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RESEARCH

Geometric Studies and Algorithmic Modeling of the Double Staircase of Palazzo Carignano

Roberta Spallone[1] · Marco Vitali[1] · Michele Russo[2] · Fabrizio Natta[1]

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Abstract
This paper examines the relationship between geometry and architecture in Guarino Guarini's design and building activities. The study was performed using methodologies that integrate and interconnect analyses, redrawings, and interpretations. A case study of the double staircase of Palazzo Carignano was performed by comparing autograph drawings with a building survey. Redrawing of the original plates and visual programming language modeling of the point cloud section enabled thorough comparison between the ideation and construction phases. The analyses shed new light on different original drawings to explore the Guarinian design process. The link between geometry and architecture is evident in not only his theoretical works, but also his design method and realized buildings.

Keywords Geometry · Digital survey · Reconstructive drawing · Algorithmic modelling · Palazzo Carignano · Guarino Guarini

Introduction

Geometry and architecture appear inextricably connected in the theoretical and design work of Guarino Guarini (1624–1683), who was an abbot of the Theatine order, an architect, and a treatise writer in not only the field of architecture, but

✉ Roberta Spallone
roberta.spallone@polito.it

Marco Vitali
marco.vitali@polito.it

Michele Russo
m.russo@uniroma1.it

Fabrizio Natta
fabrizio.natta@polito.it

1 Politecnico di Torino, Turin, Italy

2 Sapienza Università di Roma, Rome, Italy

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also in mathematics, philosophy, and astronomy. A common thread that links the discussion of geometric principles and architectural models in his treatises on architecture is reflected in his designs and buildings, where geometry and architecture are intertwined from the general concept to the formal definition of the spaces, including their interconnection. This paper explores this link using methodologies that develop and interlace analyses, redrawings, and interpretations. The focus is on the design and construction of the double staircase of Palazzo Carignano and comparison of autograph drawings with a building survey. Digital redrawing has facilitated the verification of hypothesized geometric constructions with respect to the archival documents, while algorithmic modeling allows for the testing of different solutions aimed at the best possible adherence to the data collected through digital surveying. This approach was motivated by observations of the formal interdependence between the rooms of the ceremonial route, which is composed of an entrance hall, atrium, double staircase, side vestibules, and vestibule leading to the main hall.

Guarinian Studies Between Geometry and Architecture

Studies on Guarini, particularly those investigating the relationship between geometry and architecture in his designs and buildings, include the seminal work by Paolo Portoghesi (1956). This work was revisited in 1970 (Portoghesi 1970) and has recently been enriched with photographs (Portoghesi 2024) in an analysis of the structures of the Teatine's language. Portoghesi identified recurring compositional operations and recognized the unitary conception of the entire structure with reference to the numerous preparatory plan drawings of Palazzo Carignano, which was centered on the cylindrical volume of the atrium and the main hall (Portoghesi 2024: 35). A few years later, Mario Passanti (1963) examined the drawings of Guarini that are kept at the *Archivio di Stato—Sezioni Riunite* in the *Azienda Savoia-Carignano* collection. The study involved interpretive redrawing and geometric constructions aimed at reconstructing the design phases with a focus on the ceremonial route that led to the current configuration.

A particular boost to Guarinian studies was provided by the conference *Guarino Guarini e l'internazionalità del Barocco* (1970), which included an annotated record of the drawings in the *Archivio di Stato* (Lange 1970) and the geometric interpretation of the master's work. The conference books also included the specific reconstruction of the geometry of the main hall and staircases of Palazzo Carignano in some original design plates (Millon 1970: 40–58) and a study (Fagiolo 1970: 191–193) later discussed by McQuillan (2009) that links the design of the steps of the staircase in Palazzo Carignano with the schematic representation of planetary motion in *Placita philosophica* (Guarini 1665: 326).

Harold Meek's studies contribute to the theme of ceremonial spaces with considerations relating to the strip plans in the two plates concerning the palace

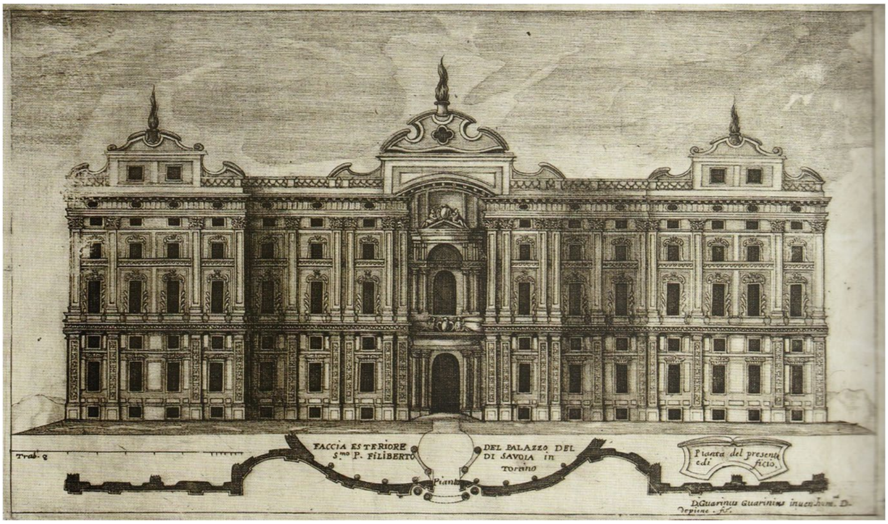

Fig. 1 Palazzo Carignano, strip plan and façade on the square. Image: Guarini (1737)

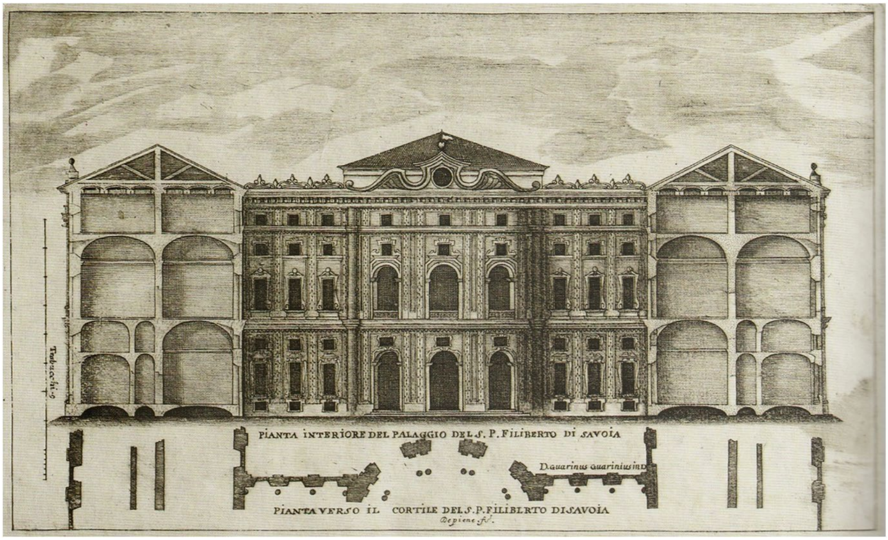

Fig. 2 Palazzo Carignano, strip plan and cross-section on the courtyard. Image: Guarini (1737)

(Meek 1988: 92) in *Architettura civile*, which was published posthumously in 1737 (Figs. 1, 2). However, no reconstructive studies of the original design drawings have linked them to the metric analysis of the existing building using current integrated digital survey techniques and algorithmic modeling interpretation.

The Double Staircase and Palazzo Carignano

The double staircase is a *unicum* in the panorama of Baroque architecture. Its shape and position are the result of a comprehensive design process that began to take form in 1679 with its construction. Among the approximately 50 drawings of the palace catalogued by Lange (1970: 317–337), many represent the general plan of the ground floor or focus on the ceremonial route. Thanks to these drawings, following Passanti (1963), Cerri (1990), and Millon (2006), we can reconstruct the evolution of the project.

