Information design for congestion minimization in transportation networks

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Information design for congestion minimization in transportation networks ⋆

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Abstract: We study an information design problem to reduce congestion in transportation networks. In presence of an uncertain network state, the central planner may sends private signals to the users, with the goal of steering the user equilibrium towards the system optimum flow. We consider private signals and provide sufficient conditions under which optimality may be achieved by information provision in networks with arbitrary number of parallel links and affine delay functions. Our results imply that optimality is more easily achieved in presence of large uncertainty of the random variables of the delay functions of the links of the network.

Keywords: Bayesian methods, Stochastic control and game theory, Control of networks, Multi-agent systems, Information processing and decision support, Urban Mobility

1. INTRODUCTION

Transportation networks are essential for the smooth operation of modern societies. However, as the number of users utilizing these infrastructures continues to grow, traffic and congestion can become significant challenges. To mitigate these issues, it is important to understand the interplay between the network and the user behavior.

Routing games are mathematical models that represent the strategic decision-making of a large number of users on a transportation network (Wardrop (1952)). In these models, each link is associated with a delay function that depends on the flow, capturing the effects of congestion. Users in such games make decisions to minimize their own travel time, which results in what is known as a user equilibrium, that is a flow where no user has an incentive to switch route. Unfortunately, these equilibria are often inefficient in terms of total travel time spent on the network (Roughgarden (2002)). Therefore, designing control and influence mechanisms is crucial to guide users toward a socially optimal use of transportation networks.

Classically, this has been done by tolls (Cole et al. (2006); Maggistro and Como (2018)) or by interventions on the network structure (Cianfanelli et al. (2023b); Farahani et al. (2013)). Instead, in presence of an uncertain state network state (that can represent uncertain weather conditions, roadworks, accidents), with the help of modern instant communication technologies, a network planner can take advantage of this uncertainty, disclosing information to the users to influence their behavior. The study on how to disclose information to the users to optimize the resulting equilibrium flow is called in the literature *information design problem*.

It is well known in the literature that, if more users are fully informed about the network state, then the total travel time spent on the network may increase, so that providing users with full information is not optimal (see, e.g., Acemoglu et al. (2018) and Wu et al. (2017, 2021)). Moreover, it has been demonstrated that public information policies are in general suboptimal with respect to private ones (Das et al. (2017), Massicot and Langbort (2022)). Motivated by these findings and by the fact that nowadays users have access to information through their personal devices, in this paper we shall focus on *private information provision*.

We formulate the problem for networks with an arbitrary number of parallel links, affine delay functions and continuous network state, extending the works of Cianfanelli et al. (2023a) and Tavafoghi and Teneketzis (2017). By applying the revelation principle (see Bergemann and Morris (2016, 2019)), we restrict the analysis to *direct signals*, namely, the signals that the planner sends to the users are recommendations about the link they should use. Moreover, we restrict the analysis to *obedient policies*, namely, such that the users have no incentive in deviating from the signal that they receive, under the assumption that the prior distribution of the network state and the information policy are publicly known.

In the literature, different studies about information design in Bayesian routing games have been done with different objectives. Zhu and Savla (2022) propose an algorithm designed to solve efficiently the problem with polynomial delay functions and arbitrary network topologies, while Doval and Skreta (2024) provide a solution strategy to solve a constrained information design problem. Other approaches consider limited rationality as in Gould and Brown (2023) and Feng et al. (2024), bilateral information asymmetry as in Sezer and Eksin (2023), dynamic infor-

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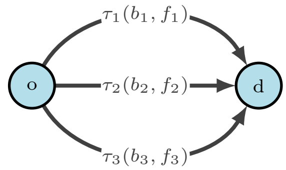

Fig. 1. Network with 3 parallel links from o to d.

mation provision as in Ouyang et al. (2024); Meigs et al. (2020) or with finite event spaces as in Griesbach et al. (2022).

Our main contribution is to find necessary and sufficient conditions on the delay functions of the network under which the planner can align the equilibrium flow to the system optimum flow by information design. These conditions are formulated in terms of first and second moments of the random variables of the delay functions. We then provide some examples for networks with two and three parallel links highlighting that, while the variance of the uncertain parameters is beneficial for the planner, the covariance of the uncertain parameters in general is not.

The paper is organized as follows. Section 2 provides the transportation network model, the concept of information policy, and the problem formulation. Section 3 contains our main results involving the necessary and sufficient conditions for the optimality. Section 4 provides some examples. Finally, Section 5 summarizes the contribution.

