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Trade-off between computation time and solution quality for integrated generation and transmission expansion planning with N-1 security criterion

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ABSTRACT

The generation and transmission (G&T) expansion planning of large-scale systems is usually carried out hierarchically due to the high complexity of the problem. However, this hierarchical plan may be more expensive than a fully integrated (co-optimized) G&T plan that, on the other hand, requires high computation time. Therefore, the trade-off between computation time and solution quality is of great importance, especially with the integration of renewable generation. This paper proposes and assesses alternative formulations of the integrated G&T planning problem, also considering the system operation simulation under the N-1 security criterion, seeking to balance solution optimality and computational effort. The assessments are illustrated for the Chilean electrical system. The main outcome is that one of the proposed methods, in which the future cost function is maintained fixed during the generation and transmission optimization and is recalculated only in the final simulation of the system operation, achieves results very close to the fully integrated generation, transmission, and operation optimization method. This method presents a cost reduction of 8 % compared to a hierarchical approach, which represents savings of around 700 million dollars, and 50 % less computation time compared to the fully integrated method. For the same proposed method, the preliminary calculation of an optimal solution without applying the N-1 security constraint as a starting point, followed by re-optimization with active N-1 security constraints, contributes to a 65 % reduction of the computation time without significantly impacting the quality of the solution.

1. Introduction

The expansion planning problem arises from the necessary changes in power systems due to the demand growth, requiring the addition of new generators and transmission circuits to meet the demand. The decisions during the planning process consist of selecting the best generators and transmission routes that ensure demand fulfillment in the future with minimum cost for society. Usually, this decision-making process is represented by a complex optimization problem that aims to minimize the total costs and is subject to several constraints that represent economic, operational, security, and environmental aspects.

In several countries, the methodology applied to the generation and transmission (G&T) expansion planning is generally based on a *hierarchical* procedure with two stages. First, the generation expansion planning (GEP) is performed without assessing the necessary network reinforcements. Next, fixing the generation expansion plan obtained in the first stage, transmission expansion planning (TEP) is carried out to obtain the required investments in the grid to meet the demand under operational limits.

By dividing the optimization problem into two more minor problems, this approach contributes to reducing the complexity of the process. Also, as most power sectors have an unbundled generation market, where private players decide the expansion but transmission expansion is still a centralized planning, the hierarchical approach looks more adequate. However, treating the G&T expansion separately may lead to suboptimal solutions in terms of costs to society. Adding generators far from the load centers, which at first glance may seem a cheaper decision, leads to further reinforcements in the transmission system to connect them to the demand, which can culminate in a total cost higher than investing in closer generators.

This trade-off, which cannot be assessed when the expansion planning is disjointed, has become more relevant in recent years. First, the leading generation expansion sources worldwide are the variable

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Nomenclature

List of acronyms

CCGT	Combined Cycle Gas Turbines
CPU	Central Processing Unit
DDP	Dual Dynamic Programming
EENS	Expected Energy Not Supplied
FCF	Future Cost Function
G&T	Generation and Transmission
G&T&O	Generation and Transmission and Operation
GEP	Generation Expansion Planning
GEP&T	Generation Expansion Planning and Transmission
GHG	Greenhouse Gas
ICF	Immediate Cost Function
MILP	Mixed Integer Linear Programming
OCGT	Open Cycle Gas Turbines
OP	Operation Problem
OPF	Optimal Power Flow
PV	Photovoltaic
SDDP	Stochastic Dual Dynamic Programming
TEP	Transmission Expansion Planning
VRE	Variable Renewable Energy

