_j^e x_2} e^{ik(x_1 - ct)} \quad (3.20)$$



where  $A_{2j}^e$  ( $j = 1-4$ ) and  $A_{\Phi j}^e$  ( $j = 5-8$ ) are the amplitude to be obtained.  $\beta_{1j}^e$  and  $\beta_{3j}^e$  are amplitude ratios, which can be obtained from the Eq. (3.19).

### (3) Solution of the semiconductor substrate

Similarly, a homogeneous linear system of undetermined coefficients  $A_1^s$ ,  $A_2^s$ ,  $A_3^s$ ,  $A_\Phi^s$  and  $A_N^s$  in the semiconductor half-space can be obtained as:

$$\begin{aligned} (c_{11}^s - c_{66}^s \eta^{s^2} - \rho^s c^2) A_1^s + i \eta^s (c_{12}^s + c_{66}^s) A_2^s &= 0 \\ i \eta^s (c_{12}^s + c_{66}^s) A_1^s + (c_{66}^s - c_{11}^s \eta^{s^2} - \rho^s c^2) A_2^s &= 0 \\ [c_{44}^s (\eta^{s^2} - 1) + \rho^s c^2] A_3^s &= 0 \\ k^2 \varepsilon_{11}^s (\eta^{s^2} - 1) A_\Phi^s - q A_N^s &= 0 \\ n_0 \mu_{11} k (\eta^{s^2} - 1) A_\Phi^s - [D_{11} k (\eta^{s^2} - 1) + ic] A_N^s &= 0 \end{aligned} \quad (3.21)$$

Eq. (3.21)<sub>3</sub> is decoupled from the other four equations to give  $\eta^s = \pm \sqrt{1 - \rho^s c^2 / c_{44}^s}$ , and because of the radiation properties, there is  $\eta_5^s = \sqrt{1 - \rho^s c^2 / c_{44}^s}$ . In order to obtain the non-zero solution, the determinant of the coefficients of the remaining four equations of the Eq. (3.21) must be zero, and eight  $\eta^s$  can be obtained, which are the positive and negative complex roots of the four pairs. Due to the radiation conditions, only 4  $\eta^s$  with positive real parts are retained.

The complete solution of the semiconductor substrate can be obtained as:

$$\begin{Bmatrix} u_1^s \\ u_2^s \\ u_3^s \\ \varphi^s \\ \Delta n^s \end{Bmatrix} = \begin{Bmatrix} \sum_{j=1}^2 \beta_{1j}^s A_{2j}^s \\ \sum_{j=1}^2 A_{2j}^s \\ \sum_{j=5}^5 A_{3j}^s \\ \sum_{j=3}^4 A_{\Phi j}^s \\ \sum_{j=3}^4 \beta_{2j}^s A_{\Phi j}^s \end{Bmatrix} e^{-k \eta_j^s x_2} e^{ik(x_1 - ct)} \quad (3.22)$$

where  $A_{2j}^s$  ( $j = 1-2$ ) and  $A_{\Phi j}^s$  ( $j = 3-4$ ) are the amplitude to be obtained.  $\beta_{1j}^s$  and  $\beta_{2j}^s$  are amplitude ratios, which can be obtained from Eq. (3.21).

By substituting the solutions of PM layer Eq. (3.17), PE layer Eq. (3.20) and semiconductor layer Eq. (3.22) to the corresponding constitutive equations (3.3),

(3.6) and (3.9), respectively, the physical quantity expressions of each layer can be obtained in Appendix A.3.1-A.3.3 of Chapter 3.

### 3.2.2 Dispersion equations

By substituting the physical quantity expressions of each layer to the boundary conditions and continuous conditions (3.12)-(3.14), we can get the total of 25 homogeneous linear equations with unknown amplitudes about  $A_{2j}^m$  ( $j = 1-4$ ),  $A_{\Psi j}^m$  ( $j = 5-8$ ),  $A_{\Phi j}^m$  ( $j = 9-10$ ),  $A_{2j}^e$  ( $j = 11-14$ ),  $A_{\Phi j}^e$  ( $j = 15-18$ ),  $A_{\Psi j}^e$  ( $j = 19-20$ ),  $A_{2j}^s$  ( $j = 21-22$ ),  $A_{\Phi j}^s$  ( $j = 23-24$ ) and  $A_{3j}^s$  ( $j = 25$ ), which can be expressed as:

$$[N]A = 0 \quad (3.23)$$

where  $[N]$  is a  $25 \times 25$  coefficient matrix, the non-zero elements of which are given in the Appendix of this Chapter as (A.3.4a) and (A.3.4b).

## 3.3 Numerical results and discussions

In this Section, the propagation characteristics of Love and Rayleigh waves in PM/PE/semiconductor three-layer structures will be discussed in detail. In the calculation, the Terfenol-D/BaTiO<sub>3</sub>/Si structure is considered by default, and the PM material CoFe<sub>2</sub>O<sub>4</sub>, PE materials PZT-4 and PZT-5H are selected for comparison. Specifically, four material combinations are considered, namely: Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si, Terfenol-D/PZT-4/Si, and Terfenol-D/PZT-5H/Si.

Among them, the first and the second combination only replace the PM layer, which aims to study the influence of PM material on the surface waves of the composite structure, while the third and fourth material combination can explore the influence of PE layer on the waves compared with the first combination that changes the PE layer material.

Table 3.1 shows the main parameters of the two PM materials. It is worth noting that although the parameters of CoFe<sub>2</sub>O<sub>4</sub> and Terfenol-D have been given in Table 2.2 and 2.3 respectively, more material parameters will be used in this Chapter, so it is listed again in combination with other literature. Table 3.2 lists the material properties of PE materials and Si (the superscripts  $m$ ,  $e$  and  $s$  are uniformly omitted, and the  $c_{sh}$  in the last row is the shear bulk wave velocity).

Table 3.1 Material properties of Terfenol-D<sup>[165,185,186]</sup> and CoFe<sub>2</sub>O<sub>4</sub><sup>[182,183]</sup>

| Material constants                           | Terfenol-D <sup>[165,185,186]</sup> | CoFe <sub>2</sub> O <sub>4</sub> <sup>[182,183]</sup> |
|----------------------------------------------|-------------------------------------|-------------------------------------------------------|
| $\rho$ (kg/m <sup>3</sup> )                  | 9230                                | 5300                                                  |
| $c_{11}$ (10 <sup>9</sup> N/m <sup>2</sup> ) | 8.541                               | 286                                                   |
| $c_{12}$ (10 <sup>9</sup> N/m <sup>2</sup> ) | 0.654                               | 173                                                   |

|                                                                   |         |         |
|-------------------------------------------------------------------|---------|---------|
| $c_{13}$ ( $10^9$ N/m <sup>2</sup> )                              | 3.91    | 170.5   |
| $c_{33}$ ( $10^9$ N/m <sup>2</sup> )                              | 28.3    | 269.5   |
| $c_{44}$ ( $10^9$ N/m <sup>2</sup> )                              | 5.99    | 45.3    |
| $\varepsilon_{11}$ ( $10^{-11}$ C <sup>2</sup> /Nm <sup>2</sup> ) | 5       | 8       |
| $\varepsilon_{33}$ ( $10^{-11}$ C <sup>2</sup> /Nm <sup>2</sup> ) | 5       | 9.3     |
| $f_{15}$ (N/Am)                                                   | 167.66  | 550     |
| $f_{31}$ (N/Am)                                                   | -5.7471 | 580.3   |
| $f_{33}$ (N/Am)                                                   | 270.1   | 699.7   |
| $\alpha_{11}$ ( $10^{-6}$ Ns <sup>2</sup> / C <sup>2</sup> )      | 8.644   | 157     |
| $\alpha_{33}$ ( $10^{-6}$ Ns <sup>2</sup> / C <sup>2</sup> )      | 2.268   | 157     |
| $c_{sh}$ (m/s)                                                    | 1190    | 2985.08 |

Table 3.2 Material properties of BaTiO<sub>3</sub>/PZT-4/PZT-5H /Si<sup>[19,70,165,183,187-189]</sup>

| Material constants                                               | BaTiO <sub>3</sub> <sup>[183]</sup> | PZT-4 <sup>[165,183]</sup> | PZT-5H <sup>[187,188]</sup> | Si <sup>[19,70,189]</sup> |
|------------------------------------------------------------------|-------------------------------------|----------------------------|-----------------------------|---------------------------|
| $\rho$ (kg/m <sup>3</sup> )                                      | 5800                                | 7500                       | 7500                        | 2332                      |
| $c_{11}$ ( $10^9$ N/m <sup>2</sup> )                             | 166                                 | 138.5                      | 126                         | 165.7                     |
| $c_{12}$ ( $10^9$ N/m <sup>2</sup> )                             | 77                                  | 77.37                      | 79.5                        | 63.9                      |
| $c_{13}$ ( $10^9$ N/m <sup>2</sup> )                             | 78                                  | 73.64                      | 84.1                        | 63.9                      |
| $c_{33}$ ( $10^9$ N/m <sup>2</sup> )                             | 162                                 | 114.75                     | 117                         | 166                       |
| $c_{44}$ ( $10^9$ N/m <sup>2</sup> )                             | 43                                  | 25.6                       | 23                          | 79.56                     |
| $\varepsilon_{11}$ ( $10^{-9}$ C <sup>2</sup> /Nm <sup>2</sup> ) | 11.2                                | 6.46                       | 15.05                       | 10.45                     |
| $\varepsilon_{33}$ ( $10^{-9}$ C <sup>2</sup> /Nm <sup>2</sup> ) | 12.6                                | 5.62                       | 13.02                       | 10.45                     |
| $\alpha_{11}$ ( $10^{-6}$ Ns <sup>2</sup> / C <sup>2</sup> )     | 5                                   | 5                          | 5                           | /                         |
| $\alpha_{33}$ ( $10^{-6}$ Ns <sup>2</sup> / C <sup>2</sup> )     | 10                                  | 10                         | 10                          | /                         |
| $e_{15}$ (C/m <sup>2</sup> )                                     | 11.6                                | 12.7                       | 17                          | /                         |
| $e_{31}$ (C/m <sup>2</sup> )                                     | -4.4                                | -5.2                       | -6.5                        | /                         |
| $e_{33}$ (C/m <sup>2</sup> )                                     | 18.6                                | 15.08                      | 23.3                        | /                         |
| $\mu_{11}$ (cm <sup>2</sup> /Vs)                                 | /                                   | /                          | /                           | 1450                      |
| $\mu_{33}$ (cm <sup>2</sup> /Vs)                                 | /                                   | /                          | /                           | 1450                      |
| $D_{11}$ (cm <sup>2</sup> /s)                                    | /                                   | /                          | /                           | 37.7                      |
| $D_{33}$ (cm <sup>2</sup> /s)                                    | /                                   | /                          | /                           | 37.7                      |
| $c_{sh}$ (m/s)                                                   | 3079.81                             | 2596.6                     | 2372.13                     | 5840.94                   |

### 3.3.1 Love waves in the three-layer composite structure

When surface waves propagate along the  $x_1$ -axis in the PM/PE/semiconductor composite structure, Love waves and Rayleigh waves can in principle be decoupled. Therefore, we first consider the propagation of Love waves.

Since the displacement of Love waves is only the  $u_3$  component, the coupled wave equations involving  $u_1$  and  $u_2$  in each layer, the equations involving  $u_1$  and  $u_2$  in the boundary conditions, and the equations involving  $T_{21}$  and  $T_{22}$  will disappear, then the  $25 \times 25$  coefficient matrix  $[N]$  in dispersion equation (3.23) will correspondingly become a  $15 \times 15$  matrix.

If not otherwise specified, consider Terfermol-D/BaTiO<sub>3</sub>/Si, and the main parameters are  $h_1 = h_2 = 50 \mu\text{m}$ ,  $n_0 = 5 \times 10^{16} / \text{m}^3$  (the main parameters of Rayleigh waves in the following Section are also the same).

As shown in Fig. 3.2, the first three modes of Love waves in Terfermol-D/BaTiO<sub>3</sub>/Si layered structure are obtained. It can be seen that the velocity of the first mode of the Love waves begins with the shear wave velocity of the matrix Si (i.e., normalized 1) and approaches to the shear wave velocity of the covering layer Terfermol-D (i.e., normalized 0.2037). The difference is that the starting point of the second and third modes is slightly larger than  $c_{sh}^s$  (the starting point of the second and third order modes is 1.0145 and 1.0253, respectively), which is similar to the Love waves propagating in the PS film/elastic half-space with a shear wave speed slightly larger than the elastic matrix [106].

In addition, the calculated wave speeds are all real numbers without imaginary parts, indicating that the Love waves in the three-layer structure does not appear to decay similar to that in Chapter 2, which will be discussed below.

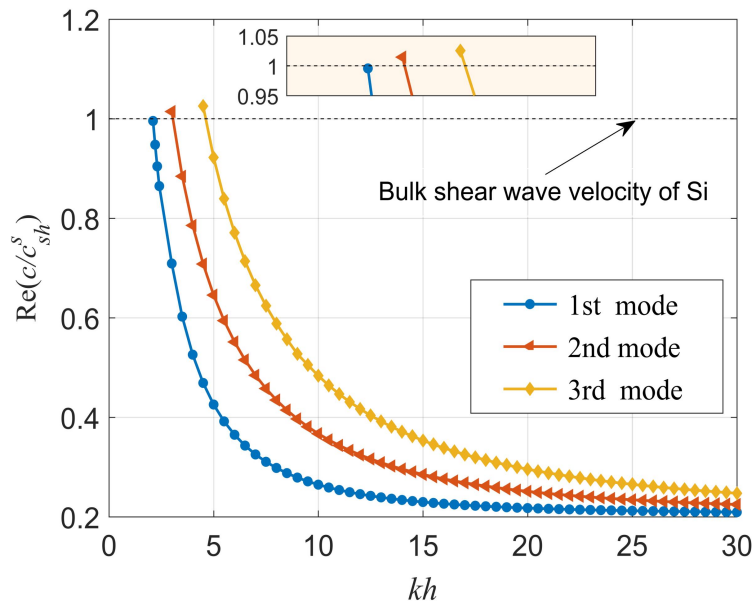


Fig. 3.2 The first three modes of Love waves in the Terfermol-D/BaTiO<sub>3</sub>/Si layered composite structure

#### 3.3.1.1 Effect of covering layer thickness on Love waves

The thickness of the covering layer in the layered half-space is an important geometric size. According to the references and the conclusion of Chapter 2, the change of the covering layer thickness has a significant effect on the wave speed and attenuation of the surface waves in the PSs. Therefore, in this Section, the influence of PM layer thickness  $h_2$  and PE layer thickness  $h_1$  on the propagation characteristics of Love waves will be considered, respectively. When considering the influence of one layer, keep the thickness of the other layer unchanged.

### (1) Effect of PM layer thickness on Love waves

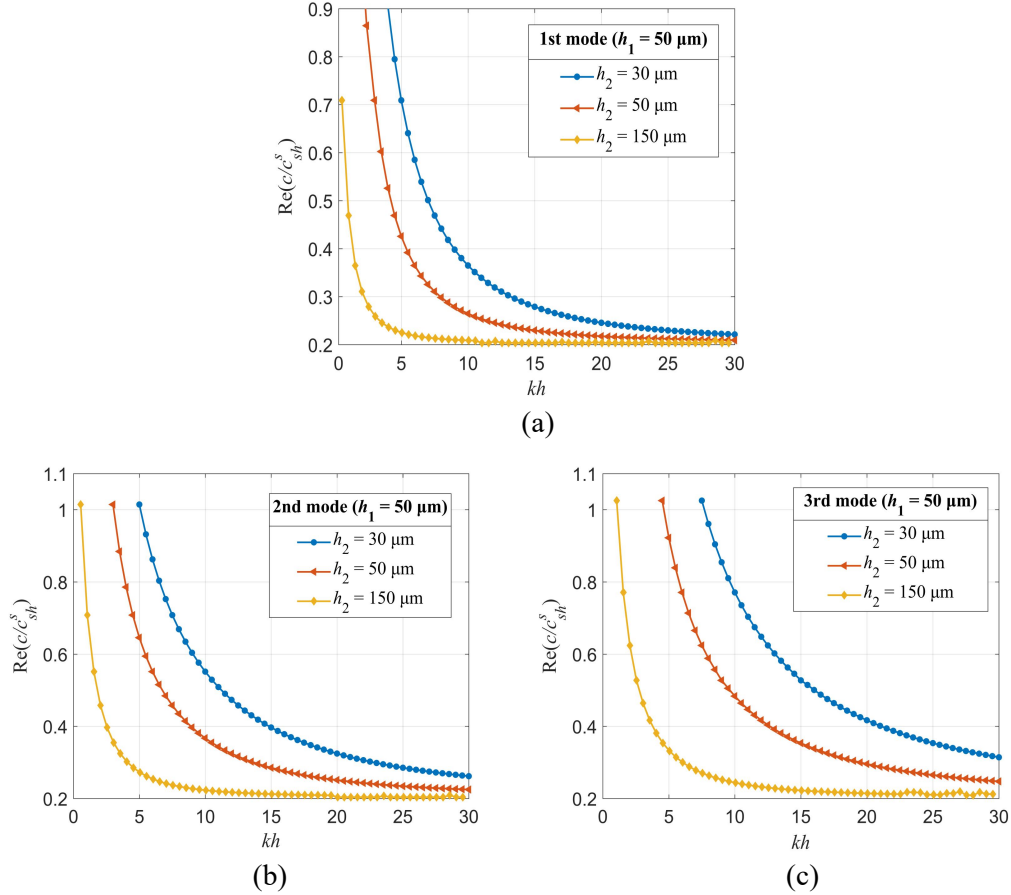


Fig. 3.3 Effect of piezomagnetic layer thickness  $h_2$  on the first three modes of Love waves: (a) effect of  $h_2$  on the first mode, (b) effect of  $h_2$  on the second mode, and (c) effect of  $h_2$  on the third mode

Figure 3.3 shows the first three modes of Love waves when the PM layer thickness  $h_2$  is  $30 \mu\text{m}$ ,  $50 \mu\text{m}$ , and  $150 \mu\text{m}$  ( $h_1 = 50 \mu\text{m}$  remains constant), respectively. According to Fig. 3.3, the PM layer thickness  $h_2$  has a significant influence on the wave speed. Regardless of modes, the wave speed decreases with the increase of  $h_2$ , which is consistent with the results in Chapter 2. Among them, the starting point of the first mode in Fig. 3.3(a) is smaller than the shear body wave velocity of the matrix Si, while the second mode in Fig. 3.3(b) and third mode in Fig. 3.3(c) are both slightly larger than  $c_{sh}^s$ , which is consistent with Fig. 3.2. In addition, the third mode in Fig. 3.3(c) has the most obvious change in wave speed.

## (2) Effect of PE layer thickness on Love waves

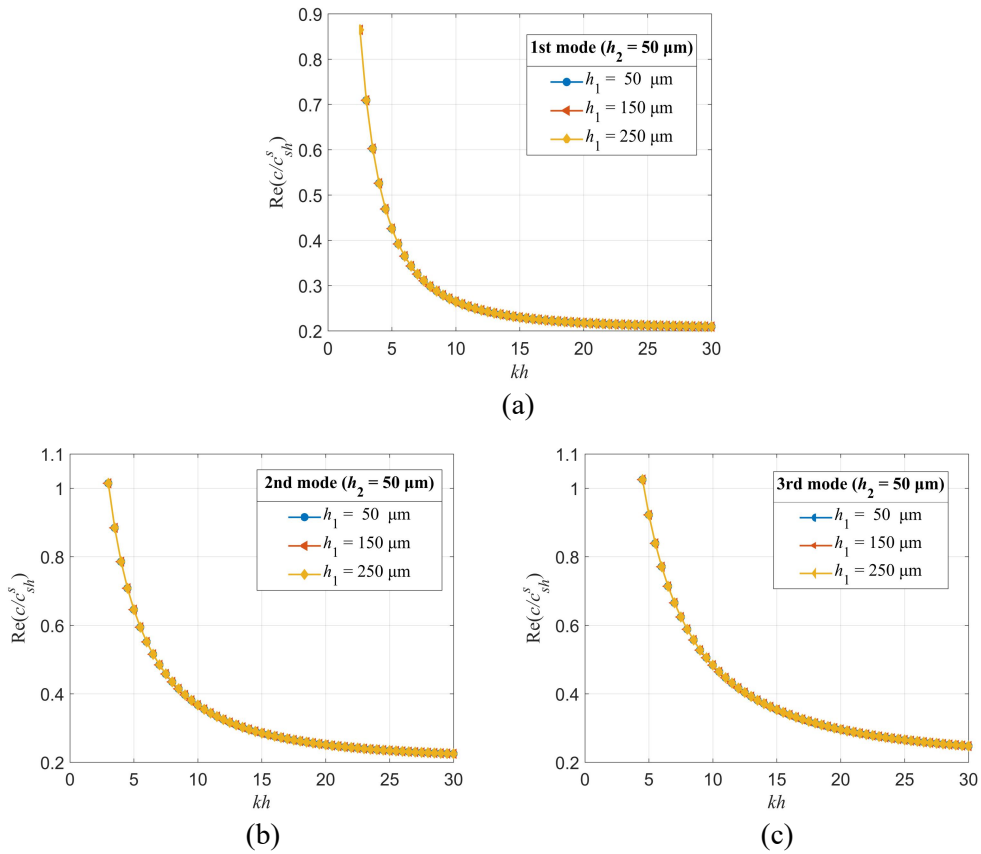
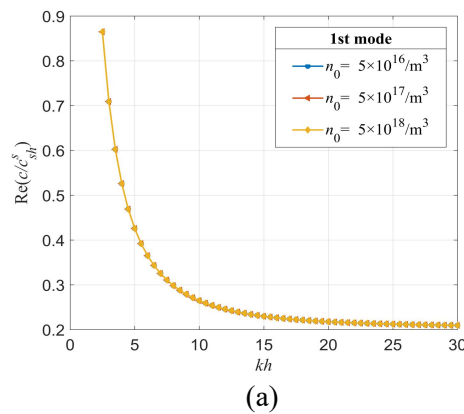


Fig. 3.4 Effect of piezoelectric layer thickness  $h_1$  on the first three modes of Love waves: (a) effect of  $h_1$  on the first mode, (b) effect of  $h_1$  on the second mode, and (c) effect of  $h_1$  on the third mode

Figure 3.4 shows the first three modes of Love waves when the PE layer thickness  $h_1$  is  $50 \mu\text{m}$ ,  $150 \mu\text{m}$ , and  $250 \mu\text{m}$  ( $h_2 = 50 \mu\text{m}$  remains constant), respectively. As can be seen from Fig. 3.4, no matter the first, second or third modes, the change of PE layer thickness  $h_1$  has no influence on the wave speed of Love waves.

### 3.3.1.2 Effect of initial carrier concentration on Love waves



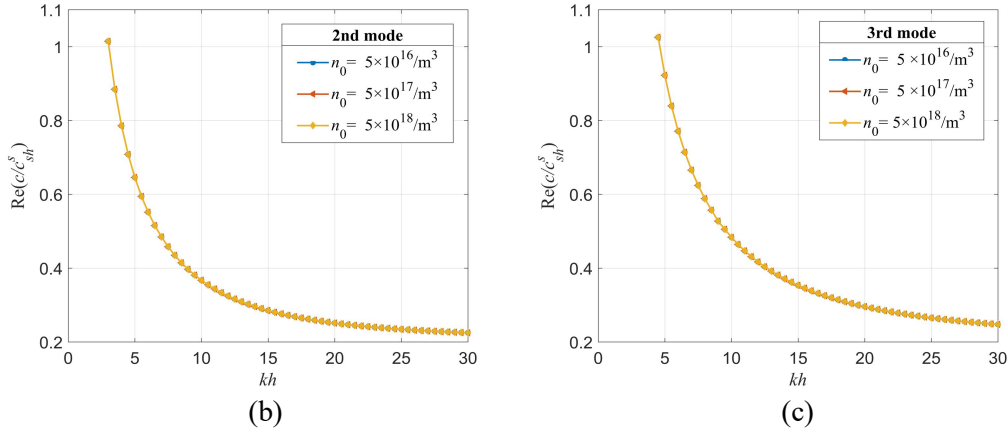


Fig. 3.5 Effect of initial carrier concentration  $n_0$  on the first three modes of Love waves: (a) effect of  $n_0$  on the first mode, (b) effect of  $n_0$  on the second mode, and (c) effect of  $n_0$  on the third mode

Similar to the PM layer/PS half-space in Chapter 2, the semiconductor exists as the substrate, and there are two covering layers on it, so the semiconductor characteristics of Si have an indirect effect on the propagation characteristics of Love waves. As shown in Fig. 3.5, different initial carrier concentrations  $n_0$  have almost no influence on the first three modes of Love waves, which is also consistent with the result that the PE layer thickness  $h_1$  has no influence on Love waves. It is worth noting that even if  $n_0$  continues to be increased, such as  $n_0 = 5 \times 10^{20}/\text{m}^3$  and  $n_0 = 5 \times 10^{22}/\text{m}^3$ , the wave speed corresponding to the third mode of the Love waves does not change (Fig. 3.5 does not repeat this).

In addition, unlike the complex phase velocities in Chapter 2, the phase velocities of Love waves obtained in this Chapter are all real numbers, indicating that Love waves do not decay in the PM/PE/semiconductor three-layer structure. This phenomenon is easy to understand, the PE layer as the second covering layer has almost no influence on the Love waves, and Si as the substrate is even more difficult to affect the propagation of Love waves.

### 3.3.1.3 Effect of piezomagnetic constant on Love waves

As the PM layer is the main covering layer which has direct influence on Love waves, the influence of PM constant  $f_{15}$  in the PM layer on Love wave is further studied.

Similar to the results of Love waves in PM layer/PS half-space (Fig. 2.11), PM constant  $f_{15}$  also has a significant influence on wave speed as shown in Fig. 3.6, but the influence is slightly smaller than that in PM layer/PS half-space. In calculation, as in Chapter 2,  $f_{15}$  also selects 100 N/Am, 167.66 N/Am and 200 N/Am. In the same  $kh$ , no matter in which mode, the wave speed will decrease with the increase of  $f_{15}$ , and the wave speed will decrease more slowly with the increase of  $kh$ . The change trend of the three modes in Fig. 3.6 is the same, with only a slight change in value.

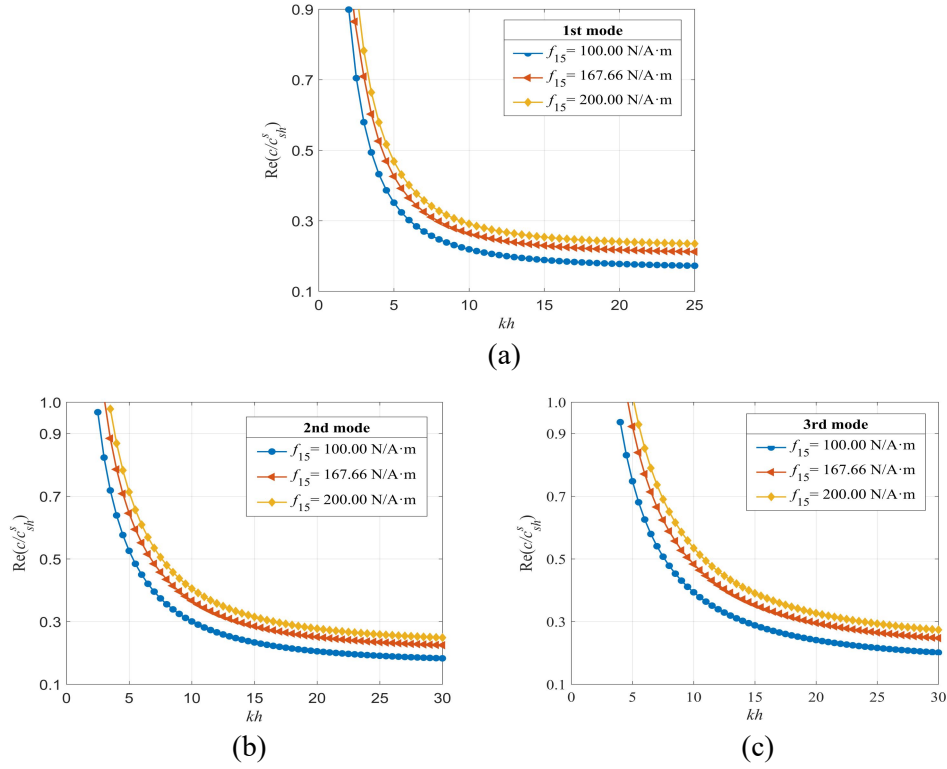


Fig. 3.6 Effect of piezomagnetic constant  $f_{15}$  on the first three modes of Love waves: (a) effect of  $f_{15}$  on the first mode, (b) effect of  $f_{15}$  on the second mode, and (c) effect of  $f_{15}$  on the third mode

### 3.3.1.4 Love waves propagating in different material combinations

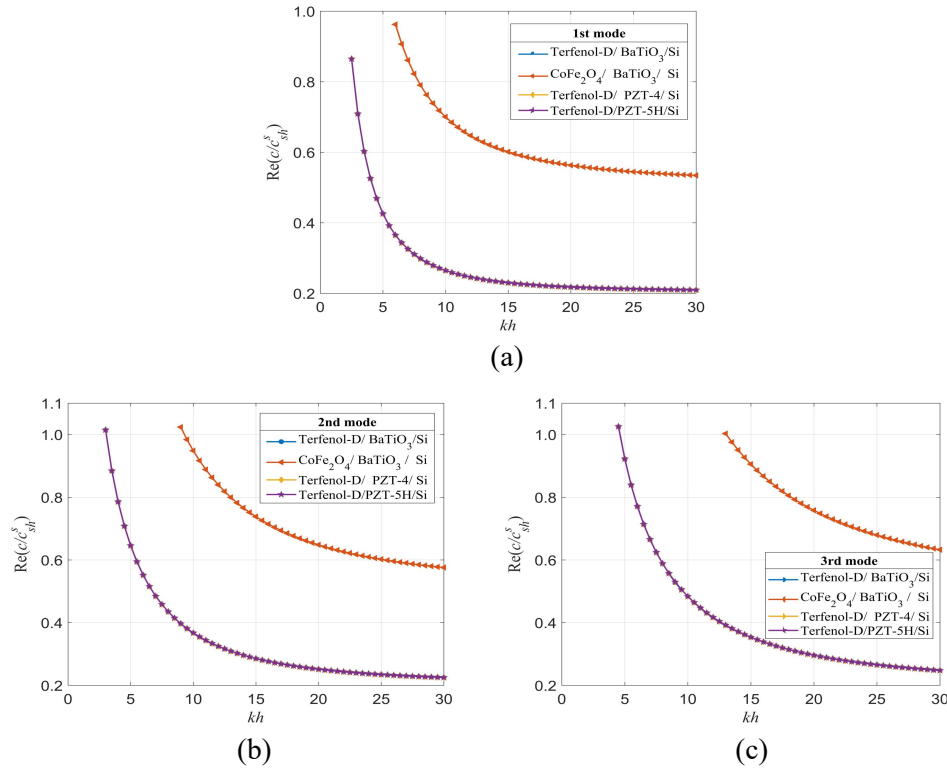


Fig. 3.7 The first three modes of Love waves in four different material combinations: (a) comparison of the first mode, (b) comparison of the second mode, and (c) comparison of the third mode



Considering four different material structures of Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si, Terfenol-D/PZT-4/Si and Terfenol-D/PZT-5H/Si (keeping the substrate Si unchanged). The propagation characteristics of the first three modes of Love waves of different material combinations are obtained, as shown in Fig. 3.7. According to Figs. 3.7(a)-3.7(c), the three groups of curves of the first, third and fourth material combinations coincide, indicating that different PE layer materials BaTiO<sub>3</sub>, PZT-4 and PZT-5H have no influence on the first three modes of Love waves. Compared with Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si has changed the PM layer material, resulting in a significant change in wave speed. The wave speed of first three modes of CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si (red solid line) are correspondingly much larger than those of the other three structures.

In order to further study the influence of the PM layer, the first three modes of Love waves in Terfenol-D/BaTiO<sub>3</sub>/Si and CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si are obtained, as shown in Fig. 3.8. The three solid lines on the left are the first three modes of Love waves in Terfenol-D/BaTiO<sub>3</sub>/Si, while the three dotted lines on the right are the results of CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si. It can be easily found that the wave speed of first three modes of the former are significantly smaller than that of the latter.

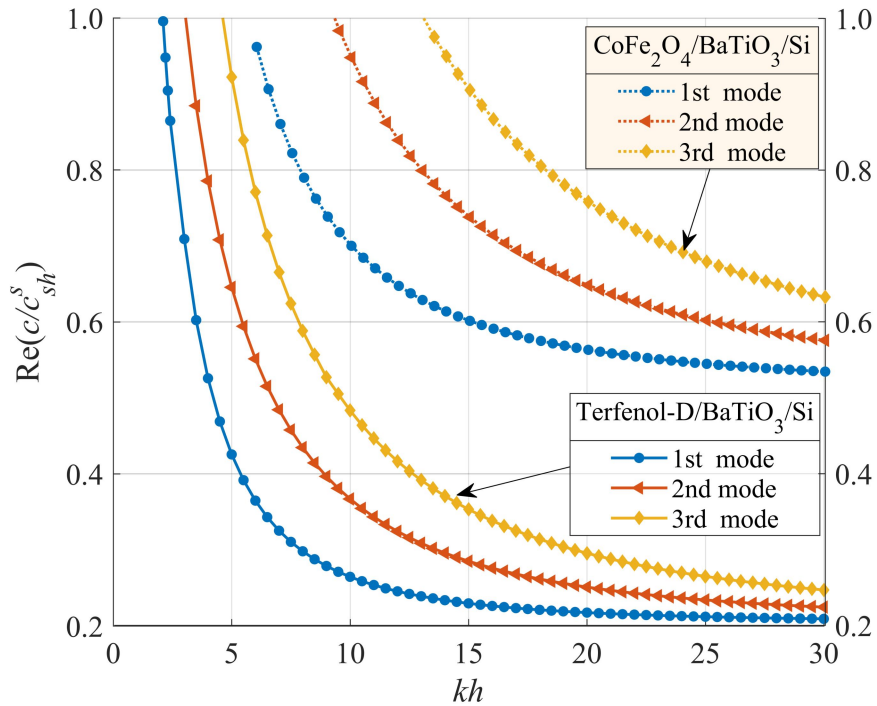


Fig. 3.8 The first three modes of Love waves in the Terfenol-D/BaTiO<sub>3</sub>/Si and CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si layered structures

### 3.3.2 Rayleigh waves in the three-layer composite structure

In Section 3.3.1 above, we discussed the propagation of Love waves in composite structures. Here, we will continue to consider the propagation of Rayleigh waves. In principle, the Rayleigh waves correspond to the two displacement components  $u_1$  and  $u_2$  in the coordinate system of Fig. 3.1. As long as the coupled wave equations involving  $u_3$  in each layer and the equations involving  $u_3$  and  $T_{23}$  in the

boundary conditions are removed, the  $25 \times 25$  coefficient matrix  $[N]$  in the dispersion equation (3.23) will correspondingly become a  $20 \times 20$  matrix, which can be solved. In fact, it can not be solved, mainly because  $A_{\Phi}^e$ ,  $A_{\Psi}^e$  and  $A_1^e$ ,  $A_2^e$  in coupled wave equations of PE layer are decoupled, and the magnetic-electric-semiconductor coupling cannot be realized at this time.

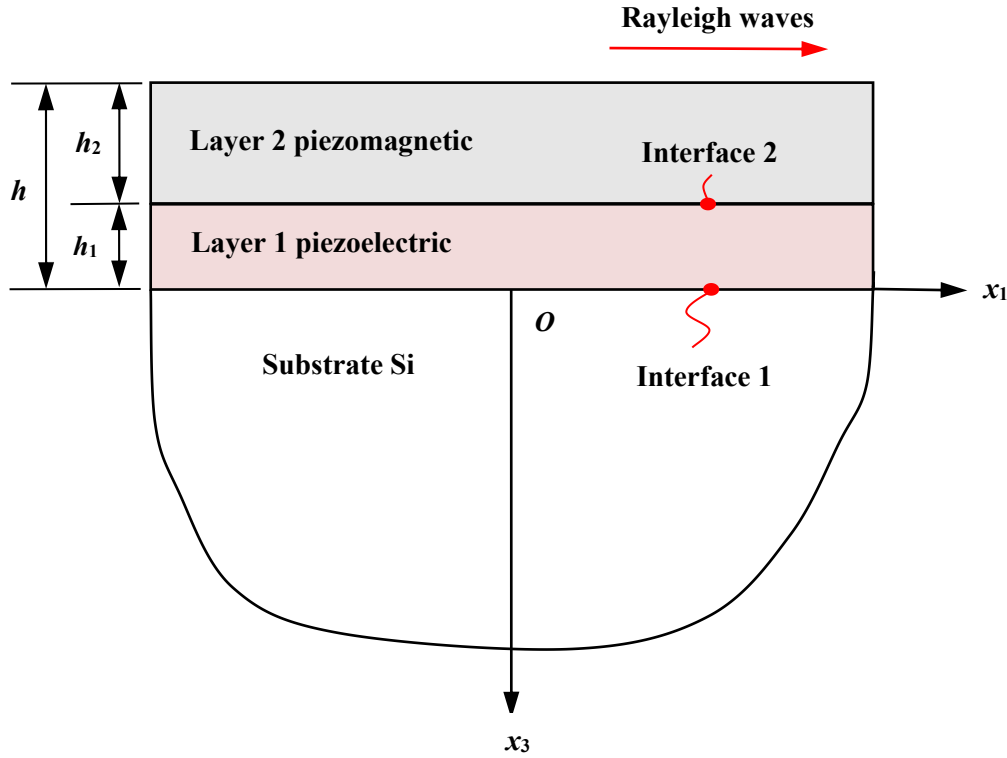


Fig. 3.9 The piezomagnetic/piezoelectric/semiconductor laminates after the change of coordinate system

Therefore, when studying Rayleigh waves propagation, the coordinate system is considered to be changed, that is, the vertical axis  $x_2$  of the thickness in the original Fig. 3.1 is replaced by the  $x_3$ -axis, the polarization direction is still along the  $x_3$ -axis, and the Rayleigh wave propagates along the  $x_1$ -axis. In fact, if the coordinate system in Fig. 3.1 is unchanged, converting the polarization direction from the  $x_3$ -axis to the  $x_2$ -axis should have the same effect. This Chapter does not focus on the influence of polarization direction, so it is not emphasized.

As in the case of Love waves, the Terfermol-D/BaTiO<sub>3</sub>/Si composite structure with covering layer thickness  $h_1 = h_2 = 50 \mu\text{m}$  and initial carrier concentration  $n_0 = 5 \times 10^{16}/\text{m}^3$  will continue to be considered unless otherwise specified.

Through the solving process similar to Love waves, the phase velocity equation and dispersion equation of Rayleigh waves after the change of coordinate system can be easily obtained, where the dispersion equation is about a  $A_{1j}^m$  ( $j = 1-6$ ),  $A_{\Phi j}^m$  ( $j = 7-8$ ),  $A_{1j}^e$  ( $j = 1-6$ ),  $A_{\Psi j}^e$  ( $j = 7-8$ ) and  $A_{3j}^s$  ( $j = 1-2$ ),  $A_{\Phi j}^s$  ( $j = 3-4$ ), a total of 20 homogeneous linear equations with unknown amplitudes are expressed as:

$$[M]A = 0 \quad (3.24)$$

where  $[M]$  is a coefficient matrix of  $20 \times 20$ , and its solution process is shown in the (A.3.5)-(A.3.19) of Appendix to this Chapter.

In order to fully compare with Love waves, the first five modes of Rayleigh waves are also obtained in this Section, and then the influences of covering layer thickness, initial carrier concentration, PM constant and different material combinations on Rayleigh waves are studied (the order of discussion is consistent with that of Love waves in Section 3.3.1).

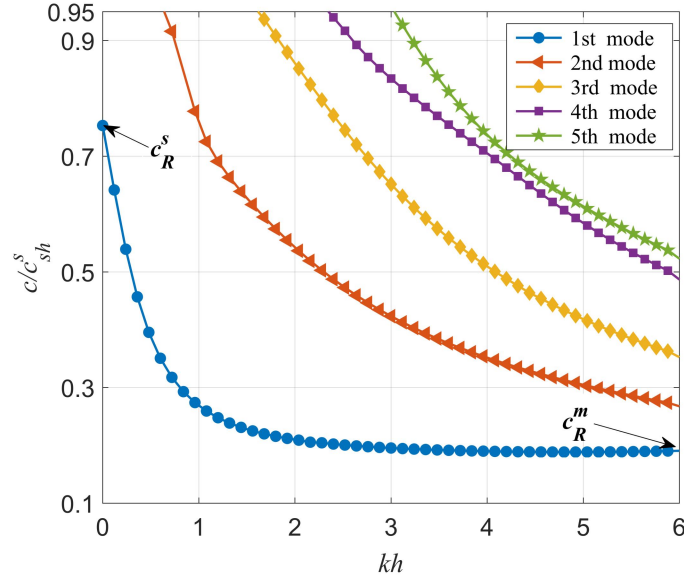


Fig. 3.10 The first five modes of Rayleigh waves in the Terferrol-D/BaTiO<sub>3</sub>/Si composite structure

Figure 3.10 shows the first five modes of Rayleigh waves in the composite structure. Similar to Fig. 3.2 of Love waves, the wave speed decreases with the increase of  $kh$  regardless of the modes, and there will be a cutoff frequency for the second to fifth modes, and the shear wave speed starts from the semiconductor substrate. The difference is that the first mode of Rayleigh waves is the basic mode, which does not start from  $c_R^s$ , but starts from the Rayleigh wave speed of semiconductor, and tends to the Rayleigh wave speed of the PM covering layer. As can be seen from Fig. 3.10, the curve of the fourth mode is very close to that of the fifth mode.

### 3.3.2.1 Effect of covering layer thickness on Rayleigh waves

Consistent with the study in Section 3.3.1 on the effect of the covering layer thickness on the Love waves, when considering the impact of one covering layer, the thickness of the other covering layer is kept unchanged. Therefore, we obtain the effects of  $h_1$  and  $h_2$  on Rayleigh waves, respectively.

#### (1) Effect of PM layer thickness on Rayleigh waves

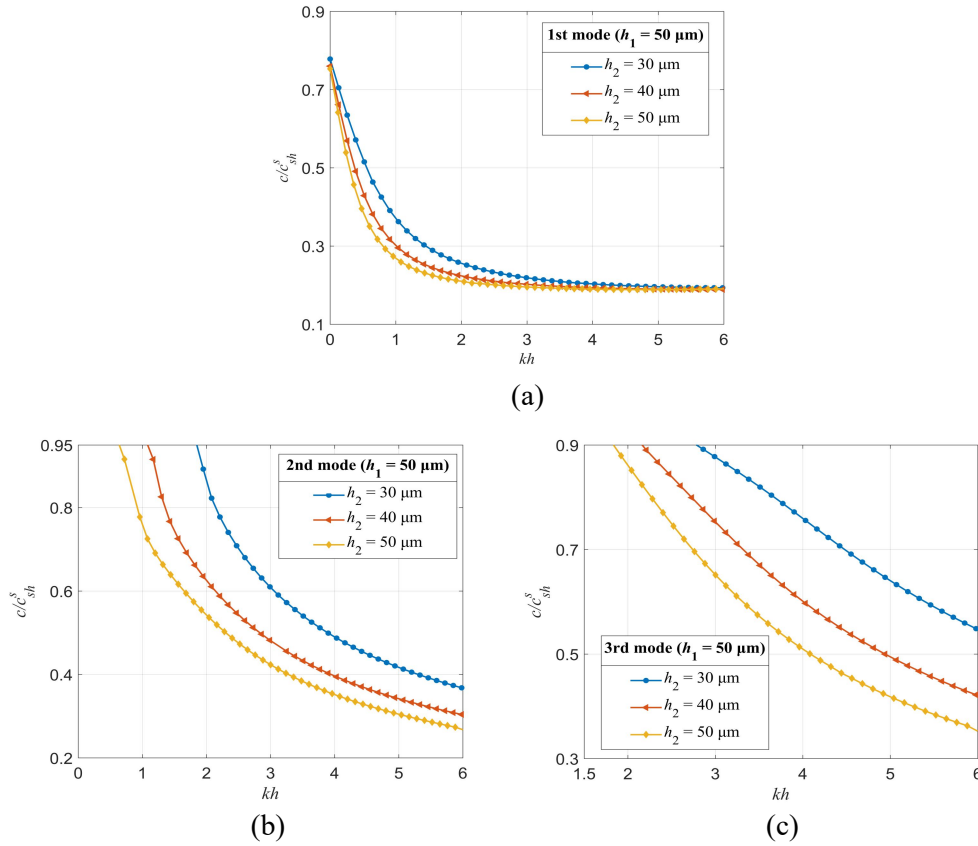
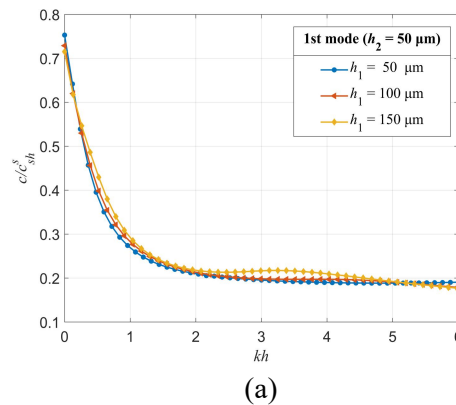


Fig. 3.11 Effect of piezomagnetic layer thickness  $h_2$  on the first three modes of Rayleigh waves: (a) effect of  $h_2$  on the first mode, (b) effect of  $h_2$  on the second mode, and (c) effect of  $h_2$  on the third mode

Figure 3.11 shows the first three modes of Rayleigh waves when the PM layer thickness  $h_2$  is  $30\mu\text{m}$ ,  $40\mu\text{m}$  and  $50\mu\text{m}$  ( $h_1 = 50\mu\text{m}$  remains unchanged), respectively. Similar to the dispersion curve of Love waves in Fig. 3.3,  $h_2$  also has a significant influence on the wave speed of Rayleigh waves, and the wave speed also decreases with the increase of  $h_2$ . Among them, Figure 3.11(a) as the basic mode has the same starting point and ending point of the wave speed in the three curves. In Fig. 3.11(c), the wave speed of the third mode changes most obviously.

## (2) Effect of PE layer thickness on Rayleigh waves

Figure 3.12 shows the first three modes of Rayleigh waves when the PE layer thickness  $h_1$  is  $50 \mu\text{m}$ ,  $100 \mu\text{m}$ , and  $150 \mu\text{m}$  ( $h_2 = 50 \mu\text{m}$  remains unchanged), respectively.



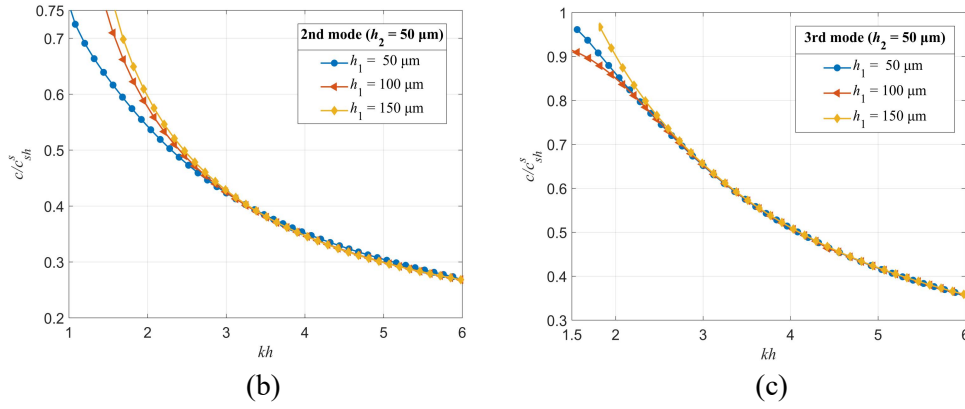


Fig. 3.12 Effect of piezoelectric layer thickness  $h_1$  on the first three modes of Rayleigh waves: (a) effect of  $h_1$  on the first mode, (b) effect of  $h_1$  on the second mode, and (c) effect of  $h_1$  on the third mode

Unlike in Fig. 3.4 where  $h_1$  has no effect on the Love waves,  $h_1$  in Fig. 3.12 has a certain influence on the Rayleigh wave speed, especially when  $kh$  is small (long waves) for the second and third mode.

According to Figs. 3.12(b)-(c), the influence of  $h_1$  on Rayleigh waves does not follow a certain rule, that is, the larger  $h_1$  is, the larger or smaller the wave speed is. However, when  $kh$  is small and  $h_1 = 150 \mu\text{m}$ , the wave speed corresponding to the orange lines will be the largest.