1.1 Notation

Given a vector x, we let x' denote its transpose. $\mathbf{1}$ and $\mathbf{0}$ denote respectively the vector of all ones and the matrix of all zeros, whose dimension may be inferred from the context. Given a natural number n, Δ_n denotes the simplex on $\mathbb{R}^n$, i.e.

$$\Delta_n = \{x \in \mathbb{R}^n_+ : \mathbf{1}'x = 1\}.$$

For a real number x, we let $\lfloor x \rfloor$ denote its integer part.

2. INFORMATION DESIGN PROBLEM

In this section we model the transportation network, define information policies, and formulate the problem.

2.1 Transportation network model

Consider a transportation network with n parallel links from an origin o to a destination d, and a unitary mass of non-atomic users travelling from o to d. Every link $i = 1, 2, \ldots, n$ is endowed with a deterministic parameter $\alpha_i > 0$ that accounts for congestion effects, as well as a nonnegative-valued random variable B_i that describes the free-flow travel time of the link. The vector B that collects all the B_i's will be referred to as the *network state*. We assume that B is defined on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, that it takes values in a non-empty measurable set $\mathcal{B} \subseteq \mathbb{R}^n_+$, and that its entries B_i have bounded first and second moment, i.e., that

$$\mathbb{E}[B_i] < +\infty\,, \qquad \mathbb{E}[B_i^2] < +\infty\,, \qquad \forall i = 1, 2, \ldots, n\,.$$

Notice that we do not make any assumption of independence of the random variables B_i, while of course the case when they are independent can be recovered as a special case in our framework.

Let the travel time on each link $= 1, 2, \ldots, n$ be

$$\tau_i(B_i, f_i) = \frac{f_i}{\alpha_i} + B_i\,, \tag{1}$$

where f_i represents the flow over link i. A network flow is a function $f : \mathcal{B} \to \Delta_n$ that assigns to every realization of the network state b in $\mathcal{B}$ a vector $f(b)$ in Δ_n. The *system cost* of a network flow is then defined as the expected travel time on the network, i.e.

$$\begin{aligned} C(f) &= \mathbb{E}\left[\sum_{i=1}^n f_i(B)\tau_i(B_i, f_i(B))\right] \\ &= \int_{\mathcal{B}} \sum_{i=1}^n f_i(b)\tau_i(b_i, f_i(b))d\mathbb{P}(b)\,. \end{aligned} \tag{2}$$

In particular, the *system optimum* f^* is the network flow that minimizes the system cost, namely,

$$f^* = \underset{f:\mathcal{B}\to\Delta_n}{\arg\min}\, C(f)\,, \tag{3}$$

which is unique because the delay functions are strictly increasing in the second argument.

2.2 Information policies

We assume that an omniscient planner observes the realization of the network state and sends private signals to the users to influence their behaviour. After receiving the signal, the users update their belief about the network state and take a link with minimum expected delay. The goal of the planner is to convey information to the users to align the resulting equilibrium flow to the system optimum flow.

We consider *direct* signals, namely, signals are recommendations about the link to take. Moreover, we focus on policies that are *obedient*, namely, such that no user has incentive to deviate from the received recommendation. These two assumptions are without loss of generality because of the revelation principle (Bergemann and Morris (2019, 2016)), which states that, for every information policy, there always exists a direct obedient policy that achieves the same cost.

A direct obedient *information policy* is a map $\pi : \mathcal{B} \to \Delta_n$ that assigns to every network state the exact fraction of users that will receive the recommendation to go on each link of the network. The planner chooses a policy π before the network state realization. After the realization of the network state, the planner draws a random variable in order to select the signal received by each user as follows. Let r in $[0, 1]$ denote a non-atomic user, and let u be a random variable defined on the probability space $(\Omega, \mathcal{A}, \mathbb{P})$, independent from the network state B, and uniformly on the interval $[0, 1]$. Then, we define the map

$$S_u : [0, 1] \to \{1, \cdots, n\}\,,$$

that maps each user r to the signal

$$S_u(r) = i \quad \text{if} \quad \sum_{j=1}^{i-1} \pi_j(b) \le u + r - \lfloor u + r \rfloor < \sum_{j=1}^{i} \pi_j(b)\,,$$

that she receives.

The users know the prior distribution of the network state and observe their private signal, without observing r and the realization of the network state. We let $\mathbb{P}_i$ denote the posterior belief on the network state B of users that receive signal i.