Variables

$a_{j,t}$	Water inflow of hydro plant j at time step t
$c_{j,t}$	Operating cost of thermal plan j at time step t
d_t	Demand at time step t
$d_{i,t}$	Demand in bus i at time step t
$f_{k,t}$	Power flow in circuit k at time step t
$\overline{f_k}$	Maximum capacity of circuit k
$\overline{f_{ke}}$	Maximum emergency capacity of circuit k
$g_{j,t}$	Generation of power plant j at time step t
$g^{*}_{j,t}$	Dispatch decision for power plant j at time step t
$\overline{g_j}$	Maximum generation of thermal plant j
$p_{j,t}$	Investment cost of power plant j at time step t
$p_{k,t}$	Investment cost of circuit k at time step t
r_t	Deficit at time step t
$r_{i,t}$	Deficit in bus i at time step t
$s_{p_{j,t}}$	Spilled outflow of hydro plant j at time step t
$u_{j,t}$	Turbined outflow of hydro plant j at time step t
$\overline{u_j}$	Maximum turbined outflow of hydro plant j
$x_{j,t}$	Investment decision of power plant j at time step t
$x^{\mu}_{j,t}$	Investment decision at the μ-th iteration for candidate j at time step t
$x_{k,t}$	Investment variable for the transmission candidate k at time step t
$x^{\mu}_{k,t}$	Investment decision for the transmission candidate k at time step t at the μ-th iteration
M_k	Disjunctive constant
T	Number of time steps
α	Approximation of the operating cost
γ_k	Susceptance of circuit k
δ	Deficit cost
$v_{j,t}$	Initial storage of hydro plant j at time step t
$v_{j,t+1}$	Final storage of hydro plant j at time step t
$\overline{v_j}$	Maximum storage of hydro plant j
$\pi^{\mu}_{j,\tau}$	Dual variable associated with constraint (3.12) and candidate j at time step τ at the μ-th iteration
$\pi^{\mu,v}_{\tau,j}$	Dual variable associated with constraint (3.13) and candidate j at time step τ at the μ-th iteration
$\pi^{\mu,u}_{\tau,j}$	Dual variable associated with constraint (3.14) and candidate j at time step τ at the μ-th iteration
$\pi^{\mu,\gamma}_{\tau,k}$	Dual variable associated with constraint (3.30) and candidate k at time step τ at the μ-th iteration
$\pi^{\mu,f}_{\tau,k}$	Dual variable associated with constraint (3.25) and candidate k at time step τ at the μ-th iteration
ρ_j	Production coefficient of hydro plant j
φ	Current iteration
$w(x^{\mu})$	Value of the objective function of the operating subproblem at the μ-th iteration
$\Delta\theta_{k,t}$	Angular difference between the terminal buses of circuit k at time step t

Sets

$\boldsymbol{I}$	Set of buses
$\boldsymbol{J}_{\mathrm{H}}$	Set of hydro plants,$\boldsymbol{J}_{\mathrm{H}} = \boldsymbol{J}_{\mathrm{HE}} \cup \boldsymbol{J}_{\mathrm{HC}}$
$\boldsymbol{J}_{\mathrm{HC}}$	Set of candidate hydro plants
$\boldsymbol{J}_{\mathrm{HE}}$	Set of existing hydro plants
$\boldsymbol{J}_{\mathrm{R}}$	Set of renewable plants,$\boldsymbol{J}_{\mathrm{R}} = \boldsymbol{J}_{\mathrm{RE}} \cup \boldsymbol{J}_{\mathrm{RC}}$
$\boldsymbol{J}_{\mathrm{RC}}$	Set of candidate renewable plants
$\boldsymbol{J}_{\mathrm{RE}}$	Set of existing renewable plants
$\boldsymbol{J}_{\mathrm{T}}$	Set of thermal plants,$\boldsymbol{J}_{\mathrm{T}} = \boldsymbol{J}_{\mathrm{TE}} \cup \boldsymbol{J}_{\mathrm{TC}}$
$\boldsymbol{J}_{\mathrm{TC}}$	Set of candidate thermal plants
$\boldsymbol{J}_{\mathrm{TE}}$	Set of existing thermal plants
$\boldsymbol{K}$	Set of circuits,$\boldsymbol{K} = \boldsymbol{K}_{\mathrm{C}} \cup \boldsymbol{K}_{\mathrm{E}}$
$\boldsymbol{K}_{\mathrm{C}}$	Set of candidate circuits
$\boldsymbol{K}_{\mathrm{E}}$	Set of existing circuits
$\boldsymbol{N}$	Set of single contingencies
Φ_i	Set of generators connected to bus i
$\boldsymbol{\Pi}_j$	Set of hydro plants immediately upstream of hydro plant j
Ω^{+}_{i}	Set of circuits which the terminal bus i is the TO bus
Ω^{-}_{i}	Set of circuits which the terminal bus i is the FROM bus

renewable energy (VRE) plants, such as utility-scale solar photovoltaic (PV) and wind, typically located far from the load centers. Also, this domination is boosted by the rapid increase of net zero greenhouse gas (GHG) emissions pledges. So, it is essential to assess if investing in resources far from the load center is indeed the cheapest decision, as this requires further reinforcements in the network. In addition, the different construction times between the VRE plants (usually three years) and the transmission lines (usually five years or more) [1] harm the coordination of the expansions when treating them separately. By comparing cases with different VRE integrations in the grid, the investment cost in electricity transmission has not been considered an economic limitation for integrating high VRE shares in the Spanish system [2] or in the Brazilian hydrothermal generation expansion planning [3].