### 3.3.2.2 Effect of initial carrier concentration on Rayleigh waves

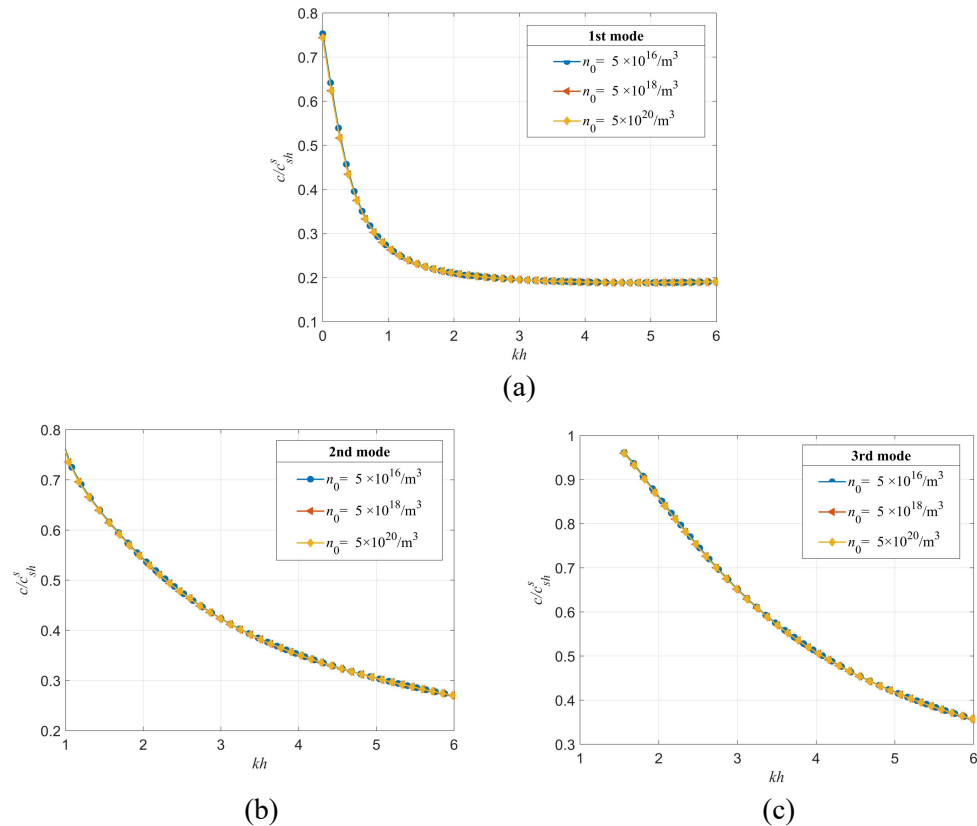


Fig. 3.13 Effect of initial carrier concentration  $n_0$  on the first three modes of Rayleigh waves: (a) effect of  $n_0$  on the first mode, (b) effect of  $n_0$  on the second mode, and (c) effect of  $n_0$  on the third mode

In Fig. 3.13, when the initial carrier concentrations are chosen as  $n_0 = 5 \times 10^{16} / \text{m}^3$ ,  $n_0 = 5 \times 10^{18} / \text{m}^3$ , and  $n_0 = 5 \times 10^{20} / \text{m}^3$  respectively, the phase velocities of the first three modes of Rayleigh waves remain almost unchanged. This is also consistent with the case of Love waves shown in Fig. 3.5.

### 3.3.2.3 Effect of piezomagnetic constant on Rayleigh waves

In Fig. 3.6, the PM constant  $f_{15}$  has a significant impact on the first three modes of Love waves, with the phase velocity increasing as  $f_{15}$  increases. However, as shown in Fig. 3.14, the effect of  $f_{15}$  on Rayleigh waves is limited, with only slight changes visible through local magnification. The reason is that in study Love waves, the PM constant only involves  $f_{15}$ . In contrast, while in the study of Rayleigh waves, three PM constants  $f_{15}$ ,  $f_{31}$ , and  $f_{33}$  are involved, so  $f_{15}$  cannot play a full role.

Additionally, as observed in Figs. 3.14(a)-(c), when  $kh$  is small, a smaller  $f_{15}$  results in a greater phase velocity. Conversely, when  $kh$  is large, local magnification reveals that a larger  $f_{15}$  results in a greater phase velocity, which is the opposite trend observed when  $kh$  is small. However, the influence of  $f_{15}$  remains very limited, especially in the third mode shown in Fig. 3.14(c).

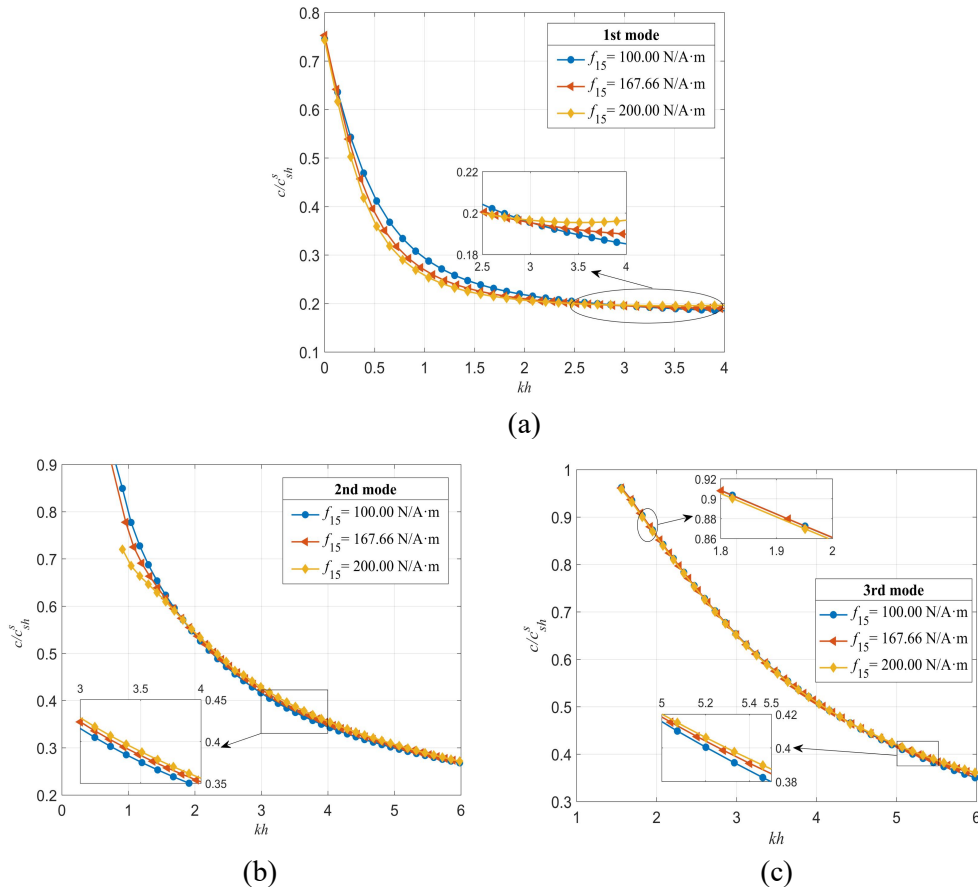


Fig. 3.14 Effect of piezomagnetic constant  $f_{15}$  on the first three modes of Love waves: (a) effect of  $f_{15}$  on the first mode, (b) effect of  $f_{15}$  on the second mode, and (c) effect of  $f_{15}$  on the third mode

### 3.3.2.4 Rayleigh waves propagating in different material combinations

Figure 3.15 shows the first three modes of Rayleigh waves for four different material combinations: Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si, Terfenol-D/PZT-4/Si, and Terfenol-D/PZT-5H/Si. Similar to the Love waves in Fig. 3.7, compared with Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si replaced the PM layer material, and the wave speed corresponding to the first three modes is significantly higher than that of the other three structures (the red line in Fig. 3.15 is far above the other three curves).

In Fig. 3.7, the curves for the three structures with different PE layer materials completely overlap, indicating that the different PE materials BaTiO<sub>3</sub>, PZT-4, and PZT-5H do not affect the phase velocities of the first three modes of Love waves. However, in Fig. 3.15, the three curves corresponding to different PE layer materials do not completely overlap. Specifically, the phase velocity of Terfenol-D/BaTiO<sub>3</sub>/Si is greater than that of Terfenol-D/PZT-4/Si, and Terfenol-D/PZT-5H/Si exhibits the lowest phase velocity. This difference is particularly noticeable in the second mode in Fig. 3.15(b) and the third mode in Fig. 3.15(c).

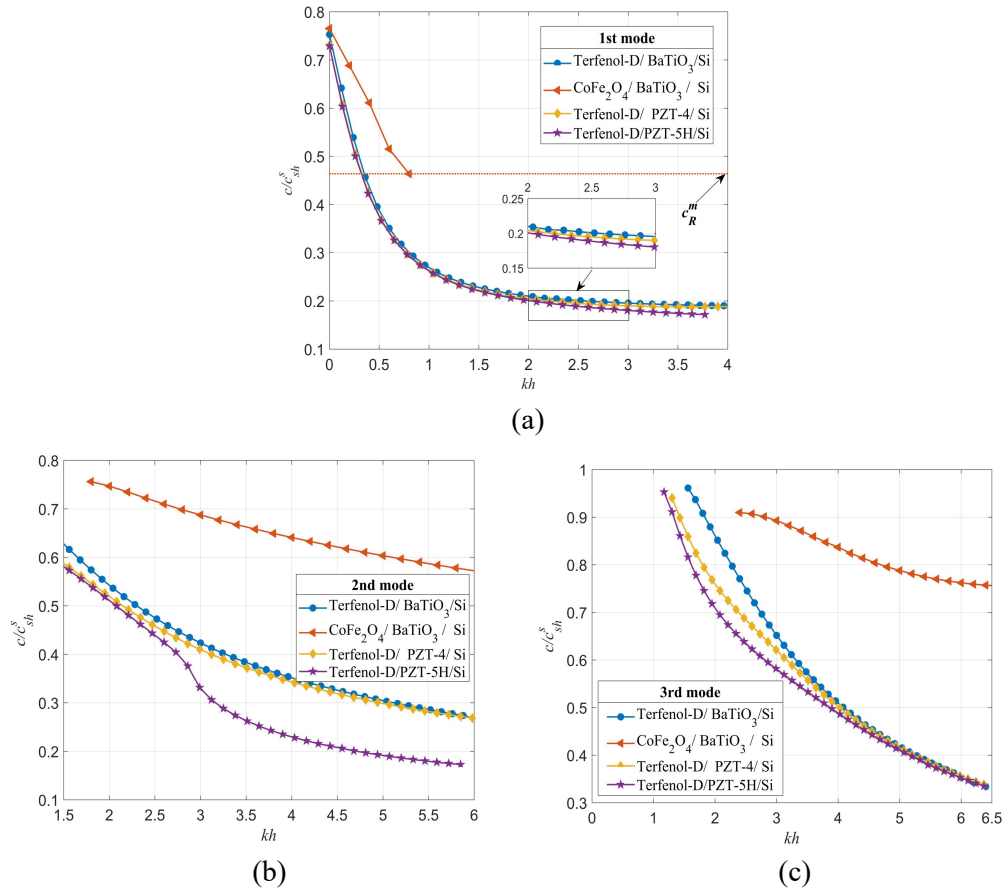


Fig. 3.15 The first three modes of Rayleigh waves in four different material combinations: (a) comparison of the first mode, (b) comparison of the second mode, and (c) comparison of the third mode

Similar to the Love waves in Section 3.3.1, the first three modes of Rayleigh waves in Terfenol-D/BaTiO<sub>3</sub>/Si and CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si are obtained in order to further determine the effect of PM materials on the wave speed, as shown in Fig.3.



16. The three curves of the solid line in Fig. 3.16 are the first three modes of Rayleigh waves in the former, while the three dashed lines correspond to the latter.

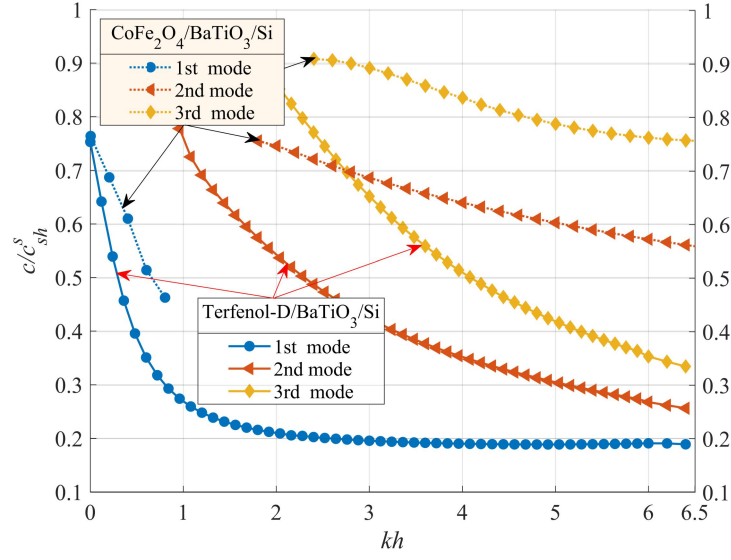


Fig. 3.16 The first three modes of Rayleigh waves in the Terfenol-D/BaTiO<sub>3</sub>/Si and CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si layered structures

As can be seen from Fig. 3.16, no matter which modes, the wave speed in the former is correspondingly smaller than that in the latter, where the third mode curve in Terfenol-D/BaTiO<sub>3</sub>/Si crosses with the second mode curve in CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si.

### 3.4 Summary

By constructing a PM/PE/semiconductor composite structure, the magneto-electro-semiconductor multi-field coupling is realized indirectly. In this Chapter, the propagation of Love waves and Rayleigh waves in the three-layer structure is deeply analyzed. The wave field equations of each layer are derived from the basic equations, and the dispersion equation is obtained by combining the boundary conditions and the continuity conditions. Based on the analytical solution, the effects of covering layer thickness, initial carrier concentration, PM constant and material combinations on Love waves and Rayleigh waves are studied. The results show that:

(1) Different from the bilayer structure of the PS/PM half-space in Chapter 2, the Love and Rayleigh waves in this three-layer structure only have dispersion without attenuation. The higher-order mode wave speed of Love waves and Rayleigh waves both start from the shear wave velocity of semiconductor substrate and have cutoff frequency. The difference is that the first mode of Rayleigh waves is the basic mode starting from the semiconductor Rayleigh wave.

(2) In the three-layer structure, the PM layer thickness  $h_2$  has a great influence on the first three modes of Love waves and Rayleigh waves, and the wave speed decreases with the increase of  $h_2$ . The thickness of the PE layer  $h_1$  has no effect on



the Love waves, but when the  $kh$  is small (long waves), it has a slight effect on the second and third modes of the Rayleigh waves. The  $n_0$  of the semiconductor substrate has no effect on the first three modes of the Love waves and Rayleigh waves, which is the main reason why the surface waves do not decay in this structure.

(3) The equations of Love waves in this structure only involve PM constant  $f_{15}$ , so  $f_{15}$  has a great influence on the Love waves, and the wave speed will decrease with the increase of  $f_{15}$ . The analytical solution of Rayleigh waves is related to the PM constants  $f_{15}$ ,  $f_{31}$  and  $f_{33}$ , and the influence of  $f_{15}$  on Rayleigh waves is relatively limited.

(4) For four different material combinations of Terfenol-D/BaTiO<sub>3</sub>/Si, CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si, Terfenol-D/PZT-4/Si and Terfenol-D/PZT-5H/Si, the results show that the Love wave speed and Rayleigh wave speed in CoFe<sub>2</sub>O<sub>4</sub>/BaTiO<sub>3</sub>/Si is much higher than that of the other three structures. In addition, different PE layer materials BaTiO<sub>3</sub>, PZT-4 and PZT-5H in Love waves do not affect the first three modes, while the three curves corresponding to Rayleigh waves do not coincide.

## 3.5 Appendix

### (1) Phase velocity equations

By substituting the solutions of PM layer, PE layer and semiconductor layer to the corresponding constitutive equations (3.3), (3.6) and (3.9), the phase velocity equations of each layer in the composite structure can be obtained. The expressions of stress, electric displacement and magnetic induction intensity in the PM layer (the  $e^{-k\eta_j^m x_2} \cdot e^{ik(x_1 - ct)}$  term is uniformly omitted) are:

$$\begin{aligned}
 T_{11}^m &= \sum_{j=1}^4 (ic_{11}^m \beta_{1j}^m - c_{12}^m \eta_j^m) k A_{2j}^m, \quad T_{22}^m = \sum_{j=1}^4 (ic_{12}^m \beta_{1j}^m - c_{11}^m \eta_j^m) k A_{2j}^m \\
 T_{33}^m &= \sum_{j=1}^4 (i\beta_{1j}^m - \eta_j^m) c_{13}^m k A_{2j}^m, \quad T_{23}^m = \sum_{j=5}^8 -(c_{44}^m \beta_{2j}^m + f_{15}^m) k \eta_j^m A_{\Psi j}^m \\
 T_{13}^m &= \sum_{j=5}^8 (c_{44}^m \beta_{2j}^m + f_{15}^m) i k A_{\Psi j}^m, \quad T_{12}^m = \sum_{j=1}^4 (-\eta_j^m \beta_{1j}^m + i) c_{66}^m k A_{2j}^m \\
 D_1^m &= -ik \epsilon_{11}^m \sum_{j=9}^{10} A_{\Phi j}^m, \quad D_2^m = k \epsilon_{11}^m \sum_{j=9}^{10} \eta_j^m A_{\Phi j}^m \\
 B_1^m &= \sum_{j=5}^8 (f_{15}^m \beta_{2j}^m - \alpha_{11}^m) i k A_{\Psi j}^m, \quad B_2^m = \sum_{j=5}^8 (-f_{15}^m \beta_{2j}^m + \alpha_{11}^m) k \eta_j^m A_{\Psi j}^m \\
 B_3^m &= \sum_{j=1}^4 (i\beta_{1j}^m - \eta_j^m) f_{31}^m k A_{2j}^m
 \end{aligned} \tag{A.3.1}$$

The expressions of stress, electric displacement and magnetic induction in the PE layer are as follows:

$$\begin{aligned}
T_{11}^e &= \sum_{j=1}^4 (ic_{11}^e \beta_{1j}^e - c_{12}^e \eta_j^e) k A_{2j}^e, \quad T_{22}^e = \sum_{j=1}^4 (ic_{12}^e \beta_{1j}^e - c_{11}^e \eta_j^e) k A_{2j}^e \\
T_{33}^e &= \sum_{j=1}^4 (i \beta_{1j}^e - \eta_j^e) c_{13}^e k A_{2j}^e, \quad T_{23}^e = \sum_{j=5}^8 -(c_{44}^e \beta_{2j}^e + e_{15}^e) k \eta_j^e A_{\Phi j}^e \\
T_{13}^e &= \sum_{j=5}^8 (c_{44}^e \beta_{2j}^e + e_{15}^e) i k A_{\Phi j}^e, \quad T_{12}^e = \sum_{j=1}^4 (-\eta_j^e \beta_{1j}^e + i) c_{66}^e k A_{2j}^e \\
D_1^e &= \sum_{j=5}^8 (e_{15}^e \beta_{2j}^e - \varepsilon_{11}^e) i k A_{\Phi j}^e, \quad D_2^e = \sum_{j=5}^8 (-e_{15}^e \beta_{2j}^e + \varepsilon_{11}^e) k \eta_j^e A_{\Phi j}^e \\
D_3^e &= \sum_{j=1}^4 (i \beta_{1j}^e - \eta_j^e) e_{31}^e k A_{2j}^e, \quad B_1^e = -i k \alpha_{11}^e \sum_{j=9}^{10} A_{\Psi j}^e, \quad B_2^e = k \alpha_{11}^e \sum_{j=9}^{10} \eta_j^e A_{\Psi j}^e
\end{aligned} \tag{A.3.2}$$

Similarly, the expressions of stress, electrical displacement and current density of the semiconductor substrate can be obtained as:

$$\begin{aligned}
T_{11}^s &= \sum_{j=1}^2 (ic_{11}^s \beta_{1j}^s - c_{12}^s \eta_j^s) k A_{2j}^s, \quad T_{22}^s = \sum_{j=1}^2 (ic_{12}^s \beta_{1j}^s - c_{11}^s \eta_j^s) k A_{2j}^s \\
T_{33}^s &= \sum_{j=1}^2 (i \beta_{1j}^s - \eta_j^s) c_{13}^s k A_{2j}^s, \quad T_{23}^s = -k c_{44}^s \sum_{j=5}^5 \eta_j^s A_{3j}^s \\
T_{13}^s &= i k c_{44}^s \sum_{j=5}^5 A_{3j}^s, \quad T_{12}^s = \sum_{j=1}^2 (-\eta_j^s \beta_{1j}^s + i) c_{66}^s k A_{2j}^s \\
D_1^s &= -i k \varepsilon_{11}^s \sum_{j=3}^4 A_{\Phi j}^s, \quad D_2^s = k \varepsilon_{11}^s \sum_{j=3}^4 \eta_j^s A_{\Phi j}^s \\
J_1 &= \sum_{j=3}^4 (-n_0 \mu_{11} + D_{11} \beta_{2j}^s) i k q \beta_{2j}^s A_{\Phi j}^s \\
J_2 &= \sum_{j=3}^4 (n_0 \mu_{11} - D_{11} \beta_{2j}^s) k q \eta_j^s A_{\Phi j}^s
\end{aligned} \tag{A.3.3}$$

## (2) The nonzero elements of the coefficient matrix in the dispersion equation

Considering the three displacement components, the non-zero elements of the coefficient matrix  $[N]$  in the dispersion equation (3.23) are:

$$\begin{aligned}
N_{1j} &= (i - \eta_j^m \beta_{1j}^m) e^{k \eta_j^m (h_1 + h_2)}, \quad N_{2j} = (ic_{12}^m \beta_{1j}^m - c_{11}^m \eta_j^m) e^{k \eta_j^m (h_1 + h_2)}, \quad j=1-4 \\
N_{3j} &= (c_{44}^m \beta_{2j}^m + f_{15}^m) \eta_j^m e^{k \eta_j^m (h_1 + h_2)}, \quad j=5-8; \quad N_{4j} = \eta_j^m e^{k \eta_j^m (h_1 + h_2)}, \quad j=9-10 \\
N_{5j} &= e^{k \eta_j^m (h_1 + h_2)}, \quad j=5-8; \quad N_{6,23} = \beta_{2,23}^s, \quad N_{6,24} = \beta_{2,24}^s \\
N_{7j} &= \beta_{1j}^e, \quad N_{7,21} = -\beta_{1,21}^s, \quad N_{7,22} = -\beta_{1,22}^s, \quad j=11-14 \\
N_{8j} &= 1, \quad N_{8,21} = N_{8,22} = -1, \quad j=11-14; \quad N_{9j} = \beta_{2j}^e, \quad N_{9,25} = -1, \quad j=15-18 \\
N_{10j} &= c_{66}^e (i - \eta_j^e \beta_{1j}^e), \quad N_{10,21} = c_{66}^e (\eta_{21}^s \beta_{1,21}^s - i), \quad N_{10,22} = c_{66}^e (\eta_{22}^s \beta_{1,22}^s - i), \quad j=11-14 \\
N_{11j} &= ic_{12}^e \beta_{1j}^e - c_{11}^e \eta_j^e, \quad N_{11,21} = -(ic_{12}^s \beta_{1,21}^s - c_{11}^s \eta_{21}^s), \quad N_{11,22} = -(ic_{12}^s \beta_{1,22}^s - c_{11}^s \eta_{22}^s), \quad j=11-14 \\
N_{12j} &= (c_{44}^e \beta_{2j}^e + e_{15}^e) \eta_j^e, \quad N_{12,25} = -c_{44}^s \eta_{25}^s, \quad j=15-18; \quad N_{13j} = 1, \quad N_{13,23} = N_{13,24} = -1, \quad j=15-18
\end{aligned} \tag{A.3.4a}$$

$$\begin{aligned}
N_{14j} &= (\varepsilon_{11}^e - e_{15}^e \beta_{2j}^e) \eta_j^e, N_{14,23} = -\varepsilon_{11}^s \eta_{23}^s, N_{14,24} = -\varepsilon_{11}^s \eta_{24}^s, j = 15-18 \\
N_{15j} &= (e_{15}^e \beta_{2j}^e - \varepsilon_{11}^e) i k c \eta_j^e, j = 15-18 \\
N_{15,23} &= (i k c \varepsilon_{11}^s - q n_0 \mu_{11} + q D_{11} \beta_{2,23}^s) \eta_{23}^s, N_{15,24} = (i k c \varepsilon_{11}^s - q n_0 \mu_{11} + q D_{11} \beta_{2,24}^s) \eta_{24}^s \\
N_{16j} &= \beta_{1j}^m e^{k \eta_j^m h_1}, N_{16k} = -\beta_{1k}^e e^{k \eta_k^e h_1}, j = 1-4, k = 11-14 \\
N_{17j} &= e^{k \eta_j^m h_1}, N_{17k} = -e^{k \eta_k^e h_1}, j = 1-4, k = 11-14 \\
N_{18j} &= \beta_{2j}^m e^{k \eta_j^m h_1}, N_{18k} = -\beta_{2k}^e e^{k \eta_k^e h_1}, j = 5-8, k = 15-18 \\
N_{19j} &= c_{66}^m (i - \eta_j^m \beta_{1j}^m) e^{k \eta_j^m h_1}, N_{19k} = -c_{66}^e (i - \eta_k^e \beta_{1k}^e) e^{k \eta_k^e h_1}, j = 1-4, k = 11-14 \\
N_{20j} &= (i c_{12}^m \beta_{1j}^m - c_{11}^m \eta_j^m) e^{k \eta_j^m h_1}, N_{20k} = -(i c_{12}^e \beta_{1k}^e - c_{11}^e \eta_k^e) e^{k \eta_k^e h_1}, j = 1-4, k = 11-14 \\
N_{21j} &= (c_{44}^m \beta_{2j}^m + f_{15}^m) \eta_j^m e^{k \eta_j^m h_1}, N_{21k} = -(c_{44}^e \beta_{2k}^e + e_{15}^e) \eta_k^e e^{k \eta_k^e h_1}, j = 5-8, k = 15-18 \\
N_{22j} &= e^{k \eta_j^m h_1}, N_{22k} = -e^{k \eta_k^e h_1}, j = 9-10, k = 15-18 \\
N_{23j} &= \varepsilon_{11}^m \eta_j^m e^{k \eta_j^m h_1}, N_{23k} = -(\varepsilon_{11}^e - e_{15}^e \beta_{2k}^e) \eta_k^e e^{k \eta_k^e h_1}, j = 9-10, k = 15-18 \\
N_{24j} &= e^{k \eta_j^m h_1}, N_{24,19} = -e^{k \eta_{19}^e h_1}, N_{24,20} = -e^{k \eta_{20}^e h_1}, j = 5-8 \\
N_{25j} &= (\alpha_{11}^m - f_{15}^m \beta_{2j}^m) \eta_j^m e^{k \eta_j^m h_1}, N_{25k} = -\alpha_{11}^e \eta_k^e e^{k \eta_k^e h_1}, j = 5-8, k = 19-20
\end{aligned} \tag{A.3.4b}$$

### (3) Dispersion equations of Rayleigh waves in the composite structure

Considering the layered composite structure after changing the coordinate system as shown in Fig. 3.9, the Rayleigh waves propagating along the  $x_1$ -axis have two displacement components,  $u_1$  and  $u_3$ . The PM layer, PE layer and semiconductor layer are discussed separately. The coupled wave equations of PM layer are obtained firstly:

$$\begin{aligned}
c_{11}^m \frac{\partial^2 u_1^m}{\partial x_1^2} + c_{44}^m \frac{\partial^2 u_1^m}{\partial x_3^2} + (c_{13}^m + c_{44}^m) \frac{\partial^2 u_3^m}{\partial x_1 \partial x_3} + (f_{15}^m + f_{31}^m) \frac{\partial^2 \psi^m}{\partial x_1 \partial x_3} &= \rho \ddot{u}_1^m \\
c_{44}^m \frac{\partial^2 u_3^m}{\partial x_1^2} + c_{33}^m \frac{\partial^2 u_3^m}{\partial x_3^2} + (c_{13}^m + c_{44}^m) \frac{\partial^2 u_1^m}{\partial x_1 \partial x_3} + f_{15}^m \frac{\partial^2 \psi^m}{\partial x_1^2} + f_{33}^m \frac{\partial^2 \psi^m}{\partial x_3^2} &= \rho \ddot{u}_3^m \\
\varepsilon_{11}^m \frac{\partial^2 \varphi^m}{\partial x_1^2} + \varepsilon_{33}^m \frac{\partial^2 \varphi^m}{\partial x_3^2} &= 0 \\
f_{15}^m \frac{\partial^2 u_3^m}{\partial x_1^2} + f_{33}^m \frac{\partial^2 u_3^m}{\partial x_3^2} + (f_{15}^m + f_{31}^m) \frac{\partial^2 u_1^m}{\partial x_1 \partial x_3} - \alpha_{11}^m \frac{\partial^2 \psi^m}{\partial x_1^2} - \alpha_{33}^m \frac{\partial^2 \psi^m}{\partial x_3^2} &= 0
\end{aligned} \tag{A.3.5}$$

The coupled wave equations of the PE layer are:

$$\begin{aligned}
c_{11}^e \frac{\partial^2 u_1^e}{\partial x_1^2} + c_{44}^e \frac{\partial^2 u_1^e}{\partial x_3^2} + (c_{13}^e + c_{44}^e) \frac{\partial^2 u_3^e}{\partial x_1 \partial x_3} + (e_{15}^e + e_{31}^e) \frac{\partial^2 \varphi^e}{\partial x_1 \partial x_3} &= \rho \ddot{u}_1^e \\
c_{44}^e \frac{\partial^2 u_3^e}{\partial x_1^2} + c_{33}^e \frac{\partial^2 u_3^e}{\partial x_3^2} + (c_{13}^e + c_{44}^e) \frac{\partial^2 u_1^e}{\partial x_1 \partial x_3} + e_{15}^e \frac{\partial^2 \varphi^e}{\partial x_1^2} + e_{33}^e \frac{\partial^2 \varphi^e}{\partial x_3^2} &= \rho \ddot{u}_3^e \\
e_{15}^e \frac{\partial^2 u_3^e}{\partial x_1^2} + e_{33}^e \frac{\partial^2 u_3^e}{\partial x_3^2} + (e_{15}^e + e_{31}^e) \frac{\partial^2 u_1^e}{\partial x_1 \partial x_3} - \varepsilon_{11}^e \frac{\partial^2 \varphi^e}{\partial x_1^2} - \varepsilon_{33}^e \frac{\partial^2 \varphi^e}{\partial x_3^2} &= 0 \\
\alpha_{11}^e \frac{\partial^2 \psi^e}{\partial x_1^2} + \alpha_{33}^e \frac{\partial^2 \psi^e}{\partial x_3^2} &= 0
\end{aligned} \tag{A.3.6}$$

Similarly, we can obtain the coupled wave equations of the semiconductor layer as:

$$\begin{aligned}
 c_{11}^s \frac{\partial^2 u_1^s}{\partial x_1^2} + c_{44}^s \frac{\partial^2 u_1^s}{\partial x_3^2} + (c_{13}^s + c_{44}^s) \frac{\partial^2 u_3^s}{\partial x_1 \partial x_3} &= \rho i_1^s \\
 c_{44}^s \frac{\partial^2 u_3^s}{\partial x_1^2} + c_{33}^s \frac{\partial^2 u_3^s}{\partial x_3^2} + (c_{13}^s + c_{44}^s) \frac{\partial^2 u_1^s}{\partial x_1 \partial x_3} &= \rho i_3^s \\
 \varepsilon_{11}^s \frac{\partial^2 \varphi^e}{\partial x_1^2} + \varepsilon_{33}^s \frac{\partial^2 \varphi^e}{\partial x_3^2} &= q \Delta n \\
 -n_0 \left( \mu_{11} \frac{\partial^2 \varphi^e}{\partial x_1^2} + \mu_{33} \frac{\partial^2 \varphi^e}{\partial x_3^2} \right) + D_{11} \frac{\partial^2 \Delta n}{\partial x_1^2} + D_{33} \frac{\partial^2 \Delta n}{\partial x_3^2} &= \Delta n.
 \end{aligned} \tag{A.3.7}$$

The boundary conditions and continuity conditions are the same as Eqs. (3.12)-(3.14), so they are not repeated, but it should be noted that the  $x_2$ -axis in the original coordinate system becomes the  $x_3$ -axis.

Assume that the solution of the PM coupled wave equation (A.3.5) is:

$$\begin{Bmatrix} u_1^m \\ u_3^m \\ \varphi^m \\ \psi^m \end{Bmatrix} = \begin{Bmatrix} A_1^m \\ A_3^m \\ A_\Phi^m \\ A_\Psi^m \end{Bmatrix} e^{-k\eta^m x_3} e^{ik(x_1 - ct)} \tag{A.3.8}$$

By substituting the above solutions into the Eq. (A.3.5) and simplifying, homogeneous linear equations with undetermined coefficients  $A_1^m$ ,  $A_3^m$ ,  $A_\Phi^m$  and  $A_\Psi^m$  can be obtained:

$$\begin{aligned}
 (\rho^m c^2 + c_{44}^m \eta^{m^2} - c_{11}^m) A_1^m - i\eta^m (c_{13}^m + c_{44}^m) A_3^m - i\eta^m (f_{15}^m + f_{31}^m) A_\Psi^m &= 0 \\
 -i\eta^m (c_{13}^m + c_{44}^m) A_1^m + (\rho^m c^2 + c_{33}^m \eta^{m^2} - c_{44}^m) A_3^m + (f_{33}^m \eta^{m^2} - f_{15}^m) A_\Psi^m &= 0 \\
 (\varepsilon_{33}^m \eta^{m^2} - \varepsilon_{11}^m) A_\Phi^m &= 0 \\
 -i\eta^m (f_{15}^m + f_{31}^m) A_1^m + (f_{33}^m \eta^{m^2} - f_{15}^m) A_3^m + (\alpha_{11}^m - \alpha_{33}^m \eta^{m^2}) A_\Psi^m &= 0
 \end{aligned} \tag{A.3.9}$$

By decoupling (A.3.9)<sub>3</sub> we get  $\eta^m = \pm \sqrt{\varepsilon_{11}^m / \varepsilon_{33}^m}$ . In order to obtain non-trivial solutions for  $A_1^m$ ,  $A_3^m$  and  $A_\Psi^m$ , the determinant of the coefficient matrix for the other three equations needs to be zero:

$$\begin{vmatrix} (\rho^m c^2 + c_{44}^m \eta^{m^2} - c_{11}^m) & -i\eta^m (c_{13}^m + c_{44}^m) & -i\eta^m (f_{15}^m + f_{31}^m) \\ -i\eta^m (c_{13}^m + c_{44}^m) & (\rho^m c^2 + c_{33}^m \eta^{m^2} - c_{44}^m) & (f_{33}^m \eta^{m^2} - f_{15}^m) \\ -i\eta^m (f_{15}^m + f_{31}^m) & (f_{33}^m \eta^{m^2} - f_{15}^m) & (\alpha_{11}^m - \alpha_{33}^m \eta^{m^2}) \end{vmatrix} = 0 \tag{A.3.10}$$

The complete solution of the PM layer is obtained as:

$$\begin{Bmatrix} u_1^m \\ u_3^m \\ \varphi^m \\ \psi^m \end{Bmatrix} = \begin{Bmatrix} \sum_{j=1}^6 A_{1j}^m \\ \sum_{j=1}^6 \beta_{1j}^m A_{1j}^m \\ \sum_{j=7}^8 A_{\Phi j}^m \\ \sum_{j=1}^6 \beta_{2j}^m A_{1j}^m \end{Bmatrix} e^{-k\eta_j^m x_3} e^{ik(x_1 - ct)} \quad (\text{A.3.11})$$

where  $A_{1j}^m$  ( $j = 1-6$ ) represent the amplitude.  $\beta_{1j}^m$  and  $\beta_{2j}^m$  are amplitude ratios, specifically:

$$\beta_{1j}^m = -\left( \frac{a_{11} \cdot a_{23} - a_{13} \cdot a_{21}}{a_{12} \cdot a_{23} - a_{13} \cdot a_{22}} \right), \quad \beta_{2j}^m = -\left( \frac{a_{12}^2 - a_{11} \cdot a_{22}}{a_{12} \cdot a_{23} - a_{13} \cdot a_{22}} \right), \quad j=1-6 \quad (\text{A.3.12})$$

where  $a_{ij}$  is the element in row  $i$  and column  $j$  of the determinant (A.3.10).

For the PE layer wave field, we can obtain homogeneous linear equations for  $A_1^e$ ,  $A_3^e$ ,  $A_\Phi^e$  and  $A_\Psi^e$ :

$$\begin{aligned} & \left( \rho^e c^2 + c_{44}^e \eta^{e^2} - c_{11}^e \right) A_1^e - i\eta^e \left( c_{13}^e + c_{44}^e \right) A_3^e - i\eta^e \left( e_{15}^e + e_{31}^e \right) A_\Phi^e = 0 \\ & -i\eta^e \left( c_{13}^e + c_{44}^e \right) A_1^e + \left( \rho^e c^2 + c_{33}^e \eta^{e^2} - c_{44}^e \right) A_3^e + \left( e_{33}^e \eta^{e^2} - e_{15}^e \right) A_\Phi^e = 0 \\ & -i\eta^e \left( e_{15}^e + e_{31}^e \right) A_1^e + \left( e_{33}^e \eta^{e^2} - e_{15}^e \right) A_3^e + \left( \varepsilon_{11}^e - \varepsilon_{33}^e \eta^{e^2} \right) A_\Phi^e = 0 \\ & \left( \alpha_{33}^e \eta^{e^2} - \alpha_{11}^e \right) A_\Psi^e = 0 \end{aligned} \quad (\text{A.3.13})$$

$\eta^e = \pm \sqrt{\alpha_{11}^e / \alpha_{33}^e}$  are obtained by decoupling Eq. (A.3.13)<sub>4</sub> from the other three equations. In order to obtain nontrivial solutions for  $A_1^e$ ,  $A_3^e$  and  $A_\Phi^e$ , the determinant of the coefficients for the other three equations is zero, namely:

$$\begin{vmatrix} \left( \rho^e c^2 + c_{44}^e \eta^{e^2} - c_{11}^e \right) & -i\eta^e \left( c_{13}^e + c_{44}^e \right) & -i\eta^e \left( e_{15}^e + e_{31}^e \right) \\ -i\eta^e \left( c_{13}^e + c_{44}^e \right) & \left( \rho^e c^2 + c_{33}^e \eta^{e^2} - c_{44}^e \right) & \left( e_{33}^e \eta^{e^2} - e_{15}^e \right) \\ -i\eta^e \left( e_{15}^e + e_{31}^e \right) & \left( e_{33}^e \eta^{e^2} - e_{15}^e \right) & \left( \varepsilon_{11}^e - \varepsilon_{33}^e \eta^{e^2} \right) \end{vmatrix} = 0 \quad (\text{A.3.14})$$

Thus, the complete solution of the PE layer can be expressed as:

$$\begin{pmatrix} u_1^e \\ u_3^e \\ \varphi^e \\ \psi^e \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^6 A_{1j}^e \\ \sum_{j=1}^6 \beta_{1j}^e A_{1j}^e \\ \sum_{j=1}^6 \beta_{2j}^e A_{1j}^e \\ \sum_{j=7}^8 A_{\Psi j}^e \end{pmatrix} e^{-k\eta_j^e x_3} e^{ik(x_1 - ct)} \quad (\text{A.3.15})$$

where  $A_{1j}^e$  ( $j = 1-6$ ) are the amplitude,  $\beta_{1j}^e$  and  $\beta_{2j}^e$  the amplitude ratio:

$$\beta_{1j}^e = -\left( \frac{b_{11} \cdot b_{23} - b_{13} \cdot b_{21}}{b_{12} \cdot b_{23} - b_{13} \cdot b_{22}} \right), \quad \beta_{2j}^e = -\left( \frac{b_{12}^2 - b_{11} \cdot b_{22}}{b_{12} \cdot b_{23} - b_{13} \cdot b_{22}} \right), \quad j=1-6 \quad (\text{A.3.16})$$

where  $b_{ij}$  represents the element in row  $i$  and column  $j$  in Eq. (A.3.14). Similarly, the complete solutions of the semiconductor substrate are:

$$\begin{pmatrix} u_1^s \\ u_3^s \\ \varphi^s \\ \Delta n^s \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^2 \beta_{1j}^s A_{3j}^s \\ \sum_{j=1}^2 A_{3j}^s \\ \sum_{j=3}^4 A_{\Phi j}^s \\ \sum_{j=3}^4 \beta_{2j}^s A_{\Phi j}^s \end{pmatrix} e^{-k\eta_j^s x_3} e^{ik(x_1 - ct)} \quad (\text{A.3.17})$$

where  $A_{3j}^s$  ( $j = 1-2$ ) and  $A_{\Phi j}^s$  ( $j = 3-4$ ) are amplitudes,  $\beta_{1j}^s$  and  $\beta_{2j}^s$  the amplitude ratios:

$$\beta_{1j}^s = \frac{i\eta^s (c_{13}^s + c_{44}^s)}{(\rho^s c^2 + c_{44}^s \eta^{s^2} - c_{11}^s)}, \quad j=1-2$$

$$\beta_{2j}^s = \frac{k^2 (\varepsilon_{33}^s \eta^{s^2} - \varepsilon_{11}^s)}{q}, \quad j=3-4 \quad (\text{A.3.18})$$

By substituting the solution of each layer to the corresponding constitutive equations, the physical quantity expressions can be obtained, which are then substituted to the boundary and continuity conditions, and finally the dispersion equation (3.24) is obtained, that is,  $[\mathbf{M}]\mathbf{A} = \mathbf{0}$ .  $[\mathbf{M}]$  is a coefficient matrix of  $20 \times$

20, whose nonzero elements (subscripts  $i = 1-6, j = 7-8, k = 9-14, l = 15-16$  and  $m = 17-18, n = 19-20$ ) are:

$$\begin{aligned}
 M_{1i} &= \left( -c_{44}^m \eta_i^m + i c_{44}^m \beta_{1i}^m + i f_{15}^m \beta_{2i}^m \right) e^{k \eta_i^m (h_1 + h_2)}, \quad M_{2i} = \left( i c_{13}^m - c_{33}^m \eta_i^m \beta_{1i}^m - f_{33}^m \eta_i^m \beta_{2i}^m \right) e^{k \eta_i^m (h_1 + h_2)} \\
 M_{3j} &= \eta_j^m e^{k \eta_j^m (h_1 + h_2)}, \quad M_{4i} = \beta_{2i}^m e^{k \eta_i^m (h_1 + h_2)}, \quad M_{5n} = \beta_{2n}^s, \quad M_{6k} = 1, \quad M_{6m} = -\beta_{1m}^s \\
 M_{7k} &= \beta_{1k}^e, \quad M_{7m} = -1, \quad M_{8k} = -c_{44}^e \eta_k^e + i c_{44}^e \beta_{1k}^e + i e_{15}^e \beta_{2k}^e, \quad M_{8m} = c_{44}^s \eta_m^s \beta_{1m}^s - i c_{44}^s \\
 M_{9k} &= i c_{13}^e - c_{33}^e \eta_k^e \beta_{1k}^e - e_{33}^e \eta_k^e \beta_{2k}^e, \quad M_{9m} = c_{33}^s \eta_m^s - i c_{13}^s \beta_{1m}^s, \quad M_{10k} = \beta_{2k}^e, \quad M_{10n} = -1 \\
 M_{11k} &= i e_{31}^e - e_{33}^e \eta_k^e \beta_{1k}^e + \varepsilon_{33}^e \eta_k^e \beta_{2k}^e, \quad M_{11n} = -\varepsilon_{33}^s \eta_n^s \\
 M_{12k} &= i k c \left( i e_{31}^e - e_{33}^e \eta_k^e \beta_{1k}^e + \varepsilon_{33}^e \eta_k^e \beta_{2k}^e \right), \quad M_{12n} = \left( q n_0 \mu_{33} - i k c \varepsilon_{33}^s - q D_{33} \beta_{2n}^s \right) \eta_n^s \\
 M_{13i} &= e^{k \eta_i^m h_1}, \quad M_{13k} = -e^{k \eta_k^e h_1}, \quad M_{14i} = \beta_{1i}^m e^{k \eta_i^m h_1}, \quad M_{14k} = -\beta_{1k}^e e^{k \eta_k^e h_1} \\
 M_{15i} &= \left( -c_{44}^m \eta_i^m + i c_{44}^m \beta_{1i}^m + i f_{15}^m \beta_{2i}^m \right) e^{k \eta_i^m h_1}, \quad M_{15k} = \left( c_{44}^e \eta_k^e - i c_{44}^e \beta_{1k}^e - i e_{15}^e \beta_{2k}^e \right) e^{k \eta_k^e h_1} \\
 M_{16i} &= \left( i c_{13}^m - c_{33}^m \eta_i^m \beta_{1i}^m - f_{33}^m \eta_i^m \beta_{2i}^m \right) e^{k \eta_i^m h_1}, \quad M_{16k} = \left( -i c_{13}^e + c_{33}^e \eta_k^e \beta_{1k}^e + e_{33}^e \eta_k^e \beta_{2k}^e \right) e^{k \eta_k^e h_1} \\
 M_{17j} &= e^{k \eta_j^m h_1}, \quad M_{17k} = -\beta_{2k}^e e^{k \eta_k^e h_1}, \quad M_{18j} = \varepsilon_{33}^m \eta_j^m e^{k \eta_j^m h_1}, \quad M_{18k} = -\left( i e_{31}^e - e_{33}^e \eta_k^e \beta_{1k}^e + \varepsilon_{33}^e \eta_k^e \beta_{2k}^e \right) e^{k \eta_k^e h_1} \\
 M_{19i} &= \beta_{2i}^m e^{k \eta_i^m h_1}, \quad M_{19l} = -e^{k \eta_l^e h_1}, \quad M_{20i} = \left( i f_{31}^m - f_{33}^m \eta_i^m \beta_{1i}^m + \alpha_{33}^m \eta_i^m \beta_{2i}^m \right) e^{k \eta_i^m h_1}, \quad M_{20l} = -\alpha_{33}^e \eta_l^e e^{k \eta_l^e h_1}
 \end{aligned}
 \tag{A.3.19}$$

## Chapter 4

# SH waves in multiferroic composite semiconductors with initial stress

In the production and use of devices, the initial stress will inevitably occur in the structures due to the influence of the process and environmental changes. On the other hand, in order to prevent the brittle fracture of the materials, prestress is usually applied in the manufacturing. For layered structures, due to the discontinuity of material parameters between layers, the difference of temperature and preparation processes, initial stress is often present in the covering layer and at the interfaces. Previous studies have shown that the effect of initial stress can change the surface waves propagation. From the early studies on the influence of initial stress on elastic waves [190], to the research on SH waves and Love waves in PE and PM layered structures with initial stress [169,171,191,192], the propagation of elastic waves in layered structures with initial stress has been extensively studied. Considering the non-uniform initial stress, Su et al. [170] used the transfer matrix method to analyze the propagation of Love waves in ZnO/SiO<sub>2</sub>/Si. Singh et al. [172] studied the propagation characteristics of Love waves in composite materials under irregular initial stress from an analytical perspective.

In recent years, there has been an increasing amount of research on the propagation of elastic waves in PSs. Scholars have also gradually studied SH-type surface waves in PSs with initial stress. Chen and Yang et al. [88,89] conducted an in-depth analysis of the propagation characteristics of SH waves in a PS half-space and a PS plate with initial stress. Sahu et al. [112] studied the reflection and transmission of quasi-plane waves at the PS interface (AlN/ZnO) with initial stress. Wang et al. [90] assumed that the initial stress and material parameters vary exponentially along the  $x$ -axis, and obtained the SH waves propagation in the FGPS with initial stress. Salah et al. [91] used a simplified ODE method to study the influence of initial stress on electron and hole drift in PS plates and the



influence of semiconductor characteristics on the dispersion curve of SH waves. Abd-alla et al. [114] explained the SH waves propagation behavior of the layered structure during rotation and initial stress by constructing a semiconductor/PE half-space. As far as is known, surface waves in MCSs with initial stress have not been studied.

Based on the macrotheory of PM and PS mentioned above, this Chapter will establish layered models with initial stress to study the effects of initial stress on the SH wave speed and attenuation. Two models are considered, one is the infinite ZnO PS plate without composite PM material, and the phase velocity equations and the dispersion equations with initial stress are obtained. The numerical examples show that similar to semiconductor properties, sufficiently large initial stress can also lead to a decrease in wave speed and an increase in attenuation. The second is to study the effect of initial stress on SH waves in PS/PM bilayer plate, and the influence of initial carrier concentration, PS and PM layer thickness on SH waves is obtained. In addition, in the second model, the PM plate is replaced by PM half-space, that is, the PS/PM layered half-space is established, and the results similar to those in the composite plate can be obtained.

## 4.1 SH waves in a piezoelectric semiconductor plate with initial stress

In this Section, we first consider a PS plate without PM material, and theoretically study its SH waves with initial stress. The effects of boundary conditions, initial carrier concentration, plate thickness and initial stress on SH waves are analyzed systematically. The results in this Section are instructive for the design of PS devices considering the initial stress.