The project began as a traditional layout with a rectangular atrium flanked by two separate staircases overlooking the courtyard (one oval and the other rectangular) (Fig. 3) and later developed into an oval atrium around which the double staircase is wrapped (Fig. 4). It was then changed to a symmetrical zigzag staircase that leads to the square, shaping the façade with a mixtilinear outline and beginning to partially

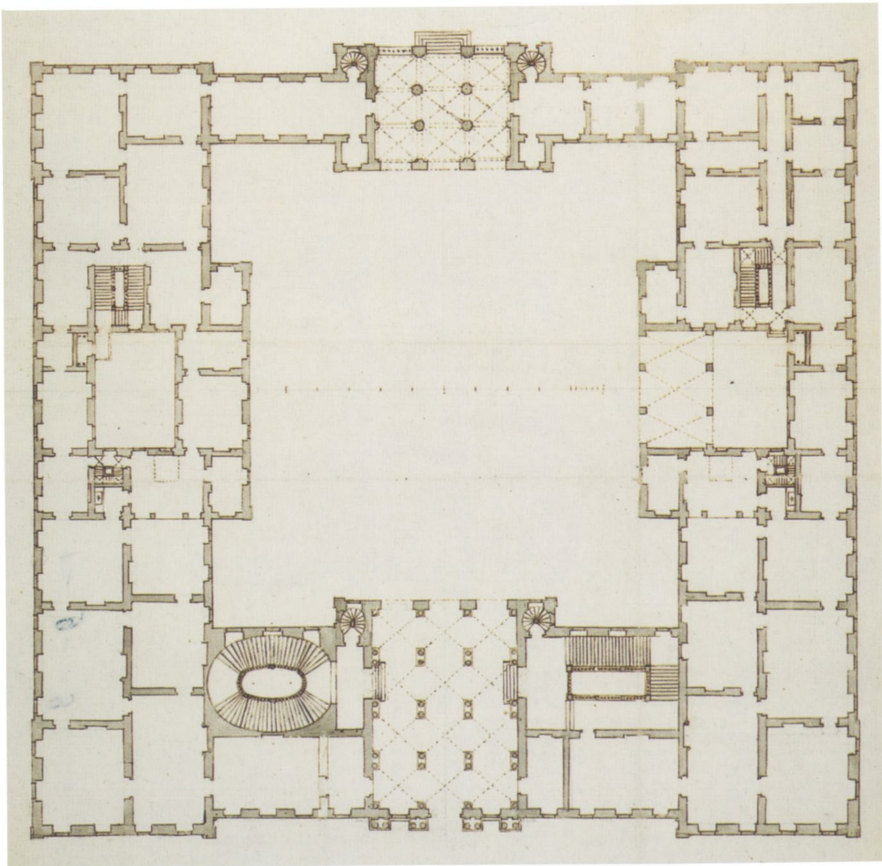

Fig. 3 Palazzo Carignano, design drawing of the ground plan with different staircases facing the courtyard on the rectangular atrium sides. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 51, m. 1, fasc. 9, n. 9

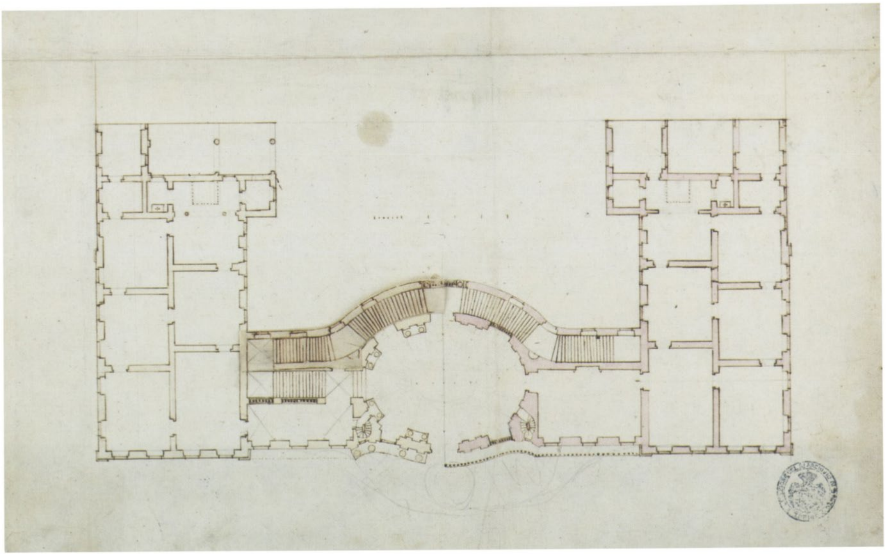

Fig. 4 Palazzo Carignano, design drawing of the ground plan with double staircase facing the courtyard and oval atrium. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 7

detach itself from the oval atrium (Fig. 5). Its final position on the façade involves a curved plan that is geometrically independent of the oval atrium. The concave and convex curvatures of the steps appear only in this final drawing (Fig. 6).

Research Methodology and Pipeline

The research involved analyses, redrawings, and interpretations applied in parallel to archival documents and digital surveys. Deep research into the bibliographical references related to Guarini and the palace was performed in the source-based analysis. They included Guarini's treatise, *Architettura civile,* and other treatises that deal with stairs in civil architecture, such as those by Serlio (1540) and Palladio (1570).

Redrawing involved selecting archival drawings that are useful for defining the geometric construction, dimensions, and morphological relationships between the staircase and adjacent rooms. The interpretive phase has enabled us to reach a plausible reconstructive hypothesis about the final design of the stairs in relation to the entrance hall, atrium, and vestibules on the ground floor and to examine hypotheses about the reconstruction of the traces on the ground. These endeavors were made possible by the interpretation of point cloud sections, which allowed us to highlight the variations between design and construction. The parallel phases of reality-based research have been performed through digital surveys using terrestrial laser scanning, the processing of point clouds to obtain plan sections, and interpretation using algorithmic modeling tools for managing the concatenation between geometric figures that characterize the Guarinian design method.

Archival Drawings Selection and Graphical Analysis

All the original documents have been examined directly at the *Archivio di Stato di Torino* to identify pencil tracings and compass holes, and annotations and dimensions were read when present. Digitized plates were compared with the original ones and redrawn by superimposition using the *oncia* as a unit of measurement. Guarini refers to the local units of measurement: *trabucchi*, *piedi liprandi,* and *once* (1 *trabucco* is equal to 3.0825 m and is composed of 6 *piedi liprandi*; 1 *piede liprando* contains 12 *once*; therefore, 1 *trabucco* is equal to 72 *once*). A detailed drawing related to the final design (Lange 1970: 264–265) could be considered the primary source for discovering the design process of the staircase and was a focal point for later analyses. By consulting Lange's catalog, three other drawings that feature a scalebar have been identified, which help to understand the geometric construction of the main hall and its relationship to that of the double staircase.

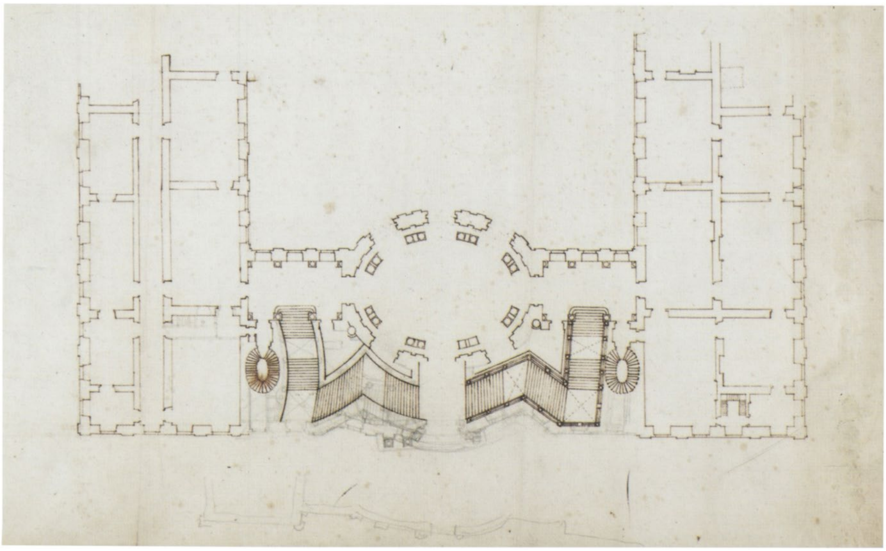

Fig. 5 Palazzo Carignano, design drawing of the ground plan with zigzag double staircase facing the square and oval atrium. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 6

Millon (1970) created geometric reconstructions of these drawings, which primarily focused on the oval shape of the main hall, but he did not relate them to measurements inferred from the scalebar. His schemes were not superimposed on Guarini's plates, which sometimes makes it difficult to interpret his work. For this reason, Millon's tracings have been digitally redrawn on the digitized original drawings while taking care to scale them using the scalebar so that measurements could be taken.