In the literature, most of the proposed methodologies treat GEP and TEP separately due to the high computational complexity of solving both jointly [4,5]. Historically, the first models that co-optimized G&T expansion were developed in the 1960 s. However, these early approaches have a very simplified representation of constraints, particularly concerning transmission network modeling [6].

An alternative to the hierarchical procedure is to use an *integrated* approach, expanding the generation system and transmission network in the same optimization problem. In this way, the trade-off between investing in more competitive generators, such as VRE plants, and plants more likely to be installed near load centers, such as thermal plants, can be evaluated accordingly. Such an approach can be directly applied in a vertical environment, as the G&T investment decisions are taken centrally. Furthermore, transmission planners may use this concept of integrated planning in an unbundled electric system for “anticipative planning,” in which they could project how grid reinforcements change the incentives for private generating companies to invest [6,7]. Combined generation and transmission planning that considers the evolution of the energy market and decarbonization policies is also considered to assist the government in making strategic decisions in the context of energy transition [8]. Also, coordination of generation and transmission planning has been studied considering the effects of demand response, which could delay the construction of generation and transmission assets, obtaining savings in the investment costs, as shown for some test systems in [9]. Incorporating energy storage systems and demand response in coordinated expansion planning reduces the effect of intermittent VRE generation, enabling demand shifting out of peak load periods [10].

In the coordination of generation and transmission investments, some challenges have been identified in the differences between the construction times of generation and transmission assets, with harmful delays in transmission projects [11], the uncertainty in the future market conditions, the implementation of specific regulations, and the risk associated to the possibility that the counterpart does not undertake the investments needed to coordinate the development of the assets [12].

Nevertheless, the optimization problem, considering an integrated approach, becomes more extensive as the number of constraints and variables of the problem increases significantly, which can cause exponential growth in the computational time needed to solve the problem [7]. Also, some network constraints may enhance the complexity of the problem and disturb the solution process.

Many elaborated methodologies that coordinate the GEP and TEP were developed recently, motivated by the above points [13]. In some cases, the expansion planning refers to the optimization of renewable generation and transmission assets, as in the bi-level programming model focused on wind generation and TEP [14], the two-stage min–max-min model solved with the column-and-constraint generation algorithm [15], and the closed-loop bi-level programming model for TEP with wind generation aimed at minimizing the real system costs, solved through the reformulation and decomposition technique [16].

In other research, the authors propose centrally co-optimizing the GEP and TEP. Pereira *et al.* [17] decompose the G&T expansion problem into two optimization subproblems and solve them iteratively using the Benders decomposition. The authors demonstrate that the network can be represented by a transportation model or a complete DC power flow model in this algorithm. Jenabi *et al.* [18] present a bi-level model composed of upper-level and lower-level optimizations. The upper level represents a centralized transmission operator that can calculate the optimal TEP, aiming for maximum social welfare or profit. The lower level represents a generation market that calculates the optimal GEP, maximizing total social welfare. Jin *et al.* [19] developed a three-level model to represent the TEP (centralized decision), individual GEP of the generation companies, and market operation.

GEP and TEP are naturally affected by uncertainties. Liu *et al.* [20] present an integrated GEP and TEP model that represents uncertainty through a scenario tree with hourly or sub-hourly resolution. This allows the representation of renewable production variability, ramp constraints, and short-term storage devices. Ferreira *et al.* [21] present minimax-cost and minimax-regret approaches to solve the TEP problem considering uncertainty in the market-based generation expansion. Ferreira *et al.* [22] incorporate the uncertainty in the implementation times of transmission lines into the TEP, aiming for better coordination with the GEP. García-Cerezo *et al.* [23] propose a new formulation of the risk-averse two-stage generation and transmission expansion planning, with an acceleration technique applied to the constraint generation-based algorithm that facilitates the combined use of scenarios and representative days, with results shown on a test system.