### 4.1.1 Basic equations with initial stress

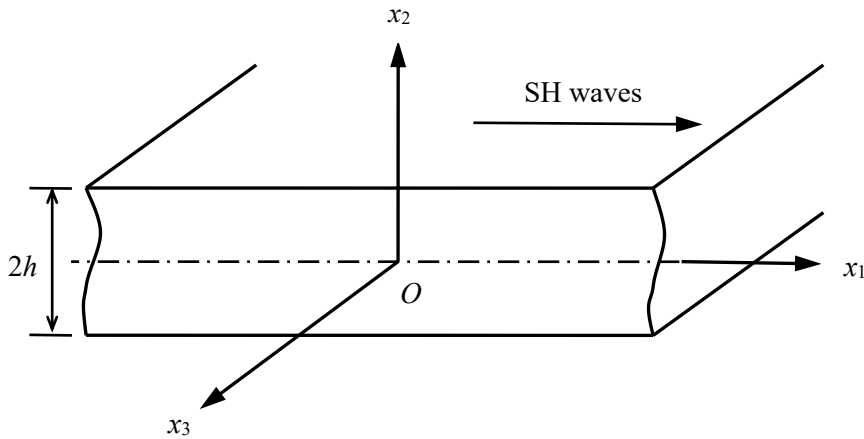


Fig. 4.1 Infinite piezoelectric semiconductor plate with initial stress

Considering an infinite n-type PS plate with a thickness of  $2h$  as shown in Fig. 4.1, the SH waves propagate along the  $x_1$ -axis and the polarization direction is  $x_3$ -axis.

It is assumed that there is a constant initial stress in the PS plate and there is no traction on the upper and lower surfaces.

The equilibrium equations with initial stress are slightly different from the equations without initial stress mentioned above, as follows:

$$\begin{aligned} T_{ji,j} + (u_{i,k} T_{kj}^0)_{,j} &= \rho \ddot{u}_i \\ D_{i,i} &= qn \\ q\dot{n} + J_{i,i} &= 0 \end{aligned} \quad (4.1)$$

where  $T_{kj}^0$  is the initial stress. The electric displacement and initial magnetic induction intensity in the initial state are not considered in Eq. (4.1). Assuming that there are only initial stress components along the  $x_1$ -axis, the coupled wave equations can be obtained as follows:

$$\begin{aligned} c_{44} \nabla^2 u_3 + T_1^0 u_{3,11} + e_{15} \nabla^2 \varphi &= \rho \ddot{u}_3 \\ e_{15} \nabla^2 u_3 - \varepsilon_{11} \nabla^2 \varphi &= qn \\ \dot{n} - n_0 \mu_{11} \nabla^2 \varphi - d_{11} \nabla^2 n &= 0 \end{aligned} \quad (4.2)$$

In order to study the propagation characteristics of waves in PS plates, it is necessary to combine the boundary conditions of the upper and lower surfaces of the plates. Mechanically, assuming that the surface stress of the plate is free, there are:

$$T_{23}(x_1, h) = 0, \quad T_{23}(x_1, -h) = 0 \quad (4.3)$$

For electrical boundary conditions, it is divided into electrical short circuit and open circuit, respectively:

**(1) Short-circuit boundary BC1**

$$\varphi(x_1, h) = 0, \quad \varphi(x_1, -h) = 0 \quad (4.4)$$

**(2) Open-circuit boundary BC2**

$$\dot{D}_2(x_1, h) + J_2(x_1, h) = 0, \quad \dot{D}_2(x_1, -h) + J_2(x_1, -h) = 0 \quad (4.5)$$

The perturbation of the PS carrier concentration disappears at the boundary of the plate, which is an additional boundary condition compared to the PE insulation material, namely:

$$n(x_1, h) = 0, \quad n(x_1, -h) = 0 \quad (4.6)$$

### 4.1.2 Solutions of the problem

For plane strain waves propagating along the  $x_1$ -axis, the solution to the coupled wave equation (4.2) can be assumed to be:

$$\begin{aligned} u_3(x_1, x_2, t) &= A_3 e^{-k\eta x_2} e^{ik(x_1 - ct)} \\ \varphi(x_1, x_2, t) &= A_\Phi e^{-k\eta x_2} e^{ik(x_1 - ct)} \\ n(x_1, x_2, t) &= A_N e^{-k\eta x_2} e^{ik(x_1 - ct)} \end{aligned} \quad (4.7)$$

Among them,  $A_3$ ,  $A_\Phi$  and  $A_N$  are undetermined coefficients, which represent the amplitude of displacement, PS potential and perturbation carrier concentration, respectively.

Substituting Eq. (4.7) into Eq. (4.2), a system of homogeneous linear equations with respect to the undetermined coefficients  $A_3$ ,  $A_\Phi$  and  $A_N$  can be obtained:

$$\begin{aligned} [c_{44}(\eta^2 - 1) + \rho c^2 - T_1^0] A_3 + e_{15}(\eta^2 - 1) A_\Phi &= 0 \\ e_{15}k^2(\eta^2 - 1) A_3 - \varepsilon_{11}k^2(\eta^2 - 1) A_\Phi - qA_N &= 0 \\ n_0\mu_{11}k(\eta^2 - 1) A_\Phi + [d_{11}k(\eta^2 - 1) + ic] A_N &= 0 \end{aligned} \quad (4.8)$$

From the Eq. (4.8), take the determinant of the coefficient matrix of  $3 \times 3$ . The condition that  $A_3$ ,  $A_\Phi$  and  $A_N$  have non-trivial solutions is that the determinant of their coefficients is equal to zero.

For a given wave velocity  $c$ , six undetermined coefficients  $\eta$  can be obtained, which are three pairs of positive and negative complex roots,  $\eta_j$  ( $j = 1-6$ ) depending on the wave speed  $c$  and the wave number  $k$  (two of which have roots  $\eta = \pm 1$ ). Therefore, the PS plate displacement, potential, and perturbation carrier concentration (4.7) can be rewritten as:

$$\begin{aligned} u_3 &= \sum_{j=1}^6 \beta_{1j} A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \\ \varphi &= \sum_{j=1}^6 A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \\ n &= \sum_{j=1}^6 \beta_{2j} A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \end{aligned} \quad (4.9)$$

where the unknown amplitude  $A_{3j} = \beta_{1j} A_{\Phi j}$ ,  $A_{Nj} = \beta_{2j} A_{\Phi j}$ . The amplitude ratio  $\beta_{1j}$  and  $\beta_{2j}$  can be determined by Eq. (4.8).

By substituting Eq. (4.9) into the constitutive equation, the expressions of stress, electrical displacement and current density of the PSs can be obtained (see

Appendix (A.4.1) ).

From the stress-free condition, the short-circuit boundary BC1 and the open-circuit boundary BC2, the dispersion equations of waves at two boundaries can be obtained, respectively (Appendix (A.4.2)-(A.4.4)).

### 4.1.3 Numerical results and discussions

For the above model, ZnO is selected as the material of PS plate, and its material properties are shown in Table 2.1. In addition, CdSe is selected for comparative calculation, and its material parameters [103] are shown in Table 4.1. In the numerical, except for the calculation part of comparing two different PSs, the more common ZnO is used for the rest.

In the calculation, the SH wave speed of the non-conductive PE plate corresponding to the two materials is introduced as the normalized velocity. The SH wave speed  $c_{SH}^1 \approx 2841.66 \text{ m/s}$  in ZnO PE plate and the SH wave speed  $c_{SH}^2 \approx 1562.10 \text{ m/s}$  in CdSe PE plate. At the same time, the normalized wavenumber is  $kh/2\pi$ .

Table 4.1 Material properties of the CdSe<sup>[103]</sup>

| $c_{44}$<br>( $\times 10^9 \text{ N/m}^2$ ) | $e_{15}$<br>( $\text{C/m}^2$ ) | $\varepsilon_{11}$<br>( $\times 10^{-11} \text{ C}^2/\text{Nm}^2$ ) | $\mu_{11}$<br>( $\text{m}^2/\text{V}$ ) | $d_{11}$<br>( $\text{m}^2/\text{s}$ ) | $\rho$<br>( $\text{kg/m}^3$ ) |
|---------------------------------------------|--------------------------------|---------------------------------------------------------------------|-----------------------------------------|---------------------------------------|-------------------------------|
| 13.20                                       | -0.138                         | 8.26                                                                | 4                                       | 0.104                                 | 5504                          |

#### 4.1.3.1 Effect of boundary conditions on SH waves

The effects of short-circuit boundary BC1 and open-circuit boundary BC2 on SH waves are studied, respectively. At this time, the thickness of the PS plate is set as  $2h = 2 \text{ mm}$ , the initial carrier concentration  $n_0 = 1 \times 10^{15} \text{ /m}^3$ , and the initial stress  $|T_1^0| = 1 \text{ GPa}$ .

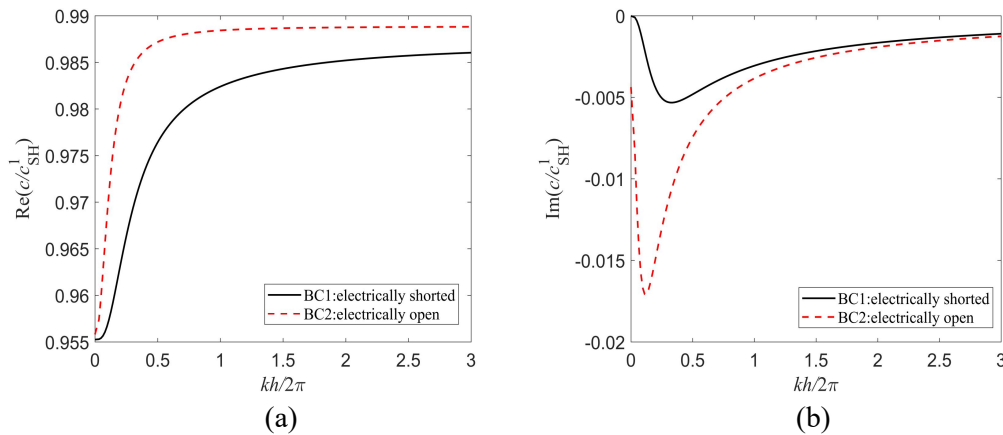


Fig. 4.2 Effect of boundary condition on SH waves propagation: (a) effect of boundary condition on phase velocity, and (b) effect of boundary condition on attenuation

It can be seen from Fig. 4.2(a) that the phase velocity increases with the increase of wave number and gradually flattens out. In addition, as shown in Fig. 4.2(b), the curves of both imaginary parts are below zero, indicating that the SH waves are attenuated by carrier concentration and initial stress. Regardless of the real or imaginary part, the absolute value of the short circuit is correspondingly smaller than the absolute value of the open circuit. The open boundary will be studied below.

#### 4.1.3.2 Effect of initial carrier concentration on SH waves

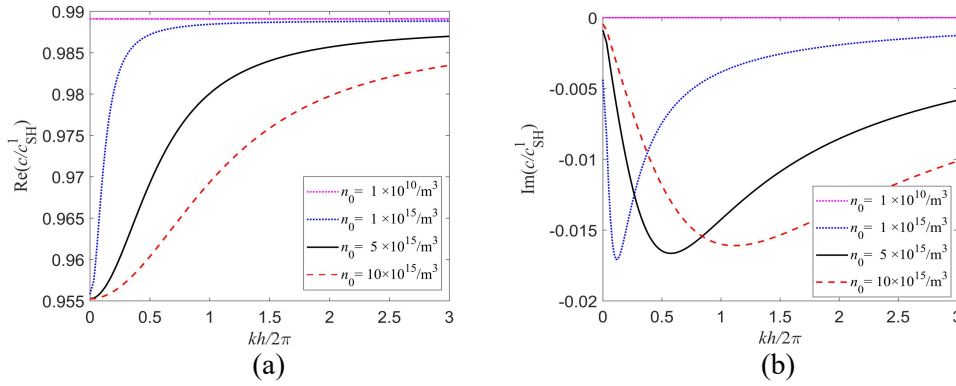


Fig. 4.3 Effect of initial carrier concentration on SH waves propagation: (a) effect of  $n_0$  on phase velocity, and (b) effect of  $n_0$  on attenuation

As shown in Fig. 4.3, when  $n_0 = 1 \times 10^{10}/\text{m}^3$ , both the real and imaginary parts of the SH waves speed are horizontal lines, indicating that the phase velocity is constant and the SH wave does not attenuate. This suggests that when the initial carrier concentration is relatively low, the ZnO PS plate can be regarded as a PE plate (i.e., semiconductor characteristics need not be considered).

When  $n_0 \geq 1 \times 10^{15}/\text{m}^3$ , as shown in Fig. 4.3(a), the phase velocity increases with increasing wave number and tends to flatten out. Meanwhile, the wave velocity decreases with increasing  $n_0$ . As depicted in Fig. 4.3(b), the attenuation of the wave continuously strengthens until reaching its maximum at long waves. With increasing wave number, the attenuation weakens, and the minimum point of attenuation shifts to the right with increasing  $n_0$ .

#### 4.1.3.3 Effect of plate thickness on SH waves

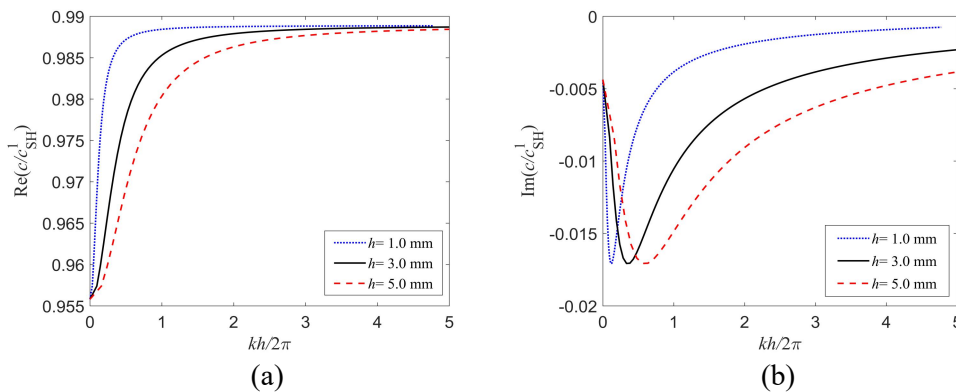


Fig. 4.4 Effect of plate thickness on SH waves propagation: (a) effect of  $h$  on phase velocity, and (b) effect of  $h$  on attenuation

From Fig. 4.4, it can be observed that when the initial stress  $|T_1^0| = 1\text{GPa}$ , the phase velocity of SH waves decreases with increasing  $h$ , and the minimum point of attenuation shifts to the right with increasing  $h$ .

#### 4.1.3.4 Effect of initial stress on SH waves

According to Fig. 4.5, when  $|T_1^0| = 0\text{GPa}$ , the normalized phase velocity gradually approaches 1, and the presence of the initial carrier concentration causes the SH waves to be dispersed and attenuated. When the initial stress is changed, the change of the real part of the wave speed is obvious, but the change of the imaginary part is small, and the slight change can be seen by local amplification.

Figure 4.5(a) shows the change of the real part of the wave speed with different initial stresses. When the wave number is small, the phase velocity increases with the increase of the wave number. The phase velocity decreases with the increase of the initial stress. In Fig. 4.5(b), the imaginary part curves almost coincide when four different initial stresses are applied. It can be found through local amplification that the imaginary part increases with the increase of the initial stress, indicating that the sufficiently large initial stress will strengthen the attenuation.

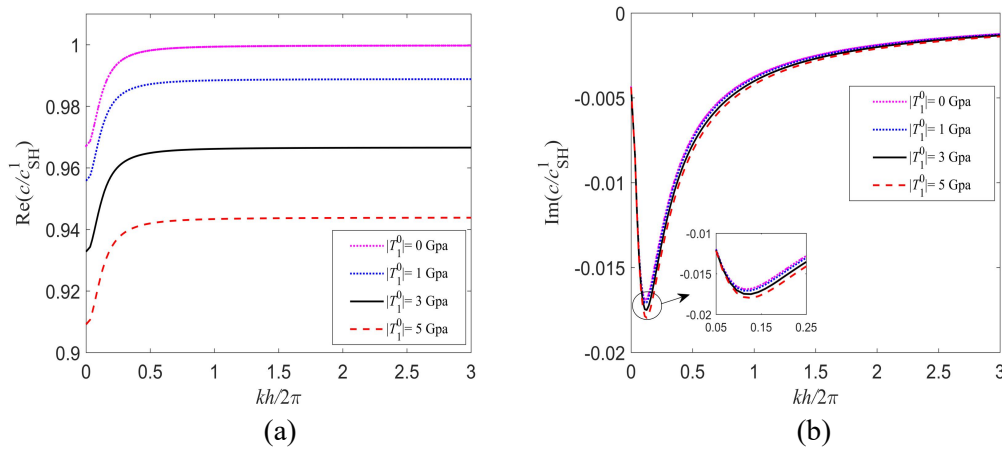


Fig. 4.5 Effect of initial stress on SH waves propagation: (a) phase velocity varies with initial stress, and (b) attenuation varies with initial stress

Considering different wave numbers, the effect of initial stress on SH waves is obtained as shown in Fig. 4.6. According to Fig. 4.6(a), when  $|T_1^0| < 100\text{MPa}$ , the initial stress has little effect on the phase velocity, while when  $|T_1^0| \geq 100\text{MPa}$ , the phase velocity decreases sharply with the increase of the initial stress. The local magnification of Fig. 4.6(a) shows that the larger the wave number, the larger the phase velocity. It can be seen from Fig. 4.6(b) that attenuation will strengthen with the increase of initial stress, and  $|T_1^0| = 1\text{GPa}$  can be regarded as the ‘boundary line’ of the impact of initial stress on attenuation. Under the same initial stress, the larger the wave number is, the weaker the attenuation will be, which is consistent with the previous results.

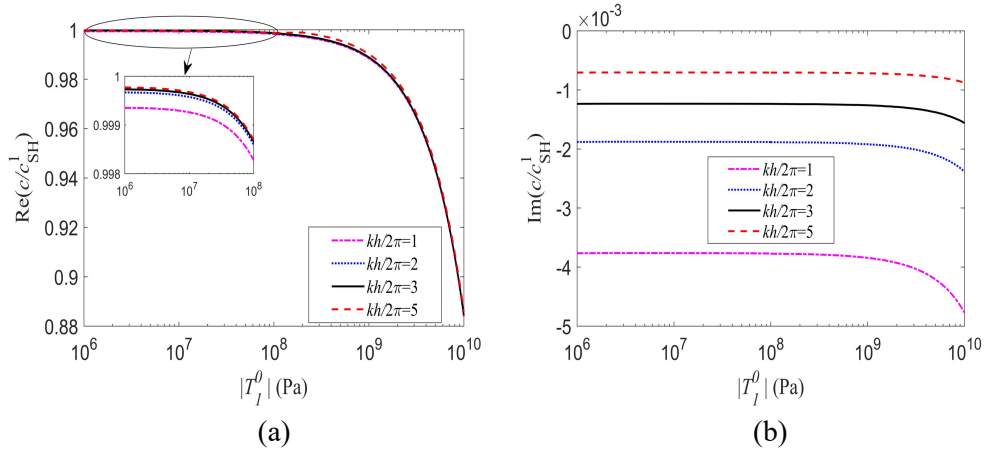


Fig. 4.6 The variation trend of phase velocity and attenuation with initial stress under different wave numbers: (a) phase velocity varies with wave number, and (b) attenuation varies with wave number

#### 4.1.3.5 SH waves propagation characteristics of two different materials

The above considers the more common ZnO material, in fact, there are many PS materials, and the material characteristics have a greater impact on the surface waves. Therefore, CdSe is selected for comparative discussion. Figure 4.7 shows the real and imaginary curves of SH wave speed for two different PS materials. As shown in Fig. 4.7(a), the phase velocity of SH waves in CdSe plate is much smaller than that of ZnO plate with initial stress. At the same time, Fig. 4.7(b) shows that the variation of SH waves attenuation in CdSe plate is also much weaker.

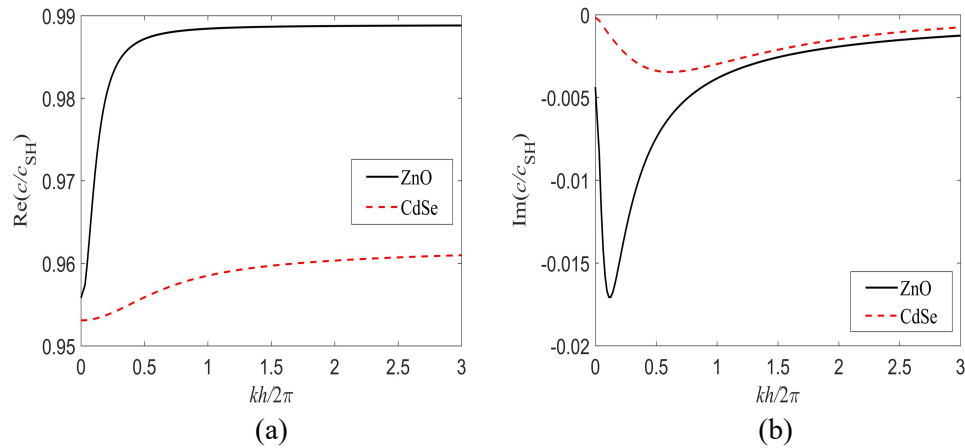


Fig. 4.7 The SH wave propagation characteristics of two different materials: (a) phase velocity variation of SH wave in ZnO and CdSe plates, and (b) attenuation of ZnO and CdSe plates

In summary, an infinite PS plate with initial stress is constructed in this Section, and its SH waves propagation characteristics with initial stress are studied. The results show that when the initial carrier concentration is small, the PS plate can be regarded as a PE plate. The phase velocity will decrease with the increase of thickness, and the lowest attenuation point will shift to the right with the increase of thickness. When the initial stress is greater than 100MPa, the phase

velocity will decrease sharply with the increase of the initial stress, and the attenuation will be strengthened when the initial stress is large enough.

## 4.2 SH waves in a piezomagnetic and piezoelectric semiconductor bilayer plate with initial stress

In the previous Section, we studied SH waves propagation in a PS plate with initial stress and no coupled PM material. In this Section, we will continue to discuss SH waves in the MCSs with initial stress, mainly for the PS/PM composite plate, and consider the PS/PM half-space for comparison, so as to analyze the influence of initial stress on SH waves in the magneto-electro-semiconductor coupling field.

Consider a PM/PS bilayer plate as shown in Fig. 4.8, where the first layer is an n-type PS with thickness  $h_1$  and the second layer is a PM layer with thickness  $h_2$ , both of which are infinite in horizontal direction, and the interface is assumed to be perfectly connected. In the composite plate, the SH waves propagate along the  $x_1$ -axis, and the polarization of both materials are along the  $x_3$ -axis.

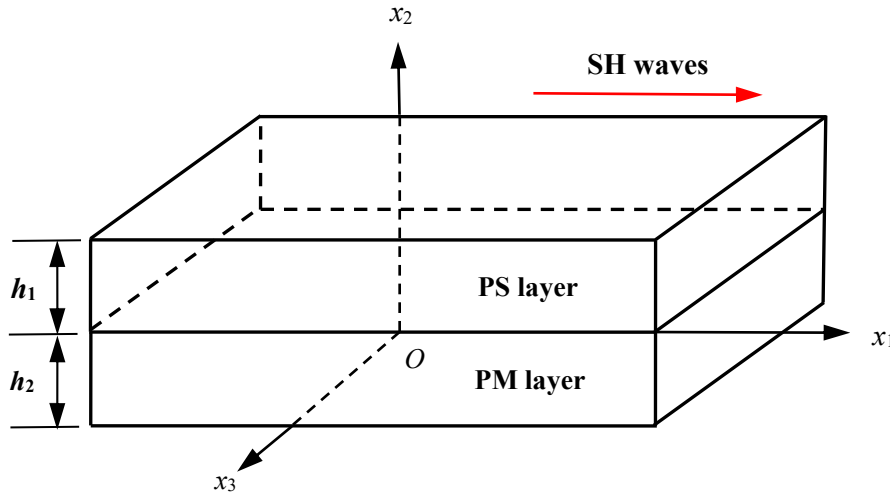


Fig. 4.8 Piezoelectric semiconductor (PS)/piezomagnetic (PM) bilayer plate with initial stress

### 4.2.1 Basic equations

Consider the PS/PM composite plate, the governing equation with initial stress is therefore one more equation  $B_{i,i} = 0$  than Eq. (4.1), i.e. :

$$\begin{aligned} T_{ji,j} + (u_{i,k} T_{kj}^0)_{,j} &= \rho \ddot{u}_i \\ D_{i,i} &= qn \\ B_{i,i} &= 0 \\ q\dot{n} + J_{i,i} &= 0 \end{aligned} \quad (4.10)$$

where  $T_{kj}^0$  is the initial stress as same as in Section 4.1, it is also assumed that



there is only one initial stress component  $T_1^0$  along the  $x_1$ -axis. Therefore, the coupled wave equation of the PS layer can be obtained as:

$$\begin{aligned} & \left[ c_{44}(\eta^2 - 1) + \rho c^2 - T_1^0 \right] A_3 + e_{15}(\eta^2 - 1) A_\Phi = 0 \\ & e_{15}k^2(\eta^2 - 1) A_3 - \varepsilon_{11}k^2(\eta^2 - 1) A_\Phi - qA_N = 0 \\ & (\eta^2 - 1) A_\Psi = 0 \\ & n_0\mu_{11}k(\eta^2 - 1) A_\Phi + \left[ d_{11}k(\eta^2 - 1) + ic \right] A_N = 0 \end{aligned} \quad (4.11)$$

Similarly, the coupled wave equation of the PM layer can be obtained:

$$\begin{aligned} & c_{44}^m \nabla^2 u_3^m + T_{1,3,11}^0 u_{3,11}^m + f_{15}^m \nabla^2 \psi^m = \rho \ddot{u}_3^m \\ & \nabla^2 \phi^m = 0 \\ & f_{15}^m \nabla^2 u_3^m - \alpha_{11}^m \nabla^2 \psi^m = 0 \end{aligned} \quad (4.12)$$

where the superscript  $m$  represents the material properties and field quantity in the PM layer.

In order to study the waves propagation characteristics in composite plates, it is necessary to combine the boundary conditions of the upper and lower surfaces of the plates. Assuming that the stress on the surface of the plate is free, there are:

$$\begin{aligned} T_{23}(x_1, h_1) &= 0 \\ T_{23}^m(x_1, -h_2) &= 0 \end{aligned} \quad (4.13)$$

On the upper surface of the PS layer:

$$\begin{aligned} n(x_1, h_1) &= 0, n(x_1, 0) = 0 \\ \phi(x_1, h_1) &= \phi^0(x_1, h_1), \psi(x_1, h_1) = \psi^0(x_1, h_1) \\ \dot{D}_2(x_1, h_1) + J_2(x_1, h_1) &= \dot{D}_2^0(x_1, h_1) \end{aligned} \quad (4.14)$$

On the lower surface of the PM layer, there are:

$$\begin{aligned} D_2^m(x_1, -h_2) &= 0 \\ \psi^m(x_1, -h_2) &= 0 \end{aligned} \quad (4.15)$$

Since the interfaces are perfectly connected, the continuity conditions at the interface are:

$$\begin{aligned} u_3(x_1, 0) &= u_3^m(x_1, 0), T_{23}(x_1, 0) = T_{23}^m(x_1, 0) \\ \phi(x_1, 0) &= \phi^m(x_1, 0), \psi(x_1, 0) = \psi^m(x_1, 0) \\ \dot{D}_2(x_1, 0) + J_2(x_1, 0) &= \dot{D}_2^m(x_1, 0) \end{aligned} \quad (4.16)$$

## 4.2.2 Solutions of the problem and dispersion equations

### 4.2.2.1 PS layer wave fields

For the SH waves propagating along the  $x_1$ -axis, the trial solution of the PS layer coupling wave equation (4.11) is assumed to be:

$$\begin{Bmatrix} u_3 \\ \varphi \\ \psi \\ n \end{Bmatrix} = \begin{Bmatrix} A_3 \\ A_\Phi \\ A_\Psi \\ A_N \end{Bmatrix} e^{-k\eta x_2} e^{ik(x_1 - ct)} \quad (4.17)$$

where  $A_3$ ,  $A_\Phi$ ,  $A_\Psi$ , and  $A_N$  are unknown amplitudes.

Substituting Eq. (4.17) into Eq. (4.11) and eliminating the common factors, a homogeneous linear system of undetermined coefficients  $A_3$ ,  $A_\Phi$ ,  $A_\Psi$ , and  $A_N$  can be obtained:

$$\begin{aligned} [c_{44}(\eta^2 - 1) + \rho c^2 - T_1^0] A_3 + e_{15}(\eta^2 - 1) A_\Phi &= 0 \\ e_{15}k^2(\eta^2 - 1) A_3 - \varepsilon_{11}k^2(\eta^2 - 1) A_\Phi - qA_N &= 0 \\ (\eta^2 - 1) A_\Psi &= 0 \\ n_0\mu_{11}k(\eta^2 - 1) A_\Phi + [d_{11}k(\eta^2 - 1) + ic] A_N &= 0 \end{aligned} \quad (4.18)$$

The Eq. (4.18)<sub>3</sub> is decoupled from the other three equations to obtain  $\eta = \pm 1$ , and in order to obtain non-trivial solutions for  $A_3$ ,  $A_\Phi$ , and  $A_N$ , the determinant of the coefficient matrix for the other three equations must be zero. Therefore, the PS plate displacement, potential, and perturbation carrier concentration solutions (4.17) can be rewritten as:

$$\begin{aligned} u_3 &= \sum_{j=1}^6 \beta_{1j} A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \\ \varphi &= \sum_{j=1}^6 A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \\ \psi &= \sum_{j=7}^8 A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \\ n &= \sum_{j=1}^6 \beta_{2j} A_{\Phi j} e^{-k\eta_j x_2} e^{ik(x_1 - ct)} \end{aligned} \quad (4.19)$$

where  $A_{\Phi j}$  ( $j = 1-8$ ) is the unknown amplitude. The amplitude ratio  $\beta_{1j}$  and  $\beta_{2j}$  can be determined by Eq. (4.18).

#### 4.2.2.2 PM layer wave fields

Similarly, the homogeneous linear equation for the undetermined coefficients  $A_3^m$ ,  $A_\Phi^m$  and  $A_\Psi^m$  in the PM layer is obtained:

$$\begin{aligned} & \left[ c_{44}^m (\eta^{m^2} - 1) + \rho^m c^2 - T_1^0 \right] A_3^m + f_{15}^m (\eta^{m^2} - 1) A_\Psi^m = 0 \\ & (\eta^{m^2} - 1) A_\Phi^m = 0 \\ & f_{15}^m (\eta^{m^2} - 1) A_3^m - \alpha_{11}^m (\eta^{m^2} - 1) A_\Psi^m = 0 \end{aligned} \quad (4.20)$$

The Eq. (4.20)<sub>2</sub> is decoupled from the other two equations to obtain  $\eta = \pm 1$ , and in order to obtain non-trivial solutions for  $A_3^m$  and  $A_\Psi^m$ , the determinant of the coefficient matrix with respect to the other two equations must be zero. For a given wave speed  $c$ , four undetermined coefficients  $\eta^m$  can be obtained, which are the positive and negative complex roots of two pairs. Together with  $\eta^m = \pm 1$  obtained from Eq. (4.20)<sub>2</sub>, a total of six  $\eta^m$  need to be considered.

In addition, the complete solution of PM layer displacement, potential and magnetic potential can be expressed as:

$$\begin{aligned} u_3^m &= \sum_{j=1}^4 \beta_{1j}^m A_{\Psi j}^m e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)} \\ \varphi^m &= \sum_{j=5}^6 A_{\Psi j}^m e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)} \\ \psi^m &= \sum_{j=1}^4 A_{\Psi j}^m e^{-k\eta_j^m x_2} e^{ik(x_1 - ct)} \end{aligned} \quad (4.21)$$

where  $A_{\Psi j}^m$  ( $j = 1-6$ ) is the unknown amplitude to be found. The amplitude ratio  $\beta_{1j}^m$  can be determined by Eq. (4.20).

The solution of the potential and magnetic potential satisfying the radiation condition in vacuum is:

$$\varphi^0 = \sum_{j=1}^6 A_{\Phi j} e^{-kx_2} e^{ik(x_1 - ct)}, \quad \psi^0 = \sum_{j=7}^8 A_{\Phi j} e^{-kx_2} e^{ik(x_1 - ct)} \quad (4.22)$$

By substituting Eqs. (4.19), (4.21) and (4.22) to the constitutive equations of PS layer, PM layer and vacuum layer, respectively, the expressions of each layer can be obtained in Appendix (A.4.5)-(A.4.7) of this Chapter.

Combining the boundary conditions and the continuity conditions, the homogeneous linear equations for the unknown amplitudes  $A_{\Phi j}$  ( $j = 1-8$ ) and  $A_{\Psi j}^m$  ( $j = 1-6$ ) are finally obtained, as follows:

$$[M]N = 0 \quad (4.23)$$

where  $N = \{ A_{\Phi 1}, A_{\Phi 2}, A_{\Phi 3}, A_{\Phi 4}, A_{\Phi 5}, A_{\Phi 6}, A_{\Phi 7}, A_{\Phi 8}, A_{\Psi 1}^m, A_{\Psi 2}^m, A_{\Psi 3}^m, A_{\Psi 4}^m, A_{\Psi 5}^m, A_{\Psi 6}^m \}$ ,  $[M]$  is the coefficient matrix of  $14 \times 14$ . The expression for the non-zero elements of the matrix is shown in Appendix (A.4.8).

In order to obtain a non-zero solution to the dispersion equation (4.23), the determinant of its coefficients must be zero, i.e.  $\text{Det}[M] = 0$ . Therefore, transcendental equations in the complex domain can be obtained for phase velocity  $c$  and wave number  $k$  or phase velocity  $c$  and frequency  $\omega$ .

### 4.2.3 Numerical results and discussions

In the numerical calculation, ZnO/CoFe<sub>2</sub>O<sub>4</sub> and CdSe/CoFe<sub>2</sub>O<sub>4</sub> material combinations are considered, and ZnO/CoFe<sub>2</sub>O<sub>4</sub> is used in the calculation except for the comparison of different material combinations.

In addition, for the convenience of comparison with Section 4.1, the main parameters are consistent with the previous Section, specifically  $h_1 = h_2 = 1$  mm,  $n_0 = 1 \times 10^{15}/\text{m}^3$ ,  $|T_1^0| = 1$  GPa. At the same time, the SH wave speed  $c_{SH}^1$  of ZnO and the SH wave velocity  $c_{SH}^2$  of CdSe are introduced as the normalized speed, and the normalized wave number is represented by  $kh/2\pi$ , where  $h = h_1 + h_2$ .

#### 4.2.3.1 Effect of initial carrier concentration on SH waves in composite plate

Figure 4.9 shows the effect of  $n_0$  on SH waves propagation characteristics of PS/PM composite plate when  $h_1 = h_2 = 1$  mm and  $|T_1^0| = 1$  GPa. When  $n_0 = 1 \times 10^{15}/\text{m}^3$ , the real and imaginary parts of the wave speed are horizontal lines, indicating that the semiconductor characteristics have no influence on the SH waves at this time, that is, the PS layer can be regarded as the PE layer. When  $n_0 \geq 1 \times 10^{15}/\text{m}^3$ , the wave speed as shown in Fig. 4.9(a) will decrease with the increase of  $n_0$ , but the wave speed of the three curves tends to be constant. Figure 4.9(b) shows that when the wave number is small, the attenuation will continue to strengthen until the maximum, then will weaken and approach zero as the wave number continues to increase.

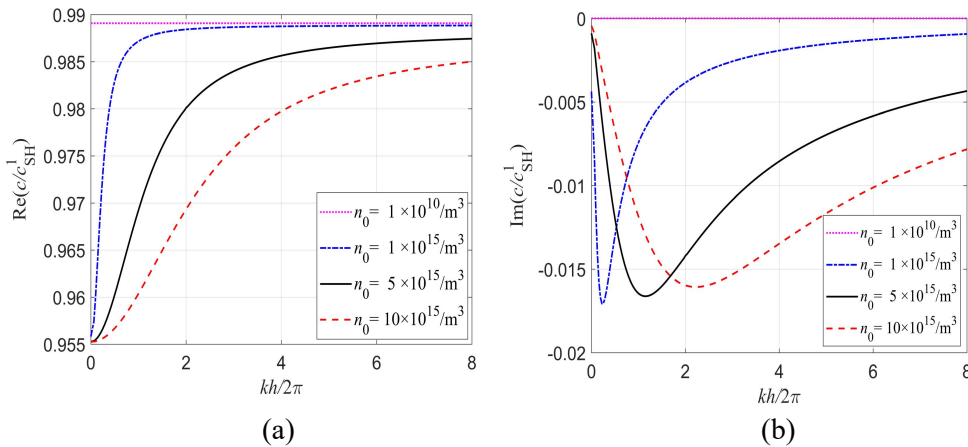


Fig. 4.9 Effect of initial carrier concentration on SH waves propagation: (a) effect of  $n_0$  on phase velocity, and (b) effect of  $n_0$  on attenuation

By comparing Fig. 4.9 and Fig. 4.3, it is found that the real and imaginary trends of the two sets of graphs are the same, indicating that SH waves in the PS/PM composite plate with initial stress are similar to SH waves in the PS plate. After careful comparison, there will be differences in the values, especially in the maximum values of the imaginary parts of Fig. 4.9(b) and Fig. 4.3(b). This is due to the influence of the PM layer, but the influence is relatively limited.

#### 4.2.3.2 Effect of composite plate thickness on SH waves

The effect of the initial carrier concentration on the SH waves in the composite plate is similar to that in the PS plate, indicating that the effect is limited after the PM layer is combined. Therefore, the influence of PS layer thickness  $h_1$  and PM thickness  $h_2$  on SH wave is further studied.

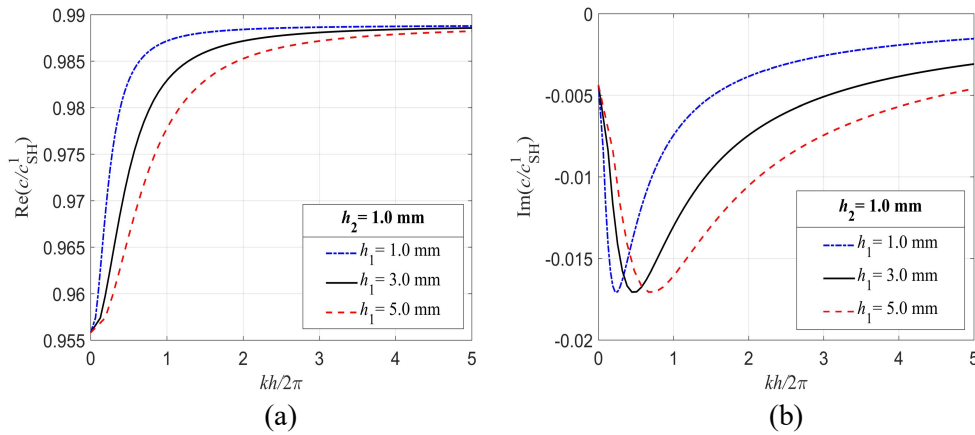


Fig. 4.10 Effect of  $h_1$  on SH waves: (a) effect of  $h_1$  on phase velocity, and (b) effect of  $h_1$  on attenuation

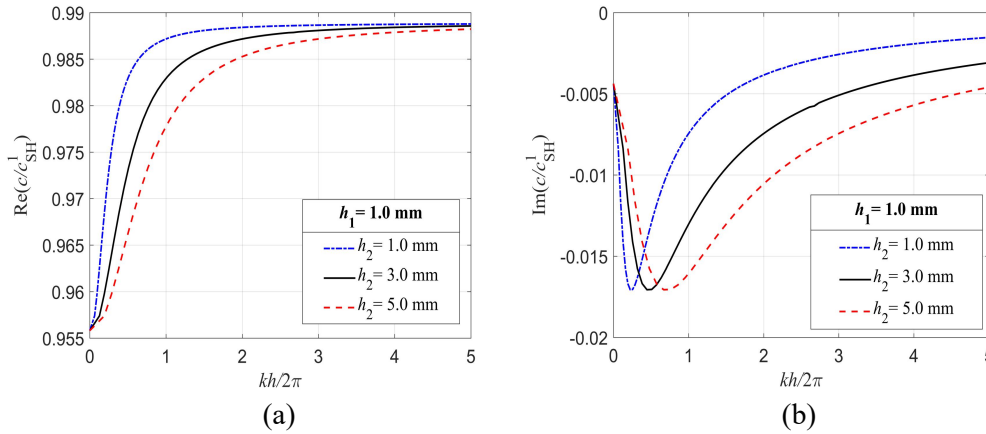


Fig. 4.11 Effect of PM layer thickness  $h_2$  on SH waves: (a) effect of  $h_2$  on phase velocity, and (b) effect of  $h_2$  on attenuation

Figure 4.10 and Fig. 4.11 show the influence of  $h_1$  and  $h_2$  on the wave speed and attenuation of SH waves, respectively. When the influence of  $h_1$  is studied,  $h_2 = 1$  mm is kept unchanged and vice versa. According to Fig. 4.10, when the initial stress  $|T_1^0| = 1$  GPa, the SH wave speed will decrease with the increase of  $h_1$ , and the lowest attenuation point will shift to the right with the increase of  $h_1$ . This is similar to the PS plate in Fig. 4.4, but a careful comparison between Fig. 4.10 and

Fig. 4.4 will still find a slight difference, especially the corner difference of the real part curve.

Figure 4.11 is almost consistent with Fig. 4.10, indicating that the PM layer thickness in the composite plate has a similar effect on SH waves as that of the PS layer thickness.

#### 4.2.3.3 Effect of initial stress on SH waves in composite plate

Figure 4.12 and Fig. 4.13 respectively show the wave speed and wave speed variation curves of SH waves with different initial stresses. Figure 4.12(a) shows that the normalized phase velocity tends to 1 when  $|T_1^0| = 0 \text{ GPa}$ . With the increase of initial stress, the phase velocity decreases obviously. In Fig. 4.12(b), the four imaginary curves almost coincide, and local magnification shows that the attenuation increases with the increase of the initial stress. It is worth noting that although Fig. 4.12 is similar to Fig. 4.5, the values of  $|T_1^0|$  are not exactly the same. The initial stress in Fig. 4.12 is 0 GPa, 0.5 GPa, 1.0 GPa and 1.5 GPa, while the initial stress in Fig. 4.5 is 0 GPa, 1.0 GPa, 3.0 GPa and 5.0 GPa, respectively. This is because if the larger initial stress of 3.0 GPa and 5.0 GPa is considered in the composite plate in Fig. 4.12, the real part of the wave speed obtained by the MATLAB program is horizontal lines, indicating that the initial stress can not be calculated normally when it is too large.

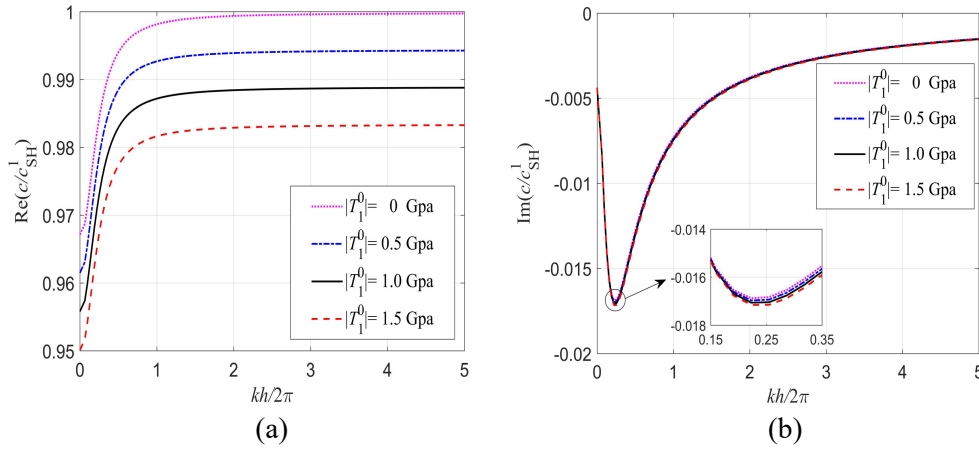


Fig. 4.12 Effect of initial stress on SH waves: (a) phase velocity varies with initial stress, and (b) attenuation varies with initial stress

In addition, since the curves in Fig. 4.12(b) almost coincide, in order to better observe the impact of initial stress on attenuation, the curves of real part variation  $\Delta\text{Re}(c)$  and imaginary part variation  $\Delta\text{Im}(c)$  of phase velocity with different initial stress are further obtained, as shown in Fig. 4.13.  $\Delta\text{Re}(c) = \text{Re}(c)(|T_1^0| \neq 0) - \text{Re}(c)(|T_1^0| = 0)$  and  $\Delta\text{Im}(c) = \text{Im}(c)(|T_1^0| \neq 0) - \text{Im}(c)(|T_1^0| = 0)$  respectively represent the difference between the real and imaginary parts of the phase velocity when the initial stress acts and the real and imaginary parts when there is no initial stress.

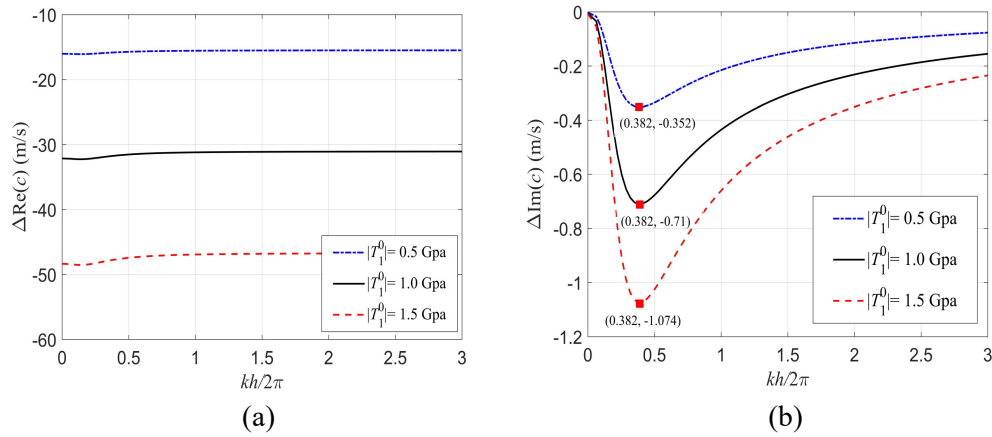


Fig. 4.13 Effect of initial stress on SH wave dispersion and attenuation characteristics: (a) initial stress effect on the variation of phase velocity, and (b) initial stress effect on the variation of attenuation

According to Fig. 4.13, both the real and imaginary parts of the phase velocity decrease with the initial stress, which is consistent with the previous results, and the variation increases with the increase of the initial stress. As shown in Fig. 4.13(b), the initial stress will strengthen the attenuation characteristics, but the attenuation first increases to the maximum and then tends to zero. Regardless of the initial stress value, all inflection points of attenuation change are at  $kh/2\pi = 0.382$ .

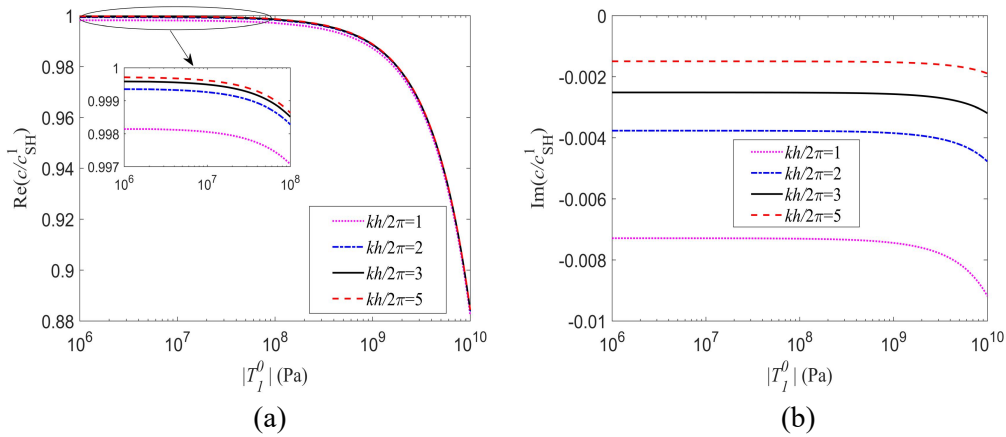


Fig. 4.14 The variation trend of phase velocity and attenuation with initial stress under different wave numbers: (a) phase velocity varies with wave number, and (b) attenuation varies with wave number

Figure 4.14 shows the influence of initial stress on SH wave when different wave numbers are considered. Figure 4.14(a) shows that when  $|T_1^0| < 100\text{MPa}$ , the initial stress has little effect on the phase velocity, while when  $|T_1^0| \geq 100\text{MPa}$ , the phase velocity decreases sharply with the increase of the initial stress. A local magnification of Fig. 4.14(a) shows that the phase velocity increases with the increase of the wave number. According to Fig. 4.14(b), attenuation is maintained at first and becomes obvious when  $|T_1^0|$  reaches a certain value. The trend of Fig.

4.14 is almost the same as that of Fig. 4.6 in the PS plate, but the comparison shows that the values of the attenuation curves in Fig. 4.14(b) and Fig. 4.6(b) are obviously different, especially when  $|T_1^0| = 1 \text{ MPa}$ , the imaginary starting points of the two groups of curves are obviously different.