The first plate attributed to Guarini (Lange 1970: 322) measures 210 mm × 285 mm and represents the main hall on the noble floor. In this

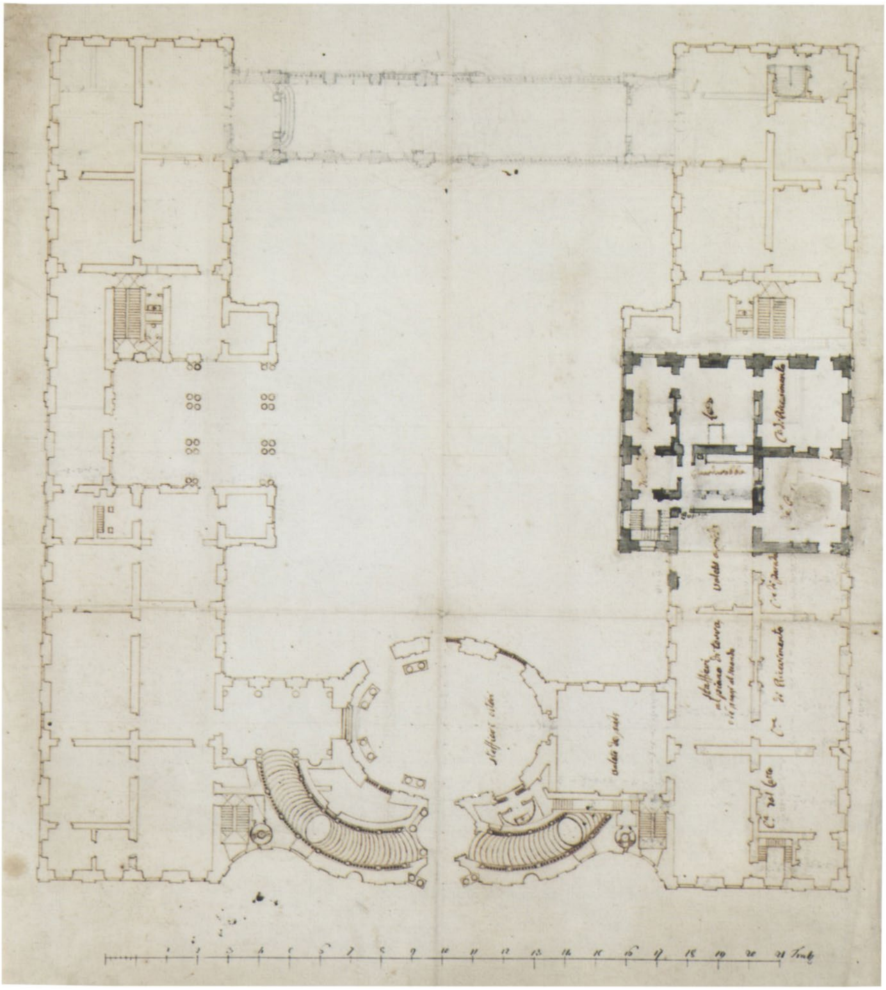

Fig. 6 Palazzo Carignano, design drawing of the ground plan with curvilinear double staircase facing the square and oval atrium. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, nn. 4, 4bis

drawing, Millon notes the use of an oval with the major axis equal to three radii of the circumference for shaping the room's internal wall (Fig. 7).

The second plate attributed to Guarini (Lange 1970: 320) measures 390 mm × 555 mm and depicts the plan of the ceremonial route on the noble floor. It includes the upper vestibule, the main hall, and the staircases. Millon's analysis reveals an attempt to determine the center of curvature of the staircase's walls, which is one of the primary issues in reconstructing the detailed drawing, as we shall see. Millon locates this center through a complex geometric construction (Fig. 8) that is not entirely convincing because it is not determined by the position of the building's elements based on the concatenation of geometric constructions strictly related to parts of the architecture, as in Guarini's method.

The third drawing is attributed by Lange to Guarini and his collaborator X (Lange 1970: 321) and also represents the plan of the ceremonial route on the noble floor. The plate measures 420 mm × 490 mm. In this drawing, Millon notes the presence of three points that are close to each other and probable centers of the curves of the northern staircase walls, while the southern staircase has only one center of curvature. This indicates that finding the center of curvature remains a challenge for

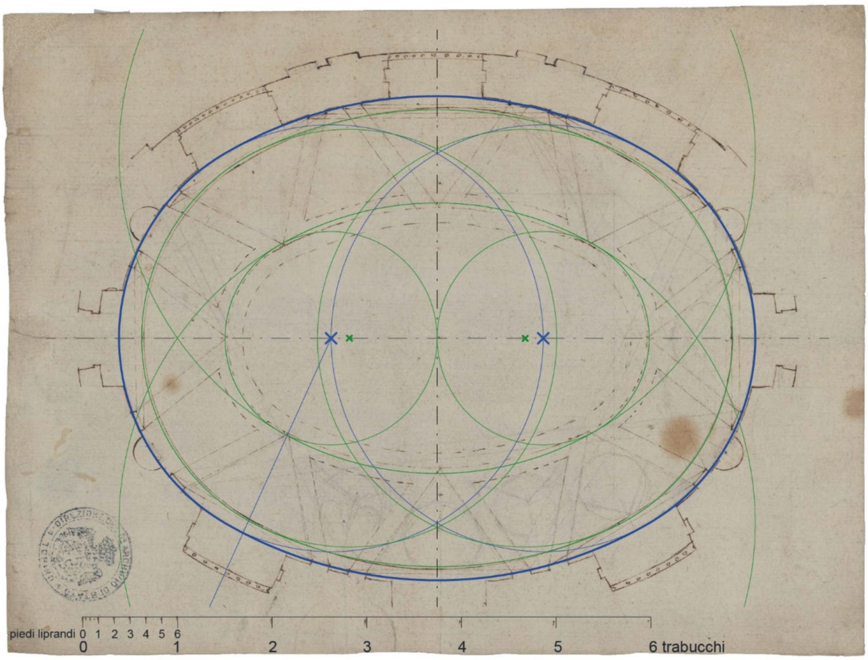

Fig. 7 Palazzo Carignano, design drawing of the main hall at the noble plan. Superimposition of the geometric construction following Millon (1970). In blue: geometric construction of the oval with the major axis equal to three radii of the circumference that shapes the interior perimeter of the main hall. In green: the other constructions by Millon for defining the central oval of the dome. Image: Archivio di Stato di Torino – Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 9. Digital redrawing: Roberta Spallone (color figure online)

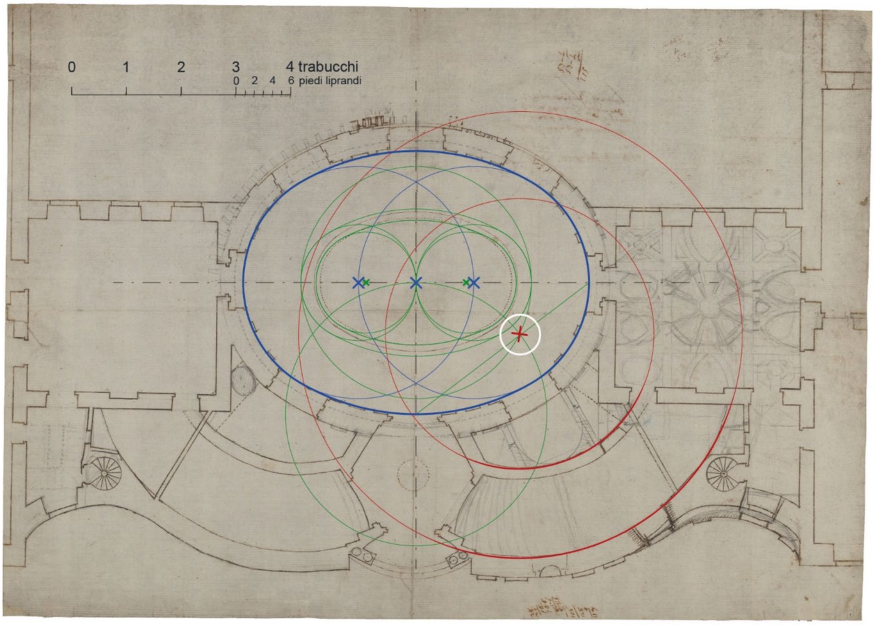

Fig. 8 Palazzo Carignano, design drawing of the main hall and the double staircase at the noble plan. Superimposition of the geometric construction following Millon (1970). In blue: geometric construction of the oval with the major axis equal to three radii of the circumference that shapes the interior perimeter of the main hall. In green: the other constructions by Millon for defining the central oval of the dome. In red: the hypothesized centre of curvature of the southern staircase. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia- Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 15. Digital redrawing: Roberta Spallone (color figure online)

his reconstructions. It is also possible to deduce that the staircase's walls could have different geometric constructions (Fig. 9).

Graphical Analysis of the Detailed Drawing

The study of Millon's constructions provided a relevant cue for investigating the constructions on the detailed drawing by Guarini. It represents a portion of the ceremonial route. Separate from the symmetry axis of the complex, the drawing shows half of the entrance hall, half of the upper vestibule, half of the atrium, the southern vestibule, and the south staircase.