Furthermore, security constraints can be introduced into the operational and expansion planning problems. Power systems can be disrupted by component failures that can even compromise complete demand fulfillment. So, operational planning commonly includes measures to ensure proper system functioning by increasing its robustness to failures. Also, the system’s expansion planning must include necessary investments to maintain adequate reliability in future operations.

The reliability criteria used in expansion and operational planning can be classified as deterministic, probabilistic, or economic. The deterministic criteria do not consider outage uncertainties and are typically represented by the N-1, N-2, or N-k criteria. The probabilistic criteria consider the probability of failures occurring to quantify reliability indexes (targets) through stochastic simulations, whereas the economic criteria try to represent an insufficiency in meeting the demand in economic terms, such as deficit costs. Some researchers incorporate reliability criteria in decision-making to ensure that the future supply system can always meet the demand. Aghaei *et al.* [24] propose a mixed integer linear programming (MILP) formulation of the GEP and TEP problem, considering reliability criteria by modeling the Expected Energy Not Supplied (EENS) as an operational constraint.

Combining the integrated G&T expansion planning approach and security constraints in the same optimization problem is challenging, as the complexity of the problem rises considerably. Tor *et al.* [25] propose a formulation that decomposes the problem into one master problem and two subproblems, where the first performs a security check, applying the N-1 criteria. If the criterion is violated, Benders cuts are generated to the master problem. Barati *et al.* [26] also incorporate the expansion of the exploration/transportation of natural gas into the problem, representing the N-1 criterion in transmission expansion. Finally, some recent studies have focused on energy management for small grids, incorporating novel technologies and AC power flow. However, they fall short in addressing integrated G&T investments [27–33].

Table 1 summarizes the characteristics of the mentioned G&T planning models, focusing on uncertainty, security criteria, network representation, and real-world system applications.

It is noticeable that, in recent literature, there is a lack of proposed methods that address the GEP and TEP problems jointly while also incorporating uncertainties and security constraints in a computationally feasible way for large-scale, real-world systems. On the other hand, a method that can represent all these aspects may face computational burden issues. Whereas planning studies typically do not require a large number of simulations within tight timeframes—as is common in operating studies—certain situations call for many sensitivity or comparative analyses to fine-tune the final expansion plan, making Central Processing Unit (CPU) efficiency an essential attribute. This need becomes even more critical as the size of the power system grows, where achieving convergence may be impossible without a computationally efficient approach.

Based on the abovementioned aspects, the main objective of this paper is to propose alternative formulations for integrated G&T expansion planning, considering uncertainty, security constraints, and the trade-off between the quality of the solution and the computational effort needed in the optimization process when applied to systems with real size. In particular:

1. One proposed formulation, called method M2, calculates a first optimal expansion planning contemplating G&T investment variables simultaneously, considering uncertainties (on hydro and VRE production), with the possibility to make simplifications in some network constraints.
2. A second proposed formulation, method M3, co-optimizes the GEP and TEP, considering uncertainties, using the DC power flow to represent the network, and avoiding excessive computational effort when applied to electrical systems of realistic size when evaluating the hydrothermal economic dispatch.
3. This paper also proposes a method to include the security constraints representing the N-1 criterion during the integrated expansion

Table 1
Literature overview.

References	*Investment decision*	*Network representation*	*Uncertainty consideration*	*Security criteria consideration*	*Real-world system application*
[8]	G&T	Transportation model	No	No	No
[9,10]	G&T	DC power flow	Yes	No	No
[14,18,19]	G&T	DC power flow	No	No	No
[15]	G&T	DC power flow	Yes	Yes	Yes
[17]	G&T	DC power flow	No	No	Yes
[20]	G&T	Transportation model	Yes	No	No
[23]	G&T	DC power flow	Yes	No	Yes
[24]	G&T	DC power flow	Yes	Yes	No
[25,26]	G&T	DC power flow	No	Yes	No
[32,33]	No	AC power flow	Yes	No	No
[28,31]	G	AC power flow	Yes	No	Yes
[30]	T	AC power flow	Yes	Yes	Yes
[27,29]	G	No	Yes	Yes	No

planning process, ensuring that the final plan meets the N-1 security constraint at minimum cost. This criterion is inserted efficiently in method M3 to avoid a significant increase in computational time or harm the convergence of the methods developed.