Lemma 1. The posterior beliefs are

$$d\mathbb{P}_i(b) = \frac{\pi_i(b)d\mathbb{P}(b)}{\int_{\mathcal{B}} \pi_i(\omega)d\mathbb{P}(\omega)}\,, \quad \forall i = 1, \cdots, n\,. \tag{4}$$

Proof. Notice that, for every user r in $[0, 1]$,

$$\mathbb{P}(S_u(r) = i|B = b) = \pi_i(b)\,,$$

because $u+r-\lfloor u+r \rfloor$ is uniformly distributed in $[0, 1]$ for every user r. Hence, (4) follows from Bayes' theorem. □

Once users have received the signal and have updated their belief, they follow the recommendations only if it is a best response for them to do so. As mentioned before, we shall focus on obedient policies, as this is without loss of generality due to the revelation principle. In other words, we shall require that each link i is optimal according to the posterior belief $\mathbb{P}_i$. The fact that a policy is obedient will be included as a constraint on the space of feasible policies in our information design problem, as outlined in the following section.

2.3 Problem formulation

In this section we formulate the information design problem.

Remark 1. It follows from (Cianfanelli et al., 2023a, Proposition 1) that, for each direct policy π, the corresponding equilibrium flow (i.e., a flow where no user wants to deviate from the link that he is using) is unique. In particular, this implies that every obedient policy π admits the equilibrium flow π only.

In light of the previous remark, it is natural to define the cost of an obedient policy π as the expected travel time under flow π, denoted $C(\pi)$. The information design problem then consists in finding the information policy π^* with minimum cost subject to the obedient constraints, namely

$$\pi^* = \underset{\pi:\mathcal{B}\to\Delta_n}{\arg\min} \int_{\mathcal{B}} \sum_{i=1}^{n} \pi_i(b)\tau_i(b_i, \pi(b))d\mathbb{P}(b)$$

subject to

$$\int_{\mathcal{B}} \tau_i(b_i, \pi_i(b))\pi_i(b)d\mathbb{P}(b) \le \int_{\mathcal{B}} \tau_j(b_j, \pi_j(b))\pi_i(b)d\mathbb{P}(b),$$

for every $i, j = 1, \cdots, n$. Notice that such an optimization problem has non-convex constraints, Therefore, it is in general hard to solve.

The problem that we solve in this paper consists in finding conditions under which the system-optimum flow can be induced by information design. In other words, we aim to find conditions under which $\pi = f^*$ is an obedient policy, and therefore $\pi^* = f^*$.

3. CONDITIONS FOR OPTIMALITY

We start by introducing some notation. Let

$$\begin{aligned} \Delta_{ij} &= \mathbb{E}[B_i - B_j], \quad \forall i, j = 1, \cdots, n, \\ K_{ij} &= \mathbb{E}[B_i B_j] - \mathbb{E}[B_i]\mathbb{E}[B_j], \quad \forall i, j = 1, \cdots, n. \end{aligned} \tag{5}$$

Before stating the results, we establish the following assumption on the support of the network state $\mathcal{B}$. This assumption ensures that, for every realization of the network state b in $\mathcal{B}$, the constraint that ensures that the system optimum flow $f^*(b)$ is non-negative is not active. The assumption is motivated to enhance the readability and interpretability of the results.

Assumption 1. Let

$$1 + \frac{1}{2}\sum_{j=1}^{n} \alpha_j(b_j - b_i) \ge 0\,,$$

for all b in $\mathcal{B}$ and $i = 1, \cdots, n$.

The following result characterizes the system optimum flow.

Lemma 2. Let Assumption 1 hold true. Then, the system optimum flow (3) is

$$f_i^*(b) = \frac{\alpha_i(1 + \frac{1}{2}\sum_j \alpha_j(b_j - b_i))}{\sum_j \alpha_j}\,, \tag{6}$$

for every $i = 1, \cdots, n$ and b in $\mathcal{B}$.

The proof of this result follows from explicit computation and may be found in the Appendix. The next result provides necessary and sufficient conditions for the system-optimum flow to be an obedient policy.

Theorem 1. Let Assumption 1 hold true. Then, $\pi^* = f^*$ if and only if

$$\Delta_{ij}(2 + \sum_{e\neq i} \alpha_e\Delta_{ei}) \le \sum_{e\neq i} \alpha_e\left[K_{ii} + K_{ej} - K_{ei} - K_{ij}\right] \tag{7}$$

for every $i, j = 1, \cdots, n$.