#### 4.2.3.4 SH waves in different material combinations

The SH waves propagation of two different PS plate materials is discussed in Section 4.1.3.5, and the SH waves propagation of two different material combinations (ZnO/CoFe<sub>2</sub>O<sub>4</sub> and CdSe/CoFe<sub>2</sub>O<sub>4</sub>) is discussed here. Similar to Fig. 4.7, as shown in Fig. 4.15, the phase velocity in CdSe/CoFe<sub>2</sub>O<sub>4</sub> is much smaller than that in ZnO/CoFe<sub>2</sub>O<sub>4</sub> under the same initial stress, and the attenuation change is not so obvious.

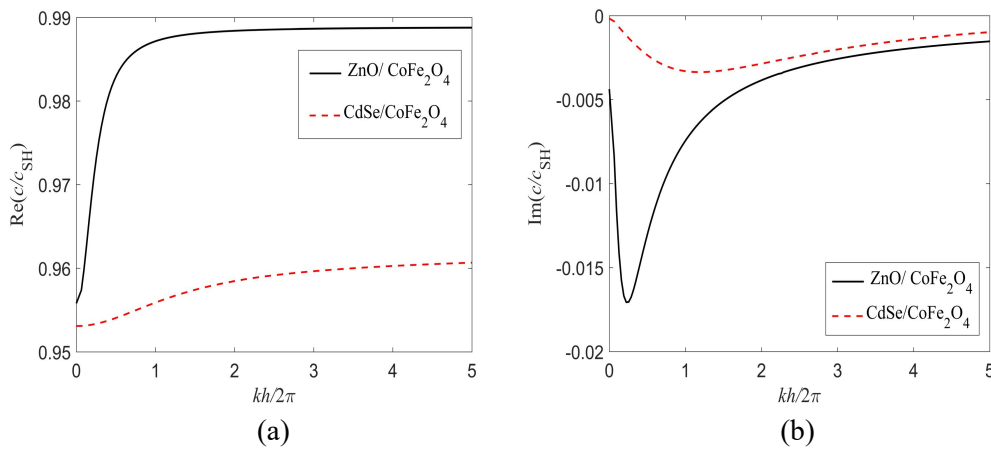


Fig. 4.15 The SH wave propagation characteristics of two different composite plates: (a) phase velocity variation of SH wave in different composite plates, and (b) SH wave attenuation in different composite plates

#### 4.2.3.5 SH waves in PS/PM half-space with initial stress

The SH waves in the PS/PM composite plate with initial stress is discussed above, and the substrate of the PM plate is considered to be replaced by a PM half-space, which is consistent with the model in Fig. 2.1, that is, the PS/PM half-space layered structure. In the composite plate model,  $h_1 = h_2 = 1 \text{ mm}$ . However, considering that  $h_2$  is much larger than  $h_1$ , the PM plate substrate can be regarded as a semi-infinite space.

Considering the PS/PM half-space, the lower surface stress free condition of the composite plate is cancelled, that is,  $T_{23}^m(x_1, -h_2) = 0$  in Eq. (4.13) is removed. Boundary conditions (4.15) regarding the lower surface of the PM layer are also removed. In addition, the radiation conditions along the negative  $x_2$ -axis need to be added, namely:

$$x_2 \rightarrow -\infty, \quad u_3^m = 0, \quad \varphi^m = 0, \quad \psi^m = 0 \quad (4.24)$$

Therefore, only 3  $\eta^m$  whose real part is negative need be retained for the 6  $\eta^m$  obtained in Section 4.2.2.2. Combining the present boundary conditions and the



continuity conditions, the homogeneous linear equations for the unknown amplitudes  $A_{\Phi_j}$  ( $j = 1-8$ ) and  $A_{\Psi_j}^m$  ( $j = 1-3$ ) can be obtained as:

$$[M]N = 0 \quad (4.25)$$

where  $N = \{ A_{\Phi_1}, A_{\Phi_2}, A_{\Phi_3}, A_{\Phi_4}, A_{\Phi_5}, A_{\Phi_6}, A_{\Phi_7}, A_{\Phi_8}, A_{\Psi_1}^m, A_{\Psi_2}^m, A_{\Psi_3}^m \}$ ,  $[M]$  is the coefficient matrix of  $11 \times 11$ . The solution process is similar to that of composite plate.

In the numerical calculation, if the thickness of the PS plate  $h_1$  and the composite plate above is consistent with 1mm, the obtained curve value will change. Since the effect of  $h_2$  in Section 4.2.3.2 is similar to that of  $h_1$ , considering  $h_1 = 2\text{mm}$  in practice, almost the same result can be obtained as the composite plate.

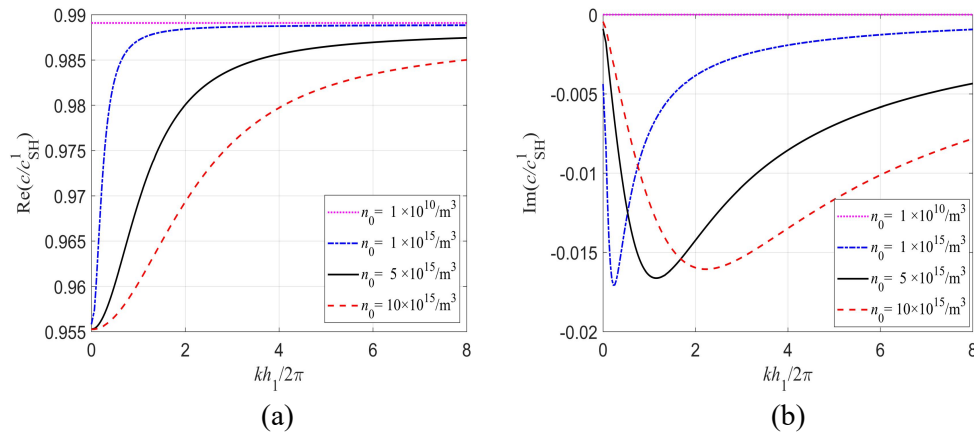


Fig. 4.16 Effect of  $n_0$  on SH wave propagation in the PS/PM half-space: (a) effect of  $n_0$  on phase velocity, and (b) effect of  $n_0$  on attenuation

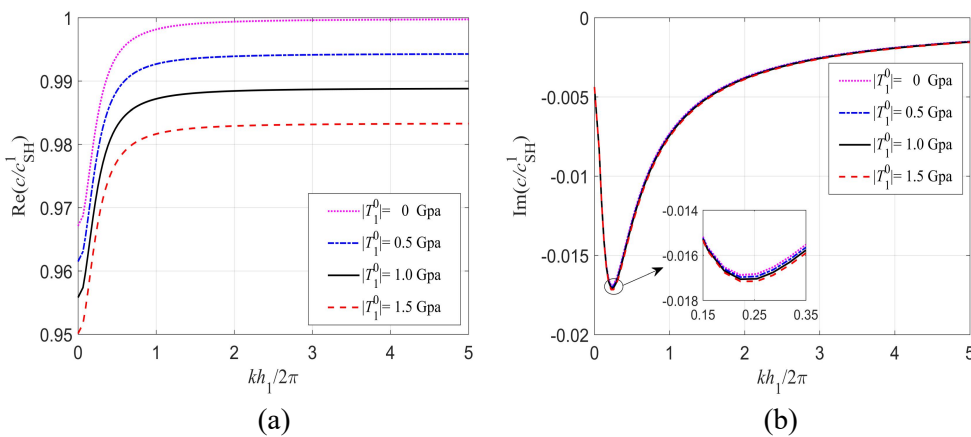


Fig. 4.17 Effect of initial stress on SH waves in the layered half-space: (a) phase velocity varies with initial stress, and (b) attenuation varies with initial stress

Specifically, Figure 4.16 is consistent with Fig. 4.9, and Fig. 4.17 is consistent with Fig. 4.12. Since the effects and curves are almost identical, further detailed discussions are unnecessary.

## 4.3 Summary

Based on the multi-field coupling theory with initial stress, Chapter 4 focuses on the influence of initial stress field on SH waves propagation in MCS structures. The effects of boundary conditions, initial carrier concentration, plate thickness and initial stress on SH waves are obtained. In addition, SH waves propagation in two different PS materials is discussed. Secondly, the influence of initial stress on SH waves is discussed for PS/PM bilayer composite plate and PS/PM half-space, respectively. The results show that:

(1) The phase velocity of SH waves in ZnO PS plate decreases and attenuates with constant initial stress. When the initial stress  $|T_1^0| \geq 100\text{MPa}$ , the phase velocity decreases sharply with the increase of the initial stress. When the initial carrier concentration is relatively small ( $n_0 = 1 \times 10^{10}/\text{m}^3$ ), the PS plate can be regarded as a PE plate. The phase velocity and attenuation corresponding to the electrical short circuit BC1 are correspondingly smaller than that of the electrical open circuit BC2.

(2) For ease of comparison, the main parameters of the PS/PM composite plate are the same as those of the ZnO PS plate. The results show that regardless of the initial carrier concentration, plate thickness, initial stress, or material influence, the propagation characteristics of SH waves in the composite plate are similar to those in the PS plate. Only slight changes occur in the real and imaginary parts of the wave speed, indicating that the PM layer has some influence on the propagation of SH waves, but the impact is limited. The effect of PM layer thickness  $h_2$  and PS layer thickness  $h_1$  on SH wave is almost the same.

(3) In the PS/PM composite plate, by analyzing the influence of the initial stress on the change of the real and imaginary parts of the phase velocity, it can be found that no matter how large the initial stress is, the inflection points of the attenuation change are all at  $kh/2\pi = 0.382$ . Under the same initial stress, the phase velocity of CdSe/CoFe<sub>2</sub>O<sub>4</sub> is much smaller than that of ZnO/CoFe<sub>2</sub>O<sub>4</sub>, and the attenuation is also much weaker. When the substrate is changed from PM plate to PM half-space, the result is almost the same as that in composite plate ( $h_1 = 1\text{mm}$  in composite plate) when the thickness of PS layer  $h_1 = 2\text{mm}$ .

## 4.4 Appendix

### (1) Phase velocity equations of PS plate with initial stress

The expressions of stress, electrical displacement and current density in the PS plate with initial stress (omit the term of  $e^{ik(x_1-ct)}$ ) are:

$$\begin{aligned} T_{13} &= \sum_{j=1}^6 ik (c_{44}\beta_{1j} + e_{15}) A_{\Phi j} e^{-k\eta_j x_2}, T_{23} = -\sum_{j=1}^6 k (c_{44}\beta_{1j} + e_{15}) \eta_j A_{\Phi j} e^{-k\eta_j x_2} \\ D_1 &= \sum_{j=1}^6 ik (e_{15}\beta_{1j} - \varepsilon_{11}) A_{\Phi j} e^{-k\eta_j x_2}, D_2 = -\sum_{j=1}^6 k (e_{15}\beta_{1j} - \varepsilon_{11}) \eta_j A_{\Phi j} e^{-k\eta_j x_2} \\ J_1 &= -\sum_{j=1}^6 ikq (n_0\mu_{11} + d_{11}\beta_{2j}) A_{\Phi j} e^{-k\eta_j x_2}, J_2 = \sum_{j=1}^6 kq (n_0\mu_{11} + d_{11}\beta_{2j}) \eta_j A_{\Phi j} e^{-k\eta_j x_2} \end{aligned} \quad (\text{A.4.1})$$

From the stress free condition (4.3), combined with the stress expression (A.4.1)<sub>1</sub>, when  $x_2 = \pm h$ :

$$\begin{aligned} T_{23}(x_1, h) &= 0, \quad \sum_{j=1}^6 (c_{44}\beta_{1j} + e_{15})\eta_j e^{-kh\eta_j} A_{\Phi j} = 0 \\ T_{23}(x_1, -h) &= 0, \quad \sum_{j=1}^6 (c_{44}\beta_{1j} + e_{15})\eta_j e^{kh\eta_j} A_{\Phi j} = 0 \end{aligned} \quad (\text{A.4.2})$$

From the electrical short-circuit boundary (4.4) and the disturbed carrier concentration condition (4.6), we can get:

$$\begin{aligned} \varphi(x_1, h) &= 0, \quad \sum_{j=1}^6 e^{-kh\eta_j} A_{\Phi j} = 0 \\ n(x_1, h) &= 0, \quad \sum_{j=1}^6 \beta_{2j} e^{-kh\eta_j} A_{\Phi j} = 0 \\ \varphi(x_1, -h) &= 0, \quad \sum_{j=1}^6 e^{kh\eta_j} A_{\Phi j} = 0 \\ n(x_1, -h) &= 0, \quad \sum_{j=1}^6 \beta_{2j} e^{kh\eta_j} A_{\Phi j} = 0 \end{aligned} \quad (\text{A.4.3})$$

In order to obtain a non-trivial solution, the coefficients of the Eqs. (A.4.2) and (A.4.3) with respect to the above unknown constants must be zero, thus obtaining the phase velocity equation of BC1: wave propagation at the short-circuit boundary.

Similarly, from the electrical open-boundary (4.5) and the disturbed carrier concentration condition (4.6), it can be obtained that:

$$\begin{aligned} \sum_{j=1}^6 (ikce_{15}\eta_j\beta_{1j} - ikc\varepsilon_{11}\eta_j + qn_0\mu_{11}\eta_j + qd_{11}\eta_j\beta_{2j}) e^{-kh\eta_j} A_{\Phi j} &= 0 \\ \sum_{j=1}^6 \beta_{2j} e^{-kh\eta_j} A_{\Phi j} &= 0 \\ \sum_{j=1}^6 (ikce_{15}\eta_j\beta_{1j} - ikc\varepsilon_{11}\eta_j + qn_0\mu_{11}\eta_j + qd_{11}\eta_j\beta_{2j}) e^{kh\eta_j} A_{\Phi j} &= 0 \\ \sum_{j=1}^6 \beta_{2j} e^{kh\eta_j} A_{\Phi j} &= 0 \end{aligned} \quad (\text{A.4.4})$$

The phase velocity equation of BC2: wave propagation at electrical open boundary is obtained.

## (2) Dispersion equation of PS/PM composite plate with initial stress

The expressions of stress, electric displacement, magnetic induction intensity and current of the PS layer ( $e^{-k\eta_j x_2} \cdot e^{ik(x_1 - ct)}$  term omitted) are:

$$\begin{aligned}
T_{13} &= ik \sum_{j=1}^6 (c_{44}\beta_{1j} + e_{15}) A_{\Phi j}, T_{23} = -k \sum_{j=1}^6 (c_{44}\beta_{1j} + e_{15}) \eta_j A_{\Phi j} \\
D_1 &= ik \sum_{j=1}^6 (e_{15}\beta_{1j} - \varepsilon_{11}) A_{\Phi j}, D_2 = k \sum_{j=1}^6 (\varepsilon_{11} - e_{15}\beta_{1j}) \eta_j A_{\Phi j} \\
B_1 &= -ik\alpha_{11} \sum_{j=7}^8 A_{\Phi j}, B_2 = k\alpha_{11} \sum_{j=7}^8 \eta_j A_{\Phi j} \\
J_1 &= -ikq \sum_{j=1}^6 (n_0\mu_{11} + d_{11}\beta_{2j}) A_{\Phi j}, J_2 = kq \sum_{j=1}^6 (n_0\mu_{11} + d_{11}\beta_{2j}) \eta_j A_{\Phi j}
\end{aligned} \tag{A.4.5}$$

The expressions of the stress, electric displacement and magnetic induction of the PM layer (the  $e^{-k\eta_j^m x_2} \cdot e^{ik(x_1 - ct)}$  term is omitted) :

$$\begin{aligned}
T_{13}^m &= ik \sum_{j=1}^4 (c_{44}^m \beta_{1j}^m + f_{15}^m) A_{\Psi j}^m, T_{23}^m = -k \sum_{j=1}^4 (c_{44}^m \beta_{1j}^m + f_{15}^m) \eta_j^m A_{\Psi j}^m \\
D_1^m &= -ik\varepsilon_{11}^m \sum_{j=5}^6 A_{\Psi j}^m, D_2^m = k\varepsilon_{11}^m \sum_{j=5}^6 \eta_j^m A_{\Psi j}^m \\
B_1^m &= ik \sum_{j=1}^4 (f_{15}^m \beta_{1j}^m - \alpha_{11}^m) A_{\Psi j}^m, B_2^m = k \sum_{j=1}^4 (\alpha_{11}^m - f_{15}^m \beta_{1j}^m) \eta_j^m A_{\Psi j}^m
\end{aligned} \tag{A.4.6}$$

The electric displacement and magnetic induction in vacuum are:

$$D_2^0 = k\varepsilon_0 \sum_{j=1}^6 e^{-kx_2} \cdot e^{ik(x_1 - ct)} A_{\Phi j}, B_2^0 = k\mu_0 \sum_{j=7}^8 e^{-kx_2} \cdot e^{ik(x_1 - ct)} A_{\Phi j} \tag{A.4.7}$$

By substituting the expressions of each layer of the composite plate into the boundary conditions and continuous conditions (4.13)-(4.16), the non-zero elements in the coefficient matrix of the dispersion equation can be obtained as:

$$\begin{aligned}
M_{1j} &= (c_{44}\beta_{1j} + e_{15})\eta_j e^{-k\eta_j \eta_j}, M_{2k} = (c_{44}^m \beta_{1k}^m + f_{15}^m) \eta_k^m e^{kh_2 \eta_k^m} \\
M_{3j} &= \beta_{2j} e^{-k\eta_j \eta_j}, M_{4j} = \beta_{2j}, M_{5j} = e^{-k\eta_j \eta_j} - e^{-k\eta_j} \\
M_{6,7} &= e^{-k\eta_j \eta_j} - e^{-k\eta_j}, M_{6,8} = e^{-k\eta_j \eta_j} - e^{-k\eta_j} \\
M_{7j} &= \left[ q(n_0\mu_{11} + d_{11}\beta_{2j}) - ikc(\varepsilon_{11} - e_{15}\beta_{1j}) \right] \eta_j e^{-k\eta_j \eta_j} + ikc\varepsilon_0 e^{-k\eta_j} \\
M_{8,13} &= \eta_{13}^m e^{kh_2 \eta_{13}^m}, M_{8,14} = \eta_{14}^m e^{kh_2 \eta_{14}^m}, M_{9k} = e^{kh_2 \eta_k^m} \\
M_{10j} &= \beta_{1j}, M_{10k} = -\beta_{1k}^m, \\
M_{11j} &= (c_{44}\beta_{1j} + e_{15})\eta_j, M_{11k} = -(c_{44}^m \beta_{1k}^m + f_{15}^m) \eta_k^m \\
M_{12j} &= 1, M_{12,13} = M_{12,14} = -1, M_{13,7} = M_{13,8} = 1, M_{13k} = -1 \\
M_{14j} &= \left[ q(n_0\mu_{11} + d_{11}\beta_{2j}) - ikc(\varepsilon_{11} - e_{15}\beta_{1j}) \right] \eta_j \\
M_{14,13} &= -\varepsilon_{11}^m \eta_{13}^m, M_{14,14} = -\varepsilon_{11}^m \eta_{14}^m
\end{aligned} \tag{A.4.8}$$

where  $j = 1-6$  and  $k = 9-12$ .

## Chapter 5

# Rotation effects on SH waves in piezoelectric semiconductor and piezomagnetic layered half-space

The effects of the biasing field will change the propagation characteristics of the SAWs, for example, the initial stress has an important effect on the propagation of the waves in the layered PE structures [169,170,191], the layered magnetoelectric structures [171,192] and the semi-infinite PSs [88]. In Chapter 4, the influence of initial stress on SH waves propagation in MCS composite structure is obtained. In addition, the rotating field can also cause changes in the vibration frequency (i.e. wave speed) of an elastomer, PE body or PS, and this principle can be used to design SAW gyroscopes [193,194] to measure the angular velocity of a rotating object. In fact, detecting the complete motion of an object requires two types of motion sensors, namely accelerometers and gyroscopes. The accelerometers are used to detect linear motion, while the gyroscopes are used to detect angular motion.

In view of the effect of rotating field on the SAWs, Fang et al. thoroughly analyzed the propagation of acoustic waves in different rotating PE structures around rotating PE crystal [195], PE half-space [196] and PE plate [197], and explained the characteristics of rotational sensitivity in detail. Yang et al. [198] studied the angular rate sensor with PE film elastic beam attached to the surface. In addition, Yang [199] also reviewed the 1D, 2D and 3D theories of rotating PE bodies, and illustrated the working principle of PE gyroscopes with numerical examples. For the semiconductor/PE composite structure, researchers obtained the dispersion equation of SH waves with rotation and initial stress, and verified that the rotation field and initial stress field will also cause the waves to disperse and decay as the semiconductor characteristics [114]. Considering the piezothermal-

elastic half space [200] and the infinite Kelvin-Voigt viscoelastic heat conduction plate [201], Sharma et al. obtained the propagation characteristics of Rayleigh waves in the half-space and Rayleigh-Lamb waves in the heat conduction plate when rotation and non-rotation respectively, providing a basis for the development of gyroscopes and rotation sensors. Singh et al. [202,203] discussed the reflection and transmission properties of plane waves in a rotating triclinic crystal half-space and in a PE-thermoelastic composite structure. Based on the surface waves gyro effect, researchers have developed SAW gyroscopes with good linearity and detection sensitivity [193,204-206].

As a kind of micromechanical gyroscope, SAW gyroscope has the advantages of passive, wireless, small size, high sensitivity and strong impact resistance. The gyroscopes based on surface wave gyro effect have strong anti-vibration ability, high reliability and wide dynamic range without additional suspension structure. In addition, the planar lithography technology greatly reduces the production cost and the device life is ideal, so it has attracted wide attention. However, compared with the gyroscopic wave problems in elastic or PE materials, there are few reports on the consideration of rotation-effect sound waves in PSs, and there are no studies on magnetically driven surface wave gyroscopes in PSs.

In this Chapter, based on the multi-field coupling theoretical including Coriolis and centrifugal force, the propagation of SH waves in the PM/PS layered structure when rotating around a fixed axis is studied, including the dispersion characteristics, attenuation characteristics and frequency drift of SH waves, and the influence of semiconductor characteristics and horizontal biasing electric field on the MCSs in the presence and absence of rotation is explored.

## 5.1 Formulation of the problem

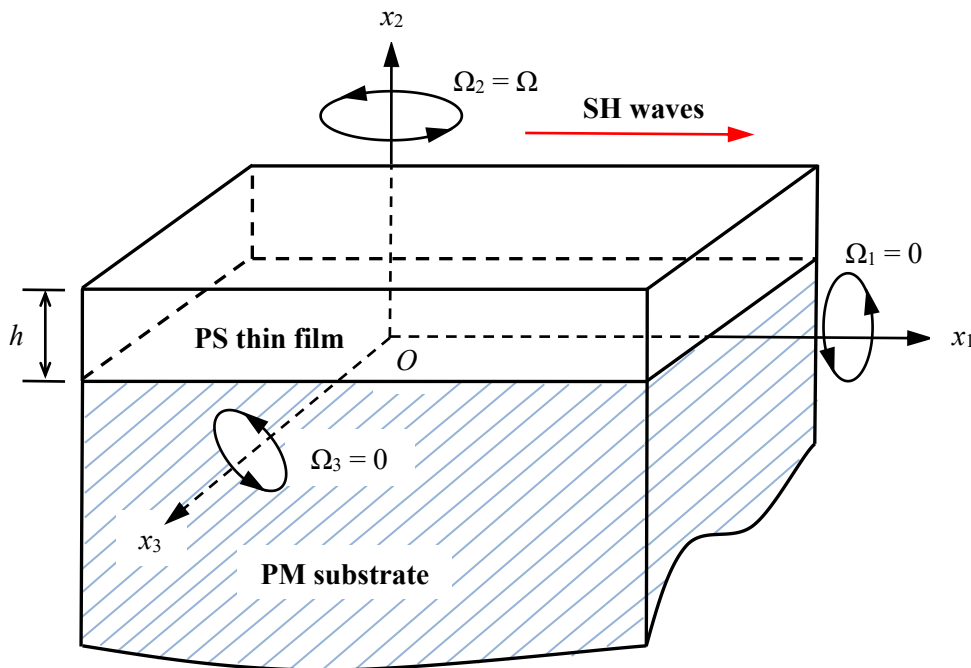


Fig. 5.1 A rotating PS and PM layered half-space

The layered composite structure as shown in Fig. 5.1 is established, where a  $n$ -type PS with thickness  $h$  is perfectly bonded on a PM half-space substrate. The PS and PM materials are assumed to be transversely isotropic. The SH waves propagate along positive  $x_1$ -axis, and the poling directions along  $x_3$ -axis. The whole structure rotates around the  $x_2$ -axis at a constant rate  $\Omega$ .

### 5.1.1 Basic equations of PS layer

According to existing studies [71,143,144,196,197], the equilibrium equations of PS layer including Coriolis and centrifugal force can be expressed as:

$$\begin{aligned} T_{ji,j} &= \rho \left[ \ddot{u}_i + 2\varepsilon_{ijk} \Omega_j \dot{u}_k + (\Omega_i \Omega_j u_j - \Omega_j^2 u_i) \right] \\ D_{i,i} &= qn \\ B_{i,i} &= 0 \\ q\dot{n} + J_{i,i} &= 0 \end{aligned} \quad (5.1)$$

where  $\varepsilon_{ijk}$  is the permutation tensor.  $\Omega_i$  represents the angular velocity, since the layered structure rotates around the  $x_2$  axis, so  $(\Omega_1, \Omega_2, \Omega_3) = (0, \Omega, 0)$ .

By substituting the Eq. (5.1) into the previous constitutive equation, the coupled wave equation of the rotating PS layer can be obtained:

$$\begin{aligned} c_{44} \nabla^2 u_3 + e_{15} \nabla^2 \varphi &= \rho \frac{\partial^2 u_3}{\partial t^2} - \rho \Omega^2 u_3 \\ e_{15} \nabla^2 u_3 - \varepsilon_{11} \nabla^2 \varphi &= qn \\ \alpha_{11} \nabla^2 \psi &= 0 \\ \frac{\partial n}{\partial t} - n_0 \mu_{11} \nabla^2 \varphi - d_{11} \nabla^2 n + \mu_{11} \bar{E}_1 \frac{\partial n}{\partial x_1} &= 0 \end{aligned} \quad (5.2)$$

### 5.1.2 Basic equations of PM substrate

For PM substrate, the PE coefficient, carrier concentration, mobility and diffusion coefficient in Eq. (5.1) are all zero, and the presence of PM coefficient is added. Therefore, the field governing equation with rotation is:

$$\begin{aligned} T_{ji,j}^m &= \rho^m \left[ \ddot{u}_i^m + 2\varepsilon_{ijk} \Omega_j \dot{u}_k^m + (\Omega_i \Omega_j u_j^m - \Omega_j^2 u_i^m) \right] \\ D_{i,i}^m &= 0 \\ B_{i,i}^m &= 0 \end{aligned} \quad (5.3)$$

where the superscript  $m$  represents the physical quantity in the PM substrate.

Similarly, by substituting the equilibrium equation (5.3) into the constitutive equation, the coupled wave equation of the rotating PM substrate can be obtained:

$$\begin{aligned}
c_{44}^m \nabla^2 u_3^m + f_{15}^m \nabla^2 \psi^m &= \rho^m \frac{\partial^2 u_3^m}{\partial t^2} - \rho^m \Omega^2 u_3^m \\
\varepsilon_{11}^m \nabla^2 \varphi^m &= 0 \\
f_{15}^m \nabla^2 u_3^m - \alpha_{11}^m \nabla^2 \psi^m &= 0
\end{aligned} \tag{5.4}$$

### 5.1.3 Boundary conditions and radiation conditions

When SH waves propagate in PM/PS layered structure, it needs to satisfy not only the governing equations, but also the boundary conditions and continuity conditions.

On the upper surface of the PS cover ( $x_2 = h$ ), assuming that the stress is free. In addition, there are boundary conditions for electricity and magnetism:

$$\begin{aligned}
T_{23}(x_1, h) &= 0 \\
\varphi(x_1, h) &= \varphi^0(x_1, h), \quad \psi(x_1, h) = \psi^0(x_1, h) \\
\dot{D}_2(x_1, h) + J_2(x_1, h) &= \dot{D}_2^0(x_1, h)
\end{aligned} \tag{5.5}$$

It is assumed that the disturbed carrier concentration disappears on the upper and lower surfaces, i.e.

$$n(x_1, h) = 0, \quad n(x_1, 0) = 0 \tag{5.6}$$

The displacement, stress, potential, magnetic potential and current at the interface should also meet the following continuous conditions:

$$\begin{aligned}
u_3(x_1, 0) &= u_3^m(x_1, 0), \quad T_{23}(x_1, 0) = T_{23}^m(x_1, 0) \\
\varphi(x_1, 0) &= \varphi^m(x_1, 0), \quad \psi(x_1, 0) = \psi^m(x_1, 0) \\
\dot{D}_2(x_1, 0) + J_2(x_1, 0) &= \dot{D}_2^m(x_1, 0)
\end{aligned} \tag{5.7}$$

The radiation conditions must be satisfied at infinity, i.e.,  $x_2 \rightarrow \pm\infty$ :

$$\begin{aligned}
x_2 \rightarrow +\infty, \quad \varphi^0 &= 0, \quad \psi^0 = 0 \\
x_2 \rightarrow -\infty, \quad u_3^m &= 0, \quad \varphi^m = 0, \quad \psi^m = 0
\end{aligned} \tag{5.8}$$

## 5.2 Solutions of the problem

### 5.2.1 Wave fields in each layer

For SH waves propagating along the  $x_1$ -axis, we can assume that the trial solution of the coupled wave equation (5.2) of the PS layer as:

$$\{u_3, \varphi, \psi, n\} = \{A_U, A_\Phi, A_\Psi, A_N\} \exp(-k\eta x_2) \cdot \exp[ik(x_1 - ct)] \tag{5.9}$$



where  $A_U$ ,  $A_\Phi$ ,  $A_\Psi$  and  $A_N$  are unknown amplitudes.

By substituting Eq. (5.9) into Eq. (5.2) and eliminating the common factors, the homogeneous linear equations with respect to the undetermined coefficients  $A_U$ ,  $A_\Phi$ ,  $A_\Psi$  and  $A_N$  can be obtained:

$$\begin{aligned} & [c_{44}k^2(\eta^2 - 1) + \rho k^2 c^2 + \rho \Omega^2] A_U + e_{15}k^2(\eta^2 - 1) A_\Phi = 0 \\ & e_{15}k^2(\eta^2 - 1) A_U - \varepsilon_{11}k^2(\eta^2 - 1) A_\Phi - q A_N = 0 \\ & \alpha_{11}(\eta^2 - 1) A_\Psi = 0 \\ & n_0 \mu_{11} k(\eta^2 - 1) A_\Phi + [d_{11}k(\eta^2 - 1) + i(c - \mu_{11} \bar{E}_1)] A_N = 0 \end{aligned} \quad (5.10)$$

Obviously, the third equation in Eq. (5.10) can be decoupled from the other three. For  $A_\Psi \neq 0$ , the parameter  $\eta$  can be easily obtained, i.e.,  $\eta = \pm 1$ , which represent the two magnetic potential wave components increasing or decaying away from the boundaries in the PS layer.

In order to obtain nontrivial solutions for  $A_U$ ,  $A_\Phi$ , and  $A_N$ , the coefficients of the matrix determinants Eqs. (5.10)<sub>1</sub>, (5.10)<sub>2</sub>, and (5.10)<sub>4</sub> must be zero. For a given wave speed  $c$ , six different  $\eta$  can be obtained, which are the positive and negative complex roots of two pairs, and  $\eta = \pm 1$ . In order to satisfy the boundary conditions, the total of eight  $\eta$  need to be considered, including the two  $\eta$  obtained from the third equation after decoupling. Therefore, the complete solution of the PS layer can be expressed as:

$$\{u_3, \phi, \psi, n\} = \left\{ \sum_{j=1}^6 \beta_{1j} A_{\Phi j}, \sum_{j=1}^6 A_{\Phi j}, \sum_{j=7}^8 A_{\Phi j}, \sum_{j=1}^6 \beta_{2j} A_{\Phi j} \right\} \exp(-k\eta_j x_2) \cdot \exp[ik(x_1 - ct)] \quad (5.11)$$

where  $A_{\Phi j}$  ( $j = 1-8$ ) are unknown amplitudes. The parameters  $\eta_j$ , amplitude ratios  $\beta_{1j}$  and  $\beta_{2j}$  corresponding to each root can be easily solved from the homogeneous linear equations. The solving process is attached at Appendix (A.5.1).

Similarly, for the PM substrate, the complete solution of displacement, electric potential and magnetic potential can be expressed accordingly as:

$$\{u_3^m, \phi^m, \psi^m\} = \left\{ \sum_{j=1}^2 \beta_{3j}^m A_{\Psi j}^m, \sum_{j=3}^3 A_{\Psi j}^m, \sum_{j=1}^2 A_{\Psi j}^m \right\} \exp(-k\eta_j^m x_2) \cdot \exp[ik(x_1 - ct)] \quad (5.12)$$

It is worth noting that only three  $\eta^m$  with negative real parts are retained due to radiation conditions (two of which have negative roots  $\eta^m = -1$ ).  $A_{\Psi j}^m$  ( $j = 1-3$ ) is the undetermined amplitude, and  $\beta_{3j}^m = A_{Uj}^m / A_{\Psi j}^m$  indicates the amplitude ratio (see Appendix (A.5.2)). The solutions for the electric and magnetic potentials in vacuum are:

$$\{\phi^0, \psi^0\} = \left\{ \sum_{j=1}^6 A_{\Phi j}, \sum_{j=7}^8 A_{\Phi j} \right\} \exp(-kx_2) \cdot \exp[ik(x_1 - ct)] \quad (5.13)$$

By substituting the complete solution Eqs. (5.11)-(5.13) of each layer to the corresponding constitutive equation, the physical quantity expressions of each layer can be obtained as shown in Appendix (A.5.3)- (A.5.5).

### 5.2.2 Dispersion equation

By substituting the physical quantity expressions of each layer into the boundary and continuous condition Eqs. (5.5)-(5.7), homogeneous linear equations of unknown amplitudes  $A_{\Phi j}$  ( $j = 1-8$ ) and  $A_{\Psi_j}^m$  ( $j = 1, 2, 3$ ) can be obtained as:

$$[M]N = 0 \quad (5.14)$$

where  $N = \{ A_{\Phi 1}, A_{\Phi 2}, A_{\Phi 3}, A_{\Phi 4}, A_{\Phi 5}, A_{\Phi 6}, A_{\Phi 7}, A_{\Phi 8}, A_{\Psi 1}^m, A_{\Psi 2}^m, A_{\Psi 3}^m \}^T$ , and  $[M]$  is a coefficient matrix of  $11 \times 11$ :

$$\begin{aligned} M_{1,j} &= (c_{44}\beta_{1j} + e_{15})\eta_j e^{-kh\eta_j} \\ M_{2,j} &= e^{-kh\eta_j} - e^{-kh} \\ M_{3,7} &= e^{-kh\eta_7} - e^{-kh}, \quad M_{3,8} = e^{-kh\eta_8} - e^{-kh} \\ M_{4,j} &= (ikce_{15}\beta_{1j} - ikc\varepsilon_{11} + qn_0\mu_{11} + qd_{11}\beta_{2j})\eta_j e^{-kh\eta_j} + ikc\varepsilon_0 e^{-kh} \\ M_{5,j} &= \beta_{2j} e^{-kh\eta_j}, \quad M_{6,j} = \beta_{2j} \\ M_{7,j} &= \beta_{1j}, \quad M_{7,9} = -\beta_{U1}^m, \quad M_{7,10} = -\beta_{U2}^m \\ M_{8,j} &= (c_{44}\beta_{1j} + e_{15})\eta_j, \quad M_{8,9} = -(c_{44}\beta_{U1}^m + f_{15}^m)\eta_1^m, \quad M_{8,10} = -(c_{44}\beta_{U2}^m + f_{15}^m)\eta_2^m \\ M_{9,j} &= 1, \quad M_{9,11} = -1 \\ M_{10,7} &= M_{10,8} = 1, \quad M_{10,9} = M_{10,10} = -1 \\ M_{11,j} &= (ikce_{15}\beta_{1j} - ikc\varepsilon_{11} + qn_0\mu_{11} + qd_{11}\beta_{2j})\eta_j, \quad M_{11,11} = -ikc\varepsilon_{11}^m \end{aligned} \quad (5.15)$$

where  $j = 1-6$ .

For a given wave number  $k$ , the phase velocity  $c$  in the corresponding complex domain can be obtained by solving  $\text{Det}[M] = 0$ . The real part of  $c$  represents the wave speed and the imaginary part denotes the attenuation of the wave amplitude. Since the dispersion equation is trivariate transcendental equation in the complex domain, the traditional dichotomy, Muller's method, perturbation method and Newton iteration method have their own limitations such as leaky roots and they can be time consuming.

In this Chapter, the solution algorithm [84,207] is adopted as previous study. For the wave dispersion equation in complex domain, the local minimal modulus points are found based on the modulus convergence and the null points are distinguished. The dispersion curves are finally determined by the rectangle scanning.

### 5.2.3 Validation of the result

From the coefficient matrix in Eq. (5.15), we can obtain the results of some special cases reported in the literature straightforwardly. In this Section, we discuss three particular cases in absence of rotation accordingly.

#### 5.2.3.1 Love waves in a PS thin film on the elastic half-space

When the considered structure consists of the PS layer over an elastic half-space, that is,  $f_{15} = 0$ , the magnetic related basic equations and boundary conditions Eqs. (5.5)<sub>3</sub> and (5.7)<sub>4</sub> no longer exist. The coefficient matrix in Eq. (5.15) reduces to the following:

$$\begin{aligned}
 M_{1j} &= (c_{44}\beta_{1j} + e_{15})\eta_j \exp(-kh\eta_j), M_{2j} = \exp(-kh\eta_j), M_{2,9} = -\exp(-kh) \\
 M_{3j} &= (ikce_{15}\beta_{1j} - ikc\varepsilon_{11} + qn_0\mu_{11} + qd_{11}\beta_{2j})\eta_j \exp(-kh\eta_j), M_{3,9} = ikc\varepsilon_0 \exp(-kh) \\
 M_{4j} &= \beta_{2j} \exp(-kh\eta_j), M_{5j} = \beta_{2j}, M_{6j} = \beta_{1j}, M_{6,7} = -1 \\
 M_{7j} &= (c_{44}\beta_{1j} + e_{15})\eta_j, M_{7,7} = -c_{44}^m \eta^m, M_{8j} = 1, M_{8,8} = -1 \\
 M_{9j} &= (ikce_{15}\beta_{1j} - ikc\varepsilon_{11} + qn_0\mu_{11} + qd_{11}\beta_{2j})\eta_j, M_{9,8} = -ikc\varepsilon_{11}^m
 \end{aligned} \tag{5.16}$$

where  $j = 1-6$ .  $[M]$  becomes a new coefficient matrix of  $9 \times 9$ , which is the Eq. (27) in [106]. Free from rotation, and by turning the n-type PS into the p-type, the Love waves in PS layer lying over on an elastic half-space can be obtained [106].

#### 5.2.3.2 Interfacial shear horizontal waves in a layered PE/PM structure

Considering a PM half-space with an overlying semi-infinite PE, the semiconductor material properties such as  $n_0$ ,  $\mu_{11}$ , and  $d_{11}$  are zero. At the interface  $x_2 = 0$ , the continuity conditions  $u_3^e = u_3^m$ ,  $\varphi^e = \varphi^m$ ,  $\psi^e = \psi^m$ ,  $T_{23}^e = T_{23}^m$ ,

$D_2^e = D_2^m$ ,  $B_2^e = B_2^m$  should be satisfied, which lead to

$$\begin{aligned}
 \frac{\varepsilon_{11}^e}{e_{15}} A_2 - \frac{\alpha_{11}^m}{f_{15}} A_5 &= 0 \\
 A_1 + A_2 - A_6 &= 0, A_3 - A_4 - A_5 = 0 \\
 e_{15} A_1 + \frac{\varepsilon_{11}^e}{e_{15}} \bar{c}_{44}^e \lambda^e A_2 + f_{15} A_4 + \frac{\alpha_{11}^m}{f_{15}} \bar{c}_{44}^m \lambda^m A_5 &= 0 \\
 \varepsilon_{11}^e A_1 + \varepsilon_{11}^m A_6 &= 0, \alpha_{11}^e A_3 + \alpha_{11}^m A_4 = 0
 \end{aligned} \tag{5.17}$$

where  $\lambda^\beta = \sqrt{1 - (c/c_{sh}^\beta)^2}$  with  $c_{sh}^\beta = \sqrt{\bar{c}_{44}^\beta / \rho^\beta}$ ,  $\beta = e, m$ .  $c_{sh}^e$  and  $c_{sh}^m$  are the velocities of the bulk shear wave in the PE and PM medias, respectively.  $\bar{c}_{44}^e = c_{44}^e + e_{15}^2 / \varepsilon_{11}^e$  and  $\bar{c}_{44}^m = c_{44}^m + f_{15}^2 / \alpha_{11}^m$  are the elastic constants with the PE and PM effects accounted for, respectively.

Hence, the dispersion relations in explicit form can be obtained by Eq. (5.17):

$$\bar{c}_{44}^e \sqrt{1 - \left(\frac{c}{c_{sh}^e}\right)^2} + \bar{c}_{44}^m \sqrt{1 - \left(\frac{c}{c_{sh}^m}\right)^2} = \frac{e_{15}^2}{\epsilon_{11}^e} \cdot \frac{\epsilon_{11}^m}{\epsilon_{11}^e + \epsilon_{11}^m} + \frac{f_{15}^2}{\alpha_{11}^m} \cdot \frac{\alpha_{11}^e}{\alpha_{11}^e + \alpha_{11}^m} \quad (5.18)$$

This expression is exact agreement with the result of Eq. (12) derived in [208]. It determines the SH interfacial waves propagating along the perfect PE and PM interface.

### 5.2.3.3 Love waves in a purely elastic layered half-space

In particular, an isotropic elastic half-space with the other elastic layer is considered. The equations in Eqs. (5.2) and (5.4) can be simplified as:

$$c_{44}^L \nabla^2 u_3^L = \rho^L \frac{\partial^2 u_3^L}{\partial t^2}, \quad c_{44}^H \nabla^2 u_3^H = \rho^H \frac{\partial^2 u_3^H}{\partial t^2} \quad (5.19)$$

where the superscripts  $L$  and  $H$  denote the involved physical quantities corresponding to the layer and half-space medium, respectively. With the  $T_{23}^L(x_1, h) = 0$ ,  $u_3^L(x_1, 0) = u_3^H(x_1, 0)$ ,  $T_{23}^L(x_1, 0) = T_{23}^H(x_1, 0)$ , one obtains:

$$c_{44}^H \sqrt{1 - \left(\frac{c}{c_{sh}^H}\right)^2} - c_{44}^L \sqrt{\left(\frac{c}{c_{sh}^L}\right)^2 - 1} \cdot \tan \left( kh \sqrt{\left(\frac{c}{c_{sh}^L}\right)^2 - 1} \right) = 0 \quad (5.20)$$

where  $c_{sh}^L = \sqrt{c_{44}^L / \rho^L}$ ,  $c_{sh}^H = \sqrt{c_{44}^H / \rho^H}$ . Eq. (5.20) is the well-known classical dispersion equation for a purely elastic layered half-space which validates the Eq. (6.4.29) in [209].

## 5.3 Numerical results and discussions

Consider the PM CoFe<sub>2</sub>O<sub>4</sub> substrate with a covered layer of PS ZnO. The material properties of CoFe<sub>2</sub>O<sub>4</sub> and ZnO are given in Tables 2.2 and 2.1. Furthermore, phase velocity and wave number are normalized, i.e.,  $C = c / c_{SH}^m$  and  $K = kh$ , where  $c_{SH}^m$  represents the velocity of PM substrate shear bulk wave.  $\lambda = \mu_{11} \bar{E}_1 / c_{SH}$  is the normalized horizontal biasing electric field, which is the ratio of electron drift velocity under the biasing electric field to the wave speed of ZnO shear bulk.  $h = 0.1$  mm,  $n_0 = 5 \times 10^{16}$  m<sup>-3</sup> and  $\lambda = 0$  are taken as basic numerical examples, in which  $n_0$  and  $\lambda$  change the value of one at a time according to the need.

### 5.3.1 Effects of rotation on SH waves

Considering the ZnO/CoFe<sub>2</sub>O<sub>4</sub> layered structure described above, the real part (dispersion characteristics) and the imaginary part (attenuation characteristics) of the phase velocity in the presence of rotation and in the absence of biasing electric

field ( $\lambda = 0$ ) are investigated. Figure 5.2 shows the relationship between the real part variation  $\Delta\text{Re}(c)$ , imaginary part variation  $\Delta\text{Im}(c)$  and the normalized wave number at different rotation rates.  $\Delta\text{Re}(c) = \text{Re}(c)(\Omega \neq 0) - \text{Re}(c)(\Omega = 0)$  represents the difference between the real part of phase velocity obtained with rotation and without rotation, while  $\Delta\text{Im}(c) = \text{Im}(c)(\Omega \neq 0) - \text{Im}(c)(\Omega = 0)$  is the difference between imaginary parts. In addition,  $\omega_0$  used to scale angular rate in the figures represents natural frequency, namely [188]:

$$\omega_0 = \frac{\pi}{2h} \sqrt{\frac{c_{44}}{\rho}} \quad (5.21)$$

Since PE gyroscopes are very small (of the order of 10 mm) and are getting miniaturized, their operating frequency  $\omega_0$  is very high. PE gyroscopes are used to measure an angular rate  $\Omega$  much smaller than its operating frequency  $\omega_0$ . Therefore, in the initial data selection of angular rate,  $\Omega = 0.05\omega_0$ ,  $\Omega = 0.1\omega_0$  and  $\Omega = 0.15\omega_0$  were considered by reference to [188,196,197].

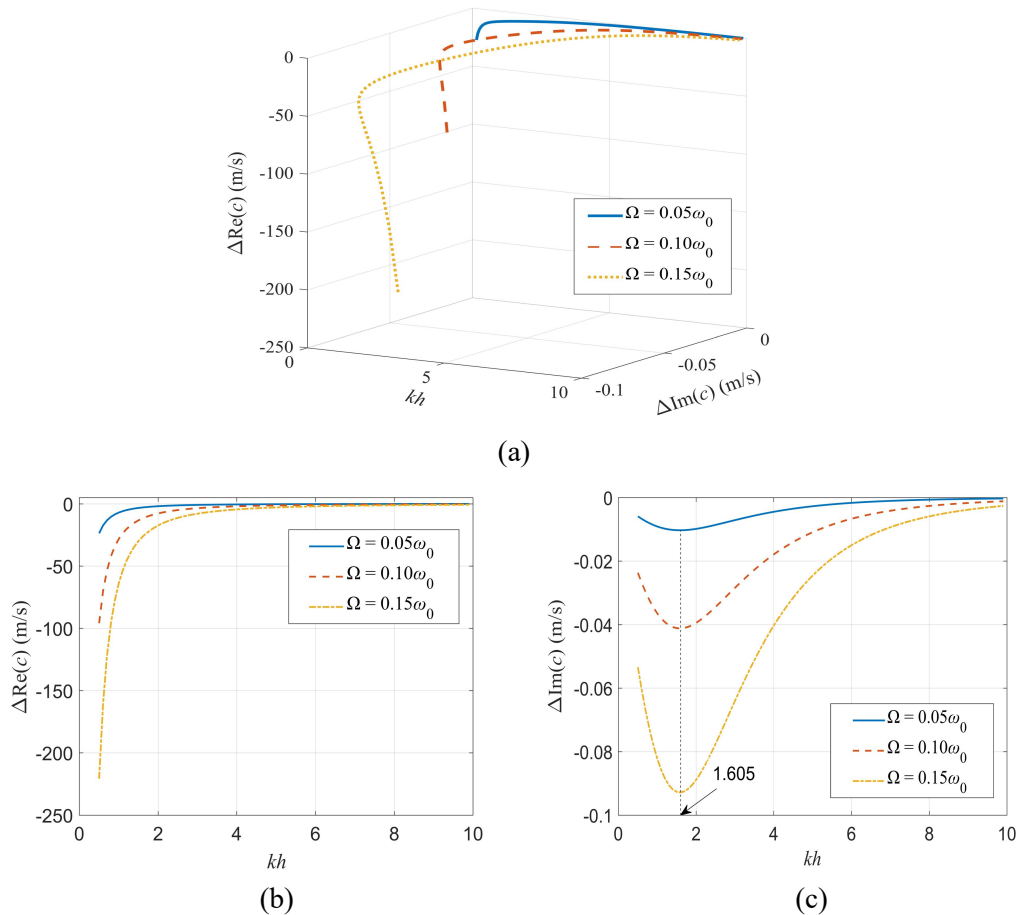


Fig. 5.2 Rotation on SH wave dispersion and attenuation characteristics: (a) the 3D curves, (b) rotation on the variation of wave speed, and (c) rotation on the variation of attenuation

Figure 5.2(a) is the 3D curves of SH waves with rotation. As shown in Figs. 5.