The hierarchical approach, executed by solving sequentially the GEP, TEP, and operation simulation, which will be referred to as method M1, will be considered as a benchmark solution of lower computational burden. Also, a complete formulation that integrates the G&T planning with the operation simulation, referred to as method M4, to provide more effective results at the expense of intensive computation, is also presented as a benchmark of the lower total cost for comparison with the proposed formulations. The N-1 criterion is also inserted in M4.

This paper is organized as follows. Section 2 presents the overall scheme of the alternatives compared for the integrated generation and expansion planning. Section 3 presents the mathematical formulation of the underlying methods used in the different alternatives. Section 4 illustrates and discusses the application of the methods in a study case of the Chilean system. Finally, Section 5 highlights the main conclusions.

2. Integrated generation and transmission expansion planning problem

The total cost that expansion planning aims to minimize can be partitioned into two components:

(i) *investment cost*, which represents the sum of the annualized investment costs in G&T, and
(ii) *operation cost*, which depends on the dispatch decision and represents the sum of costs related to fuel consumption of thermal plants and penalties for not meeting the foreseen demand.

Thus, the dispatch decision during the power system operation assessment significantly affects the expansion decision and must be well represented in the expansion planning problem. Planning problems are, by nature, multi-year types, and they typically consider annual partitions to carry out the overall analysis.

The overall scheme of the alternative methodologies for integrated G&T planning carried out in this paper is shown in Fig. 1, highlighting the variants that have been addressed. The aim is to evaluate the tradeoff between solution quality and computational effort among all methods. In each method, the solutions at the individual stages are applied sequentially. For the methods that represent the network constraints, the N-1 security criterion in the transmission can be directly represented in the methods. However, including the N-1 security criterion significantly increases the complexity of the optimization problem, so a different approach is recommended. In methods M3 and M4, given a G&T expansion plan calculated by any method without considering the N-1 criterion, the process is started once more, now contemplating the N-1 security criterion in the optimization. To do so, more constraints representing all single contingencies in the network are added to the expansion planning problem, ensuring that the final expansion plan

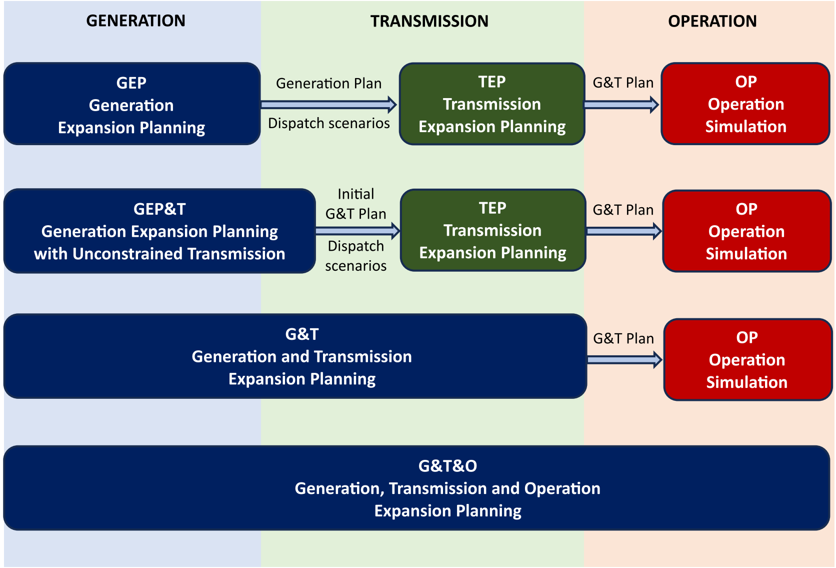

Fig. 1. Overall scheme of integrated G&T expansion planning.

meets the security criterion. The operation simulation is then executed considering the N-1 criterion.