Proof. To prove that $\pi^* = f^*$, we show that the obedience constraints are satisfied by this policy. If so, then, by definition of system optimum, any alternative policy achieves a larger cost. The flow f^* is an obedient policy if and only if

$$\int_{\mathcal{B}} [\tau_i(b_i, f_i^*(b)) - \tau_j(b_j, f_j^*(b))]f_i^*(b)d\mathbb{P}(b) \le 0\,, \tag{8}$$

for every $i, j = 1, \cdots, n$. From (1) and (6) it follows

$$\tau_i(b_i, f_i^*(b)) - \tau_j(b_j, f_j^*(b)) = \frac{b_i - b_j}{2}. \tag{9}$$

Plugging (6) and (9) into (8), we get that f^* is an obedient policy if and only if

$$\int_{\mathcal{B}} \frac{\alpha_i(1 + \frac{1}{2}\sum_e \alpha_e(b_e - b_i))}{\sum_e \alpha_e}(b_i - b_j)d\mathbb{P}(b) \le 0\,.$$

for every $i, j = 1, \cdots, n$. Using the fact that α is deterministic, this is equivalent to

$$\int_{\mathcal{B}} \Big(2 + \sum_{e\neq i} \alpha_e(b_e - b_i)\Big)(b_i - b_j)d\mathbb{P}(b) \le 0\,.$$

The latter inequalities can be rewritten using the expected value as

$$2\mathbb{E}\left[B_i - B_j\right] + \sum_{e\neq i} \alpha_e\mathbb{E}\left[(B_e - B_i)(B_i - B_j)\right] \le 0. \tag{10}$$

Now, using the definition in (5), the obedience constraint (10) becomes

$$2\Delta_{ij} + \sum_{e\neq i} \alpha_e\Delta_{ei}\Delta_{ij} \le \sum_{e\neq i} \alpha_e\left[K_{ii} + K_{ej} - K_{ei} - K_{ij}\right]\,,$$

concluding the proof. □

The necessary and sufficient condition for optimality provided by Theorem 1 may not be immediate to interpret. However, (7) shows that the obedience constraints are easier to satisfy when the diagonal entries of K are large, namely, when the variance of the unknown parameters is large. The next corollary provides some intuitive conditions that guarantee optimality.

Corollary 1. Let the expected free-flow delay of all links be equal, i.e.,

$$\Delta = \mathbf{0}\,, \tag{11}$$

and let the covariance matrix be diagonal, i.e.,

$$K_{ij} = 0\,, \quad \forall i \neq j. \tag{12}$$

Then, $\pi^* = f^*$.

Proof. The result follows immediately from Theorem 1. Indeed, (11) implies that the left hand side of (7) is equal to 0 for every $i, j = 1, \cdots, n$. On the other hand, (12), together with the fact that the diagonal of K contains the variance of the entries of B and is nonnegative by construction, implies that the right hand side of (7) is nonnegative for every $i, j = 1, \cdots, n$. Hence, the constraints (7) are satisfied and $\pi^* = f^*$. □

Remark 2. An implication of Corollary 1 is that the set of triples (α, K, Δ) that satisfy (7) is non-empty.

In the next section we shall see that the off-diagonal entries of K, representing the covariance of the entries of B, can prevent optimality to be achieved.

4. EXAMPLES

In this section we provide some examples to better understand under what conditions the system-optimum flow is an optimal policy.

The first example shows that in a network with only two parallel links, condition (11) is sufficient to achieve optimality, without requiring (12).

Example 1. Let $n = 2$, $\alpha_1 = \alpha_2 = 1$, and assume that $\Delta = \mathbf{0}$. We now show that the obedience constraints (7) are satisfied regardless the definition of K. Indeed, the constraint (7) with $i = 1, j = 2$, which ensures that users that receive signal i prefer link i over link j, reads

$$K_{12} \leq \frac{K_{11} + K_{22}}{2}, \tag{13}$$

which is satisfied for every positive semi-definite covariance matrix K. Similarly, one can prove that (7) with $i = 2, j = 1$ is also satisfied for every covariance matrix K, since the two links are symmetric.

The next example shows that Corollary 1 no longer holds true if (12) is relaxed and $n > 2$. In particular, we show how positive entries in off-diagonal elements of K can prevent optimality to be achieved.