2(b) and 5.2(c), both the real and imaginary parts of SH waves in the composite structure become smaller under the influence of rotation, and the variations increase with  $\Omega$ . When the  $kh$  is small (i.e. long wavelength), both wave speed and attenuation change markedly, and their variations tend to zero with the increase of  $kh$ , indicating that long waves (in the vicinity of the surface) are more sensitive to rotation than short waves (deep into the substrate). Figure 5.2(c) shows that rotation can enhance the attenuation, but the attenuation increases to a maximum firstly and then tends to zero. For different  $\Omega$ , the inflection points of attenuation occur at  $kh = 1.605$ .

Further, we get the relative change of SH waves phase velocity in the rotating field as shown in Fig. 5.3, that is, the relative change of real part  $|\Delta \text{Re}(c)|/\text{Re}(c) = |\text{Re}(c)(\Omega \neq 0) - \text{Re}(c)(\Omega = 0)|/\text{Re}(c)(\Omega = 0)$  and the relative change of imaginary part  $|\Delta \text{Im}(c)|/\text{Im}(c) = |\text{Im}(c)(\Omega \neq 0) - \text{Im}(c)(\Omega = 0)|/\text{Im}(c)(\Omega = 0)$ . Since  $\omega = kc$ , Figure 5.3 is actually relative changes in frequency (relative frequency drift, which will be discussed below).

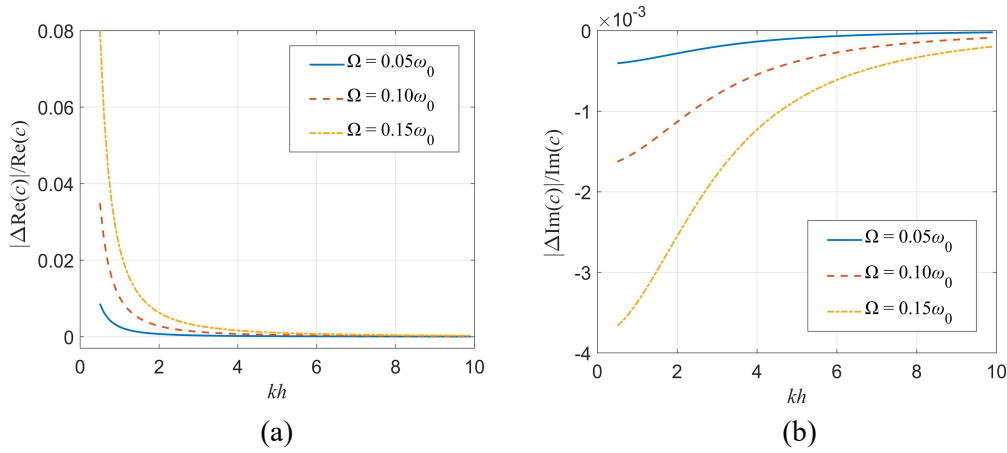


Fig. 5.3 Rotation on the relative variation of SH wave phase velocity: (a) the relative change in the real part of phase velocity, and (b) the relative change in the imaginary part of phase velocity

The trend of the real part in Fig. 5.3(a) is opposite to that in Fig. 5.2(b), where the relative change of the real part decreases as the wave number increases, but also tends to zero. In Fig. 5.3(b), the relative change in the imaginary part is still negative, because the absolute value of the change in the imaginary part is divided by the imaginary part (the attenuation) is still negative.

### 5.3.2 Changes in frequency shift and damping coefficient

The angular velocity will also cause the variation of the frequency of SAWs, namely frequency shift. Figure 5.4 shows the relationship between  $\Delta\omega$  and  $kh$  at different  $\Omega$ . As seen in the Fig. 5.4, the rotation will lead to the decrease of frequency. The greater the angular rate, the greater frequency change will be. The

frequency shifts are relatively large for small wave numbers, and they diminish as  $kh$  becomes increasingly large.

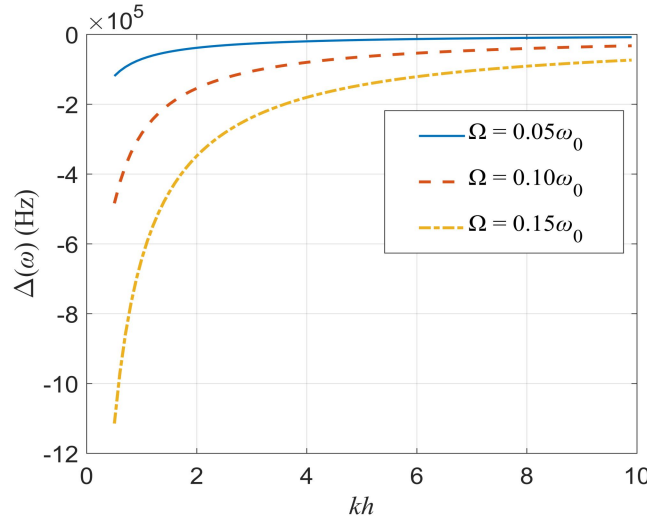


Fig. 5.4 Frequency variation (frequency shift) of SH waves with rotation

In order to better evaluate the attenuation trend, damping coefficient  $\gamma$  is introduced to describe the attenuation of SH waves.  $\gamma$  represents the attenuation value in an oscillation period [210], while the oscillation period  $T$  can be expressed as:

$$T = \frac{2\pi}{k \operatorname{Re}(c)} \quad (5.22)$$

The damping coefficient  $\gamma$  is:

$$\gamma = \exp \left[ -2\pi \frac{\operatorname{Im}(c)}{\operatorname{Re}(c)} \right] = \exp \left( -\frac{2\pi\tilde{\omega}}{\omega} \right) \quad (5.23)$$

When  $\tilde{\omega}$  is negative and  $\gamma$  is greater than 1, SH waves attenuation occurs. The larger the  $\gamma$ , the faster the decay. If  $\tilde{\omega}$  is positive and  $\gamma$  is less than 1, the amplitude of SH waves will increase along the direction of propagation.

Figure 5.5 describes the damping coefficient and its variation of SH waves with rotation. Figure 5.5(a) shows the damping coefficient at different  $\Omega$ . It can be seen that  $\gamma$  is always greater than 1, indicating that the amplitude attenuates in the rotating structure. No matter how big the angular rate is,  $\gamma$  reaches the peak at  $kh = 2.605$  and then decreases continuously. In addition, the damping coefficient curves almost coincide with each other at different  $\Omega$ . It is found through local amplification that the larger  $\Omega$  is, the larger  $\gamma$  will be, but the difference is small.

In order to better observe the difference,  $\Delta\gamma$  at different  $\Omega$  is further obtained in Fig. 5.5(b). It can be found that the  $\Delta\gamma$  also increases with the increase of  $\Omega$ . At the same time the variations are always positive, which indicates that the  $\gamma$  with

rotation is greater than that without rotation. That is, the amplitude will decay faster during rotation, which is consistent with the conclusion shown in Fig. 5.2(c) that  $\Delta \text{Im}(c)$  with rotation is negative.

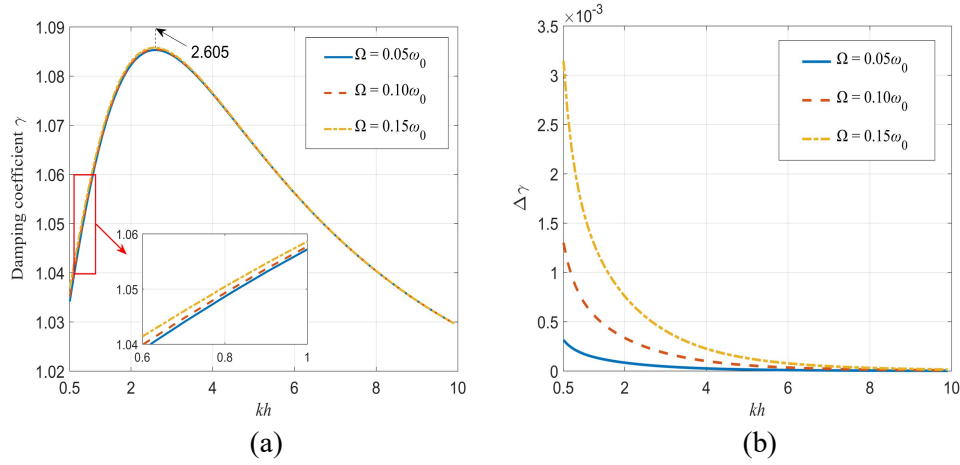


Fig. 5.5 Rotation effects on the damping coefficient and its change of SH waves: (a) damping coefficient at different angular rates, and (b) the change in the damping coefficient at different angular rates

Figure 5.6 shows the direct relationship between the wave speed and the angular rate. It can be seen from the Fig. 5.6 that the wave speed decreases with the increase of  $\Omega$ . This is consistent with the trend of  $\Delta \text{Re}(c)$  with rotation in Fig. 5.2(b). In addition, the wave speed decreases the fastest when  $kh = 3$ . With the increase of  $kh$ , the wave speed becomes larger.

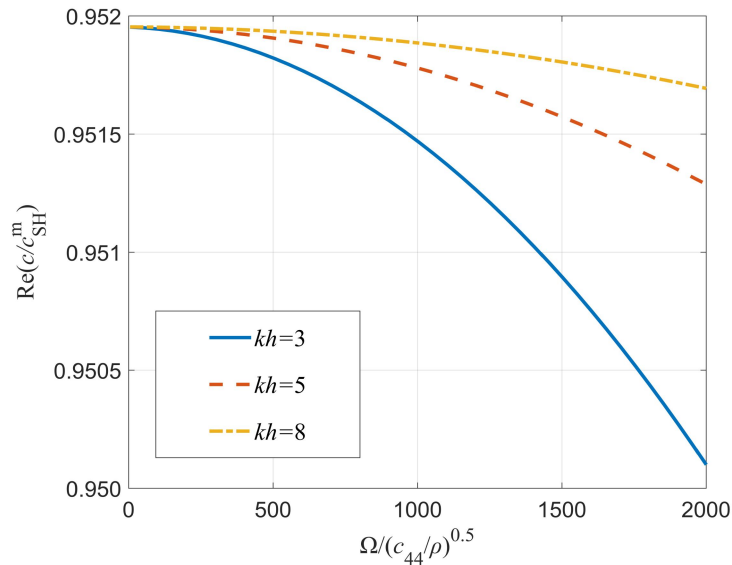


Fig. 5.6 The change of wave speed with angular rate at different  $kh$

### 5.3.3 Effects of initial carrier concentration on SH waves

As an important parameter of semiconductor materials, it is necessary to study the influence of electron doping concentration on SH waves propagation. Hence, we



obtain the influence of  $n_0$  on SH waves and phase velocity variation with rotation and without rotation in Figs. 5.7 and 5.8.

Among them, the two dotted lines in Fig. 5.7(a) represent the case without rotation, i.e.,  $\Omega = 0$ , while the two solid lines denote the case with rotation, i.e.,  $\Omega = 0.1\omega_0$ . In addition, the  $n_0$  corresponding to the two blue lines (colors are visible online) is  $5 \times 10^{16} \text{ m}^{-3}$ , and the  $n_0$  of the two red lines is  $10 \times 10^{16} \text{ m}^{-3}$ . It can be seen in Fig. 5.7(b) that in the two dotted lines without rotation, the wave speed at  $n_0 = 10 \times 10^{16} \text{ m}^{-3}$  is correspondingly less than that when  $n_0 = 5 \times 10^{16} \text{ m}^{-3}$ , which is consistent with the existing results that the carrier concentration reduces the wave speed. In the two solid lines with rotation, the wave speed also decreases as  $n_0$  increases. In addition, for two blue and two red lines with the same  $n_0$ , rotation results in a decrease in wave speed.

In Fig. 5.7(c), the left and right are  $\Omega = 0.1\omega_0$  and  $\Omega = 0$ , respectively. Their trends and values are so close that it is difficult to find the difference, so we need make a better judgment based on the  $\Delta\text{Re}(c)$  and  $\Delta\text{Im}(c)$  in Fig. 5.8.

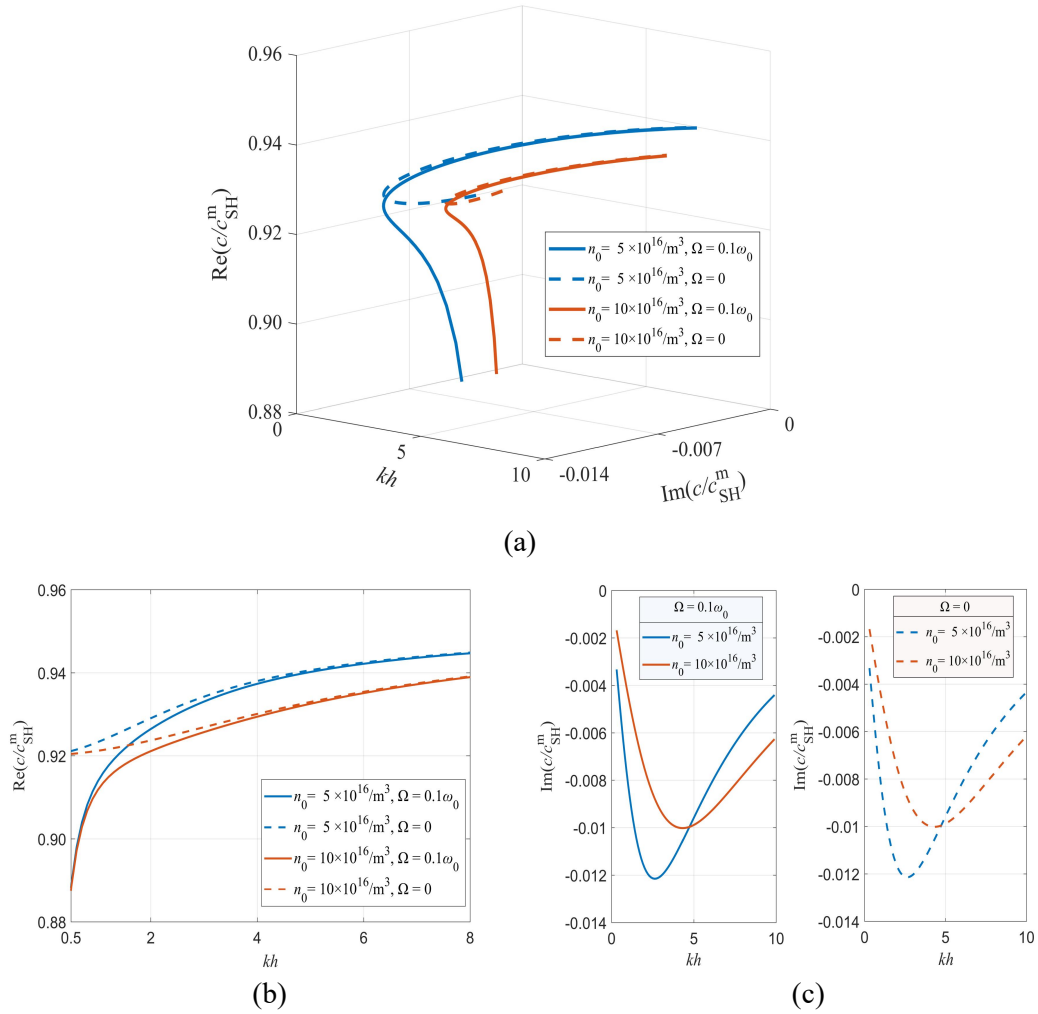


Fig. 5.7 The influence of steady-state carrier concentration  $n_0$  on SH wave propagation characteristics with or without rotation: (a) the 3D curves, (b) wave speed at different  $n_0$ , and (c) attenuation at different  $n_0$

Figure 5.8 shows the effect of three different  $n_0$  on the  $\Delta c$  at  $\Omega = 0.1\omega_0$ . Its variation is corresponding to the real and imaginary parts of the phase velocity with rotation at  $n_0$  minus the real and imaginary parts without rotation. For example, when  $n_0$  is  $3 \times 10^{16} \text{ m}^{-3}$ , the real part variation is  $\Delta \text{Re}(c) = \text{Re}(c)(\Omega = 0.1\omega_0) - \text{Re}(c)(\Omega = 0)$ . Here, the wave speeds with or without rotation are both obtained at  $n_0 = 3 \times 10^{16} \text{ m}^{-3}$ . According to Fig. 5.8(a), the three curves almost coincide, and the change in the real part increases and approaches zero with the increase of  $kh$ . The trends of the two local magnifications are inconsistent, which shows the wave speed variation does not distribute the way it is supposed to be (i.e. the larger  $n_0$  is, the larger or smaller  $\Delta \text{Re}(c)$  is). According to Fig. 5.8(b), the larger  $n_0$  is, the larger  $\Delta \text{Im}(c)$  will be, where the three points marked are the minimum values of the imaginary part variations.

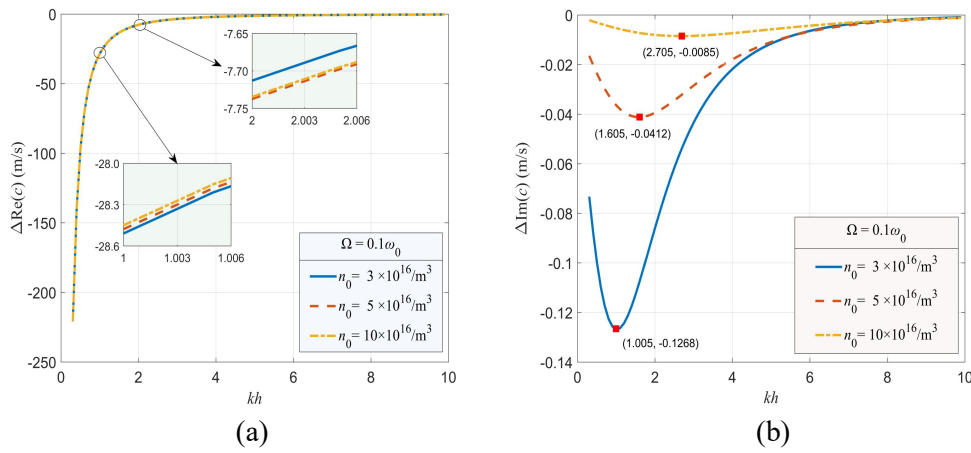


Fig. 5.8 The effect of  $n_0$  on  $\Delta c$  with rotation ( $\Omega = 0.1\omega_0$ ): (a)  $\Delta \text{Re}(c)$  at different  $n_0$ , and (b)  $\Delta \text{Im}(c)$  at different  $n_0$

### 5.3.4 Effects of biasing electric field on SH waves

The gain effect of surface waves in PSs can be achieved by applying an horizontal biasing electric field, which has been demonstrated in previous study. Therefore, Figure 5.9 first verifies the effect of normalized horizontal biasing electric field  $\lambda$  on the SH waves propagation in absence of rotation. Similar to pure PS without PM, the horizontal biasing electric field can also make the surface waves gain in PS and PM composite structure.

The results in Fig. 5.9 are similar to Figs. (3a) and (3b) in [79]. Specifically, Fig. 5.9(a) shows that the wave speed becomes smaller when  $\lambda = 1$  (solid red line), and increases significantly when  $\lambda = 3, 5$ , and  $10$  (all dotted lines). In Fig. 5.9(b), the imaginary part is still negative when  $\lambda = 0.5$  (solid red line), indicating that the wave is still decaying. When  $\lambda = 1$ , all imaginary parts are greater than zero, the SH waves begin to gain, but the gain effect is still not obvious. The gain of the SH waves becomes apparent when  $\lambda$  is 1.5 and 2, respectively.

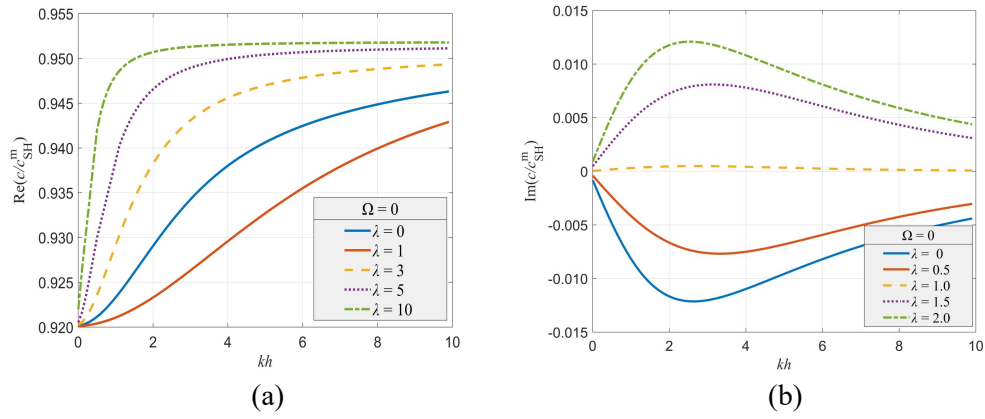


Fig. 5.9 The effect of horizontal biasing electric fields  $\lambda$  on SH waves without rotation ( $\Omega = 0$ ): (a) wave speed at different  $\lambda$  without rotation, and (b) attenuation at different  $\lambda$  without rotation

Figure 5.10 shows the effect of  $\lambda$  on SH waves propagation in the layered structure with rotation ( $\Omega = 0.1\omega_0$ ). Intuitively, Figure 5.10 is very close to Fig. 5.9 when there is no rotation, especially the imaginary part changes in Fig. 5.10(b) are almost the same as those in Fig. 5.9(b). However, the leftmost end of the real part in Fig. 10(a) is significantly inconsistent with Fig. 5.9(a).

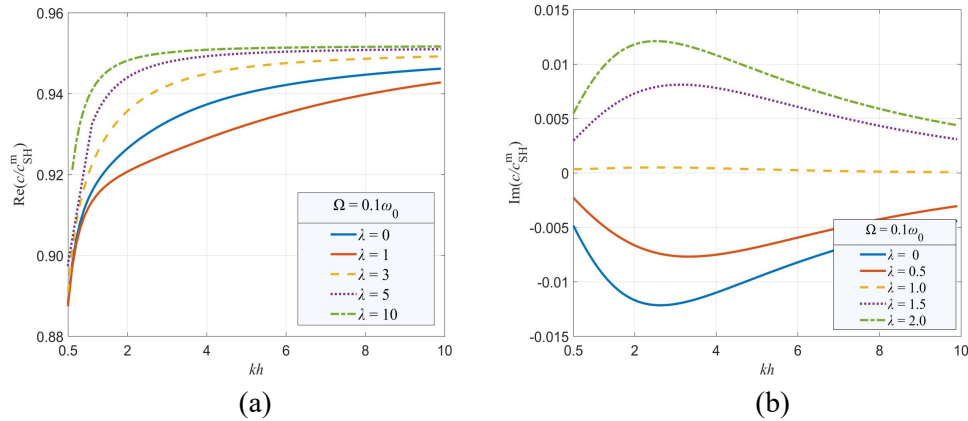


Fig. 5.10 The effect of  $\lambda$  on SH waves with rotation ( $\Omega = 0.1\omega_0$ ): (a) wave speed at different  $\lambda$  with rotation, and (b) attenuation at different  $\lambda$  with rotation

Consistent with the results in Figs. 5.2-5.3 and Figs. 5.7-5.8, the long waves are more sensitive to the rotating field and steady-state carrier concentration than the short waves, and Figs. 5.9-5.10 also show that the long waves are more sensitive to the horizontal biasing electric field.

Further, we obtain Fig. 5.11 which shows the changes of the real and imaginary parts of the phase velocity with rotation under three different  $\lambda$ . Similar to Fig. 5.8, the changes in this case are the corresponding real and imaginary parts of the phase velocity with rotation minus the values without rotation.

The three curves almost coincide in Fig. 5.11(a) which are similar to that in Fig. 5.8(a). The change in the real part increases with the increase of  $\lambda$  when locally amplified. Similarly, Figure 5.11(b) shows that the imaginary part changes also increase with the  $\lambda$ , and all of them eventually approach zero.

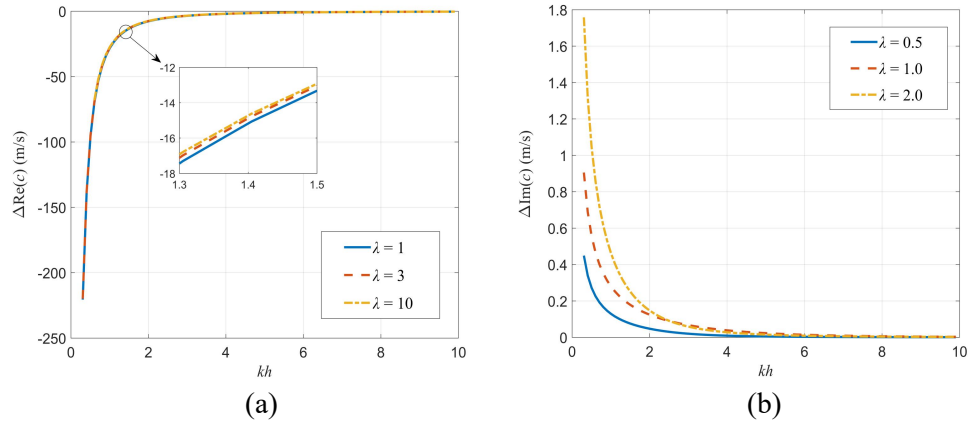


Fig. 5.11 The effect of  $\lambda$  on  $\Delta(c)$  with rotation ( $\Omega = 0.1\omega_0$ ): (a)  $\Delta \text{Re}(c)$  at different  $\lambda$ , and (b)  $\Delta \text{Im}(c)$  at different  $\lambda$

In Fig. 5.12, we can get a similar but not identical result as Fig. (5) in [106]. The influence of horizontal biasing electric field on Love waves in PS/elastic half-space layered structure is obtained in [106], and the result is the first three modes in the absence of rotation. Figure 5.12 show the variation trends of wave speed and attenuation with  $\lambda$  at fixed  $kh = 3$  under different  $\Omega$  (including  $\Omega = 0$ ).

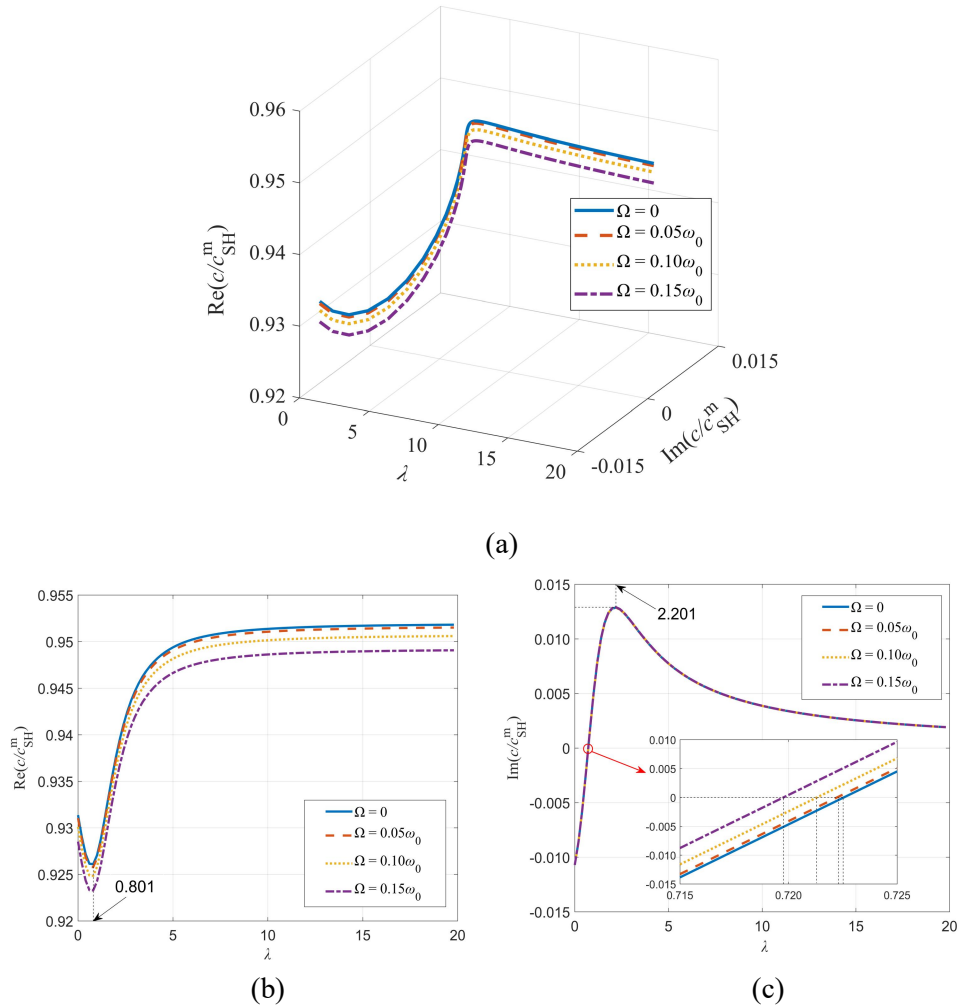


Fig. 5.12 The wave speed and attenuation vary with  $\lambda$  at different angular rates ( $kh = 3$ ): (a) the 3D curves, (b) wave speed varies with  $\lambda$ , and (c) attenuation varies with  $\lambda$

According to Fig. 5.12(b), it can be found that wave speed decreases with the increase of  $\Omega$ , which is consistent with the previous conclusions. The wave speed will decrease as  $\lambda$  increases when  $\lambda$  is relatively small, and the minimum wave speed occurs at  $\lambda = 0.801$ . Then, the wave speed increases sharply with the increase of  $\lambda$  until it becomes stable.

Figure 5.12(c) shows that no matter how big  $\Omega$  is, wave attenuation or gain may occur under the action of  $\lambda$ . The key turning points corresponding to  $\Omega = 0$ ,  $\Omega = 0.05\omega_0$ ,  $\Omega = 0.10\omega_0$  and  $\Omega = 0.15\omega_0$  are  $\lambda = 0.7226$ ,  $0.7223$ ,  $0.7213$  and  $0.7197$ , respectively. When  $\lambda$  are less than the value of these turning points, SH waves will attenuate. Otherwise, gain will occur, and the maximum gain occurs when  $\lambda = 2.201$ . As  $\lambda$  continues to increase, although the imaginary part is still greater than zero, the gain effect of the waves will be weakened.

These results show that the waves gain in rotating MCSs can also be regulated by applying a biasing electric field.

## 5.4 Summary

Based on the multi-field coupling theory with Coriolis force and centrifugal force, this Chapter studies the SH waves in PS/PM layered half-space when rotating around a fixed axis, including the dispersion characteristics, attenuation characteristics and frequency drift of SH waves, and discusses the effects of semiconductor characteristics and horizontal biasing electric field on the MCSs in rotation and non-rotation. Through calculation, the following conclusions are obtained:

(1) The rotating field will cause the waves dispersion and attenuation. With the increase of angular velocity, the wave speed and attenuation will decrease faster. The change of frequency drift and attenuation coefficient will also increase with the increase of  $\Omega$ . The rotation sensitivity of small wave number (long waves) is much greater than that of large wave number (short waves), and all the variations tend to zero with the increase of  $kh$ .

(2) The attenuation coefficient is introduced to better evaluate the attenuation trend, and the results are mutually corroborated with the attenuation characteristic curve. The semiconductor characteristics will cause the wave dispersion and attenuation, and the wave speed will decrease with the increase of steady-state carrier concentration, and the attenuation characteristics will continue to strengthen until the maximum.

(3) No matter whether the composite structure rotates or not, surface waves gain can be achieved by applying an appropriate horizontal biasing electric field. Furthermore, the wave speed will first decrease to the minimum, then increase sharply with the increase of the biasing electric field, and finally become flat at different angular rates. SH wave gain occurs when the normalized horizontal biasing electric field is greater than the critical turning point (the turning point varies slightly with different angular rates), and all the maximum gains occur when the biasing electric field is 2.201.

## 5.5 Appendix

The amplitude ratios  $\beta_{1j}$  and  $\beta_{2j}$  ( $j = 1-6$ ) in Eq. (5.11) and  $\beta_{3j}^m$  ( $j = 1-2$ ) in Eq. (5.12) are respectively:

$$\beta_{1j} = -\frac{e_{15}k^2(\eta_j^2 - 1)}{c_{44}k^2(\eta_j^2 - 1) + \rho k^2 c^2 + \rho \Omega^2}, \quad \beta_{2j} = \frac{k^2(\eta_j^2 - 1)(e_{15}\beta_{1j} - \varepsilon_{11})}{q} \quad (\text{A.5.1})$$

$$\beta_{3j}^m = \frac{A_{\psi j}^m}{A_{\psi j}^m} = -\frac{f_{15}^m k^2(\eta^{m^2} - 1)}{c_{44}^m k^2(\eta^{m^2} - 1) + \rho^m k^2 c^2 + \rho^m \Omega^2} \quad (\text{A.5.2})$$

By substituting the complete solution (5.11)-(5.13) of the wave fields of each layer into the corresponding constitutive equation, the physical quantity expression of each layer can be obtained (the term  $e^{ik(x_1 - ct)}$  is omitted below for ease).

The expressions of stress, electric displacement, magnetic induction intensity and current density of the PS overlay are:

$$\begin{aligned} \{T_{13}, T_{23}\} &= \sum_{j=1}^6 \{ic_{44}\beta_{1j} + ie_{15}, -c_{44}\eta_j\beta_{1j} - e_{15}\eta_j\} kA_{\Phi j} \exp(-k\eta_j x_2) \\ \{D_1, D_2\} &= \sum_{j=1}^6 \{ie_{15}\beta_{1j} - i\varepsilon_{11}, -e_{15}\eta_j\beta_{1j} + \varepsilon_{11}\eta_j\} kA_{\Phi j} \exp(-k\eta_j x_2) \\ \{B_1, B_2\} &= \sum_{j=7}^8 \{-i\alpha_{11}, \alpha_{11}\eta_j\} kA_{\Phi j} \exp(-k\eta_j x_2) \\ \begin{Bmatrix} J_1 \\ J_2 \end{Bmatrix} &= \sum_{j=1}^6 \begin{Bmatrix} -ikn_0\mu_{11} + \mu_{11}\bar{E}_1\beta_{2j} - ikd_{11}\beta_{2j} \\ kn_0\mu_{11}\eta_j + kd_{11}\eta_j\beta_{2j} \end{Bmatrix} qA_{\Phi j} \exp(-k\eta_j x_2) \end{aligned} \quad (\text{A.5.3})$$

The expressions of stress, electric displacement and magnetic induction strength of PM substrate are as follows:

$$\begin{aligned} \{T_{13}^m, T_{23}^m\} &= \sum_{j=1}^2 \{ic_{44}^m\beta_{3j}^m + if_{15}^m, -c_{44}^m\eta_j^m\beta_{3j}^m - f_{15}^m\eta_j^m\} kA_{\psi j}^m \exp(-k\eta_j^m x_2) \\ \{D_1^m, D_2^m\} &= -\{i, 1\} k\varepsilon_{11}^m A_{\psi 3}^m \exp(kx_2) \\ \{B_1^m, B_2^m\} &= \sum_{j=1}^2 \{if_{15}^m\beta_{3j}^m - i\alpha_{11}^m, -f_{15}^m\eta_j^m\beta_{3j}^m + \alpha_{11}^m\eta_j^m\} kA_{\psi j}^m \exp(-k\eta_j^m x_2) \end{aligned} \quad (\text{A.5.4})$$

The electric displacement and magnetic induction in vacuum are:

$$D_2^0 = k\varepsilon_0 \sum_{j=1}^6 A_{\Phi j} \exp(-kx_2), \quad B_2^0 = k\mu_0 \sum_{j=7}^8 A_{\Phi j} \exp(-kx_2) \quad (\text{A.5.5})$$

## Chapter 6

# Multi-field coupling problems in piezomagnetic and piezoelectric semiconductor structures

Compared with PS or piezotronic devices, PE devices are more widely used in electronic technology, and the development is more mature. There are many kinds of PE devices, including PE acoustic resonators (quartz crystal resonators, surface wave resonators, etc.), PE ceramic filters and transformers, PE sensors, PE gyroscopes, hydroacoustic transducers, etc. As the basic electronic component of PE devices, PE bimorphs can realize charge separation and potential difference by applying stress or electric field. For example, PE ceramic bimorphs with opposite polarization direction along the thickness are bent by the electric field along the thickness, which causes one layer to stretch and the other layer to shrink. Or when the extension of the upper and lower layers is not the same, the bimorphs will also bend and produce displacement.

PE bimorphs can be used for lead bonding, electrical switches, valves and acceleration sensors. There are many studies on PE bimorphs: Dong et al. [211] designed a toroidal piezoelectric actuator with twin/single crystal structure, which has a larger displacement output than a conventional actuator of the same size. Cui et al. [212] studied the AlN PE bimorph gyroscope with *C*-axis tilt under thickness shear mode. Michele et al. [213] designed a wearable energy harvester based on PE bimorphs by using finite element and experimental methods. In addition, there are other examples of PE bimorphs in sensors or actuators [214,215]. As far as we know, there is no research report on PS bimorphs. Therefore, this Chapter will analyze PS bimorphs as the basic bending element in piezotronic devices.

On the other hand, based on the PM and PS coupling theory, the researchers conducted a series of studies on the stretching of MCS rods under static

[71,143,144] or time-harmonic [149] magnetic fields. Wang et al. [216] studied the moving charge redistribution of composite beams in pure bending. The composite rods or beams in the above literature are three or five layers, two of which are PM materials, and the whole structure is symmetric about the geometry. Therefore, stretching and bending can be generated separately in these models without considering the coupling between them. In fact, due to the existence of multiple contact interfaces, these structures can produce undesirable effects such as interface damping. One of the effective ways to overcome interface damping is to reduce the number of layers and interfaces by using only one PM layer in a MCS composite beam, which is often referred to as a PE unimorph in the case of containing PE materials [211,217]. The single crystal structure is asymmetrical with respect to the geometry of the mid-plane, so that bending and stretching in the mid-plane occur simultaneously and are usually coupled. Obviously, this requires more complex theoretical models.

Therefore, in this Chapter, we first consider the bending problem of PS bilayer thin plate (bimorph), derive 2D plate equation from the 3D theory of PSs, then degenerate to the plate theory without shear deformation, and obtain the parameters used to characterize the moving charge and load in pure bending. The bending characteristics of bimorph under uniform load are obtained by using theoretical analytical solution and numerical solution of finite element. The influence of local self-equilibrated load on moving charge and potential barrier/potential well is also considered. Secondly, an 1D model of the flexural and tensile coupling PS/PM single crystal beam is established, and the magnetic field driven carrier redistribution in the unimorph beam is analyzed. The analytical solutions of the mechanical field, electric field and carrier concentration are derived. The influence of magnetic field and initial carrier concentration on the electromechanical fields is also discussed.

## **6.1 Interaction between bending and mobile charges in a piezoelectric semiconductor bilayer plate (bimorph)**

Here, we mainly consider the bending characteristics of bilayer PS plates with opposite c-axis. 2D plate equations including bending, electric polarization and mobile charge are derived from the macroscopic theory of 3D PSs. In the case of pure bending, a combination of physical and geometric parameters is identified which characterizes the strength of the interaction between the mechanical load and the distribution of mobile charges. In the bending of a rectangular plate under a distributed transverse mechanical load, the deflection, electric potential and carrier concentration are obtained by using the series solution of 2D plate theory and the 3D finite element numerical solution of COMSOL, respectively. The results show that the relative error of the two methods is small. In particular, when the mechanical load is local and self-balanced, the induced carrier distributions and potential barriers/wells are also localized near the loading area. The results



are fundamentally useful for mechanically manipulating mobile charges in PS devices.

### 6.1.1 Zero- and first-order plate equations for a bimorph

Consider the PS bimorph in Fig. 6.1 of crystals of class (6mm) which includes the widely used ZnO as a special case. The  $c$ -axis of the crystal is along  $x_3$  in the upper layer and opposite to  $x_3$  in the lower layer. The reversal of the  $c$ -axis causes a change of sign of the piezoelectric constants.

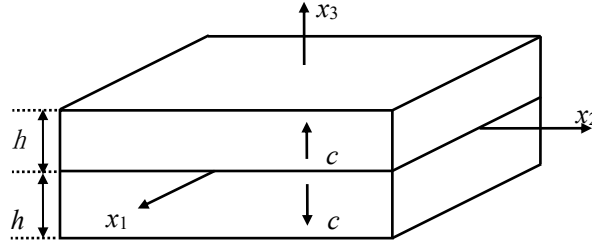


Fig. 6.1 A two-layer piezoelectric semiconductor plate (bimorph) with opposite  $c$ -axis

We derive a set of first-order plate equations first and reduce them to the special case of bending without shear deformation. For a first-order plate theory, following the notation of R.D. Mindlin [218], the displacement  $u$ , electric potential  $\varphi$ , and carrier concentration perturbations  $\Delta p$  and  $\Delta n$  are approximated by:

$$\begin{aligned} u_i(\mathbf{x}, t) &\cong u_i^{(0)}(x_1, x_2, t) + x_3 u_i^{(1)}(x_1, x_2, t) + x_3^2 u_i^{(2)}(x_1, x_2, t) \\ \varphi(\mathbf{x}, t) &\cong \varphi^{(0)}(x_1, x_2, t) + x_3 \varphi^{(1)}(x_1, x_2, t) \\ \Delta p(\mathbf{x}, t) &\cong p^{(0)}(x_1, x_2, t) + x_3 p^{(1)}(x_1, x_2, t) \\ \Delta n(\mathbf{x}, t) &\cong n^{(0)}(x_1, x_2, t) + x_3 n^{(1)}(x_1, x_2, t) \end{aligned} \quad (6.1)$$

where  $i = 1, 2, 3$ ,  $u_i^{(0)}$  and  $u_2^{(0)}$  describe in-plane extensional motion.  $u_3^{(0)}$  is the flexural displacement.  $u_1^{(1)}$  and  $u_2^{(1)}$  are the shear deformations accompanying flexure.  $u_3^{(1)}$  and  $u_3^{(2)}$  describe Poisson's effects in extension and bending, which will be eliminated later along with  $u_1^{(2)}$  and  $u_2^{(2)}$ .  $\varphi^{(0)}$  and  $\varphi^{(1)}$  describe electric potentials uniform or linearly varying along the plate thickness. They are consistent with the displacement approximations of a first-order plate theory with mechanical fields uniform or varying linearly along the plate thickness. Similarly,  $p^{(0)}$  and  $p^{(1)}$  or  $n^{(0)}$  and  $n^{(1)}$  describe charge distributions uniform or linearly varying along the plate thickness. The first-order plate equations to be derived are for  $u_i^{(0)}$ ,  $u_1^{(1)}$ ,  $u_2^{(1)}$ ,  $\varphi^{(0)}$ ,  $\varphi^{(1)}$ ,  $p^{(0)}$ ,  $p^{(1)}$ ,  $n^{(0)}$  and  $n^{(1)}$ .

#### 6.1.1.1 Bilayer plate strains and gradients

Eq (6.1) leads to the following zero- and first-order plate strains, electric fields (potential gradients), and carrier perturbation gradients:

$$\begin{aligned}
S_{11}^{(0)} &= u_{1,1}^{(0)}, \quad S_{22}^{(0)} = u_{2,2}^{(0)}, \quad S_{33}^{(0)} = u_3^{(1)} \\
2S_{23}^{(0)} &= u_{3,2}^{(0)} + u_2^{(1)}, \quad 2S_{31}^{(0)} = u_{3,1}^{(0)} + u_1^{(1)}, \quad 2S_{12}^{(0)} = u_{1,2}^{(0)} + u_{2,1}^{(0)} \\
S_{11}^{(1)} &= u_{1,1}^{(1)}, \quad S_{22}^{(1)} = u_{2,2}^{(1)}, \quad S_{33}^{(1)} = 2u_3^{(2)} \\
2S_{23}^{(1)} &= u_{3,2}^{(1)} + 2u_2^{(2)}, \quad 2S_{31}^{(1)} = u_{3,1}^{(1)} + 2u_1^{(2)}, \quad 2S_{12}^{(1)} = u_{1,2}^{(1)} + u_{2,1}^{(1)} \\
E_1^{(0)} &= -\varphi_{,1}^{(0)}, \quad E_2^{(0)} = -\varphi_{,2}^{(0)}, \quad E_3^{(0)} = -\varphi^{(1)} \\
E_1^{(1)} &= -\varphi_{,1}^{(1)}, \quad E_2^{(1)} = -\varphi_{,2}^{(1)}, \quad E_3^{(1)} = -2\varphi^{(2)} \cong 0 \\
P_1^{(0)} &= p_{,1}^{(0)}, \quad P_2^{(0)} = p_{,2}^{(0)}, \quad P_3^{(0)} = p^{(1)} \\
P_1^{(1)} &= p_{,1}^{(1)}, \quad P_2^{(1)} = p_{,2}^{(1)}, \quad P_3^{(1)} = 2p^{(2)} \cong 0 \\
N_1^{(0)} &= n_{,1}^{(0)}, \quad N_2^{(0)} = n_{,2}^{(0)}, \quad N_3^{(0)} = n^{(1)} \\
N_1^{(1)} &= n_{,1}^{(1)}, \quad N_2^{(1)} = n_{,2}^{(1)}, \quad N_3^{(1)} = 2n^{(2)} \cong 0
\end{aligned} \tag{6.2}$$

### 6.1.1.2 Bilayer plate field equations

Integrating the field equations in PSs as well as their products with  $x_3$  along the plate thickness, we obtain the following zero- and first-order plate field equations:

$$\begin{aligned}
T_{ab,a}^{(0)} + t_b^{(0)} &= 2h\rho\ddot{u}_b^{(0)} \\
T_{a3,a}^{(0)} + t_3^{(0)} &= 2h\rho\ddot{u}_3^{(0)}, \quad T_{ab,a}^{(1)} - T_{3b}^{(0)} + t_b^{(1)} = \frac{2h^3\rho}{3}\ddot{u}_b^{(1)} \\
D_{a,a}^{(0)} + d^{(0)} &= 2hq(p^{(0)} - n^{(0)}) \\
D_{a,a}^{(1)} - D_3^{(0)} + d^{(1)} &= \frac{2h^3}{3}q(p^{(1)} - n^{(1)}) \\
J_{a,a}^{p(0)} + j^{p(0)} &= -2hq\dot{p}^{(0)}, \quad J_{a,a}^{p(1)} - J_3^{p(0)} + j^{p(1)} = -\frac{2h^3}{3}q\dot{p}^{(1)} \\
J_{a,a}^{n(0)} + j^{n(0)} &= 2hq\dot{n}^{(0)}, \quad J_{a,a}^{n(1)} - J_3^{n(0)} + j^{n(1)} = \frac{2h^3}{3}q\dot{n}^{(1)}
\end{aligned} \tag{6.3}$$

where we have introduced a 2D indicial notation that  $a$  and  $b$  assume 1 and 2 but not 3. Eq. (6.3)<sub>1</sub> is for extension. Eq. (6.3)<sub>2</sub> are for flexure with shear deformations. In Eq. (6.3), the plate electromechanical resultants and moments are defined by the following integration along the plate thickness:

$$\{T_{ij}^{(m)}, D_i^{(m)}, J_i^{p(m)}, J_i^{n(m)}\} = \int_{-h}^h x_3^m \{T_{ij}, D_i, J_i^p, J_i^n\} dx_3, \quad m=0,1 \tag{6.4}$$

The electromechanical loads at the plate top and bottom surfaces are described by:

$$\{t_j^{(m)}, d^{(m)}, j^{p(m)}, j^{n(m)}\} = [x_3^m T_{3j}, x_3^m D_3, x_3^m J_3^p, x_3^m J_3^n]_h, \quad m=0,1 \tag{6.5}$$

### 6.1.1.3 Bilayer plate constitutive relations

For crystals of class (6mm), combined with the material constants polarized along the  $x_3$ -axis, we can obtain the following constitutive relationships:

$$\begin{aligned}
T_1 &= c_{11}S_1 + c_{12}S_2 + c_{13}S_3 - e_{31}E_3, \quad T_2 = c_{12}S_1 + c_{11}S_2 + c_{13}S_3 - e_{31}E_3 \\
T_3 &= c_{13}S_1 + c_{13}S_2 + c_{33}S_3 - e_{33}E_3, \quad T_4 = c_{55}S_4 - e_{15}E_2 \\
T_5 &= c_{55}S_5 - e_{15}E_1, \quad T_6 = c_{66}S_6 \\
D_1 &= e_{15}S_5 + \varepsilon_{11}E_1, \quad D_2 = e_{15}S_4 + \varepsilon_{11}E_2 \\
D_3 &= e_{31}(S_1 + S_2) + e_{33}S_3 + \varepsilon_{33}E_3
\end{aligned} \tag{6.6}$$

For thin plates we set  $T_3 = c_{13}S_1 + c_{13}S_2 + c_{33}S_3 - e_{33}E_3 = 0$ . We then solve for  $S_3$  from Eq. (6.6) to obtain:

$$\begin{aligned}
T_1 &= \bar{c}_{11}S_1 + \bar{c}_{12}S_2 - \bar{e}_{31}E_3, \quad T_2 = \bar{c}_{12}S_1 + \bar{c}_{11}S_2 - \bar{e}_{31}E_3 \\
T_3 &= 0, \quad T_4 = c_{55}S_4 - e_{15}E_2 \\
T_5 &= c_{55}S_5 - e_{15}E_1, \quad T_6 = c_{66}S_6 \\
D_1 &= e_{15}S_5 + \varepsilon_{11}E_1, \quad D_2 = e_{15}S_4 + \varepsilon_{11}E_2 \\
D_3 &= \bar{e}_{31}(S_1 + S_2) + \bar{\varepsilon}_{33}E_3
\end{aligned} \tag{6.7}$$

where

$$\begin{aligned}
\bar{c}_{11} &= c_{11} - (c_{13})^2 / c_{33}, \quad \bar{c}_{12} = c_{12} - (c_{13})^2 / c_{33} \\
\bar{e}_{31} &= e_{31} - e_{33}c_{13} / c_{33}, \quad \bar{\varepsilon}_{33} = \varepsilon_{33} + e_{33}^2 / c_{33}
\end{aligned} \tag{6.8}$$

The substitution of Eqs. (6.7), (6.8) and Eq. (6.2) into Eq. (6.4) leads to the following plate constitutive relations:

$$\begin{aligned}
T_{11}^{(0)} &= 2h(\bar{c}_{11}u_{1,1}^{(0)} + \bar{c}_{12}u_{2,2}^{(0)}), \quad T_{22}^{(0)} = 2h(\bar{c}_{12}u_{1,1}^{(0)} + \bar{c}_{11}u_{2,2}^{(0)}) \\
T_{12}^{(0)} &= 2hc_{66}(u_{1,2}^{(0)} + u_{2,1}^{(0)}), \quad T_{3a}^{(0)} = 2hc_{44}(u_{3,a}^{(0)} + u_a^{(1)}) + h^2e_{15}\varphi_{,a}^{(1)} \\
T_{11}^{(1)} &= \frac{2}{3}h^3(\bar{c}_{11}u_{1,1}^{(1)} + \bar{c}_{12}u_{2,2}^{(1)}) + h^2\bar{e}_{31}\varphi^{(1)}, \quad T_{22}^{(1)} = \frac{2}{3}h^3(\bar{c}_{12}u_{1,1}^{(1)} + \bar{c}_{11}u_{2,2}^{(1)}) + h^2\bar{e}_{31}\varphi^{(1)} \\
T_{12}^{(1)} &= \frac{2}{3}h^3c_{66}(u_{1,2}^{(1)} + u_{2,1}^{(1)})
\end{aligned} \tag{6.9}$$

$$\begin{aligned}
D_a^{(0)} &= h^2e_{15}(u_{3,a}^{(1)} + 2u_a^{(2)}) - 2h\varepsilon_{11}\varphi_{,a}^{(0)} \cong -2h\varepsilon_{11}\varphi_{,a}^{(0)} \\
D_3^{(0)} &= h^2\bar{e}_{31}(u_{1,1}^{(1)} + u_{2,2}^{(1)}) - 2h\bar{\varepsilon}_{33}\varphi^{(1)} \\
D_a^{(1)} &= h^2e_{15}(u_{3,a}^{(0)} + u_a^{(1)}) - \frac{2}{3}h^3\varepsilon_{11}\varphi_{,a}^{(1)}
\end{aligned} \tag{6.10}$$

$$\begin{aligned}
J_a^{p(0)} &= 2hq(-p_0\mu_{11}^p\varphi_{,a}^{(0)} - D_{11}^p p_{,a}^{(0)}), \quad J_3^{p(0)} = 2hq(-p_0\mu_{33}^p\varphi^{(0)} - D_{33}^p p^{(0)}) \\
J_a^{p(1)} &= \frac{2}{3}h^3q(-p_0\mu_{11}^p\varphi_{,a}^{(1)} - D_{11}^p p_{,a}^{(1)}) \\
J_a^{n(0)} &= 2hq(-n_0\mu_{11}^n\varphi_{,a}^{(0)} + D_{11}^n n_{,a}^{(0)}), \quad J_3^{n(0)} = 2hq(-n_0\mu_{33}^n\varphi^{(0)} + D_{33}^n n^{(0)}) \\
J_a^{n(1)} &= \frac{2}{3}h^3q(-n_0\mu_{11}^n\varphi_{,a}^{(1)} + D_{11}^n n_{,a}^{(1)})
\end{aligned} \tag{6.11}$$

The substitution of Eqs. (6.9)-(6.11) into Eq. (6.3) yields eleven equations for  $u_i^{(0)}$ ,  $u_a^{(1)}$ ,  $\varphi^{(0)}$ ,  $\varphi^{(1)}$ ,  $p^{(0)}$ ,  $p^{(1)}$ ,  $n^{(0)}$  and  $n^{(1)}$ . They split into two uncoupled groups.