In synthesis, the four methods have the following main characteristics:

M1) In the hierarchical approach, there are three separate stages. The first stage calculates the generation expansion plan without considering the transmission network. The second stage calculates the transmission expansion plan, considering the fixed generation expansion and the dispatch decisions made in the first stage. Finally, the Stochastic Dual Dynamic Programming (SDDP) methodology (described in Section 3.1.1) is used for the operation simulation, considering the G&T expansion plan calculated in the previous stages. This final stage is vital for validating the final expansion plan and calculating the actual operation costs. Note that the hierarchical approach differs from the integrated approach, as the generation decisions are taken at *Stage 1* for an ideal network, and then the network constraints are included in *Stage 2*. Finally, *Stage 3* is performed considering the final G&T expansion plan. The hierarchical method is used as a benchmark of CPU time for comparisons with the other methods.

M2) In the partially integrated G&T method, the first stage is no longer executed to determine only the generation plan but is extended to include some content referring to the transmission network. An initial GEP&T expansion plan considering a transmission network with no limits is solved with the power balance constraint at each node. It is then necessary to execute the transmission expansion planning, including the network limits, to complement the initial transmission expansion plan with new circuits and to make the final expansion plan meet all operational constraints. Again, *Stage 3* is performed considering the final G&T expansion plan.

M3) In the integrated G&T proposal without recalculation of the Future Cost Function (FCF, see Section 3.1.1), given the current global concern about the environmental impact of hydro plants, it is becoming less likely that new hydro plants will be built with large reservoirs. Therefore, depending on the type of candidates considered in the expansion planning (with no hydro plant projects on the list, for instance), it may be reasonable to assume that the reservoirs' operational policies will remain mostly the same in the future years. This method proposes avoiding recalculating the FCFs in the G&T iterations on these bases. In this case, the FCFs must be given at the beginning of the optimization and will remain fixed throughout the whole computational process until convergence. Therefore, this strategy tends to save CPU time in the computational process. As the network is fully represented at this point, applying the TEP methodology after defining the G&T plan is unnecessary. Still, the operation simulation at the end is necessary to calculate the FCFs consistent with the final G&T expansion plan.

M4) The fully integrated approach applies the integrated G&T expansion planning together with the simulation of the system operation without simplifying the formulation and solution of the optimization problem. This fully integrated approach is foreseen to produce the expansion plan with the lowest total cost, whose results can be compared with the other alternatives.

Table 2
Summary of the methods considered.

Method	*Stage 1*	*Stage 2*	*Stage 3*	*Notes*
M1 (hierarchical approach)	GEP no network	TEP fixed generations	Operation	Lower CPU time
M2 (partially integrated G&T)	GEP&T, network no limit	TEP full network	Operation	No network limits
M3 (integrated G&T no new FCF)	G&T fixed FCFs		Operation	No FCF update
M4 (fully integrated approach)	G&T no simplification			Lower total costs

Table 2 summarizes the proposed methods:

It must be emphasized that although the compared methods follow an open loop approach as Fig. 1, they are characterized by a refining process in terms that each stage improves the solution obtained in the previous one as a chained model.

3. Formulation of the underlying methods

The details of the four methods are presented in this section. The formulations are first shown without including the N-1 security to compare the results obtained for a first preselection of the methods to be developed further. The formulations that include the N-1 security are then presented in Section 3.2 for the most promising methods.

3.1. *Problem formulations without N-1 security constraints*

3.1.1. *Operation problem (OP)*

The operation problem involves determining each plant dispatch at all time steps to minimize the total operating cost. The presence of hydro plants with storage makes the dispatch problem more complex since hydro plants can transfer water from one time step to the next. Therefore, it has to be established whether the plant should store water in the reservoir for future use or use it promptly. The problem becomes even more complex when considering the future uncertainty related to VRE production.

Disregarding the uncertainties and the transmission network (single-node representation) by now, the hydrothermal dispatch problem can be formulated as follows:

$$\min \left\{ \sum_{t=1}^{T} \left(\sum_{j\in J_T} c_{j,t} g_{j,t} + \delta\, r_t \right) \right\} \tag{3.1}$$

s.t.

$$\sum_{j\in J_T \cup J_R} g_{j,t} + \sum_{i\in J_H} \rho_j u_{j,t} + r_t = d_t \;;\; \forall t = 1,\cdots T \tag{3.2}$$

$$v_{j,t+1} = v_{j,t} + a_{j,t} - u_{j,t} - s_{p_{j,t}} + \sum_{m\in \Pi_j} \left(u_{m,t} + s_{p_{m,t}} \right) \;;\; \forall j \in J_H \;;\; \forall t = 1,\cdots T \tag{3.3}$$

$$v_{j,t+1} \le \overline{v_j} \forall j \in J_H \;;\; \forall t = 1,\cdots T \tag{3.4}$$

$$u_{j,t} \le \overline{u_j} \forall j \in J_H \;;\; \forall t = 1,\cdots T \tag{3.5}$$

$$g_{j,t} \le \overline{g_j} \forall j \in J_T \;;\; \forall t = 1,\cdots T \tag{3.6}$$