Lange attributed the drawing to Guarini and stated that the dimensions were written by an anonymous draughtsman, who was identified as collaborator Z. The plate dimensions are 295 mm × 390 mm, and the drawing was made with pen and pencil on paper with pink backgrounds inside the sectioned walls. Pencil constructions and compass holes are visible. The dimensions annotated by collaborator Z are in *trabucchi* (T), *piedi liprandi* (P), and *once* (o). In the

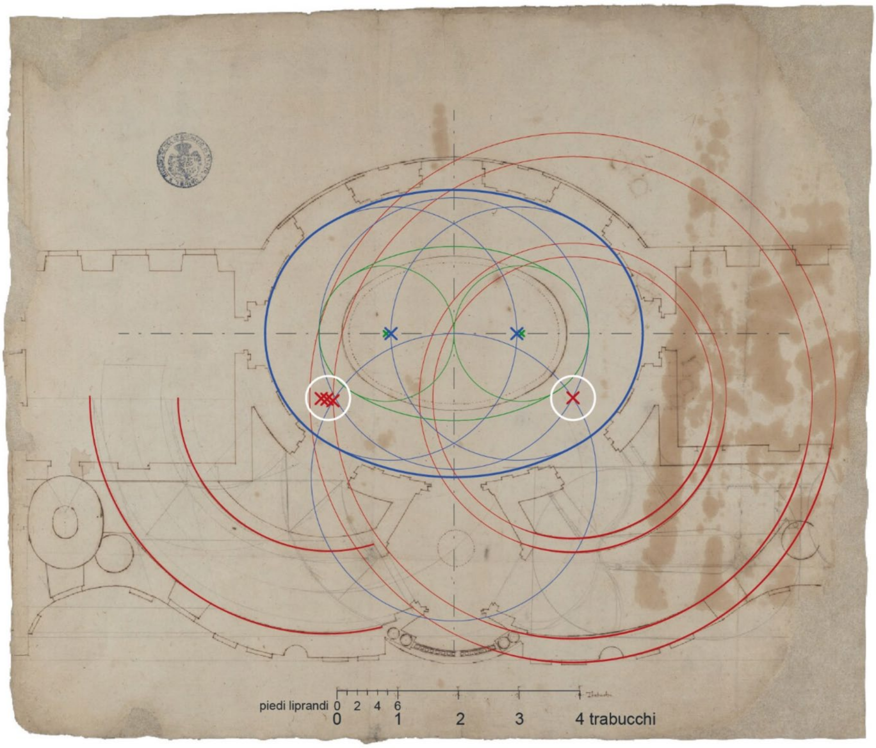

Fig. 9 Palazzo Carignano, design drawing of the atrium at the noble plan. Superimposition of the geometric construction following Millon (1970). In blue: geometric construction of the oval with the major axis equal to three radii of the circumference that shapes the interior perimeter of the main hall. In green: the other constructions by Millon for defining the central oval of the dome. In red: the hypothesized centres of curvature of the southern and northern staircases. Image: Archivio di Stato di Torino – Sezioni Riunite, Disegni Azienda Savoia- Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 30. Digital redrawing: Roberta Spallone (color figure online)

following text, the abbreviations T, P, and o are used to indicate the measurements preceding the numerical value, as done by collaborator Z.

It is very important to note that the portion of the drawing representing the staircase has been pasted on the plate containing the remaining part of the drawing. This could mean that the staircase itself was the subject of different versions. Regardless, the overlap shows only minimal inaccuracies in the gluing.

An autograph note by Guarini placed at the bottom specifies the risers' numbers and dimensions: 'Height of each riser *once* 3 ½, which multiplied by 58 risers gives 203, that is *piedi liprandi* 16, *once* 6: to arrive at 17, two risers in the door of the staircase, *once* 7' (Fig. 10). The reconstruction has been divided into two parts. One part focuses on determining the position of the internal walls of the rooms along the ceremonial route and the thickness of the staircase walls, while the other part focuses on defining the landing and steps of the staircase.

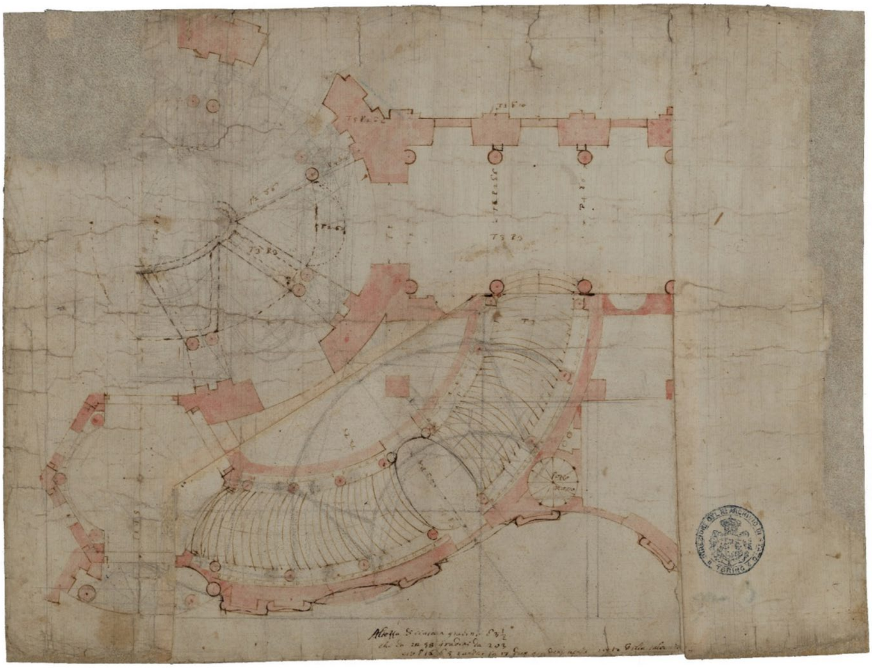

Fig. 10 Palazzo Carignano, detailed design drawing of the south sector of the ceremonial route, from the ground to the noble plan. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 3

For the position of the walls, two different reconstruction methods were adopted. The first was aimed at maximum fidelity to Guarini's drawing. The non-autographic dimensions were considered only to confirm the tracings, and in the case of discrepancies, priority was given to the autograph drawing. Conversely, the second method prioritized the dimensions in cases of discrepancies.

First Reconstruction of Spaces and Walls

Following the first method, it is possible to observe that all measures base on the *trabucco* (i.e., 72 *once*) feature some figures of the drawing: T2 for the two radii of the construction circumferences for the oval plan of the atrium, T1 for the radius of the circumference inscribed in the hexagon of the entrance hall, T2 for the width of the southern vestibule, T4 for the radius of the circumference that defines one of the walls of the staircase, T1 for the width of the stairs, and T1 for the radius of the arc of circumference of the landing and treads. The construction of the staircase walls hinges on a point near one of the atrium's columns. The lack of geometric constructions related to this point results in uncertain data for reconstruction.

Nevertheless, this point has been considered the center for a series of circumferences, which define the position and thickness of the internal wall (radii

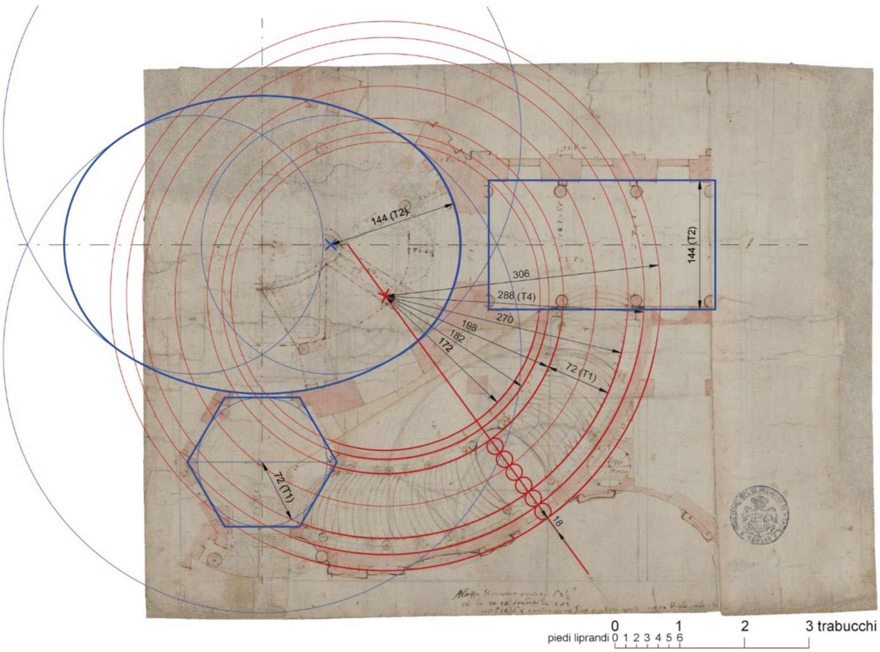

Fig. 11 Palazzo Carignano, detailed design drawing of the south sector of the ceremonial route. Superimposition of the first geometric construction. In blue: geometric constructions and interior perimeters of the entrance hall, the atrium, and the southern vestibule. In red: modularities and geometric constructions of the southern staircase's positions of the walls and balustrades. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 3. Digital redrawing: Roberta Spallone (color figure online)

of o172 and o182) and the internal and external edges of the flights (o198 and o270) from the inside to the outside. From the same point passes the landing axis, which intersects the arc axial to the flights at one point. From this point, a sequence of four modules, each o18, generates points on the axis through which circumferences pass and define the thickness of the external wall (radii of o288 and o306) (Fig. 11).

Second Reconstruction of Spaces and Walls

In the second reconstruction, attention is again focused on the uncertain point that defines the center of curvature of the staircase walls. Dimensions that did not match the drawing or were unclear in regard to the elements to which they referred were considered again. The first, T2 o6 (i.e., o150), may refer to the radius of the circumference generating the oval, unlike the original construction, although in Guarini's drawing, the radius perfectly corresponds to o144, or T2. It should also be noted that the radius of o150 would allow for construction of the oval with a major axis equal to three radii. This oval construction was presented in Serlio's treatise (Serlio 1545: 21) and is also recognizable in the other previously analyzed Guarinian drawings.