One is for extension with  $u_a^{(0)}$ ,  $\varphi^{(0)}$ ,  $p^{(0)}$  and  $n^{(0)}$ . The other is for flexure with shear deformations described by  $u_3^{(0)}$ ,  $u_a^{(1)}$ ,  $\varphi^{(1)}$ ,  $p^{(1)}$  and  $n^{(1)}$ . We are interested in the latter.

### 6.1.2 Plate theory without shear deformations

The equations for flexure with shear deformations in the previous Section is a Reissner-Mindlin type plate theory with shear deformations described by  $u_a^{(1)}$ . Next we reduce it to a Kirchhoff type plate theory without shear deformations. First we neglect the rotatory inertia described by  $2h^3/3$  on the right-hand side of Eq. (6.3)<sub>3</sub> to obtain the shear force-moment relations as:

$$T_{3b}^{(0)} = T_{ab,a}^{(1)} + t_b^{(1)} \quad (6.12)$$

which serves as the constitutive relations for the transverse shear force  $T_{3b}^{(0)}$ . The substitution of Eq. (6.12) into Eq. (6.3)<sub>1</sub> gives the following Kirchhoff bending:

$$T_{ab,ab}^{(1)} + t_{b,b}^{(1)} + t_3^{(0)} = 2h\rho\ddot{u}_3^{(0)} \quad (6.13)$$

Then we set the plate shear strains  $S_{23}^{(0)}$  and  $S_{31}^{(0)}$  to zero. This implies, from Eq. (6.2)<sub>2</sub> to obtain:

$$u_a^{(1)} = -u_{3,a}^{(0)} \quad (6.14)$$

which is used to eliminate  $u_a^{(1)}$  in the constitutive relations in Eqs. (6.9)<sub>3</sub>, (6.9)<sub>4</sub> and (6.10). We gather the relevant constitutive relations for Kirchhoff bending as:

$$\begin{aligned} M_{11} &= T_{11}^{(1)} = \frac{2}{3}h^3(-\bar{c}_{11}u_{3,11}^{(0)} - \bar{c}_{12}u_{3,22}^{(0)}) + h^2\bar{e}_{31}\varphi^{(1)} \\ M_{22} &= T_{22}^{(1)} = \frac{2}{3}h^3(-\bar{c}_{12}u_{3,11}^{(0)} - \bar{c}_{11}u_{3,22}^{(0)}) + h^2\bar{e}_{31}\varphi^{(1)} \\ M_{12} &= T_{12}^{(1)} = \frac{2}{3}h^3c_{66}(-2u_{3,12}^{(0)}) = M_{21} \end{aligned} \quad (6.15)$$

$$\begin{aligned} D_3^{(0)} &= h^2\bar{e}_{31}(-u_{3,aa}^{(0)}) - 2h\bar{e}_{33}\varphi^{(1)} \\ D_a^{(1)} &= -\frac{2}{3}h^3\varepsilon_{11}\varphi_{,a}^{(1)} \end{aligned} \quad (6.16)$$

$$\begin{aligned} J_3^{p(0)} &= 2hq(-p_0\mu_{33}^p\varphi^{(1)} - D_{33}^p p^{(1)}) \\ J_a^{p(1)} &= \frac{2}{3}h^3q(-p_0\mu_{11}^p\varphi_{,a}^{(1)} - D_{11}^p p_{,a}^{(1)}) \\ J_3^{n(0)} &= 2hq(-n_0\mu_{33}^n\varphi^{(1)} + D_{33}^n n^{(1)}) \\ J_a^{n(1)} &= \frac{2}{3}h^3q(-n_0\mu_{11}^n\varphi_{,a}^{(1)} + D_{11}^n n_{,a}^{(1)}) \end{aligned} \quad (6.17)$$

The relevant field equations of electrical origins are:

$$\begin{aligned}
 D_{a,a}^{(1)} - D_3^{(0)} + d^{(1)} &= \frac{2h^3}{3} q(p^{(1)} - n^{(1)}) \\
 J_{a,a}^{p(1)} - J_3^{p(0)} + j^{p(1)} &= -\frac{2h^3}{3} q\dot{p}^{(1)} \\
 J_{a,a}^{n(1)} - J_3^{n(0)} + j^{n(1)} &= \frac{2h^3}{3} q\dot{n}^{(1)}
 \end{aligned} \tag{6.18}$$

With successive substitutions from Eqs. (6.15)-(6.17) and Eq. (6.12), we can write Eq. (6.13) and Eq. (6.18) as four equations for  $u_3^{(0)}$ ,  $\varphi^{(1)}$ ,  $p^{(1)}$  and  $n^{(1)}$ . At the boundary of a plate with an in-plane unit exterior normal  $\mathbf{n}$  and an in-plane unit tangent  $\mathbf{s}$ , the usual boundary conditions may be:

$$\begin{aligned}
 T_{n3}^{(0)} + T_{ns,s}^{(1)} \quad \text{or} \quad u_3^{(0)} \\
 T_{nn}^{(1)} \quad \text{or} \quad u_{3,n}^{(0)} \\
 D_n^{(1)} \quad \text{or} \quad \varphi^{(1)} \\
 J_n^{p(1)} \quad \text{or} \quad p^{(1)}, \quad J_n^{n(1)} \quad \text{or} \quad n^{(1)}
 \end{aligned} \tag{6.19}$$

where Eq. (6.19)<sub>1</sub> represents the transverse shear force or deflection, Eq. (6.19)<sub>2</sub> the bending moment or rotation, Eq. (6.19)<sub>3</sub> the charge or electric potential, and Eq. (6.19)<sub>4</sub> the current densities or carrier concentrations.

### 6.1.3 Pure bending

Consider the static bending of a plate with an arbitrary and smooth boundary curve without corners. It is under a uniform bending moment  $M_0$  only per unit length along the boundary. There are no charges and currents at the boundary. The boundary conditions are:

$$T_{n3}^{(0)} + M_{ns,s} = 0, \quad M_{nn} = M_0, \quad D_n^{(1)} = 0, \quad J_n^{p(1)} = 0, \quad J_n^{n(1)} = 0 \tag{6.20}$$

In this case, from Eqs. (6.13) and (6.18), the governing equations are:

$$\begin{aligned}
 T_{ab,ab}^{(1)} &= 0, \\
 D_{1,1}^{(1)} + D_{2,2}^{(1)} - D_3^{(0)} &= \frac{2h^3}{3} q(p^{(1)} - n^{(1)}) \\
 J_{1,1}^{p(1)} + J_{2,2}^{p(1)} - J_3^{p(0)} &= 0, \quad J_{1,1}^{n(1)} + J_{2,2}^{n(1)} - J_3^{n(0)} = 0
 \end{aligned} \tag{6.21}$$

It can be verified that a pure-bending solution with constant  $u_{3,11}^{(0)}$ ,  $u_{3,22}^{(0)}$ ,  $u_{3,12}^{(0)}$ ,  $\varphi^{(1)}$ ,  $p^{(1)}$ ,  $n^{(1)}$  and hence constant  $T_{ab}^{(1)}$  satisfies Eq. (6.20) and Eq. (6.21). We are mainly interested in the bending induced potential and charge carrier distributions which are found to be:

$$\begin{aligned}
\varphi^{(1)} &= \frac{3\bar{\epsilon}_{31}}{2h^2\bar{\epsilon}_{33}(\bar{c}_{11} + \bar{c}_{12}) \left(1 + \frac{3}{2}k^2 + \frac{h^2}{3\lambda_D^2}\right)} M_0 \\
p^{(1)} &= -\frac{qP_0}{k_B T} \varphi^{(1)}, \quad n^{(1)} = \frac{qn_0}{k_B T} \varphi^{(1)} \\
k^2 &= \frac{\bar{\epsilon}_{31}^2}{\bar{\epsilon}_{33}(\bar{c}_{11} + \bar{c}_{12})}, \quad \lambda_D^2 = \frac{\bar{\epsilon}_{33}k_B T}{q^2(p_0 + n_0)}
\end{aligned} \tag{6.22}$$

where  $k$  is an electromechanical coupling coefficient and  $\lambda_D$  is the Debye-Hückel length. From Eq. (6.22) we calculate the charge in the upper half of the plate per unit area as:

$$\begin{aligned}
Q^e &= \int_0^h q(\Delta p - \Delta n) dx_3 = \int_0^h qx_3(p^{(1)} - n^{(1)}) dx_3 = \gamma M_0 \\
\gamma &= -\frac{9\bar{\epsilon}_{31}}{4(\bar{c}_{11} + \bar{c}_{12})} \frac{1}{h^2 + \left(1 + \frac{3}{2}k^2\right) 3\lambda_D^2}
\end{aligned} \tag{6.23}$$

$\gamma$  describes the charge produced by unit  $M_0$  and can be used as a measure of the strength of the interaction between the mechanical load and mobile charges in pure bending. It depends on the structural parameters of the plate only.

For numerical results we consider ZnO and AlN. The main parameters of ZnO have been given in Table 2.1 of Chapter 2, but the calculation in this Chapter will involve more elastic, piezoelectric and dielectric constants, so it is completely listed again after referring to literature [19]:

$$[c_{pq}] = \begin{pmatrix} 20.97 & 12.11 & 10.51 & 0 & 0 & 0 \\ 12.11 & 20.97 & 10.51 & 0 & 0 & 0 \\ 10.51 & 10.51 & 21.09 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.247 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.247 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.43 \end{pmatrix} \times 10^{10} \text{N/m}^2 \tag{6.24a}$$

$$[e_{ip}] = \begin{pmatrix} 0 & 0 & 0 & 0 & -0.48 & 0 \\ 0 & 0 & 0 & -0.48 & 0 & 0 \\ -0.573 & -0.573 & 1.32 & 0 & 0 & 0 \end{pmatrix} \times \text{C/m}^2 \tag{6.24b}$$

$$[\epsilon_{ij}] = \begin{pmatrix} 7.57 & 0 & 0 \\ 0 & 7.57 & 0 \\ 0 & 0 & 9.03 \end{pmatrix} \times 10^{-11} \text{F/m} \tag{6.24c}$$

The density of ZnO [19] is  $\rho = 5680 \text{ kg/m}^3$ , while the density of AlN [219] is  $\rho = 3260 \text{ kg/m}^3$ . For AlN, the complete elastic, piezoelectric and dielectric constant are [219]:

$$[c_{pq}] = \begin{pmatrix} 34.5 & 12.5 & 12 & 0 & 0 & 0 \\ 12.5 & 34.5 & 12 & 0 & 0 & 0 \\ 12 & 12 & 39.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 11.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 \end{pmatrix} \times 10^{10} \text{N/m}^2 \quad (6.25a)$$

$$[e_{ip}] = \begin{pmatrix} 0 & 0 & 0 & 0 & -0.48 & 0 \\ 0 & 0 & 0 & -0.48 & 0 & 0 \\ -0.58 & -0.58 & 1.55 & 0 & 0 & 0 \end{pmatrix} \times \text{C/m}^2 \quad (6.25b)$$

$$[\varepsilon_{ij}] = \begin{pmatrix} 8.0 & 0 & 0 \\ 0 & 8.0 & 0 \\ 0 & 0 & 9.5 \end{pmatrix} \times 10^{-11} \text{F/m} \quad (6.25c)$$

Therefore, considering the thickness of single-layer PS plate  $h = 50 \text{ nm}$  and the initial carrier concentration  $p_0 = n_0 = 10^{23} / \text{m}^3$ , the relationship curves between  $\gamma$  and  $\lambda_D$ ,  $\gamma$  and  $h$  can be obtained according to Eq. (6.23). It is easy to see from Eq. (6.23) that they are monotonically, and the obtained Fig. 6.2 also conforms to the trend of monotonically decreasing.

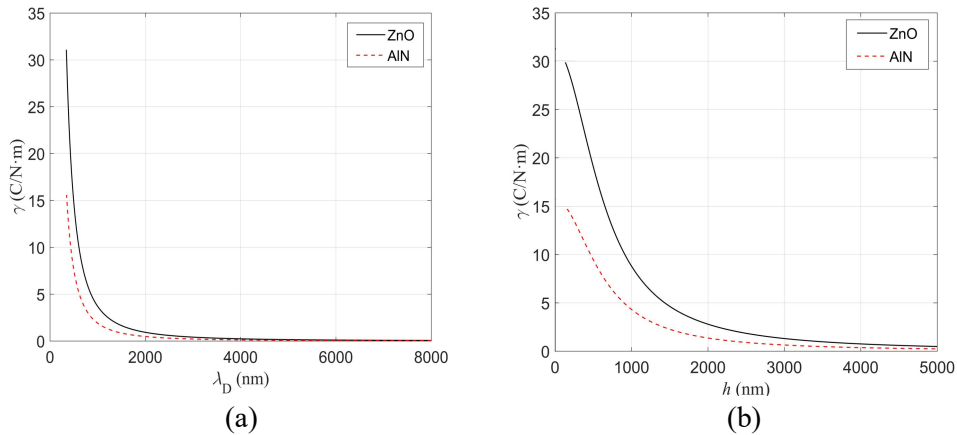


Fig. 6.2  $\gamma$  versus  $\lambda_D$  and  $h$ : (a)  $\gamma$  versus  $\lambda_D$  when  $h = 50 \text{ nm}$ , and (b)  $\gamma$  versus  $h$  when  $p_0 = n_0 = 10^{23} / \text{m}^3$

If there are many initial carriers from doping with a larger  $p_0 + n_0$ , we expect a larger  $\gamma$ . Indeed in this case  $\lambda_D$  becomes smaller according to Eq. (6.22) which leads to a larger  $\gamma$  according to Eq. (6.23) as shown in Fig. 6.22(a). We note that  $\lambda_D$  cannot go to zero because that requires unrealistically large values of the doping levels described by  $p_0$  and/or  $n_0$ . On the other hand, for very small values of  $p_0$  and  $n_0$ , there are not many mobile charges. The value of  $\lambda_D$  is large according to Eq. (6.22) and  $\gamma$  approaches zero according to Eq. (6.23) as expected which can also be seen in Fig. 6.22(a).

In Fig. 6.22(b),  $h$  cannot go to zero because in that case the plate bending stiffness is very small which leads to large deformations and the above linear analysis is invalid. When  $h$  is large, the plate is very stiff. There is not much

deformation and hence the piezoelectrically-induced electric field due to deformation is weak which cannot drive many mobile charges. Hence  $\gamma$  is small for large  $h$ .

#### 6.1.4 Bending of a rectangular plate under distributed load

As an example of nonuniform bending under a static and distributed transverse load described by  $t_3^{(0)}(x_1, x_2)$  we consider a simply-supported rectangular plate within  $0 < x_1 < a$  and  $0 < x_2 < b$ . Electrically the plate edges are grounded without charge accumulations there. The boundary conditions are:

$$\begin{aligned} T_{11}^{(1)} = 0, \quad u_3^{(0)} = 0, \quad \varphi^{(1)} = 0, \quad p^{(1)} = 0, \quad n^{(1)} = 0, \quad x_1 = 0, a \\ T_{22}^{(1)} = 0, \quad u_3^{(0)} = 0, \quad \varphi^{(1)} = 0, \quad p^{(1)} = 0, \quad n^{(1)} = 0, \quad x_2 = 0, b \end{aligned} \quad (6.26)$$

Let

$$\begin{aligned} \{t_3^{(0)}, u_3^{(0)}, \varphi^{(1)}, p^{(1)}, n^{(1)}\} = \sum_{m,n}^{\infty} \{T_{mn}, W_{mn}, \Phi_{mn}, P_{mn}, N_{mn}\} \sin(\xi_m x_1) \sin(\eta_n x_2) \\ \xi_m = \frac{m\pi}{a}, \quad \eta_n = \frac{n\pi}{b}, \quad m, n = 1, 2, 3, \dots \end{aligned} \quad (6.27)$$

where  $T_{mn}$  are known.  $W_{mn}$ ,  $\Phi_{mn}$ ,  $P_{mn}$  and  $N_{mn}$  are undetermined constants. Then the boundary conditions in Eq. (6.26) are automatically satisfied.

By substituting Eq. (6.15) in the rectangular plate into Eqs. (6.13), (6.16) and Eq. (6.17) into Eq. (6.18), four linear algebraic equations with respect to  $u_3^{(0)}$ ,  $\varphi^{(1)}$ ,  $p^{(1)}$  and  $n^{(1)}$  can be obtained with the summation form as follows:

$$\begin{aligned} -\frac{2}{3}h^3 [\bar{c}_{11}u_{3,1111}^{(0)} + 2(\bar{c}_{12} + 2c_{66})u_{3,1122}^{(0)} + \bar{c}_{11}u_{3,2222}^{(0)}] + h^2 \bar{e}_{31}(\varphi_{,11}^{(1)} + \varphi_{,22}^{(1)}) + t_3^{(0)} = 0 \\ -\varepsilon_{11}(\varphi_{,11}^{(1)} + \varphi_{,22}^{(1)}) + \frac{3}{2h}\bar{e}_{31}(u_{3,11}^{(1)} + u_{3,22}^{(1)}) + \frac{3}{h^2}\bar{e}_{33}\varphi^{(1)} = q(p^{(1)} - n^{(1)}) \\ -p_0\mu_{11}^p(\varphi_{,11}^{(1)} + \varphi_{,22}^{(1)}) - D_{11}^p(p_{,11}^{(1)} + p_{,22}^{(1)}) + \frac{3}{h^2}(p_0\mu_{33}^p\varphi^{(1)} + D_{33}^p p^{(1)}) = 0 \\ -n_0\mu_{11}^n(\varphi_{,11}^{(1)} + \varphi_{,22}^{(1)}) + D_{11}^n(n_{,11}^{(1)} + n_{,22}^{(1)}) + \frac{3}{h^2}(n_0\mu_{33}^n\varphi^{(1)} - D_{33}^n n^{(1)}) = 0 \end{aligned} \quad (6.28)$$

By substituting Eq. (6.27) in the rectangular plate under distributed load into the above Eq. (6.28), four linear algebraic equations about  $W_{mn}$ ,  $\Phi_{mn}$ ,  $P_{mn}$  and  $N_{mn}$  can be obtained, namely:



$$\begin{aligned}
& \sum_{m,n} \left\{ -\frac{2}{3} h^3 \left[ \bar{c}_{11} \xi_m^4 + 2(\bar{c}_{12} + 2c_{66}) \xi_m^2 \cdot \eta_n^2 + \bar{c}_{11} \eta_n^4 \right] W_{mn} - h^2 \bar{e}_{31} (\xi_m^2 + \eta_n^2) \Phi_{mn} + T_{mn} \right\} = 0 \\
& \sum_{m,n} \left\{ -\frac{3}{2h} \bar{e}_{31} (\xi_m^2 + \eta_n^2) W_{mn} + \left[ \varepsilon_{11} (\xi_m^2 + \eta_n^2) + \frac{3}{h^2} \bar{\varepsilon}_{33} \right] \Phi_{mn} - q (P_{mn} - N_{mn}) \right\} = 0 \\
& \sum_{m,n} \left\{ \left[ p_0 \mu_{11}^p (\xi_m^2 + \eta_n^2) + \frac{3}{h^2} p_0 \mu_{33}^p \right] \Phi_{mn} + \left[ D_{11}^p (\xi_m^2 + \eta_n^2) + \frac{3}{h^2} D_{33}^p \right] P_{mn} \right\} = 0 \\
& \sum_{m,n} \left\{ \left[ n_0 \mu_{11}^n (\xi_m^2 + \eta_n^2) + \frac{3}{h^2} n_0 \mu_{33}^n \right] \Phi_{mn} - \left[ D_{11}^n (\xi_m^2 + \eta_n^2) + \frac{3}{h^2} D_{33}^n \right] N_{mn} \right\} = 0
\end{aligned} \tag{6.29}$$

Eq. (6.29) with undetermined coefficients can be solved by MATLAB programming.  $m$  and  $n$  both range from 1 to 60 so that  $60 \times 60$  terms of the series are used with which the figures plotted become stable and do not show visible differences when more terms are used. Specifically, consider a plate under a uniform  $t_3^{(0)} = f = 1 \text{ N/m}^2$  within a rectangular region center at  $(x_0, y_0)$  as shown in Fig. 6.3. The coefficients of the trigonometric series expansion for this particular load are given by:

$$T_{mn} = \frac{4}{ab} \frac{1}{\xi_m \eta_n} [\cos(\xi_m (x_0 - c)) - \cos(\xi_m (x_0 + c))] [\cos(\eta_n (y_0 - d)) - \cos(\eta_n (y_0 + d))] \tag{6.30}$$

For numerical results consider a plate with  $h = 50 \text{ nm}$ ,  $a = 3600 \text{ nm}$ ,  $b = 4800 \text{ nm}$ ,  $c = 600 \text{ nm}$ ,  $d = 800 \text{ nm}$ ,  $x_0 = 0.5a$ ,  $y_0 = 0.5b$ , and  $p_0 = n_0 = 10^{20}/\text{m}^3$ . The mobility of holes and electrons in ZnO are given by and  $\mu^p = 180 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$  and  $\mu^n = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$  [70] which are used for both  $\mu_{11}$  and  $\mu_{33}$ .

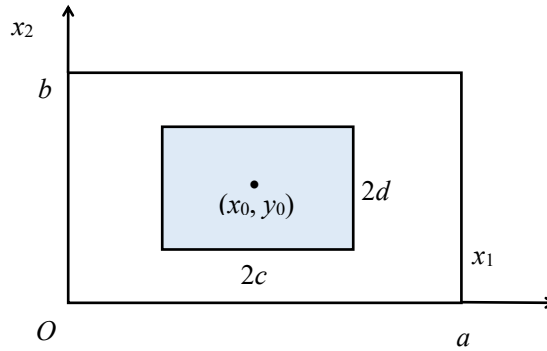


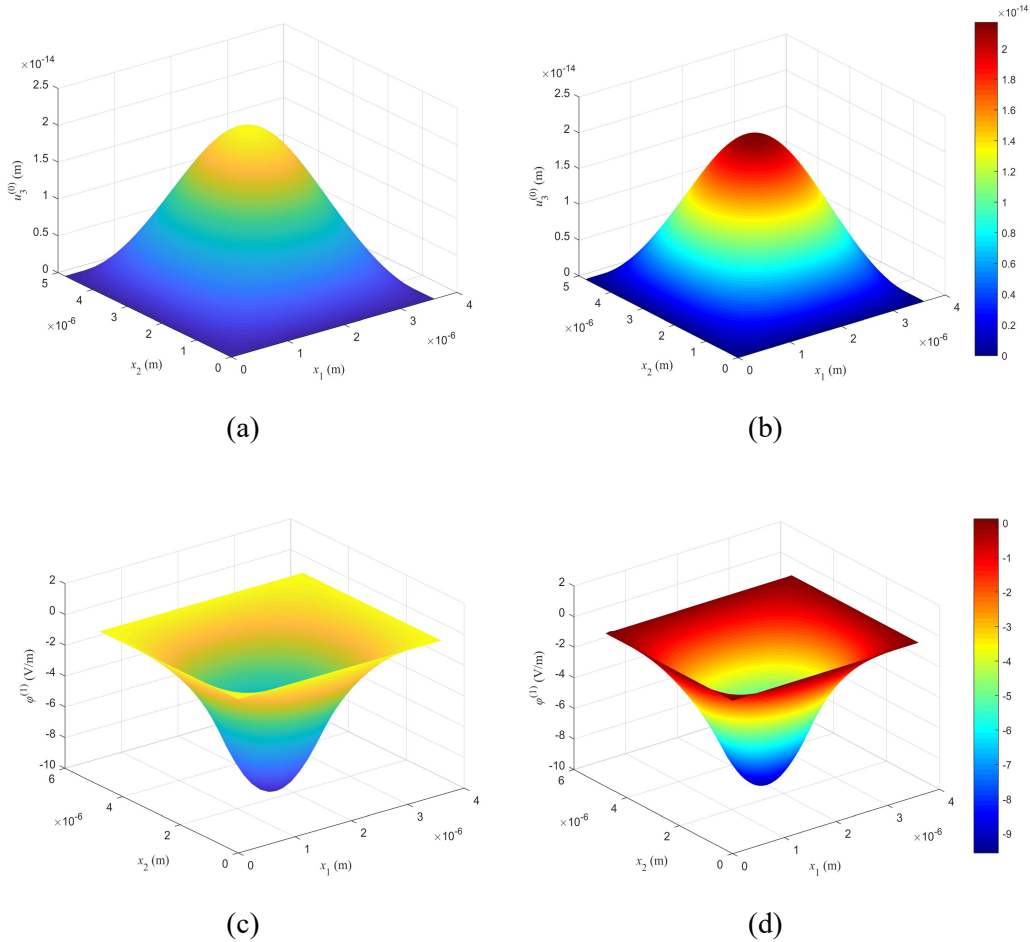
Fig. 6.3 A rectangular plate and loading area

The distributions of the deflection  $u_3^{(0)}$ , potential  $\varphi^{(1)}$ , hole and electron concentration perturbations described by  $p^{(1)}$  and  $n^{(1)}$  are shown in Fig. 6.4 including a comparison with 3D finite element numerical results from COMSOL. Qualitatively the two solutions agree well. Quantitatively a comparison is shown in Table 6.1 in which there is a small difference as expected from a 2D plate theory when compared to the 3D theory.

Table 6.1 Quantitative comparison of series and numerical solutions at  $(x_1, x_2) = (x_0, y_0)$ 

|                                | Series solution          | COMSOL solution          | Relative error |
|--------------------------------|--------------------------|--------------------------|----------------|
| $u_3^{(0)} / (\text{m})$       | $2.1786 \times 10^{-14}$ | $2.1710 \times 10^{-14}$ | 0.35%          |
| $\varphi^{(1)} / (\text{V/m})$ | -10.0377                 | -9.5641                  | 4.72%          |
| $p^{(1)} / (\text{m}^{-4})$    | $3.8606 \times 10^{22}$  | $3.6928 \times 10^{22}$  | 4.35%          |
| $n^{(1)} / (\text{m}^{-4})$    | $-3.8606 \times 10^{22}$ | $-3.6928 \times 10^{22}$ | 4.35%          |

It can be seen from Fig. 6.4 that a transverse mechanical load produces electric potential and mobile charge distributions in addition to deflection. Thus it is possible to manipulate the mobile charges mechanically. Eq. (6.15) shows that  $\varphi^{(1)}$  is closely related to the bending moment and curvature. The locally applied transverse load produces bending moments everywhere in the plate. Therefore the electromechanical fields caused by the local transverse load spread all over the plate while being large near the center where the curvature and moment are large. The charge carriers are driven by  $\varphi^{(1)}$  and therefore they assume similar distributions as shown in Figs. 6.4(e)-6.4(h) in which the holes and electrons move in opposite directions as expected.



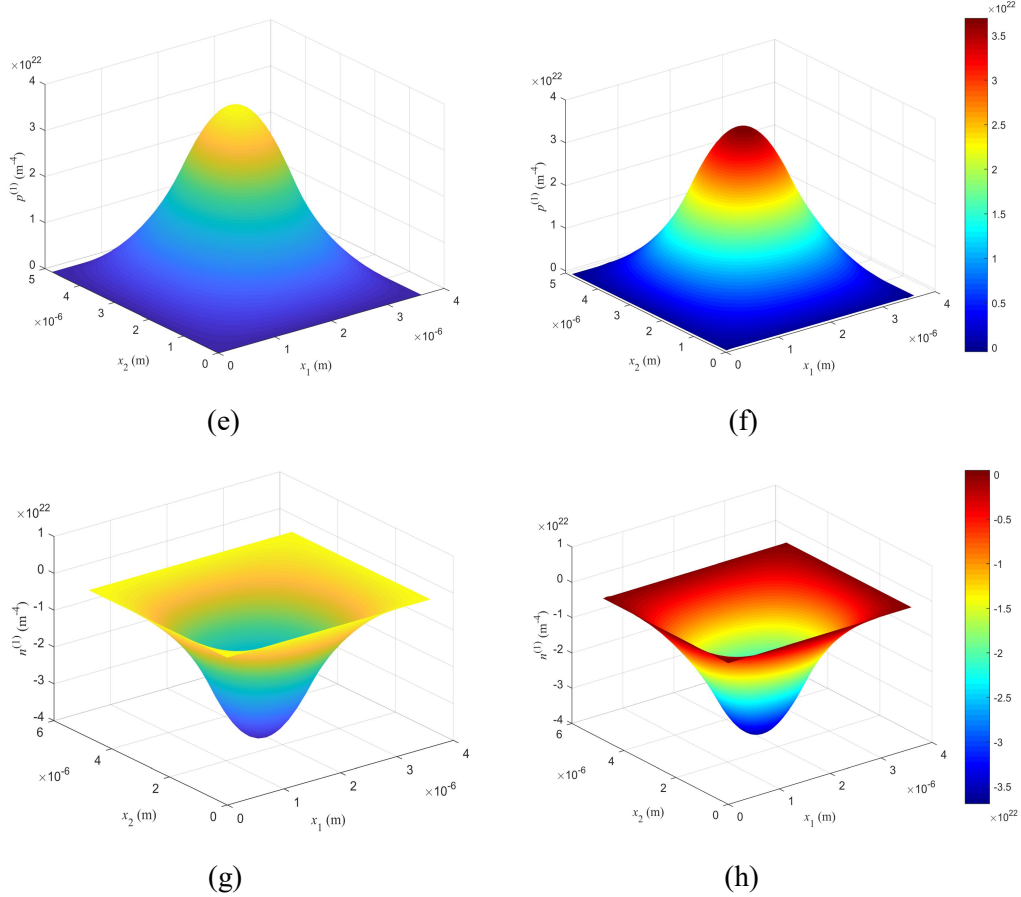


Fig. 6.4 Comparison of series and numerical solutions: (a) deflection by series solution, (b) deflection by COMSOL, (c) electric potential by series solution, (d) electric potential by COMSOL, (e) hole distribution by series solution, (f) hole distribution by COMSOL, (g) electron distribution by series solution, and (h) electron distribution by COMSOL

For the same ZnO plate as in Fig. 6.3, in addition to  $t_3^{(0)} = f = 1 \text{ N/m}^2$  within the loading area in Fig. 6.3, we apply a second locally uniform transverse load  $t_3^{(0)} = f'$  within in a smaller local region also centered at  $(x_0, y_0)$  as shown in Fig. 6.5(a). We require  $f(2c)(2d) = -f'(2c')(2d')$  so that  $f$  and  $f'$  together form a set of self-equilibrated load. We have:

$$T'_{mn} = -\frac{16}{ab} \frac{1}{\xi_m \eta_n} [\cos(\xi_m(x_0 - c')) - \cos(\xi_m(x_0 + c'))] [\cos(\eta_n(y_0 - d')) - \cos(\eta_n(y_0 + d'))] \quad (6.31)$$

In the case of  $c' = 300 \text{ nm}$ ,  $d' = 400 \text{ nm}$  for which  $f' = 4 \text{ N/m}^2$ , the distributions of the total applied load, deflection  $u_3^{(0)}$ , potential  $\varphi^{(1)}$ , and carrier concentration perturbations described by  $p^{(1)}$  and  $n^{(1)}$  are shown in Figs. 6.5(b)-6.5(f). It can be seen that in this case, qualitatively different from Fig. 6.4, the electromechanical fields produced by the mechanical load are also localized, which offers more flexibility for mechanically manipulating the mobile charges in

the plate.

The total load in Fig. 6.5(b) is described by a piecewise constant function with some finite discontinuities and thus its trigonometric series representation shows Gibbs oscillations. The other fields are continuous functions and are without Gibbs oscillations.

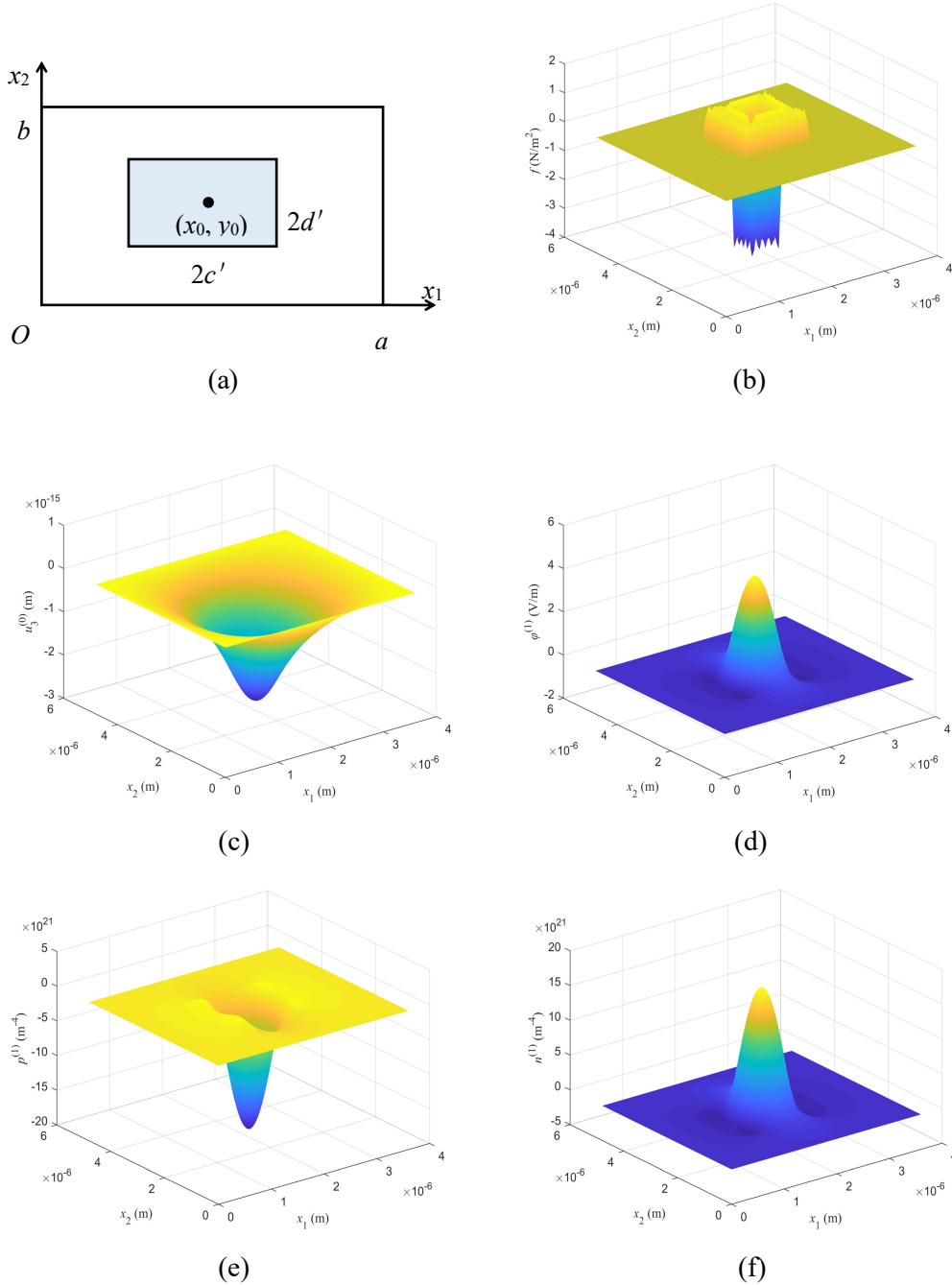


Fig. 6.5 Electromechanical fields under self-equilibrated load: (a) area of additional loading, (b) self-equilibrated load, (c) deflection, (d) electric potential, (e) hole concentration perturbation, and (f) electron concentration perturbation

For the same plate with the local and self-balanced load in Fig. 6.5(b), we examine the effects of various parameters through the distribution of the electric

potential  $\varphi^{(1)}$  in Fig. 6.6.

In Fig. 6(a), as the load intensity  $f$  increases, the deformation becomes larger with a stronger polarization and hence the higher electric potential shown. In Fig. 6(b), the bending stiffness of the plate increased with its thickness, resulting in a lower potential. In Fig. 6(c), the potential distribution becomes wider and higher as the loading area increases. In Fig. 6(d), as  $p_0$  and  $n_0$  increase, there are more mobile charges which screen the piezoelectrically-induced electric potential more severely and lead to a lower voltage. These qualitative trends are all physically correct.

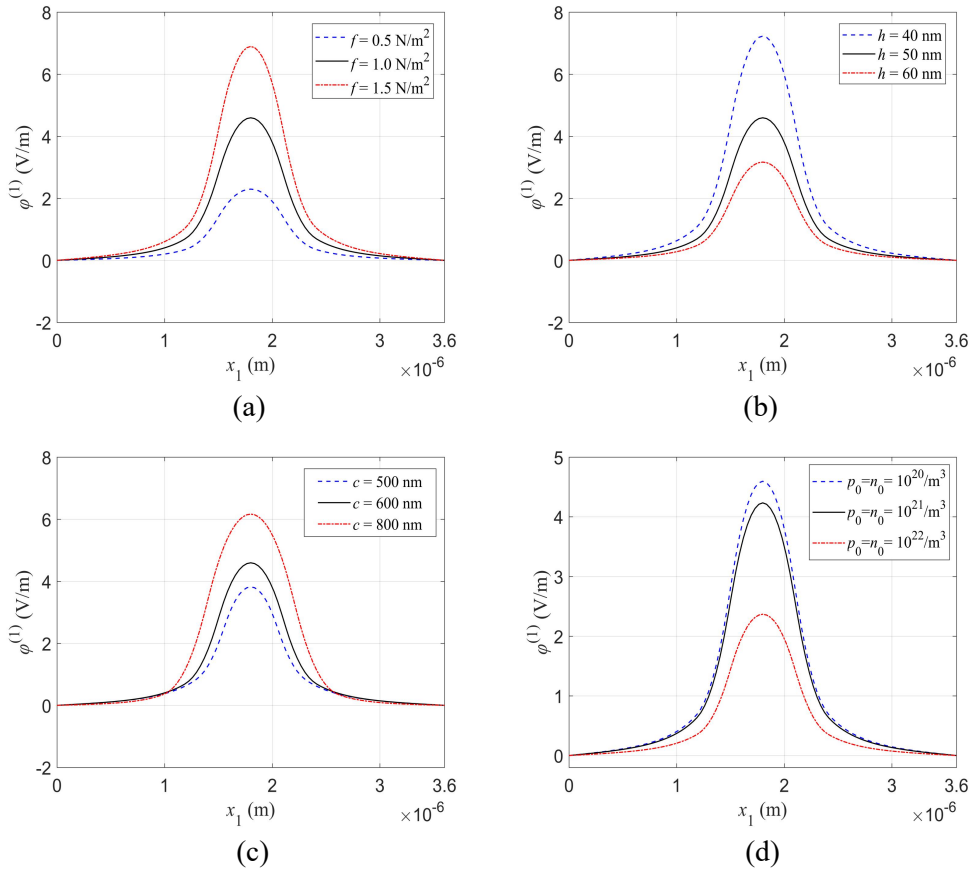


Fig. 6.6 Potential distribution along  $x_1$  under self-equilibrated load: (a) effect of the load  $f$ , (b) effect of the plate half thickness  $h$ , (c) effect of the loading area  $c$ , and (d) effect of initial carrier concentrations  $p_0$  and  $n_0$

## 6.2 Coupled bending and extension of piezoelectric semiconductor/piezomagnetic unimorphs under a transverse magnetic field

In Section 6.1 above, we studied the multi-field coupling characteristics of PS bilayer plate (bimorph) without coupling PM materials. In this Section, we continue to discuss the coupling response of the PM/PS composite beam under transverse magnetic field. First, the 1D coupling equations of bending and extension are derived according to the 3D theory, and then the analytical solutions

are obtained. The influence of the magnetic field and the initial carrier concentration on the electromechanical fields is studied, and the coupling coefficient is introduced to represent the coupling strength between PM and PS.

### 6.2.1 1D equations for unimorphs

Consider the thin beam unimorph in Fig. 6.7 consisting of a PS layer (layer (1)) such as ZnO, and a PM dielectric layer (layer (2)) such as CoFe<sub>2</sub>O<sub>4</sub> or Terfenol-D. The  $x_3$ -axis is at the geometric middle plane which experiences both axial extension and bending because of the lack of structural and material symmetry about the middle plane.

In Fig. 6.7, the primed local coordinate system is for the PM layer only. The unimorph is under a transverse magnetic field  $H$  along the 3' or the thickness direction of the PM layer.  $H$  is assumed to be known and uniform in the PM layer. It produces an axial extension in the PM layer through the piezomagnetic constant  $f_{31}$  in the local coordinate system and thus produces coupled extension and bending of the whole structure. The lateral surface of the unimorph is unelectroded. The electric field outside the unimorph is neglected as a usual approximation.

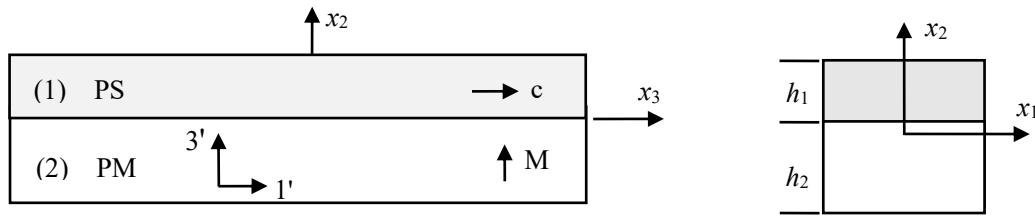


Fig. 6.7 A unimorph of a PS layer and a PM layer

Consider the extension and bending of the unimorph in the  $x_3$ - $x_2$  plane. The extensional and bending displacements of the whole structure are described approximately by the following expressions according to the conventional mechanics of materials:

$$u_2 \cong v(x_3, t), \quad u_3 \cong w(x_3, t) - x_2 v_{,3} \quad (6.32)$$

where  $v$  and  $w$  are the flexural and extensional displacements of the middle plane. So the axial strain  $S_3$  is expressed in terms of  $w$  and  $v$  as:

$$S_3 = u_{3,3} = w_{,3} - x_2 v_{,33} \quad (6.33)$$

The electric potential and field in the entire unimorph as well as the carrier concentration perturbations in the semiconductor layer only are approximated by:

$$\begin{aligned} \varphi &\cong \varphi(x_3, t), \quad E_3 = -\varphi_{,3} \\ \Delta p &\cong \Delta p(x_3, t), \quad \Delta n \cong \Delta n(x_3, t) \end{aligned} \quad (6.34)$$

For the upper ZnO layer, under the usual stress relaxation for thin beams that the transverse normal stress components  $T_1 = T_2 = 0$ , the 1D constitutive relations for the axial stress, electric displacement and current densities are:

$$\begin{aligned} T &= c^{(1)}S - e^{(1)}E = c^{(1)}(w_{,3} - x_2 v_{,33}) + e^{(1)}\varphi_{,3} \\ D &= e^{(1)}S + \varepsilon^{(1)}E = e^{(1)}(w_{,3} - x_2 v_{,33}) - \varepsilon^{(1)}\varphi_{,3} \\ J^p &\cong qp_0\mu^p E - qD^p \frac{\partial(\Delta p)}{\partial z} \\ J^n &\cong qn_0\mu^n E + qD^n \frac{\partial(\Delta n)}{\partial z} \end{aligned} \quad (6.35)$$

where  $S = S_{33}$ ,  $T = T_{33}$ ,  $E = E_3$ ,  $D = D_3$ ,  $J^p = J_3^p$ ,  $J^n = J_3^n$ ,  $\mu^p = \mu_{33}^p$ ,  $\mu^n = \mu_{33}^n$ ,  $D^p = D_{33}^p$  and  $D^n = D_{33}^n$ .  $c^{(1)}$ ,  $e^{(1)}$  and  $\varepsilon^{(1)}$  are the effective 1D material constants under  $T_1 = T_2 = 0$ . They are related to the 3D material constants through:

$$\begin{aligned} c^{(1)} &= c_{33}^{(1)} - \frac{2(c_{13}^{(1)})^2}{c_{11}^{(1)} + c_{12}^{(1)}} \\ e^{(1)} &= e_{33}^{(1)} - \frac{2c_{13}^{(1)}e_{31}^{(1)}}{c_{11}^{(1)} + c_{12}^{(1)}} \\ \varepsilon^{(1)} &= \varepsilon_{33}^{(1)} + \frac{2(e_{31}^{(1)})^2}{c_{11}^{(1)} + c_{12}^{(1)}} \end{aligned} \quad (6.36)$$

Similarly, for the lower PM layer (CoFe<sub>2</sub>O<sub>4</sub> or Terfenol-D), under the stress relaxation for thin beams that in the local coordinate system  $T_2' = T_3' = 0$ , the relevant 1D constitutive relation for the axial fields take the following form:

$$\begin{aligned} T &= c^{(2)}S - h^{(2)}H = c^{(2)}(w_{,3} - x_2 v_{,33}) - h^{(2)}H \\ D &= \varepsilon^{(2)}E \end{aligned} \quad (6.37)$$

where the effective 1D material constants are defined by:

$$\begin{aligned} c^{(2)} &= c_{11}^{(2)} + \frac{2c_{12}^{(2)}(c_{13}^{(2)})^2 - c_{33}^{(2)}(c_{12}^{(2)})^2 - c_{11}^{(2)}(c_{13}^{(2)})^2}{c_{11}^{(2)}c_{33}^{(2)} - (c_{13}^{(2)})^2} \\ h^{(2)} &= h_{31}^{(2)} - \frac{c_{12}^{(2)}c_{33}^{(2)}h_{31}^{(2)} - c_{12}^{(2)}c_{13}^{(2)}h_{33}^{(2)} + c_{13}^{(2)}c_{11}^{(2)}h_{33}^{(2)} - (c_{13}^{(2)})^2h_{31}^{(2)}}{c_{11}^{(2)}c_{33}^{(2)} - (c_{13}^{(2)})^2} \\ \varepsilon^{(2)} &= \varepsilon_{11}^{(2)} \end{aligned} \quad (6.38)$$

For the two layers together, the total axial force  $\hat{T}$ , transverse shear force  $Q$ , bending moment  $M$ , and axial electric displacement  $\hat{D}$  are defined by the following integrations over a cross section:

$$\begin{aligned}
\hat{T} &= \int T dx_1 dx_2 = \hat{c} w_{,3} - \hat{c}' v_{,33} + \hat{e} \varphi_{,3} - \hat{h} H \\
Q &= \int T_{32} dx_1 dx_2 \\
M &= \int T x_2 dx_1 dx_2 = \hat{c}' w_{,3} - \bar{D} v_{,33} + \hat{e} \frac{h_2}{2} \varphi_{,3} + \hat{h} \frac{h_1}{2} H \\
\hat{D} &= \int D dx_1 dx_2 = \hat{e} w_{,3} - \hat{e} \frac{h_2}{2} v_{,33} - \hat{\varepsilon} \varphi_{,3}
\end{aligned} \tag{6.39}$$

where

$$\begin{aligned}
\hat{c} &= c^{(1)} A^{(1)} + c^{(2)} A^{(2)}, \quad \hat{c}' = (c^{(1)} - c^{(2)}) \frac{1}{2} b h_1 h_2 \\
\hat{e} &= e^{(1)} A^{(1)}, \quad \hat{h} = h^{(2)} A^{(2)}, \quad A^{(1)} = b h_1, \quad A^{(2)} = b h_2 \\
\bar{D} &= c^{(1)} \frac{b}{24} [(h_1 + h_2)^3 - (h_2 - h_1)^3] + c^{(2)} \frac{b}{24} [(h_1 + h_2)^3 + (h_2 - h_1)^3] \\
\hat{\varepsilon} &= \varepsilon^{(1)} A^{(1)} + \varepsilon^{(2)} A^{(2)}
\end{aligned} \tag{6.40}$$

The constitutive relation for the shear force  $Q$  in Eq. (6.39)<sub>2</sub> is not needed. It will be provided by the bending moment-shear force relation below as usual in mechanics of materials.