The objective function (3.1) aims to minimize the sum of the operating cost of the thermal plants, subject to a set of constraints: load balance of the system (3.2), water balance of each hydro plant (3.3), storage (3.4) and turbined outflow (3.5) limits of the hydro plants, and generation limit for the thermal plants (3.6). In this problem, the renewable generation is predetermined.

Depending on the dimension of the system and the number of time steps, the problem could become computationally intractable. So, decomposing the large problem into several smaller one-time-step subproblems is a fair approach to solving the problem. However, to maintain coherence in the time-coupled decisions between the time steps, it is necessary to approximate a function that represents the future costs related to the dispatch decision at each time step. This function is called the Future Cost Function (FCF).

After approximating the FCFs, it is possible to evaluate the trade-off between using the water in the current and future time steps. So, the one-time step subproblems can be solved forward in time, minimizing the sum of the Immediate Cost Function (ICF), which represents the operating cost of the current time step, and the FCF. The following formulation illustrates the one-time step dispatch problem, now including the network representation, at time step t:

$$\min\left\{\sum_{j\in J_T} c_j g_{j,t} + \delta\, r_t + FCF_t\right\} \tag{3.7}$$

s.t.

(3.3) – (3.6)

$$\sum_{j\in\Phi_i\cap(J_T,J_R)} g_{j,t} + \sum_{i\in\Phi_i\cap J_H} \rho_j u_{j,t} + \sum_{k\in\Omega_i^+} f_{k,t} - \sum_{k\in\Omega_i^-} f_{k,t} + \delta\, r_{i,t} = d_{i,t}\ ;\ \forall i\in I \tag{3.8}$$

$$f_{k,t} = \gamma_k \Delta\theta_{k,t}\ ;\ \forall k\in K \tag{3.9}$$

$$-\overline{f_k} \le f_{k,t} \le \overline{f_k}\ ;\ \forall k\in K \tag{3.10}$$

where $\sum_{j\in J_T} c_j g_j + \delta\, r_t$ represents the ICF at time step t and FCF_t is the future cost function at time step t. The constraint (3.8) represents the power balance, (3.9) the DC power flow, and (3.10) the operating limits for the transmission circuits.

One technique used to approximate those functions is Dual Dynamic Programming (DDP) [34], based on the Benders decomposition theory [35]. This technique consists of an iterative method for building approximations of the FCF around interesting storage states defined by the method itself by iteratively creating linear segments around those interesting storage states, using the information on the operating cost at these states (equation (3.7)) and the FCF derivatives with respect to the storage state of each hydro plant (dual-variables of constraint (3.3)).

The iterative method of DDP can be divided into two phases: the Forward phase, which consists of solving all time steps from the first to the final time step T, passing the calculated storage levels of the current problem to the problems of subsequent time steps, and the Backward phase that solves the time steps from T to the first time step and adds linear segments around the interesting storage points generated in the Forward phase to update the approximation of the FCF of each time step. That is, dual variables are calculated in the problem associated with stage *t*+1 and sent to stage *t* as constraints that relate the marginal variation of the total operating cost from stage *t*+1 to stage *T* with the marginal variation of the primal solution provided at stage *t*.

This Forward-Backward process repeats until the convergence criterion is satisfied. This criterion consists of calculating an upper and a lower bound at each iteration, calculating the gap between them, and comparing this gap to a predefined tolerance. The upper bound is the sum of the immediate cost of all time steps, which represents the best solution at the current iteration, and the lower bound, the total cost of the first time step (ICF + FCF), which represents an approximation for the total cost of all time steps.