The second, T5 P1 (i.e., o372), could indicate one of the points on the horizontal construction line through which the axis of the landing passes. The other is given by the intersection of the two circumferences generating the oval. The third, T4 P1 o2 (i.e., o300), can be referred to as the central circumference, which does not define any element of the construction but helps to identify the center of curvature of the staircase walls by intersecting with the axis of the landing. This second reconstruction hypothesis appears to be the most convincing at the current stage of research (Fig. 12).

Reconstruction Method of Flights

Finally, the construction of the two flights seems to have started from the oval landing: the major axis lies on the axis identified in previous stages of the work, and the minor axis is slightly shifted toward the entrance hall (by about o2) with respect to the flight axis, presumably due to the curvature of the staircase. This oval

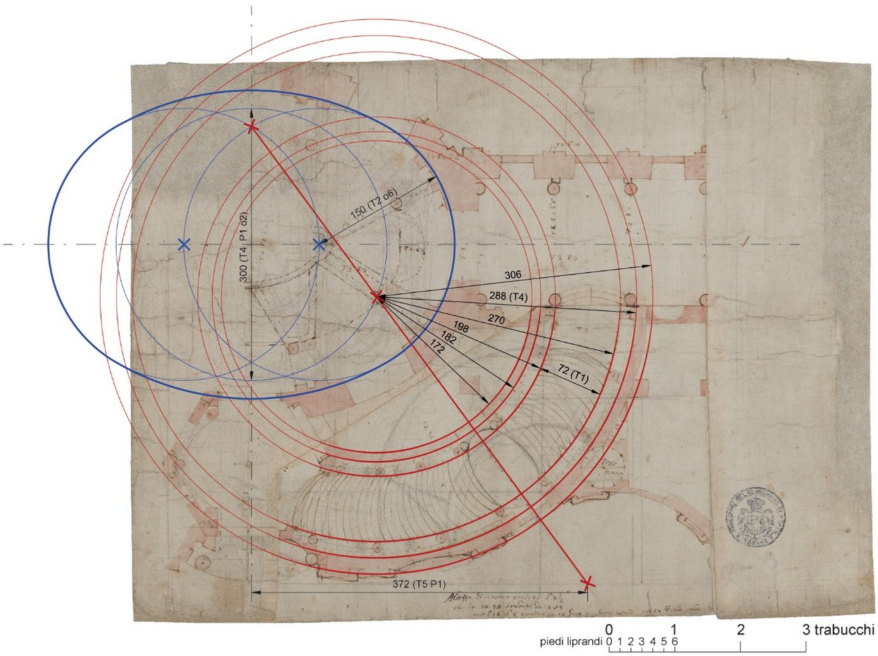

Fig. 12 Palazzo Carignano, detailed design drawing of the south sector of the ceremonial route. Superimposition of the second geometric construction. In blue: geometric constructions and interior perimeters of the atrium, and the southern vestibule. In red: geometric constructions of the southern staircase's positions of the walls and balustrades. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 3. Digital redrawing: Roberta Spallone (color figure online)

determines the position of the last riser of the first flight and the first riser of the second flight. The two flights are made by treads with the same radius of o72 (T1). In Guarini's plate, compass marks divide the perimeter arcs of each flight into 28 parts, which correspond to a total of 58 risers, as indicated in the note above.

The tracing of the treads could have been done using a template with the same radius. A similar technique has been employed in the redrawing, where the initial tread is aligned with the wall of the rectangular vestibule, while the final tread is aligned with the wall of the upper vestibule (Fig. 13). This method digitally replicates the construction logic of the original drawing, with consideration of any inaccuracies due to the use of a compass, and produces a rotation of each step with respect to the axis of the flights. In this way, although each step maintains the same radius of curvature, the lengths of the curve corresponding to the horizontal edge of each step increase, from the ground to the noble floor, by approximately o2.

Finally, the positions of the columns on both sides of the flights were analyzed. In this case as well, the marks left by the compass suggest how the arcs on which the columns are arranged were drawn concentrically with the perimeter arcs divided into three equal parts that define the axis of each of them. Through the centers of

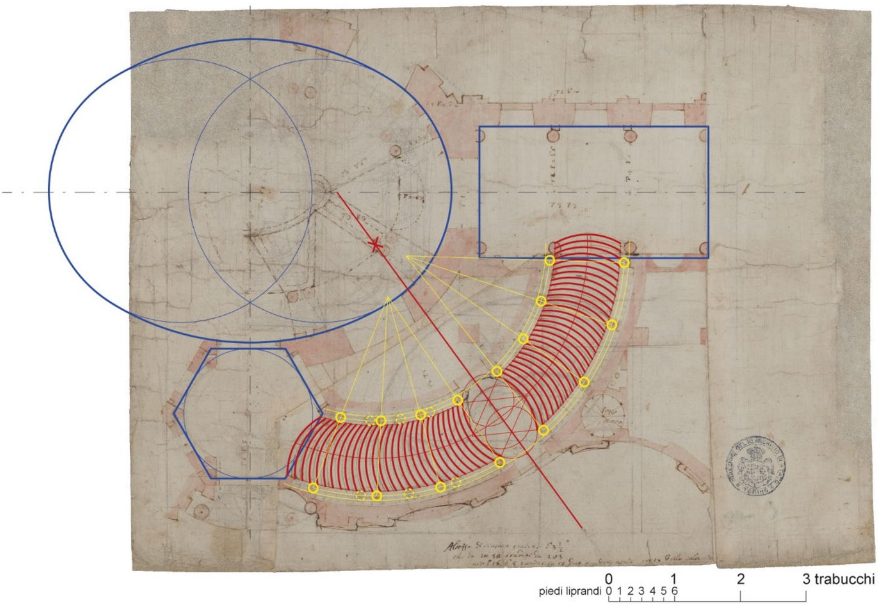

Fig. 13 Palazzo Carignano, detailed design drawing of the south sector of the ceremonial route. Superimposition of the landing and treads. In blue: geometric constructions and interior perimeters of the entrance hall, the atrium, and the southern vestibule. In yellow: geometric constructions of the balaustrades' column centers and column positions. In dashed yellow: partially erased geometric constructions of the balustrades' columns position. In red: geometric constructions of the southern staircase's landing and risers. Image: Archivio di Stato di Torino—Sezioni Riunite, Disegni Azienda Savoia—Carignano Torino Palazzo Carignano, Cat. 53, m. 1, fasc. 9, n. 3. Digital redrawing: Marco Vitali (color figure online)

the couples of columns that correspond on opposite sides of the staircase, it is also possible to draw axes that meet at a point (one point for the axes of each flight).

However, analysis of the plate does not allow them to be identified unambiguously. Defined in this way, the positions of the columns on opposite sides of the flights do not appear to be accurately aligned with the risers. The plate shows some probable design changes regarding the positioning of the columns: a freehand sketch suggests narrowing the landing on the inside of the staircase, changing its shape from oval to ovoid, and consequently moving the adjacent columns. Additionally, some very light compass holes on the second flight appear to have been erased from a previous solution, which included an extra column on each side.

Digital Survey

When the construction practice follows the design phase, there is often a process of readjustment, in which the design ideas can be deviated from according to the surrounding operating conditions, the materials used, and the skill of the construction workers. For this reason, historical architecture represents a unique and unrepeatable outcome of a complex process in which multiple variables interact. Therefore, from a theoretical point of view, case studies may sometimes differ from reality. However, theoretical studies cannot be ignored as they consolidate a single process that develops across different models of knowledge, which can lead to greater understanding.

Surveying in the sense of studying, analyzing, and interpreting existing data is primarily intended to collect such information in space to understand and interpret the built environment, which can building a bridge between design drawings and their possible interpretations and representations. This aim is complex when dealing with architecture such as Palazzo Carignano, where many difficulties must be overcome. First and foremost, the complex morphology of the structure necessitates the use of range-based 3D acquisition techniques that can provide geometric and radiometric accuracy that is consistent with the values of the structure being analyzed. Furthermore, the density of the acquired data and the distribution of the instrument stations must take into account the morphological complexity of the case study, as well as the management of the data, according to its subsequent use as a basis for 3D remodeling.

An RTC360 3D laser scanner (Leica Geosystems) was used for range-based survey. This scanner can acquire high-quality radiometric data from optically non-cooperative surfaces using HDR photography. For the double staircase, 48 scans were acquired at resolutions ranging from 3 mm@10 m to 6 mm@10 m with color data. The distribution of the scans followed a spatial development of intervisible positions to optimize data overlap and reduce drifting between point clouds. The data were checked in situ in real time using the Cyclone Field 360 application.

The clouds can be aligned automatically using a visual inertial system (VIS), and the point cloud coverage can be controlled in real time using the Cyclone Field 360 application. These capabilities minimized the number of shadow areas. Finally, the 3D laser scanner data were processed in Register software (Leica Geosystems).