Example 2. Consider $n = 3$ links and let $\alpha_1 = \alpha_2 = \alpha_3 = 1$. Assume $\Delta = \mathbf{0}$ and assume that the free-flow travel time of link 3 is deterministic, namely, $K_{i3} = K_{3i} = 0$ for every $i = 1, 2, 3$. For this case, the constraint (7) with $i = 1, j = 2$ reads

$$K_{12} \leq \frac{2K_{11} + K_{22}}{3}. \tag{14}$$

Notice that, in contrast with Example 1, this obedience constraint can be violated. Let for instance $K_{11} = 1, K_{22} = 2$, which implies $K_{12} \leq \sqrt{2}$ because K must be semi-positive definite. Hence, (14) is violated if

$$\sqrt{2} \geq K_{12} > 4/3. \tag{15}$$

If (15), then for users that receive the recommendation to travel on link 1 it is rational to deviate to link 2.

Example 1 shows that with $n = 2$ links, $\Delta = \mathbf{0}$ is a sufficient condition to achieve optimality, without additional conditions on the covariance matrix K. Notably, Example 2 shows that adding a third link with deterministic free-flow travel time can prevent optimality to be achieved. This reveals that the nature of the problem is more complicated when the number of parallel links increases from $n = 2$ to $n = 3$, as the entries of the covariance matrix play in this context a relevant role. In the next example we show that also negative correlations can prevent obedience constraints to be satisfied.

Example 3. Consider a network with $n = 3$ parallel links and assume that $\Delta = \mathbf{0}$. In particular, assume that B_2 and B_3 are maximally anti-correlated and B_1 is deterministic, that is, there exist two positive numbers c, w and a random variable η such that

$$B_1 = c, \quad B_2 = c + \eta, \quad B_3 = c - w\eta.$$

Then, (7) with $i = 1, j = 2$ reads

$$\alpha_2 K_{22} + \alpha_3 K_{32} \geq 0.$$

Notice that $K_{32} = -wK_{22}$, the constraint becomes

$$w \leq \frac{\alpha_2}{\alpha_3}.$$

We now consider the constraint (7) with $i = 1, j = 3$, that is

$$\alpha_2 K_{23} + \alpha_3 K_{33} \geq 0.$$

Using $K_{23} = -K_{33}/w$, the constraint gets

$$w \geq \frac{\alpha_2}{\alpha_3}.$$

To guarantee that users who receive the recommendation to go on link 1 are obedient (namely, both (7) with $i = 1, j = 2$ and (7) with $i = 1, j = 3$ are satisfied), we then need

$$\frac{\alpha_2}{\alpha_3} \geq w \geq \frac{\alpha_2}{\alpha_3} \implies w = \frac{\alpha_2}{\alpha_3}.$$

If $w < \alpha_2/\alpha_3$, users who receive the recommendation to go on link 1 prefer link 3. if $w > \alpha_2/\alpha_3$ users who receive the recommendation to go on link 1 prefer link 2. If $w = \alpha_2/\alpha_3$, the choice is neutral and users who receive the recommendation to go on link 1 are obedient.

5. CONCLUSION

In this paper we have studied the design of information policies in routing games with an uncertain network state. We have formulated the problem for networks with parallel links and affine delay functions, considering only private and direct policies. We have determined necessary and sufficient conditions under which the system-optimum flow can be obtained by disclosing privately information to the users. We then have provided examples that reveal how the complexity of the problem increases when considering more than two links.

Future researches will include extensions to more general network topologies and delay functions, as well as studying dynamic information provision.

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APPENDIX

Proof of Lemma 2

From the linearity of the integral, the system optimum flow $f^*(b)$ does not depend on the network state distribution and can be found separately for every value of b in $\mathcal{B}$. Moreover, Assumption 1 implies that for every value of b there exists a positive $\lambda(b)$ such that $f^*(b)$ is the unique vector in Δ_n that satisfies

$$2\frac{f_j^*(b)}{\alpha_j} + b_j = \lambda(b)\,, \quad \forall j = 1, \cdots, n\,. \tag{16}$$

Hence,

$$1 = \sum_j f_j^*(b) = \sum_j \alpha_j \frac{\lambda(b) - b_j}{2}.$$

where the first equality follows from the fact that $f^*(b)$ belongs to Δ_n, and the second one from (16). It then follows

$$\lambda(b) = \frac{2 + \sum_j \alpha_j b_j}{\sum_j \alpha_j}.$$

Plugging this expression into (16), we get

$$f_i^*(b) = \frac{\alpha_i(1 + \frac{1}{2}\sum_j \alpha_j(b_j - b_i))}{\sum_j \alpha_j}.$$

Assumption 1 implies that $f^*(b)$ belongs to the simplex, since all the components are non-negative. Therefore, f^* is the unique system optimum flow. $\square$