From the equations of motion of the differential element of the unimorph in Fig. 6.8 in the  $x_3$ - and  $x_2$ -directions as well as its moment equation, we have:

$$\begin{aligned}
\hat{T}_{,3} &= b(\rho^{(1)} h_1 + \rho^{(2)} h_2) \ddot{w} + b(\rho^{(2)} - \rho^{(1)}) \frac{h_1 h_2}{2} \ddot{v}_{,3} \\
Q_{,3} &= b(\rho^{(1)} h_1 + \rho^{(2)} h_2) \ddot{v} \\
M_{,3} - Q &= 0
\end{aligned} \tag{6.41}$$

where the rotatory inertia has been neglected. Eq. (6.41)<sub>3</sub> is the so-called bending moment-shear force relation in mechanics of materials. Eq. (6.41)<sub>3</sub> and Eq. (6.39)<sub>3</sub> together imply the following constitutive relation for the transverse shear force:

$$Q = M_{,3} = \hat{c}' w_{,33} - \bar{D} v_{,333} + \hat{e} \frac{h_2}{2} \varphi_{,33} \tag{6.42}$$

Similar to the derivation of Eq. (6.41), by considering the differential element in Fig. 6.8 under electrical loads, the charge equation of electrostatics can be written as:

$$\hat{D}_{,3} = q(\Delta p - \Delta n) A^{(1)} \tag{6.43}$$

The 1D conservation of holes and electrons are reduced as  $q \frac{\partial}{\partial t}(\Delta p) = -J_{,3}^p$  and  $q \frac{\partial}{\partial t}(\Delta n) = J_{,3}^n$ .



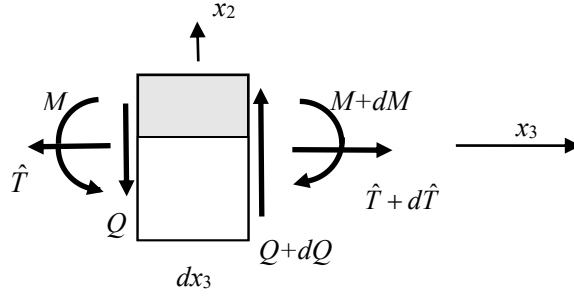


Fig. 6.8 A differential element of the unimorph under mechanical loads

The substitution of Eq. (6.35)<sub>3,4</sub>, Eq. (6.39)<sub>1</sub>, Eq. (6.39)<sub>4</sub>, Eq. (6.42) into Eq. (6.41)<sub>1,2</sub> and Eq. (6.43) gives five equations for  $w$ ,  $v$ ,  $\varphi$ ,  $\Delta p$  and  $\Delta n$ .

### 6.2.2 Analytical solutions

To exhibit the effects of  $H$  on charge carriers in the simplest manner, consider the static bending and extension of a unimorph within  $[-L, L]$  under a uniform  $H$  only without other electromechanical loads. The PS layer is doped into an n-type semiconductor with  $p_0 \approx 0$  and  $\Delta p \approx 0$ . The above five equations for  $w$ ,  $v$ ,  $\varphi$ ,  $\Delta p$  and  $\Delta n$  can be reduced to four equations for  $w$ ,  $v$ ,  $\varphi$  and  $\Delta n$ , namely:

$$\begin{aligned}
 \hat{T}_{,3} &= 0, & \hat{c}w_{,33} - \hat{c}'v_{,333} + \hat{e}\varphi_{,33} &= 0 \\
 Q_{,3} &= 0, & \hat{c}'w_{,333} - \bar{D}v_{,3333} + \hat{e}\frac{h_2}{2}\varphi_{,333} &= 0 \\
 \hat{D}_{3,3} &= q(-\Delta n)A^{(1)}, & \hat{e}w_{,33} - \hat{e}\frac{h_2}{2}v_{,333} - \hat{\varepsilon}\varphi_{,33} &= q(-\Delta n)A^{(1)} \\
 J_{3,3}'' &= 0, & -qn_0\mu^n\varphi_{,33} + qD''(\Delta n)_{,33} &= 0
 \end{aligned} \tag{6.44}$$

The boundary conditions for this problem are:

$$\hat{T}(\pm L) = 0, M(\pm L) = 0, Q(\pm L) = 0, \hat{D}(\pm L) = 0, J''(\pm L) = 0 \tag{6.45}$$

Since there are no boundary conditions prescribed directly on the bending and extensional displacements as well as the electric potential, the solutions of the mechanical fields have rigid-body translations and rotation of the unimorph. At the same time, the electric potential may have an arbitrary constant which does not make any difference in the electric field it produces. To make the mechanical displacements and the electric potential unique, we set:

$$w(0) = 0, v(\pm L) = 0, \varphi(0) = 0 \tag{6.46}$$

Eqs. (6.44)-(6.46) represent the boundary-value problem of a system of linear ordinary differential equations with constant coefficients. Its solution can be obtained in a systematic manner. The procedure of solving the equations is shown in Eqs. (A.6.1)-(A.6.9) of Appendix. The result is found to be:

$$\begin{aligned}
w &= \frac{\left(\bar{D} - \hat{c}' \frac{h_2}{2}\right) \hat{e}^2 \hat{h}}{(\hat{c} \bar{D} - \hat{c}'^2) \tilde{\varepsilon}} \frac{H}{\Delta} \sinh(kx_3) + \frac{\left(\bar{D} + \hat{c}' \frac{h_1}{2}\right) \hat{h} H}{\hat{c} \bar{D} - \hat{c}'^2} x_3 \\
v &= \frac{\left(\hat{c}' \hat{e} - \hat{c} \hat{e} \frac{h_2}{2}\right) \hat{e} \hat{h}}{(\hat{c} \bar{D} - \hat{c}'^2) k \tilde{\varepsilon}} \frac{H}{\Delta} \cosh(kx_3) + \frac{\left(\hat{c} \frac{h_1}{2} + \hat{c}'\right) \hat{h} H}{2(\hat{c} \bar{D} - \hat{c}'^2)} x_3^2 \\
&+ \frac{\left[\hat{c} \hat{e}^2 h_2 \cosh(kL) - 2\hat{c}' \hat{e}^2 \cosh(kL) - kL^2 \hat{c} \tilde{\varepsilon} \frac{h_1}{2} \Delta - kL^2 \hat{c}' \tilde{\varepsilon} \Delta\right] \hat{h}}{2(\hat{c} \bar{D} - \hat{c}'^2) k \tilde{\varepsilon}} \frac{H}{\Delta}
\end{aligned} \tag{6.47}$$

$$\begin{aligned}
S &= \frac{\left(\bar{D} - \hat{c}' \frac{h_2}{2}\right) \hat{e}^2 \hat{h} k}{(\hat{c} \bar{D} - \hat{c}'^2) \tilde{\varepsilon}} \frac{H}{\Delta} \cosh(kx_3) + \frac{\left(\bar{D} + \hat{c}' \frac{h_1}{2}\right) \hat{h} H}{\hat{c} \bar{D} - \hat{c}'^2} \\
&- \frac{\left(\hat{c}' \hat{e} - \hat{c} \hat{e} \frac{h_2}{2}\right) \hat{e} \hat{h} k}{(\hat{c} \bar{D} - \hat{c}'^2) \tilde{\varepsilon}} \frac{H}{\Delta} \cosh(kx_3) x_2 + \frac{\left(\hat{c} \frac{h_1}{2} + \hat{c}'\right) \hat{h} H}{(\hat{c} \bar{D} - \hat{c}'^2)} x_2
\end{aligned} \tag{6.48}$$

$$\varphi = -\frac{\hat{e} \hat{h}}{\tilde{\varepsilon}} \frac{H}{\Delta} \sinh(kx_3), \quad \Delta n = -\frac{n_0 \mu^n}{D^n} \frac{\hat{e} \hat{h}}{\tilde{\varepsilon}} \frac{H}{\Delta} \sinh(kx_3) \tag{6.49}$$

where  $k^2 = \frac{qn_0 \mu^n A^{(1)}}{D^n \tilde{\varepsilon}}$ ,  $A = A^{(1)} + A^{(2)}$ ,  $\tilde{\varepsilon} = \hat{\varepsilon} + \frac{\hat{e}^2}{\hat{c}}$ ,  $\frac{1}{\hat{c}} = \frac{\hat{c} \frac{h_2^2}{4} - \hat{c}' h_2 + \bar{D}}{\hat{c} \bar{D} - \hat{c}'^2}$ . In addition,

$$\Delta = \frac{k(\hat{c} \bar{D} - \hat{c}'^2) \cosh(kL)}{\hat{c} \frac{h_1 h_2}{4} + \hat{c}' \frac{h_2}{2} - \hat{c}' \frac{h_1}{2} - \bar{D}}.$$

Then the axial electric field, electric displacement and electric polarization can be calculated as:

$$\begin{aligned}
E &= \frac{\hat{e} \hat{h} k}{\tilde{\varepsilon}} \frac{H}{\Delta} \cosh(kx_3) \\
D &= \hat{D}/A = \frac{\hat{e} \hat{h} k}{A} \frac{H}{\Delta} [\cosh(kx_3) - \cosh(kL)] \\
P &= P_3 = D_3 - \varepsilon_0 E_3 = D - \varepsilon_0 E = \frac{(\tilde{\varepsilon} - \varepsilon_0 A) \hat{e} \hat{h} k}{\tilde{\varepsilon} A} \frac{H}{\Delta} \cosh(kx_3) - \frac{\hat{e} \hat{h} k}{A} \frac{H}{\Delta} \cosh(kL) \\
\rho^P &= -P_{,i} = -P_{3,3} = -P_{,3} = \frac{(\varepsilon_0 A - \tilde{\varepsilon}) \hat{e} \hat{h} k^2}{\tilde{\varepsilon} A} \frac{H}{\Delta} \sinh(kx_3)
\end{aligned} \tag{6.50}$$

The total charge in the right half of the unimorph due to the motion of electrons can be calculated and written as:

$$Q^e = bh_1 \int_0^L (-q \Delta n) dx_3 = -\gamma H \tag{6.51}$$

where  $\gamma = \frac{\hat{e}\hat{h}}{\hat{c}} \frac{\bar{D} - \hat{c}h_1h_2/4 - \hat{c}'h_2/2 + \hat{c}'h_1/2}{\bar{D} - \hat{c}'^2/\hat{c}} \left(1 - \frac{1}{\cosh(kL)}\right)$ .  $\gamma$  represents the amount

of charges produced by a unit magnetic field and may be used as a measure of the strength of the magneto-semiconduction coupling. It is a structural property depending on the physical and geometric parameters. The contributions from PM and PE couplings appear explicitly as a product effect. The effects of the parameters of semiconduction appear as a combination through  $k$ .

### 6.2.3 Numerical results and discussions

In the numerical calculation, the material constants of the PS layer ZnO are shown in Eqs. (6.24a)-(6.24c). The material constants of the PM layer CoFe<sub>2</sub>O<sub>4</sub> and Terfenol-D are listed in Table 3.1. Consider a unimorph of ZnO and CoFe<sub>2</sub>O<sub>4</sub> with  $L = 0.6 \mu\text{m}$ ,  $h_1 = 0.03 \mu\text{m}$ ,  $h_2 = 0.05 \mu\text{m}$ ,  $b = 0.2 \mu\text{m}$ ,  $n_0 = 10^{21} / \text{m}^3$ , and  $H = 10^3 \text{ A/m}$  as our basic example. Some of these parameters and the PM material may be varied one at a time below.

Figures 6.9 and 6.10 show the basic behavior of the structure, i.e., the development of various electromechanical fields in the unimorph under a magnetic field.

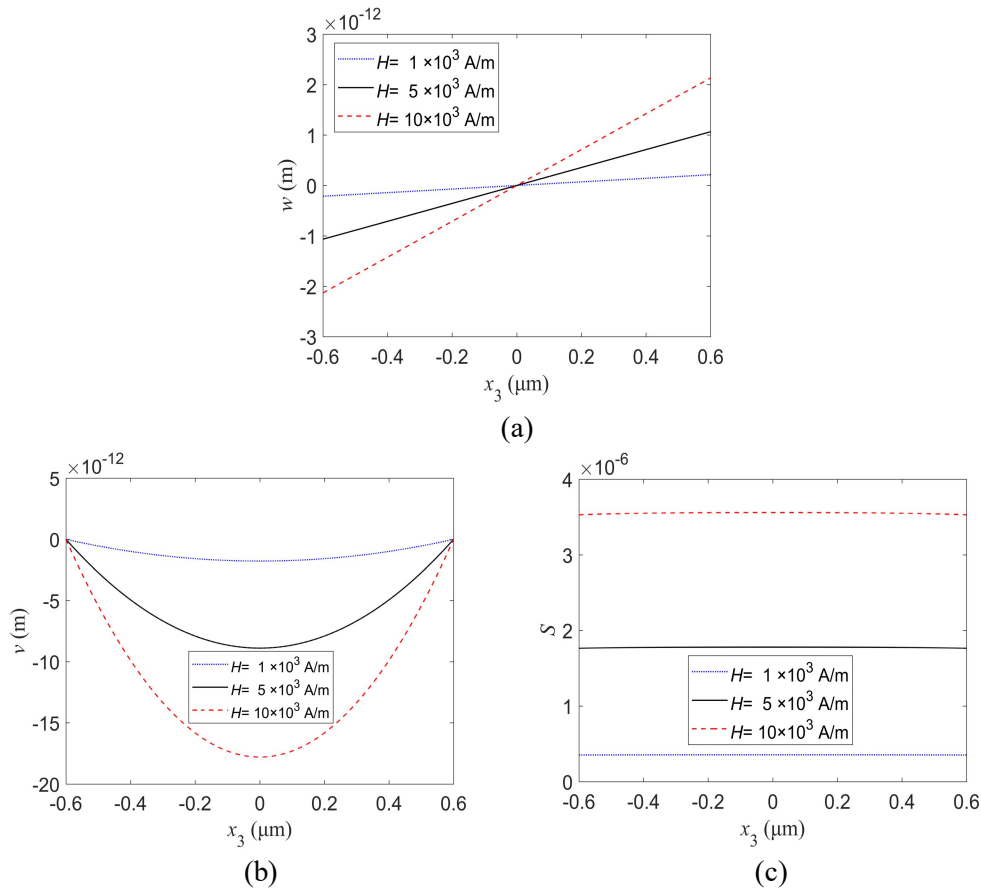


Fig. 6.9 Effects of  $H$  on electromechanical fields (mechanical physical quantities): (a) effect of  $H$  on extensional displacement, (b) effect of  $H$  on flexural displacement, and (c) effect of  $H$  on strain

Among them, Fig. 6.9 shows the distribution of electromechanical field (mechanical physical quantities) under the magnetic field  $H$ . Figures 6.9(a) and 6.9(b) are the familiar extensional and flexural displacements in the mechanics of materials produced by PM coupling. Figure 6.9(c) shows the corresponding strain. It can be seen that they all increase with the increase of magnetic field intensity, indicating that the magnetic field regulation effect is obvious.

These mechanical fields in turn produce the electric potential in Fig. 6.10(a), electric field in Fig. 6.10(b), electric displacement in Fig. 6.10(c), polarization in Fig. 6.10(d), and effective polarization charge in Fig. 6.10(e) through PE coupling.

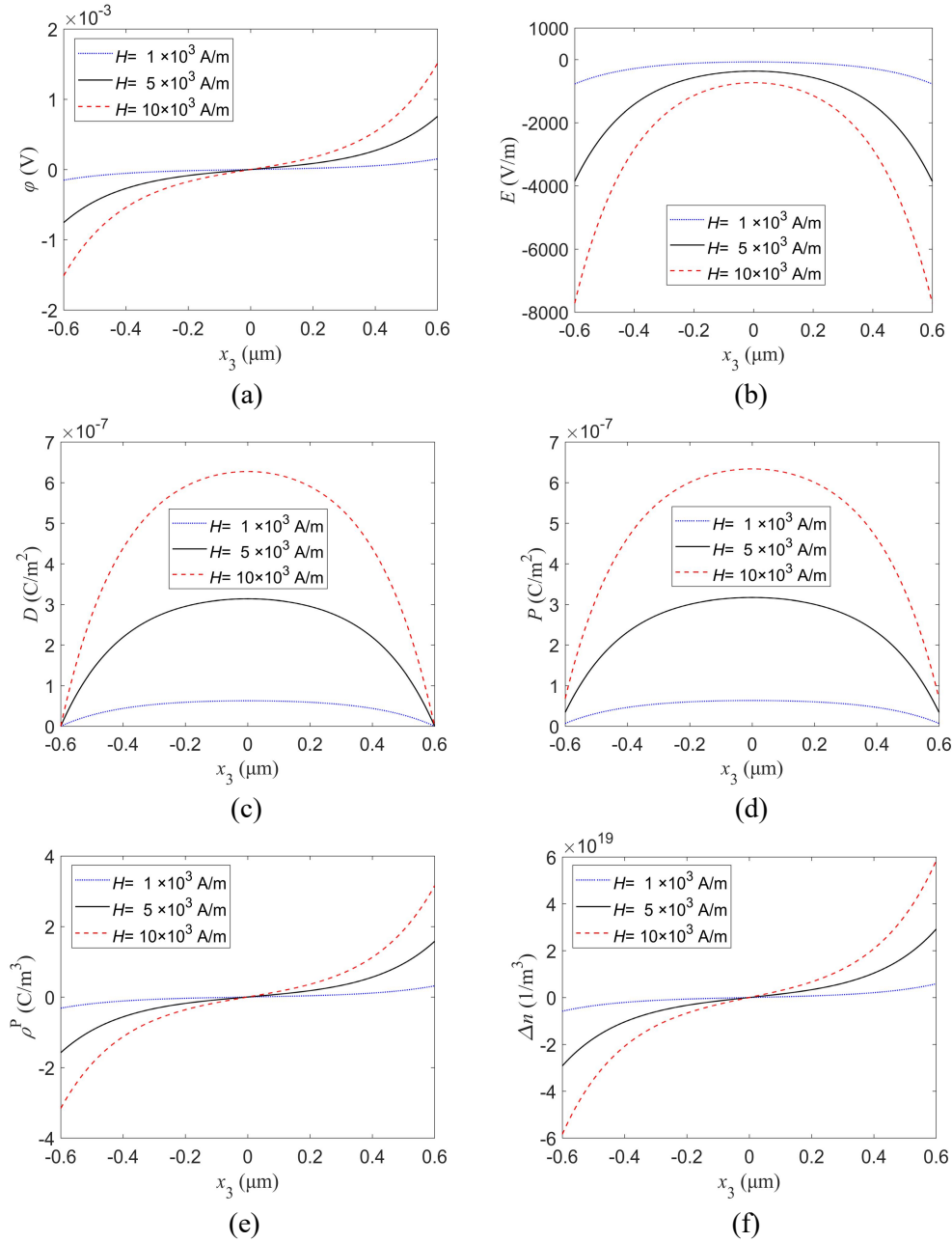


Fig. 6.10 Effects of  $H$  on electromechanical fields (electrical physical quantities and carrier distributions): (a) electric potential, (b) axial electric field, (c) electric displacement, (d) electric polarization, (e) effective polarization charge density, and (f) electron concentration perturbation

Since the PE layer is semiconducting, the electric field or the effective polarization charge drives the electrons to move and redistribute themselves to screen the effective polarization charge as shown in Fig. 6.10(f). As expected, these electromechanical fields become stronger when  $H$  increases.

If the doping level of the electrons described by  $n_0$  is varied, the effects are shown in Figs. 6.11-6.12. The mechanical fields in Fig. 6.11(a) and Fig. 6.11(b) are mainly produced by  $H$  and are insensitive to  $n_0$ .

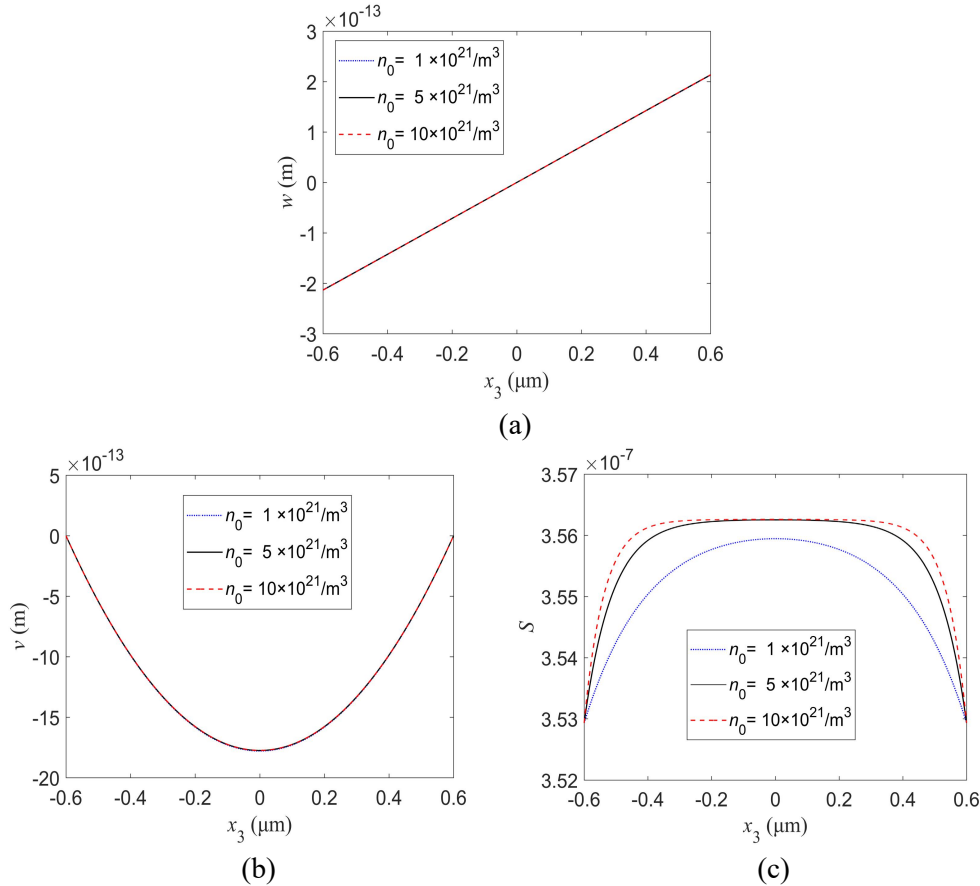


Fig. 6.11 Effects of  $n_0$  on electromechanical fields (mechanical physical quantities): (a) effect of  $n_0$  on extensional displacement, (b) effect of  $n_0$  on flexural displacement, and (c) effect of  $n_0$  on strain

Similar to Fig. 6.10, Figure 6.12 shows the distribution of electric potential, axial electric field, electric displacement, electric polarization, effective polarized charge density and electron concentration perturbation at different  $n_0$ . Different from the mechanical fields in Figs. 6.11(a) and 6.11(b) that are insensitive to  $n_0$ , the other fields in Figs 6.12(a)-(f) are of electrical origin and are sensitive to the mobile charges represented by  $n_0$ . This is because the initial electron concentration affects the electric fields directly. However, its effect on the mechanical fields is indirect, through PE coupling which is not particularly strong in ZnO.

For larger values of  $n_0$ , it can be seen from  $k^2 = \frac{qn_0\mu^n A^{(1)}}{D^n \epsilon}$  that  $k$  increases

and hence all hyperbolic functions in Eqs. (6.49)-(6.50) change more rapidly, which agrees with Figs 6.12(a)-(f).

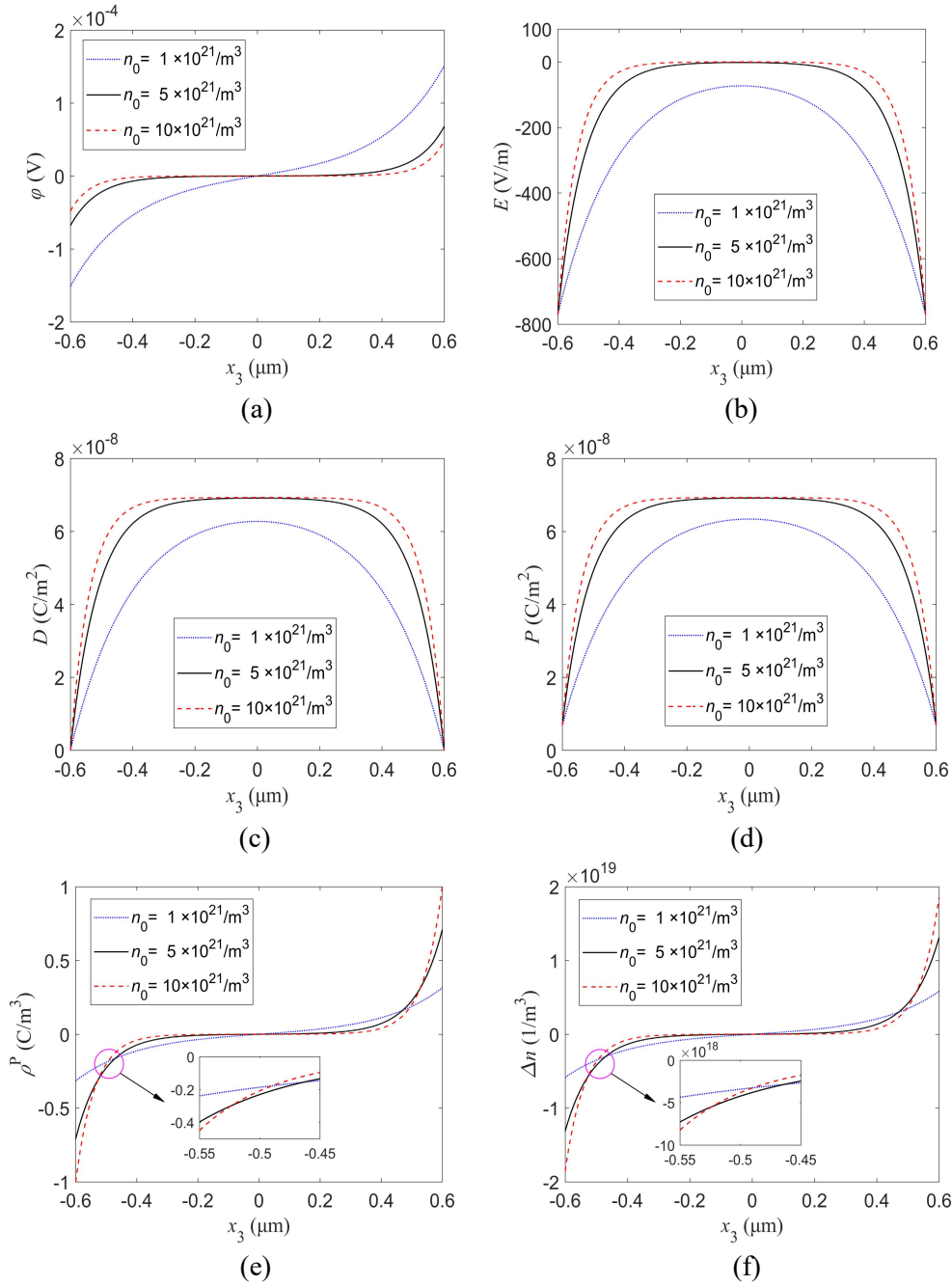


Fig. 6.12 Effects of  $n_0$  on electromechanical fields (electrical physical quantities and carrier distributions): (a) electric potential, (b) axial electric field, (c) electric displacement, (d) electric polarization, (e) effective polarization charge density, and (f) electron concentration perturbation

Figure 6.13 shows the magneto-semiconduction coupling coefficient  $\gamma$  versus the ratio of the PS layer thickness over the total thickness, which is effectively the volume fraction of the PS layer. When the PS layer is very thin and the volume fraction is close to zero, there are not many electrons and hence  $\gamma$  is small. In the other extreme case when the PS layer is very thick and the volume fraction is

close to one, there is not much PM material to deform the structure under  $H$ , and  $\gamma$  is also small.

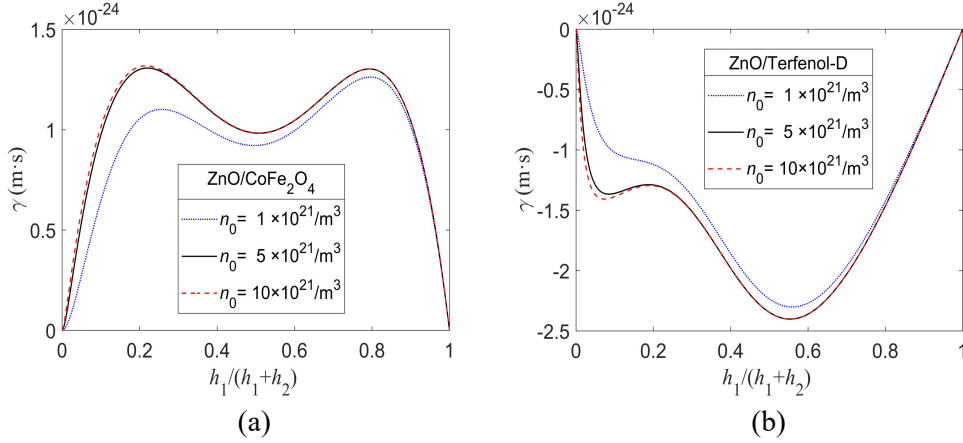


Fig. 6.13  $\gamma$  versus the volume fraction of the PS layer: (a) ZnO/CoFe<sub>2</sub>O<sub>4</sub> composite beam, and (b) ZnO/Terfenol-D composite beam

In Fig. 6.13, when the volume fraction is in the medium range between zero and one, the neutral axis of the unimorph where there is zero axial strain may be within the PS layer. Above and below the neutral axis the axial polarization changes its sign which has a canceling effect on the next axial polarization. This has an adverse effect on the net effective polarization charge and tends to lower the coupling coefficient. This may be responsible for that the curves in Fig. 6.13 are somewhat complicated with two peaks in most cases.

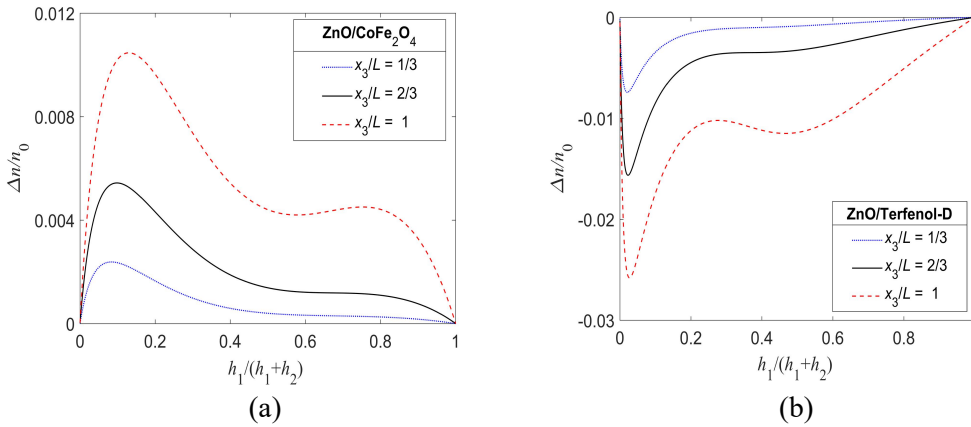


Fig. 6.14  $\Delta n/n_0$  versus the volume fraction of the PS layer at different locations along the unimorph: (a) ZnO/CoFe<sub>2</sub>O<sub>4</sub> unimorph, and (b) ZnO/Terfenol-D unimorph

Further, we obtained the relationship curves between  $n/n_0$  and the volume fraction  $h_1/(h_1 + h_2)$  of the PS layer at different positions of the unimorph as shown in Fig. 6.14. Similar to Fig. 6.13(a), the curves of ZnO/CoFe<sub>2</sub>O<sub>4</sub> unimorph in Fig. 6.14(a) are all above 0, while the curves of ZnO/Terfenol-D unimorph in Fig. 6.14(b) are all below 0. The perturbation carrier concentration of  $x_3/L = 1$  at the end of the unimorph will change more dramatically regardless of the

combination of materials, because the moving charge of the PS layer is driven to the two ends of the beam under the magnetic field.

### 6.3 Summary

Based on the magneto-mechanical-electric-carrier multi-field coupling, the bending coupling characteristics of PS bilayer plate (bimorph) and the electromechanical response of PS/PM composite unimorph are studied respectively.

The 2D plate equations are derived from the 3D theory for the PS bimorph with opposite c-axis. The electromechanical fields of the rectangular plate under uniform load are obtained by using the 2D plate theoretical series solution and COMSO finite element numerical solution. The distribution of electromechanical field and potential under local self-equilibrated load is also discussed.

In addition, the influence and regulation of transverse magnetic field on the coupling response of composite unimorph composed of PM layer and PS layer are studied. Also based on the 3D theory, the 1D coupling model of bending and extension of composite beam is established. By solving the boundary value problem of ordinary differential equations (ODEs) to determine the unknown constants, the analytical solutions are obtained. The effects of magnetic field intensity and initial carrier concentration on electromechanical field are analyzed. At the same time, the coupling coefficient  $\gamma$  is introduced to characterize the coupling strength. The results show that:

(1) The 2D plate equations of PS bimorph exhibit complex bending, electric polarization and moving charge coupling characteristics. The physical quantity  $\gamma$ , which represents the interaction between bending and moving charge, decreases with increasing  $h$  and increases with increasing  $p_0$  and  $n_0$ .

(2) In the PS bimorph, if the uniform load in rectangular region is considered, the results obtained by 2D plate theoretical series solution and 3D finite element numerical solution are qualitatively and quantitatively close. In particular, when the uniform load is a locally self-equilibrated load, the rectangular loading region is induced to generate local charge distributions and potential barriers/potential wells.

(3) The composite beam composed of PS layer and PM layer has no symmetry in the geometry, so the coupling of bending and extension occurs in the unimorph mid-plane. The results show that the moving charge of the semiconductor layer will be redistributed under the magnetic field and move to the two ends of the beam to shield the effective polarized charge. The greater the magnetic field intensity, the larger the mechanical field and the electrical physical quantity will be, and the change is obvious. However, the effect of the  $n_0$  on the mechanical field is not obvious, and the effect on the electrical physical quantity is greater.



## 6.4 Appendix

The process of solving the analytical solution of the PS/PM composite beam under the transverse magnetic field, i.e., Eqs. (6.44)-(6.46), as described in Section 6.3.2, is actually a boundary value solution problem for a system of ODEs. First, consider that the current of the unimorph is always zero, that is,  $J^n \cong -qn_0\mu^n\varphi_{,3} + qD^n(\Delta n)_{,3} = 0$ . By integrating the equation, we get:

$$-q\Delta n = -\frac{qn_0\mu^n}{D^n}\varphi + C_1 \quad (\text{A.6.1})$$

In addition, according to shear force equal to zero there are:

$$Q = M_{,3} = \hat{c}'w_{,33} - \bar{D}v_{,333} + \hat{e}\frac{h_2}{2}\varphi_{,33} = 0 \quad (\text{A.6.2})$$

According to Eq. (6.44)<sub>1</sub> and Eq. (A.6.2), the expressions of  $w_{,33}$ ,  $v_{,333}$  can be obtained as follows:

$$w_{,33} = \frac{-\bar{D}\hat{e} + \hat{c}'\hat{e}\frac{h_2}{2}}{\hat{c}\bar{D} - \hat{c}'^2}\varphi_{,33}, \quad v_{,333} = \frac{-\hat{c}'\hat{e} + \hat{c}\hat{e}\frac{h_2}{2}}{\hat{c}\bar{D} - \hat{c}'^2}\varphi_{,33} \quad (\text{A.6.3})$$

By substituting Eq. (A.6.3) to Eq. (6.44)<sub>3</sub>, a second-order non-homogeneous linear differential equation with constant coefficients for  $\varphi$  can be obtained:

$$\frac{\hat{c}\hat{e}^2\frac{h_2^2}{4} - \hat{c}'\hat{e}^2h_2 + \bar{D}\hat{e}^2 - \hat{c}'^2\hat{e} + \hat{c}\bar{D}\hat{e}}{\hat{c}\bar{D} - \hat{c}'^2}\varphi_{,33} - \frac{qn_0\mu^n A^{(1)}}{D^n}\varphi = -A^{(1)}C_1 \quad (\text{A.6.4})$$

namely:

$$\varphi_{,33} - k^2\varphi = -\frac{A^{(1)}}{\tilde{\varepsilon}}C_1 \quad (\text{A.6.5})$$

where  $k^2 = \frac{qn_0\mu^n A^{(1)}}{D^n \tilde{\varepsilon}}$  and  $\tilde{\varepsilon} = \hat{e} + \frac{\hat{e}^2}{\tilde{c}}$  are given above. The general solution of Eq. (A.6.5) is:

$$\varphi = kC_2 \exp(kx_3) - kC_3 \exp(-kx_3) + \frac{A^{(1)}}{k^2 \tilde{\varepsilon}}C_1 \quad (\text{A.6.6})$$

According to the antisymmetric distribution of the potential  $\varphi$  of the unimorph,  $\varphi$  is an odd function in the range of  $[-L, L]$ . Therefore, we get  $\varphi(-x_3) = -\varphi(x_3)$ , and when combined with  $\varphi(0) = 0$ , we get  $C_1 = 0$  and  $C_2 = C_3$ .

Therefore, the general solution of  $\varphi$  is reduced to  $\varphi = 2kC_2 \sinh(kx_3)$ . By substituting the simplified general solution into the Eq. (A.6.3), the extension displacement and bending displacement components  $w$  and  $v$  are:

$$w = \frac{-\bar{D}\hat{e} + \hat{e}'\frac{h_2}{2}}{\hat{e}\bar{D} - \hat{e}'^2} \varphi + C_4 x_3 + C_5, \quad v = \frac{-\hat{e}'\hat{e} + \hat{e}\hat{e}'\frac{h_2}{2}}{\hat{e}\bar{D} - \hat{e}'^2} [2C_2 \cosh(kx_3)] + \frac{1}{2}C_6 x_3^2 + C_7 x_3 + C_8 \quad (\text{A.6.7})$$

Obviously, a total of 8 unknown constants  $C_1$ - $C_8$  can be obtained to determine  $w$  and  $v$ , and then the expressions of other physical fields can be determined accordingly. By substituting the Eq. (A.6.7) into the boundary conditions  $w(0) = 0$  and  $v(\pm L) = 0$ ,  $C_5 = C_7 = 0$  is obtained.

Considering that some boundary conditions are satisfied automatically, and  $\hat{D}$  and  $v$  are even functions, then using the boundary conditions  $\hat{T} = 0$ ,  $M(L) = 0$ ,  $\hat{D}(L) = 0$  and  $v(L) = 0$ , we can obtain:

$$\begin{aligned} \hat{T} = 0, \quad \hat{T} &= \hat{e}C_4 - \hat{e}'C_6 - \hat{h}H = 0 \\ M(L) = 0, \quad M &= \hat{e}'C_4 - \bar{D}C_6 + \hat{h}\frac{h_1}{2}H = 0 \\ \hat{D}(L) = 0, \quad \hat{D}(L) &= -\tilde{\varepsilon} \cdot 2k^2 C_2 \cosh(kL) + \hat{e}C_4 - \hat{e}'\frac{h_2}{2}C_6 = 0 \\ v(L) = 0, \quad v(L) &= \frac{-\hat{e}'\hat{e} + \hat{e}\hat{e}'\frac{h_2}{2}}{\hat{e}\bar{D} - \hat{e}'^2} [2C_2 \cosh(kL)] + \frac{1}{2}C_6 L^2 + C_8 = 0 \end{aligned} \quad (\text{A.6.8})$$

Then, the unknown constants  $C_2$ ,  $C_4$ ,  $C_6$  and  $C_8$  are respectively:

$$\begin{aligned} C_2 &= \frac{\left( \bar{D} + \hat{e}'\frac{h_1}{2} - \hat{e}'\frac{h_2}{2} - \hat{e}\frac{h_1 h_2}{4} \right) \hat{e} \hat{h} H}{2k^2 \tilde{\varepsilon} (\hat{e}\bar{D} - \hat{e}'^2) \cosh(kL)} = -\frac{\hat{e} \hat{h}}{2k \tilde{\varepsilon}} \frac{H}{\Delta} \\ C_4 &= \frac{\left( \hat{e}'\frac{h_1}{2} + \bar{D} \right) \hat{h} H}{\hat{e}\bar{D} - \hat{e}'^2}, \quad C_6 = \frac{\left( \hat{e}\frac{h_1}{2} + \hat{e}' \right) \hat{h} H}{\hat{e}\bar{D} - \hat{e}'^2} \\ C_8 &= \frac{\left[ \hat{e}\hat{e}'^2 h_2 \cosh(kL) - 2\hat{e}'\hat{e}^2 \cosh(kL) - kL^2 \hat{e} \tilde{\varepsilon} \frac{h_1}{2} \Delta - kL^2 \hat{e}' \tilde{\varepsilon} \Delta \right] \hat{h}}{2(\hat{e}\bar{D} - \hat{e}'^2) k \tilde{\varepsilon}} \frac{H}{\Delta} \end{aligned} \quad (\text{A.6.9})$$

Therefore, by solving the unknown constants  $C_1$ - $C_8$ , the analytical solution of the electromechanical fields can be finally obtained, as shown in Eqs. (6.47)-(6.50) in Section 6.3.2.

# Chapter 7

## Conclusions and prospects

### 7.1 Conclusions

PSs have both piezoelectric and semiconductor properties, and have important application prospects in the fields of communication, sensing, energy harvesting, etc. When a PM material is added or embedded in PSs, a more complex three-phase coupling of piezoelectric, semiconductor and magnetic properties can be achieved. The characteristics of PS devices can be controlled remotely through external biasing field, which becomes a good choice for the development of PS devices. In this thesis, based on the macroscopic theory of magneto-mechanical-electric-carrier multi-field coupling, the propagation characteristics of SAWs in PM and PS composite structures are studied, including SH-type surface waves in PM/PS layered half-space, dispersion and attenuation characteristics of Love and Rayleigh waves in PM/PE/semiconductor composite structures, the regulation of SH waves in MCSs by initial stress and rotation. In addition, the multi-field coupling behavior of PS bimorph and MCS unimorph is studied in detail. The main conclusions are as follows:

(1) Based on the 3D linear theory, the analytical solution of the dispersion equation is obtained for the PS/PM layered half-space. Numerical examples show that the phase velocity of SH wave decreases with the increase of initial carrier concentration, covering thickness and PE constant, but the wave speed and attenuation do not depend on the change of PM constant. The influence of semiconductor properties on the real and imaginary parts of the mechanical and electricity related physical quantities is large, while the influence on the magnetic related quantities is relatively small as it is indirect. On the other hand, when the PM layer/PS half-space is considered, the analysis shows that the Love wave speed and attenuation of the first three order in the open circuit are correspondingly smaller than that in the short circuit, and the propagation characteristics of the Love waves depend significantly on the thickness of the PM layer.

(2) The propagation characteristics of Love waves and Rayleigh waves are obtained by using a three-layer composite model of PM, PE and semiconductor. Unlike the bilayer structure of the PS/PM half-space, the Love and Rayleigh waves in this composite structure only dispersion without attenuation. The higher-order mode wave speed starts from the shear wave velocity of the semiconductor substrate, and all have cutoff frequencies, but the first-order mode of Rayleigh waves is the basic mode starting from the substrate Rayleigh wave velocity. The wave speed of the first three modes of Love wave and Rayleigh wave decreases obviously with the increase of PM layer thickness, but does not change with the initial carrier concentration of the substrate. Only the wave speed of the second and third modes of Rayleigh wave changes slightly with the change of PE layer thickness. The wave speed of Love waves decreases with the increase of  $f_{15}$ , and the analytical solution of Rayleigh waves is simultaneously related to the three PM constants ( $f_{15}$ ,  $f_{31}$ , and  $f_{33}$ ), so the influence of  $f_{15}$  on Rayleigh waves is relatively limited. The influence of different material combination on Rayleigh waves is greater than that on Love waves.

(3) The effect of initial stress on SH waves in PS plate and PS/PM composite plate is studied. In the PS plate, the phase velocity of SH wave decreases and the attenuation increases with the constant initial stress. The critical point where the phase velocity decreases sharply with the increase of initial stress is at  $|T_1^0| = 100\text{MPa}$ . The phase velocity decreases with the increase of the initial carrier concentration and plate thickness. When  $n_0 = 1 \times 10^{10} / \text{m}^3$ , the PS plate can be regarded as a PE plate. The phase velocity and attenuation in short circuit are smaller than that in open circuit. The effects of carrier concentration, plate thickness, initial stress and material properties on SH waves in the PS/PM composite plate with initial stress are similar to those in the PS plate, but the values of the real and imaginary parts of the wave speed change slightly, indicating that the PM layer has limited influence on SH waves in this model. The inflection point of attenuation change is at  $kh/2\pi = 0.382$  regardless of the initial stress.

(4) Based on the coupling theory with Coriolis and centrifugal force, the propagation characteristics of SH waves in MCSs rotating around the fixed axis are obtained. Wave speed and attenuation decrease faster with increasing angular velocity, and frequency drift and attenuation coefficient increase with increasing angular velocity. The rotational sensitivity of long waves is much higher than that of short waves. The presence of semiconductor properties causes the SH waves to disperse and attenuate regardless of rotation. When the normalized horizontal biasing electric field  $\lambda \leq 0.801$ , the wave speed will decrease with the increase of  $\lambda$ , and when  $\lambda \geq 0.801$ , the wave speed will increase sharply with the increase of  $\lambda$  until it becomes stable.

(5) The deflection, electric potential, hole and electron distributions under uniform loading in rectangular domain are obtained by means of series solution for a 2D PS bilayer plate (bimorph) with opposite  $c$ -axis. The electromechanical fields are larger near the center with larger curvature and moment. The 3D finite

element solution obtained by COMSOL is compared with the series solution, and it is found that the two methods are qualitatively close, and the relative error is small in quantitative. In particular, the local self-equilibrated load will induce the local potential barriers/wells in the loading area, and the potential distribution will become wider and higher as the loading area increases. The electric potential will decrease with the increase of plate thickness and initial carrier concentration. On the other hand, the coupling behavior of bending and extension of a beam under transverse magnetic field is obtained for a 1D beam composed of PS layer and PM layer. Under the magnetic field, the moving charge of the PS layer will be redistributed and move towards the two ends of the beam. When the magnetic field is increased, the mechanical fields such as extensional and flexural displacements, electric fields such as electric potential, electric displacement and polarized charge density, and the change of the disturbed electron concentration will be more obvious. The initial carrier concentration has little effect on the mechanical fields, but has great effect on the electrical quantity and the carrier concentration.