The pre-alignments were verified in situ, and multiple outliers (mainly due to public interference) were eliminated. The final cloud was then sub-sampled at different resolution levels (0.5 cm, 1 cm, and 2 cm) to provide a range of detail for management during the interpretative reconstruction phase.

Two final aspects ensured the usability and reliability of the output data. An external reference system that is consistent with the spatial distribution of the building system (the X-axis along the main façade) was defined, which allowed for a system that could be implemented over time and easily managed in the 3D modelling phase. To verify the quality of the collected data, a 3D topographic network was created using a precision total station (Zoom 35 Pro, Geomax) with a closed loop along the main staircase of the building, which yielded a closure error of less than 3 mm in both planimetry and elevation. With the station at the network's vertices, 61 control points were acquired, which correspond to the same number of architectural details.

The coordinates were then roto-translated into the same reference system as the range-based data, which yielded an average residual of 16 mm. This demonstrates both the precision of the range-based coordinates and the accuracy of the entire model. Therefore, the final point cloud can serve as a reliable basis for the 3D modelling process (Fig. 14).

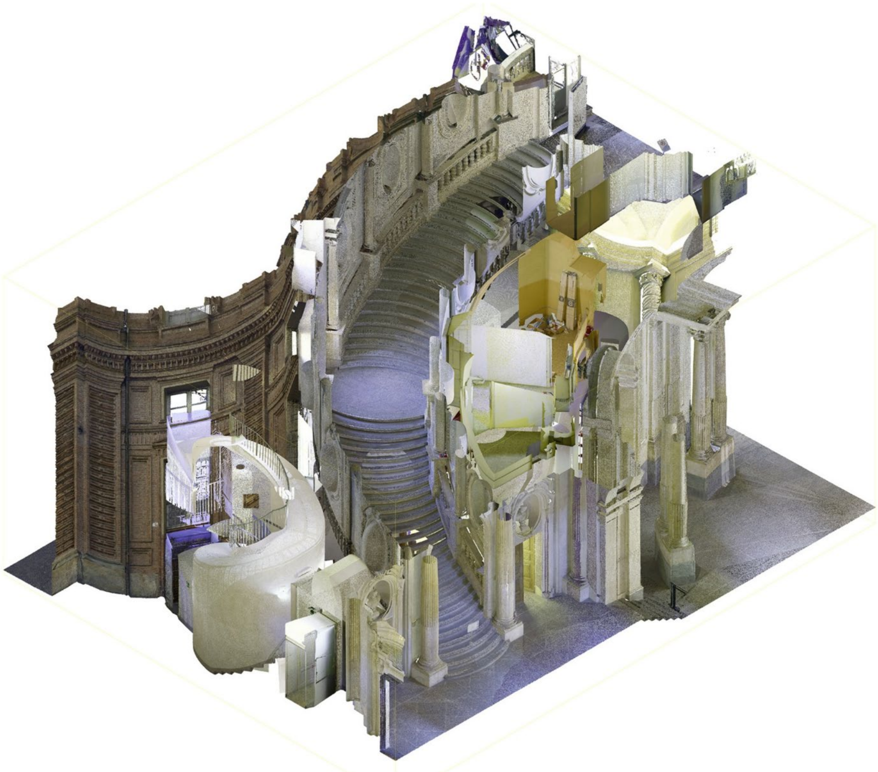

Fig. 14 Sectioned point cloud of the southern staircase. Point cloud processing: Michele Russo

Parametric Workflow from Design Drawings to the Built Palace

Geometric reconstructions of the Guarini's detailed drawing were then translated into algorithmic rule sets in a visual programming language (VPL) environment to compare the source's interpretation with the point cloud section. The choice of VPL was motivated by the capabilities of an algorithmic approach, where processes are defined as sequences of explicit logical steps to create a parametric model in which geometry is governed by editable parameters and their relationships (Paris and Valenti 2015; Giovannini 2023). The model was validated through an iterative feedback loop where the concatenation of geometric figures generated by the script was digitally overlaid onto the point cloud, and any deviations were analyzed to identify the best-fitting geometric constructions. The final output is a parametrically defined model that is geometrically coherent, governed by a logical rule set, and metrically accurate thanks to the digital survey of the building (Fig. 15).

Parametric Reconstruction and Geometric Analysis of the Southern Staircase

The parametric reconstruction of the southern staircase using the *once* as units of measurement was initiated by modeling the plan of the rooms surrounding the stairs starting from the atrium (Fig. 16). The digital survey confirmed its regular oval plan. The generative algorithm for the oval was identified as a subdivision of the major axis (which measures o453) into three equal parts (o151 each), which define the centers of the arcs of the oval. The adjacent entrance hall was modeled based on a regular hexagon circumscribed to a circle with a radius of o72 (T1), while the vestibule was defined by a rectangle (o144 by o246). The parametric definition of these spaces established the geometric framework necessary for situating the staircase walls and connecting the rooms.

A central element of the analysis involved testing different hypotheses for the geometric construction of the staircase's walls. The first hypothesis postulated a single-center curve and was modeled algorithmically. This model produced significant deviations when compared to the point cloud, particularly at the junctures with the landings, where the generated curve failed to match the slight shift in curvature revealed by the survey.

This led to the successful validation of a more complex hypothesis, which refers to two centers of curvature being necessary to define the staircase's walls. The first center C1 is located on the symmetry axis of the landing and can be positioned near a column's base vertex in the atrium. This point serves as the center of the concentric arcs of the upper flight walls (with radii of 186, 197, 274, and 286 *once*). The second center, C2, is on the same axis at a distance of o14 from C1 and defines the arcs of the lower flight walls, which have the same thicknesses.

Both flights feature a constant width of o77, and the individual treads maintain a radius of curvature of o72 (1 T). The southern staircase comprises 51 risers, which are divided into a first flight of 27 and a second of 24 risers. These geometric

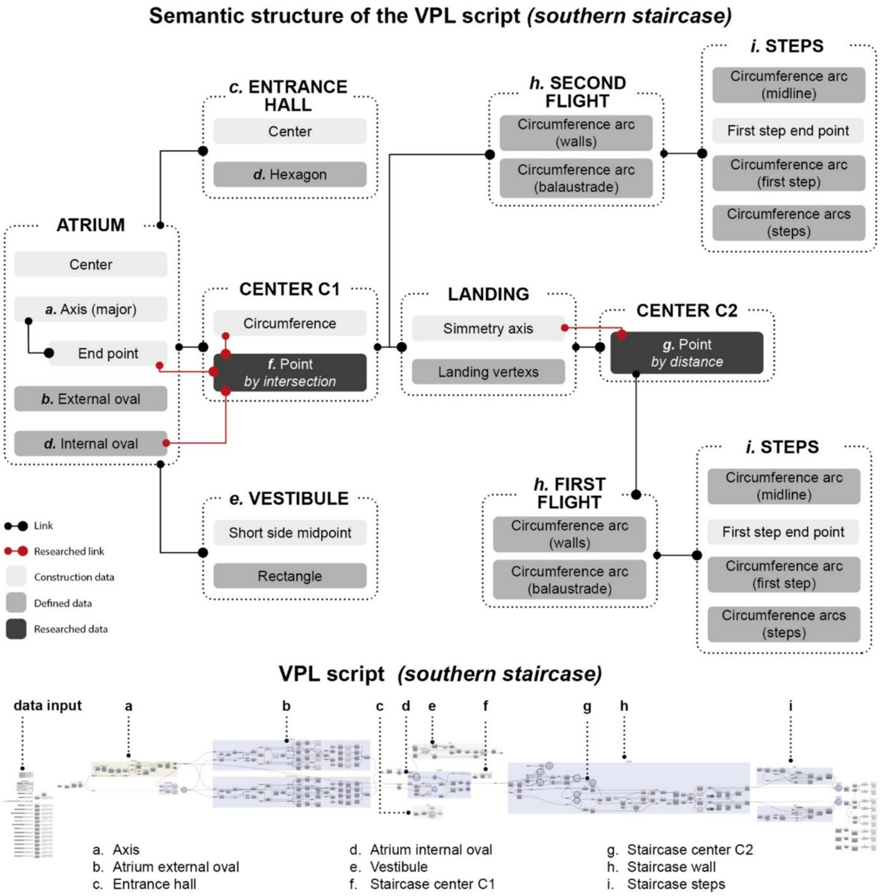

Fig. 15 Southern Staircase VPL: semantic structure and script organized by computational groups (**a**–**i**). VPL processing: Fabrizio Natta

features were integrated as input parameters in the VPL script. The parametric model generated from the defined set of algorithmic rules is geometrically consistent with the built staircase (Fig. 17).

Comparative Analysis and Parametric Adaptation for the Northern Staircase

The investigation was extended to the northern staircase to assess the complex's overall geometric coherence. The plan's apparent symmetry suggested that the parametric model of the southern flight could be mirrored along the atrium's central axis. The mirroring transformation was then overlaid on the point cloud

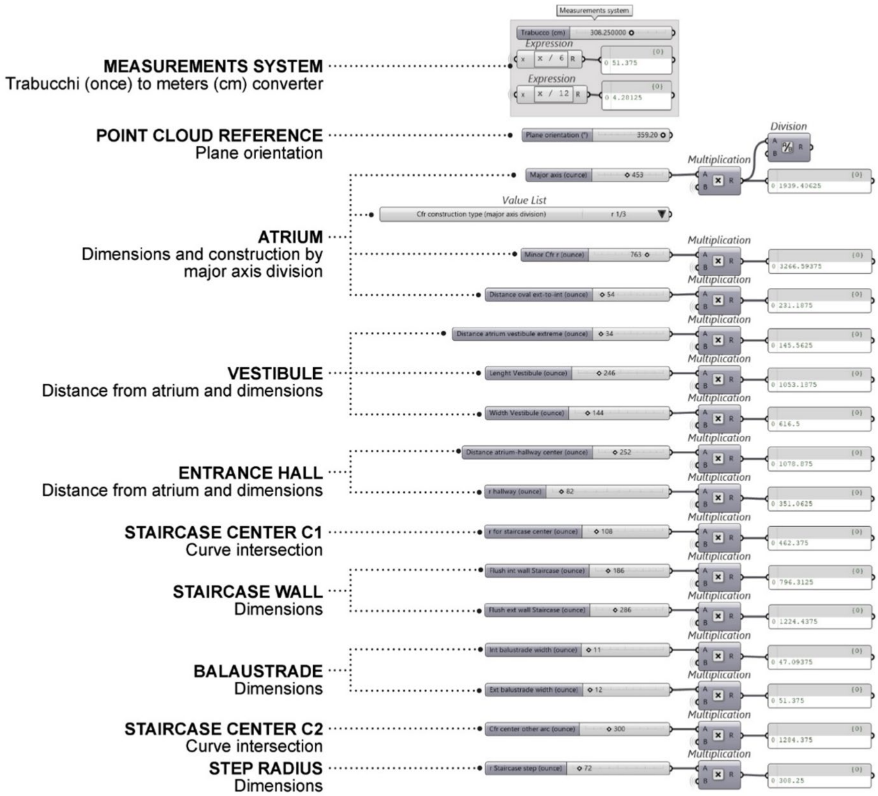

Fig. 16 Input parameters for the VPL script, showing the historical unit conversion system and the main geometric values in once. VPL processing: Fabrizio Natta

section of the northern flight. The results revealed that the mirrored geometry failed to align with the built walls and treads of the northern flight.

The parametric workflow's adaptability proved essential in this phase when the VPL script for the southern staircase was duplicated and recalibrated. The underlying algorithmic logic was preserved in terms of the sequence of geometric operations defining the polycentric curves, the treads, and their relationships (Fig. 18). The focus shifted to adjusting the key input parameters using the southern model as a flexible template to align the new model with the geometric features of the northern flight. This approach transformed the model from a simple reconstruction tool into a sensitive instrument for comparative analysis.

To achieve geometric consistency from the model to the survey for the northern staircase, a series of targeted modifications was implemented, which were each informed by iterative comparison with the point cloud.

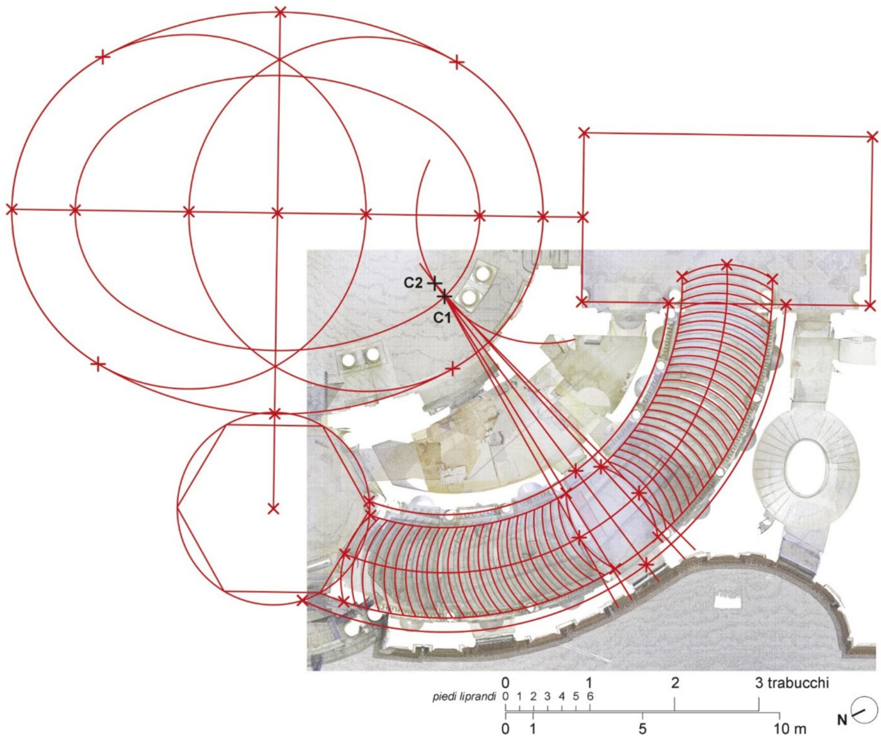

Fig. 17 Superimposition of the geometric reconstruction (centers C1, C2) on the southern staircase point cloud. Point cloud: Michele Russo; VPL processing: Fabrizio Natta

- Geometric centers and radii: While the polycentric feature was confirmed, the specific parameters required adjustment. The inner wall's radius originating from the northern flight's first geometric center (C3, the counterpart to C1) was modified from o186 to o184. The outer wall's radius of o286, however, remained constant, indicating asymmetry that also affected the inner balustrade's layout.
- Landing step orientation: The angles defining the rotation of the landing steps were recalibrated from 30° to 29° and from 45.50° to 44.50° to match the orientation of the actual landing.
- Second flight step generation: The polar array angle, a parameter in the algorithm that governs the angular spacing of the steps for the second flight, was reduced from 50° to 49° to align with the tread layout.

The parametric adaptation highlights the VPL's capacity to describe slight yet significant variations (Fig. 19). The ability to model both distinct flights by modifying a limited set of parameters in a unique algorithmic structure underscores the model's robustness. It effectively quantifies the asymmetries of the existing double staircase and provides insight into the dialogue between the desirable symmetrical geometry and the execution.

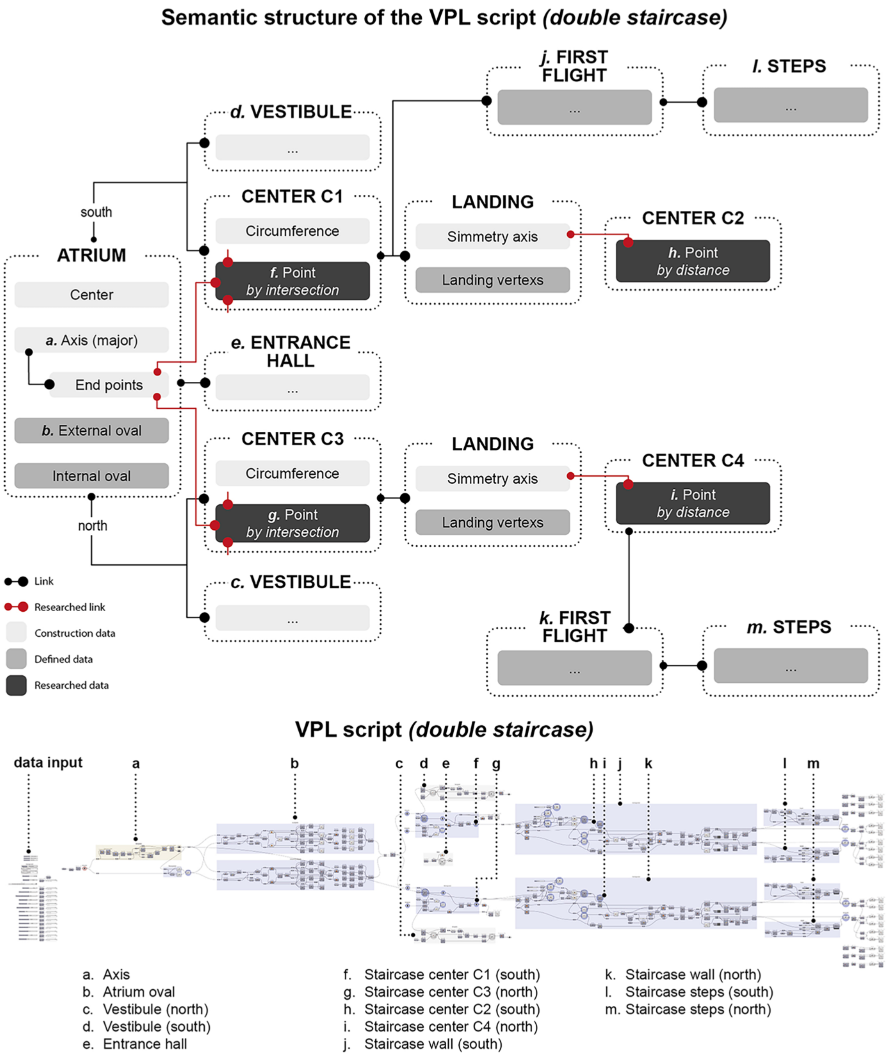

Fig. 18 Double staircase VPL: semantic structure and script organized by computational groups (**a**–**m**). VPL processing: Fabrizio Natta