In summary, based on the magneto-mechanical-electro-carrier coupling theory, the thesis obtains the dispersion and attenuation characteristics of surface waves in the PS/PM half-space, PM/PS half-space, PM/PE/semiconductor composite structures, respectively. The regulation of SH waves in MCSs by initial stress, rotating field and biasing electric field is also discussed. In addition, for the bilayer structures, the bending coupling characteristics of the PS bimorph and the electromechanical coupling response of the PS/PM composite beam are obtained, and the effects of the mechanical load and magnetic field on the electromechanical fields is revealed. The results obtained are innovative in the understanding of multi-field coupling of specific functional materials/structures, and provide a powerful theoretical basis for novel magnetic field controlled SAW devices and PS devices.

## 7.2 Future work

In this thesis, the surface waves propagation and coupling fields in PM and PS composite structures are studied. The research in related fields is developing rapidly, and there are many problems worth further exploration:

(1) The effects of initial stress and rotation effect on SH waves in MCSs are studied in this thesis. Further, the influence of functional gradient or viscoelastic material as the covering layer or non-ideal interface on the surface waves in MCSs can be considered.

(2) The effects of surface effects, interface effects and non-local effects on elastic waves in nano MCSs can be considered in the near future.

(3) For the artificial periodic composite structure composed of PM materials and PS materials, the SAWs propagation in MCS phonon crystals can be further studied.

(4) The bending coupling in PS bimorph with n-type doping is studied in this thesis, and the coupling characteristics of PS bimorph with p-type and n-type doping respectively can be further studied to form PN junction at the interface. At the same time, for 1D MCS beams, it can also be developed to 2D plates and coupling problems with PN junction.

(5) Based on the nonlinear coupling theory of MCSs, the nonlinear solutions will be obtained by using commercial finite element software such as COMSOL. In addition, the non-commercial finite element method of Carrera Unified Formulation (CUF), which has been widely proven to be effective and cost-effective in the literature, can be used to solve the typical wave problems in the layered structures of PM and PS. The development of theory needs experimental verification, in order to speed up the engineering application, experimental studies are also necessary.

# References

- [1] HICKERNELL, F S. The piezoelectric semiconductor and acoustoelectronic device development in the sixties. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 2005, 52(5): 737-745.
- [2] WEINREICH G. Acoustodynamic effects in semiconductors. *Physical Review*, 1956, 104(2): 321-324.
- [3] HANEBREKKE H, INGEBRIGTSEN K A. Acoustoelectric amplification of surface waves in structure of cadmium-selenide film on lithium niobate. *Electronics Letters*, 1970, 6(16): 520-521.
- [4] LUO J T, LUO P X, XIE M, et al. A new type of glucose biosensor based on surface acoustic wave resonator using Mn-doped ZnO multilayer structure. *Biosensors and Bioelectronics*, 2013, 49: 512-518.
- [5] KALETTA U C, WIPF C, FRASCHKE M, et al. AlN/SiO<sub>2</sub>/Si<sub>3</sub>N<sub>4</sub>/Si(100)-based COMS compatible surface acoustic wave filter with-12.8-dB minimum insertion loss. *IEEE Transactions on Electron Devices*, 2015, 62(3): 764-768.
- [6] TRIVEDI S, NEMADE H B. ZnO nanorod-based Love wave delay line for high mass sensitivity: A finite element analysis. *IET Science, Measurement & Technology*, 2019, 13(9): 1245-1253.
- [7] GHOSH S, KHARE P. Acoustic wave amplification in ion-implanted piezoelectric semiconductor. *Indian Journal of Pure & Applied Physics*, 2006, 44: 183-187.
- [8] WILLATZEN M, CHRISTENSEN J. Acoustic gain in piezoelectric semiconductors at  $\epsilon$ -near-zero response. *Physical Review B*, 2014, 89(4): 041201.
- [9] WANG Z L. Nanobelts, nanowires, and nanodiskettes of semiconducting oxides-from materials to nanodevices. *Advanced Materials*, 2003, 15(5): 432-436.
- [10] WEN X N, WU W Z, DING Y, et al. Piezotronic effect in flexible thin-film based devices. *Advanced Materials*, 2013, 25, 3371-3379.
- [11] HEO Y W, TIEN L C, NORTON D P, et al. Pt/ZnO nanowire Schottky diodes. *Applied Physics Letters*, 2004, 85(15): 3107-3109.
- [12] HUANG H L, ZHANG H F, CAO Y Q, et al. High-temperature three-dimensional GaN-based hall sensors for magnetic field detection. *Journal of*

- Physics D: Applied Physics*, 2020, 54(7): 075003.
- [13] FRÖMLING T, YU R M, MINTKEN M, et al. Piezotronic sensors. *MRS Bulletin*, 2018, 43(12): 941-945.
- [14] WANG Z L, SONG J H. Piezoelectric nanogenerators based on zinc oxide nanowire arrays. *Science*, 2006, 312(5771): 242-246.
- [15] KAUR J, SINGH H. Fabrication and analysis of piezoelectricity in 0D, 1D and 2D Zinc Oxide nanostructures. *Ceramics International*, 2020, 46: 19401-19407.
- [16] WANG X D, ZHOU J, SONG J H, et al. Piezoelectric field effect transistor and nanoforce sensor based on a single ZnO nanowire. *Nano Letters*, 2006, 6(12): 2768-2772.
- [17] WANG Z L. Piezopotential gated nanowire devices: Piezotronics and piezophotonics. *Nano Today*, 2010, 5: 540-552.
- [18] BUYUKKOSE S, HERNANDEZ-MINGUEZ A, VRATZOV B, et al. High-frequency acoustic charge transport in GaAs nanowires. *Nanotechnology*, 2014, 25(13): 135204.
- [19] AULD B A. *Acoustic fields and waves in solids*. New York: John Wiley & Sons, 1973.
- [20] PIERRET R F. *Semiconductor fundamentals*. 2nd ed. New York: Addison Wesley, 1988.
- [21] ZHANG C L, WANG X Y, CHEN W Q, et al. An analysis of the extension of a ZnO piezoelectric semiconductor nanofiber under an axial force. *Smart Materials and Structures*, 2017, 26(2): 025030.
- [22] ZHANG C L, LUO Y X, CHENG R R, et al. Electromechanical fields in piezoelectric semiconductor nanofibers under an axial force. *MRS Advances*, 2017, 2(56): 1-6.
- [23] LIANG Y X, YANG W L, YANG J S. Transient bending vibration of a piezoelectric semiconductor nanofiber under a suddenly applied shear force. *Acta Mechanica Solida Sinica*, 2019, 32: 688-697.
- [24] DAI X Y, ZHU F, QIAN Z H, et al. Electric potential and carrier distribution in a piezoelectric semiconductor nanowire in time-harmonic bending vibration. *Nano Energy*, 2018, 43: 22-28.
- [25] WAUER J, SUHERMAN S. Thickness vibrations of a piezo-semiconducting plate layer. *International Journal of Engineering Science*, 1997, 35(15): 1387-1404.
- [26] LI P, JIN F, YANG J S. Effects of semiconduction on electromechanical energy conversion in piezoelectrics. *Smart Materials and Structures*, 2015, 24(2): 025021.
- [27] HU Y T, ZENG Y, YANG J S. A mode III crack in a piezoelectric semiconductor of crystals with 6 mm symmetry. *International Journal of Solids and Structures*, 2007, 44(11-12): 3928-3938.
- [28] FAN C Y, YAN Y, XU G T, et al. Piezoelectric-conductor iterative method for analysis of cracks in piezoelectric semiconductors via the finite element method. *Engineering Fracture Mechanics*, 2016, 165: 183-196.



- [29] ZHAO M H, PAN Y B, FAN C Y, et al. Extended displacement discontinuity method for analysis of cracks in 2D piezoelectric semiconductors. *International Journal of Solids and Structures*, 2016, 94-95: 50-59.
- [30] ZHAO M H, LI X F, LU C S, et al. Nonlinear analysis of a crack in 2D piezoelectric semiconductors with exact electric boundary conditions. *Journal of Intelligent Material Systems and Structures*, 2020, 32(3): 1-8.
- [31] HUTSON A R, WHITE D L. Elastic wave propagation in piezoelectric semiconductors. *Journal of Applied Physics*, 1962, 33(1): 40-47.
- [32] YANG J S. *Analysis of piezoelectric semiconductor structures*. Switzerland: Springer, 2020.
- [33] WHITE D L. Amplification of ultrasonic waves in piezoelectric semiconductors. *Journal of Applied Physics*, 1962, 33(8): 2547-2554.
- [34] WHITE R M. Surface elastic-wave propagation and amplification. *IEEE Transactions on Electron Devices*, 1967, 14(4): 181-189.
- [35] YANG J S, YANG X M, TURNER A. Amplification of acoustic waves in laminated piezoelectric semiconductor plates. *Archive of Applied Mechanics*, 2004, 74(3): 288-298.
- [36] YANG J S, ZHOU H G. Amplification of acoustic waves in piezoelectric semiconductor plates. *International Journal of Solids and Structures*, 2005, 42(11-12): 3171-3183.
- [37] ÖZGÜR Ü, ALIVOV Y I, LIU C L, et al. A comprehensive review of ZnO materials and devices. *Journal of Applied Physics*, 2005, 98(4): 041301.
- [38] WANG Z L. Nanopiezotronics. *Advanced Materials*, 2007, 19(6), 889-892.
- [39] WANG Z L. The new field of nanopiezotronics. *Materials Today*, 2007, 10(5): 20-28.
- [40] YANG Q, GUO X, WANG W H, et al. Enhancing sensitivity of a single ZnO micro-/nanowire photodetector by piezo-phototronic effect. *ACS Nano*, 2010, 4(10): 6285-6291.
- [41] HU Y F, CHANG Y L, FEI P, et al. Designing the electric transport characteristics of ZnO micro/nanowire devices by coupling piezoelectric and photoexcitation effects. *ACS Nano*, 2010, 4(2): 1234-1240.
- [42] WANG Z L. *Piezotronics and piezo-phototronics*. Beijing Science Press, 2012 (in Chinese).
- [43] ZHOU J, GU Y D, FEI P, et al. Flexible piezotronic strain sensor. *Nano Letters*, 2008, 8(9): 3035-3040.
- [44] LIU W H, LEE M, DING L, et al. Piezopotential gated nanowire-nanotube hybrid field-effect transistor. *Nano Letters*, 2010, 10(8): 3084-3089.
- [45] HE JR H, HSIN C L, LIU J, et al. Piezoelectric gated diode of a single ZnO nanowire. *Advanced Materials*, 2007, 19(6): 781-784.
- [46] WU W Z, WANG Z L. Piezotronic nanowire-based resistive switches as programmable electromechanical memories. *Nano Letters*, 2011, 11(7): 2779-2785.

- [47] YANG Y, GUO W X, ZHANG Y, et al. Piezotronic effect on the output voltage of P3HT/ZnO micro/nanowire heterojunction solar cells. *Nano Letters*, 2011, 11(11): 4812-4817.
- [48] YANG Q, WANG W H, XU S, et al. Enhancing light emission of ZnO microwire-based diodes by piezo-phototronic effect. *Nano Letters*, 2011, 11(9): 4012-4017.
- [49] WANG L F. Piezophotonics and its applications. *Scientia Sinica Technologica*, 2023, 53(6): 881-891 (in Chinese).
- [50] WANG Z L. Progress in piezotronics and piezo-phototronics. *Advanced Materials*, 2012, 24(34): 4632-4646.
- [51] WANG Z L, Wu W Z, Falconi C. Piezotronics and piezo-phototronics with third-generation semiconductors. *MRS Bulletin*, 2018, 43(12): 922-927.
- [52] LI Z T, WANG Z L. Air/liquid-pressure and heartbeat-driven flexible fiber nanogenerators as a micro/nano-power source or diagnostic sensor. *Advanced Materials*, 2011, 23(1): 84-89.
- [53] PIERRET R F. *Semiconductor device fundamentals*. 2nd ed. New York: Addison Wesley, 1996.
- [54] ZHANG K F, HU H F. *Introduction to partial differential equations in semiconductors*. Beijing Science Press, 2016 (in Chinese).
- [55] LORENZI H G, TIERSTEN H F. On the interaction of the electromagnetic field with heat conducting deformable insulators. *Journal of Mathematical Physics*, 1975, 13(3): 938-957.
- [56] MAUGIN G A, DAHER N. Phenomenological theory of elastic semiconductors. *International Journal of Engineering Science*, 1986, 24(5): 703-731.
- [57] EITENNE P, CREUZET G, FRIEDERICH A, et al. Giant magnetoresistance of (001)Fe/(001)Cr magnetic superlattices. *Physical Review Letters*, 1988, 61(21): 2472-2475.
- [58] LIU L S, LIU S, WANG W X, et al. Research progress and application of semiconductor spintronics, 2007, 30(3): 1125-1128 (in Chinese).
- [59] ZHANG T R. *Electrons, Electrons! Who will save the Moore's Law?* Beijing: Tsinghua University Press, 2014 (in Chinese).
- [60] LIU Y D, GUO J M, YU A F, et al. Magnetic-induced-piezopotential gated MoS<sub>2</sub> field-effect transistor at room temperature. *Advanced Materials*, 2018, 30(8): 1704524.
- [61] WONG M C, CHEN L, TSANG M L, et al. Magnetic-induced luminescence from flexible composite laminates by coupling magnetic field to piezophotonic effect. *Advanced Materials*, 2015, 27(30): 4488-4495.
- [62] HUANG L B, BAI G X, WONG M C, et al. Magnetic-assisted noncontact triboelectric nanogenerator converting mechanical energy into electricity and light emissions. *Advanced Materials*, 2016, 28(14): 2744-2751.
- [63] PENG M Z, ZHANG Y, LIU Y D, et al. Magnetic-mechanical-electrical-optical coupling effects in GaN-based led/rare-earth Terfenol-D structures. *Advanced Materials*, 2014, 26(39): 6767-6772.

- [64] Cui N Y, Wu W W, ZHAO Y, et al. Magnetic force driven nanogenerators as a noncontact energy harvester and sensor. *Nano Letters*, 2012, 12(7): 3701-3705.
- [65] LI G J, LI X M, ZHU Q X, et al. Integration of BaTiO<sub>3</sub>/CoFe<sub>2</sub>O<sub>4</sub> multiferroic heterostructure on GaN semiconductor. *CrystEngComm*, 2019, 21(43): 6545-6551.
- [66] THOMAS R, SCOTT J F, BOSE D, et al. Multiferroic thin-film integration onto semiconductor devices. *Journal of Physics: Condensed Matter*, 2010, 22(42): 423201.
- [67] XU M, JI H X. Brief introduction to Spintronics and its development of study. *College Physics*, 2006, 25(11): 12-17 (in Chinese).
- [68] YU X X, ZI J, WANG B, et al. Research status and trends of spintronics. *Science and technology of China*, 2018, 5: 7-10 (in Chinese).
- [69] LANDAU L D, LIFSHITZ E M, PITAEVSKII L P. *Electrodynamics of continuous media*. 2nd ed. Beijing: World Publishing Corporation, 1999.
- [70] SZE S M, KWOK K NG. *Physics of semiconductor devices*. 3rd ed. New Jersey: John Wiley & Sons, 2007.
- [71] CHENG R R, ZHANG C L, ZHANG C Z, et al. Magnetically controllable piezotronic responses in a composite semiconductor fiber with multiferroic coupling effects. *Physica Status Solidi (A) Applications and Materials*, 2019, 217(2): 1900621.
- [72] WEI P J. *Elastic Wave Theory*. Beijing Science Press, 2021 (in Chinese).
- [73] WEINREICH G, SANDERS T M, WHITE H G. Acoustoelectric effect in *n*-type Germanium. *Physical Review*, 1959, 114(1): 33-44.
- [74] HUTSON A R, MCFEE J H. Ultrasonic amplification in CdS. *Physical Review Letters*, 1961, 7(6): 237-239.
- [75] KINO G S. Acoustoelectric interactions in acoustic-surface-wave devices. *Proceedings of IEEE*, 1976, 64 (5): 724-748.
- [76] HEYMAN J S. Phase insensitive acoustoelectric transducer. *The Journal of the Acoustical Society of America*, 1978, 64(1): 243-249.
- [77] BUSSE L J, MILLER J G. Response characteristics of a finite aperture, phase insensitive ultrasonic receiver based upon the acoustoelectric effect. *The Journal of the Acoustical Society of America*, 1981, 70(5): 1370-1376.
- [78] DIETZ D R, BUSSE L J, FIFE M J. Acoustoelectric detection of ultrasound power with composite piezoelectric and semiconductor devices. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 1988, 35(2):146-151.
- [79] GU C L, JIN F. Shear-horizontal surface waves in a half-space of piezoelectric semiconductors. *Philosophical Magazine Letters*, 2015, 95(2): 92-100.
- [80] CAO X S, HU S M, LIU J J, et al. Generalized Rayleigh surface waves in a piezoelectric semiconductor half space. *Meccanica*, 2019, 54(1-2): 271-281.
- [81] WANG W H, LI L, LAN M, et al. Surface wave speed of functionally gradient piezoelectric semiconductors. *Archive of Applied Mechanics*, 2022,

- 92(1): 1905-1912.
- [82] TIAN R, LIU J X, PAN E N, et al. Some characteristics of elastic waves in a piezoelectric semiconductor plate. *Journal of Applied Physics*, 2019, 126(12): 125701.
- [83] ZHANG C L, WANG X Y, CHEN W Q, et al. Propagation of extensional waves in apiezoelectric semiconductor rod. *AIP Advances*, 2016, 6(4): 045301.
- [84] ZHU F, JI S H, ZHU J Q, et al. Study on the influence of semiconductive property for the improvement of nanogenerator by wave mode approach. *Nano Energy*, 2018, 52(5): 474-484.
- [85] LIANG Y X, HU Y T. Effect of interaction among the three time scales on the propagation characteristics of coupled waves in a piezoelectric semiconductor rod. *Nano Energy*, 2019, 68: 104345.
- [86] JIAO F Y, WEI P J, ZHOU X L, et al. The dispersion and attenuation of the multi-physical fields coupled waves in a piezoelectric semiconductor. *Ultrasonics*, 2018, 92(1): 68-78.
- [87] JAHANGIR A, ZENKOUR A M, KAYNAT A. Effect of free surface boundary condition on reflected waves in a half-space piezoelectric semiconductor medium. *Indian Journal of Physics*, 2023, 97(12): 3523-3530.
- [88] CHEN Y M, YANG L, FANG X, et al. Shear-horizontal surface waves in a half-space of piezoelectric semiconductor with initial stresses. *Acta Physica Polonica A*, 2022, 142(2): 272-279.
- [89] CHEN Y M, YANG L. SH wave propagation in piezoelectric semiconductor plate with initial stress. *Chinese Journal of Solid Mechanics*, 2022, 43(4): 467-476 (in Chinese).
- [90] WANG W H, LI L, LAN M, et al. SH waves in a half-space model of functionally gradient piezoelectric semiconductors with initial stresses. *Ferroelectrics Letters Section*, 2023, 50(4-6): 91-101.
- [91] SALAH I B, TAKALI F, OTHMANI C, et al. SH waves in a stressed piezoelectric semiconductor plates: Electron and hole drift phenomenon. *International Journal of Mechanical Sciences*, 2022, 223: 107281.
- [92] CHEN T, ZHANG X M, ZHOU H M, et al. Characteristics of complete circumferential guided wave in a piezoelectric semiconductor cylindrical shell. *Journal of Intelligent Material Systems and Structures*, 2022, 34(8): 733-748.
- [93] WEI Z B, WEI P J, XU C Y, et al. Influences of piezoelectric positive-negative junction on the multi-field coupled waves propagation in the piezoelectric semiconductor. *The Journal of the Acoustical Society of America*, 2022, 152(3): 1883-1900.
- [94] WEI Z B, WEI P J, XU C Y, et al. Equivalent imperfect interface model of PN junction of piezoelectric semiconductor for the multi-field coupled waves propagation. *Acta Mechanica*, 2023, 235(8): 1-18.
- [95] KONG D J, GUO Y T, ZHANG C L. Manipulation of a piezoelectric p-n junction with an elastic longitudinal wave. *Mechanics of Solids*, 2023, 58(3): 903-911.

- [96] LI D Z, LI S P, ZHANG C L, et al. Propagation characteristics of shear horizontal waves in piezoelectric semiconductor-surface effect. *International Journal of Mechanical Sciences*, 2023, 247(2): 108201.
- [97] LI D Z, ZHANG C L, ZHANG S F, et al. Propagation of terahertz elastic longitudinal waves in piezoelectric semiconductor rods. *Ultrasonics*, 2023, 132(10): 106964.
- [98] YANG J S, YANG X M, TUNER J A. Amplification of acoustic waves in piezoelectric semiconductor shells. *Journal of Intelligent Material Systems and Structures*, 2005, 16(7): 613-621.
- [99] YANG J S, ZHOU H G. Wave propagation in a piezoelectric ceramic plate sandwiched between two semiconductor layers. *International Journal of Applied Electromagnetics and Mechanics*, 2005, 22(1): 97-109.
- [100] OTHMANI C, TAKALI F, NJEH A. Theoretical study on the dispersion curves of Lamb waves in piezoelectric-semiconductor sandwich plates GaAs. *Superlattices and Microstructures*, 2007, 106: 86-101.
- [101] OTHMANI C, TAKALI F, NJEH A. Modeling of phase velocity and frequency spectrum of guided Lamb waves in piezoelectric-semiconductor multilayered structures. *Superlattices and Microstructures*, 2007, 111: 396-404.
- [102] OTHMANI C, TAKALI F, NJEH A, et al. Numerical simulation of lamb waves propagation in a functionally graded piezoelectric plate composed of GaAs-AlAs materials using Legendre polynomial approach. *Optik*, 2017, 142: 401-411.
- [103] SHARMA J N, SHARMA K K, KUMAR A. Surface waves in a piezoelectric-semiconductor composite structure. *International Journal of Solids and Structures*, 2010, 47(6): 816-826.
- [104] SHARMA J N, SHARMA K K, KUMAR A. Modelling of acoustodiffusive surface waves in piezoelectric-semiconductor composite structures. *Journal of Mechanics of Materials and Structures*, 2011, 6(6): 791-812.
- [105] SHARMA J N, SHARMA K K, KUMAR A. Acousto-diffusive waves in a piezoelectric-semiconductor-piezoelectric sandwich structure. *World Journal of Mechanics*, 2011, 1(5): 247-255.
- [106] TIAN R, NIE G Q, LIU J X, et al. Love waves in a piezoelectric semiconductor thin film on an elastic dielectric half-space. *Acta Mechanica Sinica*, 2022, 36(1): 45-54.
- [107] TIAN R, NIE G Q, LIU J X, et al. On Rayleigh waves in a piezoelectric semiconductor thin film over an elastic half-space. *International Journal of Solids and Structures*, 2021, 204(1): 106565.
- [108] TIAN R, LIU J X, PAN E N, et al. SH waves in multilayered piezoelectric semiconductor plates with imperfect interfaces. *European Journal of Mechanics/A Solids*, 2020, 81(34): 103961.
- [109] GUO X, WEI P J. Dispersion relations of in-plane elastic waves in nano-scale one dimensional piezoelectric semiconductor piezoelectric dielectric

- phononic crystal with the consideration of interface effect. *Applied Mathematical Modelling*, 2021, 96: 189-214.
- [110] GUO X, WEI P J, XU M X, et al. Dispersion relations of anti-plane elastic waves in micro-scale one dimensional piezoelectric semiconductor phononic crystals with the consideration of interface effect. *Mechanics of Materials*, 2021, 161: 104000.
- [111] JIAO F Y, WEI P J, ZHOU Y H, et al. Wave propagation through a piezoelectric semiconductor slab sandwiched by two piezoelectric half-spaces. *European Journal of Mechanics/A Solids*, 2019, 75(5): 70-81.
- [112] SAHU S A, NIRWAL S, MONDAL S. Reflection and transmission of quasi-plane waves at the interface of piezoelectric semiconductors with initial stresses. *Applied Mathematics and Mechanics (English Edition)*, 2021, 42(1): 951-968.
- [113] ZHAO L N, WEI P J, HUANG M S, et al. Electro-Thermo-Mechanical multiple fields coupled wave propagation through piezoelectric semiconductor sandwich structure. *Composite Structures*, 2022, 288(6): 115358.
- [114] ABD-ALLA A N, OTHMAN M, HASBULLAH N F. Shear horizontal waves in composite materials: Behavior under rotation and initial stress. *Mathematics and Mechanics of Solids*, 2017, 24(1): 85-97.
- [115] XU C Y, WEI P J, WEI Z B, et al. Rayleigh wave in layered piezoelectric semiconductor with consideration of PN junction effects. *Mathematics and Mechanics of Solids*, 2022, 28(8): 1-17.
- [116] XU C Y, WEI P J, WEI Z B, et al. Shear horizontal wave in a piezoelectric semiconductor with consideration of Schottky junction effects. *Applied Mathematical Modelling*, 2022, 109: 509-518.
- [117] XU C Y, WEI P J, WEI Z B, et al. Effects of corrugated boundaries on Rayleigh waves in a piezoelectric semiconductor substrate covered with a metal layer. *Applied Mathematical Modelling*, 2024, 125: 110-124.
- [118] XU C Y, WEI P J, WEI Z B, et al. Shear horizontal wave propagation on a piezoelectric semiconductor substrate under slight ridge or thin metal strip gratings. *Thin-Walled Structures*, 2023, 191: 111059.
- [119] QU Y L, ZHU F, PAN E N, et al. Analysis of wave-particle drag effect in flexoelectric semiconductor plates via Mindlin method. *Applied Mathematical Modelling*, 2023, 118: 541-555.
- [120] JAAFAR M M, WEE M F M R, NGUYEN H T N, et al. Surface acoustic wave propagation of GaN/Sapphire integrated with a gold guiding layer. *Sensors*, 2023, 23(5): 2464.
- [121] ZHANG C L, WANG X Y, CHEN W Q, et al. Carrier distribution and electromechanical fields in a free piezoelectric semiconductor rod. *Journal of Zhejiang University-Science A: Applied Physics & Engineering*, 2016, 17(1): 37-44.
- [122] FAN S Q, LIANG Y X, XIE J M, et al. Exact solutions to the electromechanical quantities inside a statically-bent circular ZnO nanowire

- by taking into account both the piezoelectric property and the semiconducting performance: Part I-Linearized analysis. *Nano Energy*, 2017, 40: 82-87.
- [123] FAN S Q, YANG W L, HU Y T. Adjustment and control on the fundamental characteristics of a piezoelectric PN junction by mechanical-loading. *Nano Energy*, 2018, 52: 416-421.
- [124] FAN S Q, HU Y T, YANG J S. Stress-induced potential barriers and charge distributions in a piezoelectric semiconductor nanofiber. *Applied Mathematics and Mechanics (English Edition)*, 2019, 40(5): 591-600.
- [125] QU Y L, JIN F, YANG J S. Stress-induced electric potential barriers in thickness-stretch deformations of a piezoelectric semiconductor plate. *Acta Mechanica*, 2021, 232(11): 4533-4543.
- [126] QU Y L, JIN F, YANG J S. Electromechanical interactions in a composite plate with piezoelectric dielectric and nonpiezoelectric semiconductor layers. *Acta Mechanica*, 2022, 233(9): 3795-3812.
- [127] CHENG R R, ZHANG C L, CHEN W Q, et al. Piezotronic effects in the extension of a composite fiber of piezoelectric dielectrics and nonpiezoelectric semiconductors. *Journal of Applied Physics*, 2018, 124(6): 064506.
- [128] JU S, ZHANG H F, YANG J S. Stress induced potential barriers in composite piezoelectric semiconductor fibers in extension. *Ferroelectrics Letters Section*, 2021, 48(4-6): 72-82.
- [129] HE J L, DU J K, YANG J S. Stress effects on electric currents in antiplane problems of piezoelectric semiconductors over a rectangular domain. *Acta Mechanica*, 2022, 233(3): 1173-1185.
- [130] YANG Y Z, YANG W L, WANG Y B, et al. Performance of piezoelectric semiconductor bipolar junction transistors and the tuning mechanism by mechanical loadings. *Journal of Applied Physics*, 2023, 133: 145701.
- [131] GUO Z W, CHEN J B, ZHANG G Y, et al. Exact solutions for plane stress problems of piezoelectric semiconductors: Tuning free-carrier motions by various mechanical loadings. *European Journal of Mechanics/A Solids*, 2023, 101(5): 105073.
- [132] CHENG R R, ZHANG C L, YANG J S. Thermally induced carrier distribution in a piezoelectric semiconductor fiber. *Journal of Electronic Materials*, 2019, 48(8): 4939-4946.
- [133] CHENG R R, ZHANG C L, CHEN W Q, et al. Electrical behaviors of a piezoelectric semiconductor fiber under a local temperature change. *Nano Energy*, 2019, 66: 104081.
- [134] CHENG R R, ZHANG C L, CHEN W Q, et al. Temperature effects on PN junctions in piezoelectric semiconductor fibers with thermoelastic and pyroelectric couplings. *Journal of Electronic Materials*, 2020, 45(5): 3140-3148.
- [135] CHENG R R, ZHANG C L, CHEN W Q, et al. Temperature effects on mobile charges in extension of composite fibers of piezoelectric dielectrics and non-piezoelectric semiconductors. *International Journal of Applied*

- Mechanics*, 2019, 11(9): 1950088.
- [136] QU Y L, JIN F, YANG J S. Temperature effects on mobile charges in thermopiezoelectric semiconductor plates. *International Journal of Applied Mechanics*, 2021, 13(03): 2150037.
- [137] QU Y L, JIN F, YANG J S. Temperature-induced potential barriers in piezoelectric semiconductor films through pyroelectric and thermoelastic couplings and their effects on currents. *Journal of Applied Physics*, 2022, 131: 94502.
- [138] QU Y L, PAN E N, ZHU F, et al. Modeling thermoelectric effects in piezoelectric semiconductors: New fully coupled mechanisms for mechanically manipulated heat flux and refrigeration. *International Journal of Engineering Science*, 2023, 182(1):103775.
- [139] JIN Z H, YANG J S. Analysis of a sandwiched piezoelectric semiconducting thermoelectric structure. *Mechanics Research Communications*, 2019, 98(8): 31-36.
- [140] JU S, YANG J S, ZHANG H F. Effects of mobile charges on interface thermal stresses in a piezoelectric semiconductor composite rod. *Archive of Applied Mechanics*, 2022, 92(3): 1-9.
- [141] QIAO Y P, ZHAO M H, QIN G S, et al. The influence of temperature on the electrical conductivity of GaN piezoelectric semiconductors. *AIP Advances*, 2023, 13(1): 015105.
- [142] YANG Z, ZHANG Z C, LIU C B, et al. Analysis of a hollow piezoelectric semiconductor composite cylinder under a thermal loading. *Mechanics of Advanced Materials and Structures*, 2023, 30(9): 1-10.
- [143] WANG G L, LIU J X, FENG W J, et al. Magnetically induced carrier distribution in a composite rod of piezoelectric semiconductors and piezomagnetics. *Materials*, 2020, 13(14): 3115.
- [144] LIANG C, ZHANG C L, CHEN W Q, et al. Electrical response of a multiferroic composite semiconductor fiber under a local magnetic field. *Acta Mechanica Solida Sinica*, 2020, 33(5): 663-673.
- [145] LIANG C, ZHANG C L, CHEN W Q, et al. Effects of magnetic fields on PN junctions in piezomagnetic-piezoelectric semiconductor composite fibers. *International Journal of Applied Mechanics*, 2020, 12(14): 2050085.
- [146] YANG L, DU J K, WANG J, et al. An analysis of piezomagnetic-piezoelectric semiconductor unimorphs in coupled bending and extension under a transverse magnetic field. *Acta Mechanica Solida Sinica*, 2021, 34(5): 743-753.
- [147] WANG Y, HUANG Y F, ZHANG C L, et al. Bending analysis of multiferroic semiconductor composite beam towards smart cement-based materials. *Materials*, 2023, 16(1): 421.
- [148] LIANG C, ZHANG C L. Analysis of multi-field coupling responses of piezomagnetic/piezoelectric semiconductor cylindrical shell under a constant magnetic field. *Chinese Journal of Solid Mechanics*, 2020, 41(3): 206-215 (in Chinese).



- [149] KONG D J, CHENG R R, ZHANG C L, et al. Dynamic manipulation of piezotronic behaviors of composite multiferroic semiconductors through time-dependent magnetic field. *Journal of Applied Physics*, 2020, 128: 064503.
- [150] ZHANG Q Y, TAN N, WANG B B, et al. Study on nonlinear dynamic responses of the p-n GaN/AlN heterojunction in piezoelectric semiconductor composite fibers. *Physica Status Solidi (A) Applications and Materials*, 2023, 220(21): 2300578.
- [151] WANG W J, JIN F, HE T H, et al. Nonlinear magneto-mechanical-thermo coupling characteristic analysis for transport behaviors of carriers in composite multiferroic piezoelectric semiconductor nanoplates with surface effect. *Applied Mathematics and Mechanics (English Edition)*, 2022, 43(9): 1323-1338.
- [152] WANG W J, JIN F, HE T H, et al. Size-dependent and nonlinear magneto-mechanical coupling characteristics analysis for extensional vibration of composite multiferroic piezoelectric semiconductor nanoharvester with surface effect. *European Journal of Mechanics/A Solids*, 2022, 96(5): 104708.
- [153] ZHAO L K, DENG T, JIN F. Two-dimensional analysis on the magnetic field adjusted electrical behaviors in composite semiconductor structures. *Composite Structures*, 2023, 309(5771): 116732.
- [154] CHI M S, ZHAO Y L, ZHANG X, et al. A piezotronic and magnetic dual-gated ferroelectric semiconductor transistor. *Advanced Functional Materials*, 2023, 33(46): 2307901.
- [155] HUANG F, ZHANG W X, LI L, et al. MEMS surface acoustic wave resonator based on AlN/Si/Fe-Co-Si-B structure for magnetic field sensing. *IEEE Sensors Journal*, 2022, 22(23): 22510-22518.
- [156] MIAO H C, LI F X. Shear horizontal wave transducers for structural health monitoring and nondestructive testing: A review. *Ultrasonics*, 2021, 114: 106355.
- [157] MANDAL D, BANERJEE S. Surface acoustic wave (SAW) sensors: Physics, materials, and applications. *Sensors*, 2022, 22(3): 820.
- [158] GOLUB M V, ZHANG C Z, WANG Y S. SH-wave propagation and resonance phenomena in a periodically layered composite structure with a crack. *Journal of Sound and Vibration*, 2011, 330(13): 3141-3154.
- [159] GOLUB M V, ZHANG C Z, WANG Y S. SH-wave propagation and scattering in periodically layered composites with a damaged layer. *Journal of Sound and Vibration*, 2012, 331: 1829-1843.
- [160] OSIKA M, ZIAJA-SUJDAK A, RADECKI R, et al. Nonlinear modes in shear horizontal wave propagation-analytical and numerical analysis. *Journal of Sound and Vibration*, 2022, 540(6): 117247.
- [161] LOVE A E H. *Some problems of geodynamics*. London: Cambridge University Press, 1911.
- [162] WANG Q, QUEK S T, VARADAN V K. Love waves in piezoelectric

- coupled solid media. *Smart Materials and Structures*, 2001, 10(2): 380-388.
- [163] ZAKHARENKO A A. Love-type waves in layered systems consisting of two cubic piezoelectric crystals. *Journal of Sound and Vibration*, 2005, 285(4-5): 877-886.
- [164] DU J K, JIN X Y, WANG J. Love wave propagation in layered magneto-electro-elastic structures. *Science in China Series G Physics Mechanics and Astronomy*, 2008, 51(6): 617-631.
- [165] LIU J X, FANG D N, WEI W Y, et al. Love waves in layered piezoelectric-piezomagnetic structures. *Journal of Sound and Vibration*, 2008, 315: 146-156.
- [166] EZZIN H, AMOR M B, GHOZLEN M H B. Love waves propagation in a transversely isotropic piezoelectric layer on a piezomagnetic half-space. *Ultrasonics*, 2016, 69: 83-89.
- [167] YANG W J, LIANG X, SHEN S P. Love waves in layered flexoelectric structures. *Philosophical Magazine*, 2017, 97(33): 1378825.
- [168] HRYTSYNA O, SLADEK J, SLADEK V, et al. Love waves propagation in layered waveguide structures including flexomagneticity/flexoelectricity and micro-inertia effects. *Mechanics of Advanced Materials and Structures*, 2022, 30(3): 1-19.
- [169] LIU H, WANG Z K, WANG T J. Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. *International Journal of Solids and Structures*, 2001, 38: 37-51.
- [170] SU J, KUANG Z B, LIU H. Love wave in ZnO/SiO<sub>2</sub>/Si structure with initial stresses. *Journal of Sound and Vibration*, 2005, 286: 981-999.
- [171] DU J K, JIN X Y, WANG J. Love wave propagation in layered magneto-electro-elastic structures with initial stress. *Acta Mechanica*, 2007, 192: 169-189.
- [172] SINGH A K, MISTRI K C, CHATTOPADHYAY A. Love-type wave propagation in an irregular prestressed composite sandwiched layer. *International Journal of Geomechanics*, 2016, 16(2): 04015060.
- [173] DU J K, JIN X Y, WANG J, et al. Love wave propagation in functionally graded piezoelectric material layer. *Ultrasonics*, 2007, 46: 13-22.
- [174] ESKANDARI M, SHODJA H M. Love waves propagation in functionally graded piezoelectric materials with quadratic variation. *Journal of Sound and Vibration*, 2008, 313: 195-204.
- [175] CAO X S, JIN F, JEON I, et al. Propagation of Love waves in a functionally graded piezoelectric material (FGPM) layered composite system. *International Journal of Solids and Structures*, 2009, 46: 4123-4132.
- [176] KIEŁCZYŃSKI P, SZALEWSKI M, BALCERZAK A. Effect of a viscous liquid loading on Love wave propagation. *International Journal of Solids and Structures*, 2012, 49: 2314-2319.
- [177] BILLON F, BAROUDI A E. Mathematical modelling of Love waves propagation in viscoelastic waveguide loaded with complex fluids. *Applied*

- Mathematical Modelling*, 2021, 96: 559-569.
- [178] JAKOBY B, VELLEKOOP M J. Properties of Love waves: Applications in sensors. *Smart Materials and Structures*, 1997, 6(6): 668-679.
  - [179] CHÁVEZ P A S, BONHOMME J, BELLAREDJ M L F, et al. Love wave sensor with high penetration depth for potential application in cell monitoring. *Biosensors-Basel*, 2022, 12(2): 61.
  - [180] DRBOHLAVOVÁ L, FEKETE L, BOVTUN V, et al. Love-wave devices with continuous and discrete nanocrystalline diamond coating for biosensing applications. *Sensors and Actuators A Physical*, 2019, 298(1): 111584.
  - [181] TIERSTEN H F, SHAM T L. On the necessity of including electrical conductivity in the description of piezoelectric fracture in real materials. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 1998, 45(1): 1-3.
  - [182] PANG Y, LIU J X, WANG Y S, et al. Propagation of Rayleigh-type surface waves in a transversely isotropic piezoelectric layer on a piezomagnetic half-space. *Journal of Applied Physics*, 2008, 103: 074901.
  - [183] RAMIREZ F, HEYLIGER P R, PAN E N. Free vibration response of two-dimensional magneto-electro-elastic laminated plates. *Journal of Sound and Vibration*, 2006, 292: 626-644.
  - [184] NAN C W, BICHURIN M I, DONG S X, et al. Multiferroic magnetoelectric composites: Historical perspective, status, and future directions. *Journal of Applied Physics*, 2008, 103: 031101.
  - [185] LIU T J C, CHUE C H. On the singularities in a bimaterial magneto-electro-elastic composite wedge under antiplane deformation. *Composite Structures*, 2006, 72: 254-265.
  - [186] KUO H Y, YANG L H, HUANG P C, et al. Wave characteristics in magneto-electro-elastic laminated composites with different layering directions. *Acta Mechanica*, 2023, 234(9): 1-19.
  - [187] JIANG S N, JIANG Q, LI X F, et al. Piezoelectromagnetic waves in a ceramic plate between two ceramic half-spaces. *International Journal of Solids and Structures*, 2006, 43: 5799-5810.
  - [188] YANG J S. *Analysis of piezoelectric devices*. Singapore: World Scientific, 2006.
  - [189] NAKAHATA H, HACHIGO A, HIGAKI K, et al. Theoretical study on SAW characteristics of layered structures including a diamond layer. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 1995, 42(2): 362-375.
  - [190] BIOT M A. The influence of initial stress on elastic waves. *Journal of Applied Physics*, 1940, 11(8): 522-530.
  - [191] QIAN Z H, JIN F, KISHIMOTO K, et al. Effect of initial stress on the propagation behavior of SH-waves in multilayered piezoelectric composite structures. *Sensors and Actuators A Physical*, 2004, 112: 368-375.
  - [192] ZHAO X, QIAN Z H, ZHANG S, et al. Effect of initial stress on propagat-

- ion behaviors of shear horizontal waves in piezoelectric/piezomagnetic layered cylinders. *Ultrasonics*, 2015, 63: 47-53.
- [193] LEE S W, RHIM J W, PARK S W, et al. A micro rate gyroscope based on the SAW gyroscopic effect. *Journal of Micromechanics and Microengineering*, 2007, 17(11): 2272-2279.
- [194] OH H, LEE K J, YANG S S, et al. Development of novel dual-axis sensing gyroscope using surface acoustic wave. *Microelectronic Engineering*, 2012, 97: 259-264.
- [195] FANG H Y, YANG J S, JIANG Q. Rotation-perturbed surface acoustic waves propagating in piezoelectric crystals. *International Journal of Solids and Structures*, 2000, 37: 4933-4947.
- [196] FANG H Y, YANG J S, JIANG Q. Surface acoustic waves propagating over a rotating piezoelectric half-space. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 2001, 48(4): 998-1004.
- [197] FANG H Y, YANG J S, JIANG Q. Rotation sensitivity of waves propagating in a rotating piezoelectric plate. *International Journal of Solids and Structures*, 2002, 39: 5241-5251.
- [198] YANG J S, FANG H Y. Analysis of a rotating elastic beam with piezoelectric films as an angular rate sensor. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 2002, 49(6): 798-804.
- [199] YANG J S. A review of analyses related to vibrations of rotating piezoelectric bodies and gyroscopes. *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control*, 2005, 52(5): 698-706.
- [200] SHARMA J N, WALIA V. Effect of rotation on Rayleigh waves in piezothermoelastic half space. *International Journal of Solids and Structures*, 2007, 44: 1060-1072.
- [201] SHARMA J N, OTHMAN M I A. Effect of rotation on generalized thermo-viscoelastic Rayleigh-Lamb waves. *International Journal of Solids and Structures*, 2007, 44: 4243-4255.
- [202] SINGH P, SINGH A K, CHATTOPADHYAY A. Reflection of three-dimensional plane waves at the free surface of a rotating triclinic half-space under the context of generalized thermoelasticity. *Applied Mathematics and Mechanics (English Edition)*, 2021, 42(9): 1363-1378.
- [203] SINGH P, SINGH A K, PASWAN B, et al. Mathematical study on reflection and transmission of plane waves in a rotating piezo thermo elastic composite structure. *Mechanics of Advanced Materials and Structures*, 2022, 30(14): 2941-2952.
- [204] WANG W, XU F Q, HE S T, et al. A new micro-rate sensor based on shear horizontal surface acoustic wave gyroscopic effect. *Japanese Journal of Applied Physics*, 2010, 49: 096602.
- [205] WANG W, LIU J L, XIE X, et al. Development of a new surface acoustic wave based gyroscope on a X-112°Y LiTaO<sub>3</sub> substrate. *Sensors*, 2011, 11: 10894-10906.
- [206] WANG W, SHAO X T, LIU J L, et al. Enhanced sensitivity of surface

- acoustic wave-based rate sensors incorporating metallic dot arrays. *Sensors*, 2014, 14: 3908-3920.
- [207] ZHU F, WANG B, QIAN Z H. A numerical algorithm to solve multivariate transcendental equation sets in complex domain and its application. *Acta Mechanica*, 2019, 230: 1303-1321.
- [208] SOH A K, LIU J X. Interfacial shear horizontal waves in a piezoelectric-piezomagnetic bi-material. *Philosophical Magazine Letters*, 2006, 86(1): 31-35.
- [209] GRAFF K F. *Wave motion in elastic solids*. New York: Dover, 1975.
- [210] THOMSON W T. *Theory of vibration with applications*. 4th ed. New Jersey: Prentice Hall, 1993.
- [211] DONG S X, BOUCHILLOUX P, DU X H, et al. Ring type uni/bimorph piezoelectric actuators. *Journal of Intelligent Material Systems and Structures*, 2001, 12(9): 613-616.
- [212] CUI J, DU J K, WANG J, et al. A piezoelectric gyroscope based on thickness-shear modes of an AlN bimorph with inclined *c*-axes. *Philosophical Magazine Letters*, 2014, 94(7): 447-454.
- [213] POZZI M. Magnetic plucking of piezoelectric bimorphs for a wearable energy harvester. *Smart Materials and Structures*, 2016, 25(4): 045008.
- [214] BENASCIUTTI D, MORO L, ZELENKA S. Vibration energy scavenging via piezoelectric bimorphs of optimized shapes. *Microsyst Technol*, 2010, 16: 657-668.
- [215] VEL S S, BATRA R C. Analysis of piezoelectric bimorphs and plates with segmented actuators. *Thin-Walled Structures*, 2001, 39: 23-44.
- [216] WANG G L, NIE G Q, LIU X L, et al. Magnetically induced redistribution of mobile charges in bending of composite beams with piezoelectric semiconductor and piezomagnetic layers. *Archive of Applied Mechanics*, 2021, 91: 2949-2956.
- [217] GAO X T, SHIH W H, SHIH W Y. Induced voltage of piezoelectric unimorph cantilevers of different nonpiezoelectric/piezoelectric length ratios. *Smart Materials and Structures*, 2009, 18: 125018.
- [218] MINDLIN R D. High frequency vibrations of piezoelectric crystal plates. *International Journal of Solids and Structures*, 1972, 8: 895-906.
- [219] TSUBOUCHI K, SUGAI K, MIKOSHIBA N. AlN material constants evaluation and SAW properties on AlN/Al<sub>2</sub>O<sub>3</sub> and AlN/Si. *Proceedings of the IEEE Ultrasonics Symposium, IEEE*, 1981: 375-380.

# Appendix A

## List of publications

### A.1 Journal articles

1. **Yang L**, Zappino E, Carrera E, Du J K. Shear horizontal waves in a multiferroic composite semiconductor structure. *Ultrasonics*, 2024, 139: 107287.
2. **Yang L**, Zappino E, Carrera E, Du J K. Rotation effects on propagation of shear horizontal surface waves in piezomagnetic-piezoelectric semiconductor layered structures. *Applied Mathematical Modelling*, 2024, 129(20): 494-508.
3. **Yang L**, Du J K, Yang J S. Interaction between bending and mobile charges in a piezoelectric semiconductor bimorph. *Applied Mathematics and Mechanics (English Edition)*, 2022, 43(8): 1171-1186.
4. **Yang L**, Du J K, Wang J, Yang J S. An analysis of piezomagnetic-piezoelectric semiconductor unimorphs in coupled bending and extension under a transverse magnetic field. *Acta Mechanica Solida Sinica*, 2021, 34(5): 743-753.
5. **Yang L**, Du J K, Wang J, Yang J S. On the capacitance of piezoelectric metal-insulator-semiconductor junctions. *Ferroelectrics Letters Section*, 2021, 48(1-3): 1-12.
6. Chen Y M, **Yang L**, Lou J, Wang J, Filippi M, Carrera E, Fang X. Bio-inspired elastic metamaterial by B-form DNA: Programmable dual helix structures for low-frequency longitudinal wave prohibition. *Composite Structures*, 2025, 360: 118986.
7. Chen Y M, **Yang L**<sup>\*</sup>. SH wave propagation in piezoelectric semiconductor plate with initial stress. *Chinese Journal of Solid Mechanics*, 2022, 43(4): 467-476 (in Chinese).
8. Chen Y M, **Yang L**, Fang X, Wang J. Shear-horizontal surface waves in a half-space of piezoelectric semiconductor with initial stresses. *Acta Physica Polonica A*, 2022, 142(2): 272-279.

9. Yang G Y, **Yang L**, Du J K, Wang J, Yang J S. PN junctions with coupling to bending deformation in composite piezoelectric semiconductor fibers. *International Journal of Mechanical Sciences*, 2020, 173: 105421.

## A.2 Conference Proceeding

1. **Yang L**, Du J K, Carrera E. Love waves propagation in layered piezoelectric semiconductor and piezomagnetic composite structures, In: *International Conference on Advanced Topics in Mechanics of Materials, Structures and Construction (AToMech-1)*, Al Khobar, Saudi Arabia, 12-14 March, 2023.
2. **Yang L**, He J L, Fang C L, Du J K. Love waves propagation in layered piezoelectric semiconductor and elastic structures under biasing electric field, In: *Chinese Congress of Theoretical and Applied Mechanics (CCTAM)*, Chengdu, China, 5-10 November, 2022.
3. **Yang L**, Chen Y M, Du J K. Propagation characteristics of SH waves in piezoelectric semiconductor plate with initial stress, In: *2nd National Academic Forum on PhD Students in Mechanics*, Hangzhou, China, 15-17 October, 2021.