Synoptic Comparison

The comparison between Guarini's detailed design drawing and the actual built staircase revealed metric recurrences and good adherence of the architecture to the intentions expressed in the design, both in terms of the unitary conception and geometric concatenation of the spaces and in the definition of the characteristics of the individual rooms. The main difference lies in the geometric construction used to locate the center of the arcs that define the double staircase walls. In the interpretive

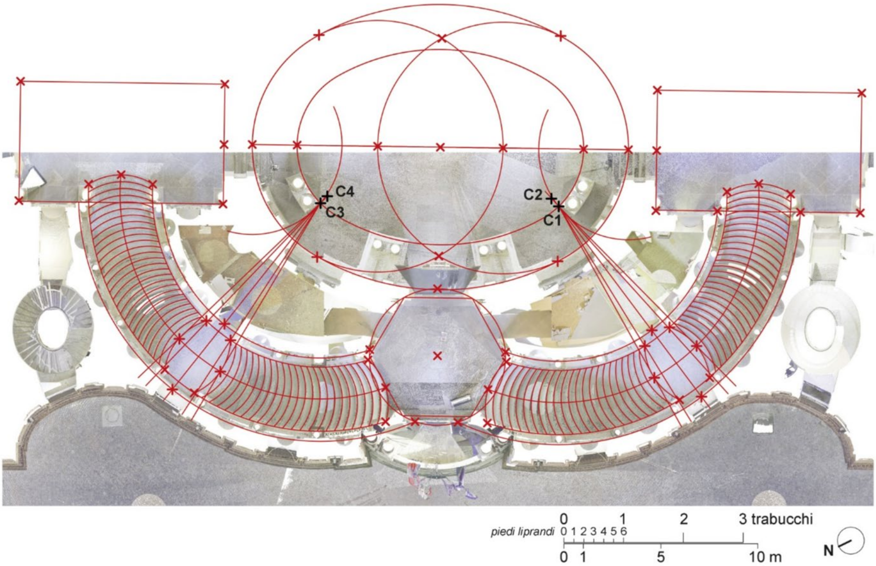

Fig. 19 Superimposition of the final geometric reconstruction on the point cloud, revealing the distinct geometric centers (C1-C4) of the double staircase. Point cloud processing: Michele Russo; VPL processing: Fabrizio Natta

redrawing of the plate, it is based on the documented dimensions and geometric constructions, while in the survey analysis, it is based on the physical identification of the symmetry axis of landings and on a column-base vertex. In addition, the layout of the walls of the two flights consists of concentric arcs in the plate and arcs with different centers for each of the two flights in the building.

Regarding the geometry of the landing, the survey data confirm the hypothesis of the shape of the landing being transformed from oval to ovoid during the design process, as sketched by Guarini. The new position assumed by the columns adjacent to the landing allows us to precisely identify C1 as the common point of the axes passing through pairs of opposite columns on the upper flight, which unfortunately is not the case on the lower flight. Another noticeable difference concerns the shape and orientation of the individual steps. In the plate, the position of the steps in relation to the axis of each flight changes continuously, which produces significant variations in the length of the horizontal edges of the individual risers. In the building, they are arranged regularly in relation to the axis, thus maintaining a constant length, which is in line with the construction advantage of making almost all the steps identical.

Regarding the number of steps in Guarini's drawing, as indicated in his note, both flights have 29 risers, for a total of 58 risers measuring o3.5 in height (approximately 15 cm). This helps to overcome a difference in height between the ground and noble floors of approximately o203, which is equal to approximately 8.73 m. In contrast, in the building, the two flights have 27 and 24 risers, respectively (with an average height of 16.5 cm), which are necessary to overcome a height difference of 8.41 m.

Conclusion

The pipeline setup allowed for the study of the geometries underlying the relationships between the spaces of the ceremonial route, particularly the atrium and the double staircase, during the design and construction phases of Palazzo Carignano. The interpretive methods applied facilitate the exploration of deductive reasoning in the re-drawing activity of the original plates and inductive reasoning in the parametric tracing of the building, respectively. The analyses carried out shed new light on the interest of different original drawings to explore the Guarinian design process, and the link between geometry and architecture is evident not only in his theoretical works, but also in his design method and realized buildings. The digital survey has provided a solid knowledge base for verifying the consistency from design to construction. VPL modelling has proven to be a valuable tool for verifying different solutions and maintaining the parametric relationships between architectural parts. The original aspects that emerged from the research consist of:

- The verification of Millon's interpretations, made possible by consulting and directly measuring the documents and by digitally redrawing and superimposing the drawings, which offered some interesting insights and revealed some problems left unresolved by the author.
- The geometric decoding of the detailed drawing of the south staircase and, above all, the identification of the center of curvature of the perimeter walls. Indeed, it is the critical node for understanding Guarini's design process that led to the final definition of the staircase in its relationship with the other elements of the ceremonial route, and in particular with the atrium.
- The relationship established between the detailed drawing and the ground layout of the staircase, mediated by digital surveying, through parametric modeling tools that allow the design model to be compared with the built staircase, highlighting similarities and differences and opening up new possibilities for geometric, historical, metrological, and constructive studies.

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Roberta Spallone architect and Ph.D., is Full Professor at the Politecnico di Torino, Department of Architecture and Design (DAD). She is a member of the School of Architecture, on the Board of the Doctorate of Architectural Heritage. Her research focusses on the fields of history and criticism of architectural drawing, with particular attention to classic treatises on architecture and the art,

fortification, and digital technologies for graphical analyses and reconstructive modeling, and, recently, the interweavings between representation, artificial intelligence and augmented reality. President of REAACH (REpresentation Advances And CHallenges) association. Chair with Kim Williams of Nexus Conference 2023 in Torino. Advisor of FORTMED (Fortification of the Mediterranean Coast) Conferences. Guest editor (with M. Vitali) of Nexus Network Journal vol. 22, no. 4 (2020) “Complex Vaulted Systems: Geometry and Architecture from Design to Construction”, vol. 25, supplement 1 (2023), and “Research Arising: The Nexus Conference”, vol. 26, n. 3 (2024). She is the author of over 250 publications, including both monographs and articles, and has participated as a speaker in many international conferences, including Nexus conferences in 2016, 2018, and 2021.

Marco Vitali architect and Ph.D. in drawing and survey for the protection of the housing stock and territorial heritage, is an associate professor of drawing at the Politecnico di Torino, Department of Architecture and Design (DAD). He is a member of the School of Architecture and of the School of Design and Communication, on the Board of the Doctorate of Architectural Heritage, and scientific supervisor of the modeling laboratories MODLab Arch and MODLab Design.He conducts research in the areas of architectural survey, descriptive geometry, graphical analyses and reconstructive modeling, algorithmic modeling, and fortifications, with thematic insights on analysis and representation of complex vaulted systems, and 3D modeling and virtual anastylosis for the study, understanding, and valorization of Cultural Heritage. Guest editor (with R. Spallone) of Nexus Network Journal vol. 22, no. 4 (2020) “Complex Vaulted Systems: Geometry and Architecture from Design to Construction”, vol. 25, supplement 1 (2023).He is the author of over 120 publications, including both monographs and articles, and has participated as a speaker in many international conferences, including Nexus conferences in 2016, 2018, and 2021.

Michele Russo architect and PhD, is an Associate Professor in the Department of History, Representation and Restoration of Architecture at Sapienza University of Rome. Member of the Faculty of Architecture and the Doctoral College, his research ranges from integrating 3D sensing techniques in different application areas to the reality-based and interpretive representation of the existing. He has participated in numerous national and international projects on surveying and modelling archaeological areas, architecture, museum apparatus, and Design products at different scales. In recent years, his interests have focused on automatic processing and content construction through Machine Learning, Deep Learning, and visualization systems based on Augmented Reality. He is secretary of REAACH Association (REpresentation Advances And CHallenges), Expert Member of CIPA. Reviewer for international journals and conferences, he is author of more than 150 proceedings, journal articles and essays in 3D Imaging and Digital Representation in multiple application domains.

Fabrizio Natta architect, PhD in Architectural and Landscape Heritage (cycle XXXVI) and currently research fellow at the Politecnico di Torino for the SSD ICAR/17 (Drawing).His research interests are on the geometry of vaulted systems in brickwork related to the study of treatises and manuals, with the analysis of archive drawings and with the survey of architectural organisms. He is currently studying the possibility of digital parameterization of complex geometry through VPL and HBIM modeling, aimed at comparing survey data, theoretical models, and construction techniques.He is responsible as a tutor in graduate programs in Architecture and Design and Communication in the courses of Descriptive Geometry, Digital Representation and Architectural Surveying.