To incorporate uncertainties, once stochastic models that capture the statistical behavior of hydrology (or VRE production) are available, it is possible to generate as many scenarios of natural inflows and VRE production as needed by the Monte Carlo method to represent this variability. The Stochastic Dual Dynamic Programming (SDDP) method [36] can be applied to consider those synthetic scenarios in determining optimal dispatch under uncertainty. SDDP is very similar to the deterministic DDP described before. In the forward phase, multiple scenarios are considered, and the solution is similar to various deterministic DDP problems. However, the Backward phase presents some differences, as the approximated FCFs must now capture the uncertainty in the future operation. For that purpose, more scenarios are contemplated in this phase.

Thus, in the SDDP method, the formulation represented by Eqs. (3.3) to (3.10) is applied at each stage for each synthetic scenario, which includes one inflow scenario and one VRE production scenario. These single-stage/scenario problems are solved iteratively through the Forward-Backward process. The FCFs are then approximated by linear segments, with coefficients equal to the average of dual variables calculated for each future scenario in the following stage during the Backward phase.

Thanks to applying this decomposition and iterative algorithm, the SDDP method can efficiently compute the optimal dispatch of real-sized systems encompassing many hydroelectric and VRE plants. However, if the operating problem includes non-convexities, such as integer variables for unit commitment constraints, the method may yield suboptimal solutions, as convexity is required for Benders decomposition to ensure optimality.

3.1.2. *Generation expansion planning (GEP)*

The GEP involves including investment variables and constraints to the operation problem to calculate the optimal generation expansion plan. At this point, the transmission network will be neglected. The multi-time step generation expansion planning problem is formulated as follows:

$$\min\left\{\sum_{t=1}^{T}\left(\sum_{j\in J_{TC}\cup J_{RC}\cup J_{HC}} p_{j,t}x_{j,t} + \sum_{j\in J_T} c_{j,t}g_{j,t} + \delta\, r_t\right)\right\} \tag{3.11}$$

s.t.

$$\begin{array}{ll} (3.2),\ (3.3) & \\ (3.4),\ (3.5) & ;\ \forall j\in J_{HE};\ \forall t\ = 1,\cdots T \\ (3.6) & ;\ \forall j\in J_{TE}\cup J_{RE};\ \forall t\ = 1,\cdots T \end{array}$$

$$g_{j,t} \le \overline{g_j}\sum_{\tau=1}^{t} x_{j,\tau}\ ;\ \forall j\in J_{TC}\cup J_{RC}\ ;\ \forall t = 1,\cdots T \tag{3.12}$$

$$v_{j,t+1} \le \overline{v_j}\sum_{\tau=1}^{t} x_{j,\tau}\ ;\ \forall j\in J_{HC}\ ;\ \forall t = 1,\cdots T \tag{3.13}$$

$$u_{j,t} \le \overline{u_j}\sum_{\tau=1}^{t} x_{j,\tau}\ ;\ \forall j\in J_{HC}\ ;\ \forall t = 1,\cdots T \tag{3.14}$$

$$x_{j,t} \in \{0,1\}\ ;\ \forall j\in J_{TC}\cup J_{RC}\cup J_{HC}\ ;\ \forall t = 1,\cdots T \tag{3.15}$$

$$\sum_{t=1}^{T} x_{j,t} \le 1\ ;\ \forall j\in J_{TC}\cup J_{RC}\cup J_{HC} \tag{3.16}$$

The objective function incorporates the investment costs in new generation plants in addition to the operating and deficit costs, aiming to minimize the total cost of the system. The operating limits of the candidate plants, multiplied by the term $\sum_{\tau=1}^{t} x_{j,\tau}$, are represented by the constraints (3.12), (3.13), and (3.14). This term, due to constraints (3.15) and (3.16), is equal to 1 at time steps from t to T, if project j is decided to commit at time step t, and 0 at time steps from 1 to $t-1$.

Since the problem now presents integer variables ($x_{j,t}$), it is characterized as a MILP problem, unlike the linear programming problems shown in the previous section. Furthermore, this structure naturally suggests using a decomposition scheme to solve the problem as the operating constraints depend on the investment decisions. In this scheme, the investment decision is made upfront and, after that, the dispatch problem can be solved by fixing those decisions. The Benders decomposition technique [35] is applied to solve this problem in this work.

The investment module aims to minimize the sum of the investment costs and an approximation of the operating cost. This approximation is represented by a piecewise linear function built by the Benders cuts generated by the operation module. At each iteration, the investment subproblem is solved, and the investment decisions are sent to the operation module.

The operation module, in turn, calculates the multi-time step optimal dispatch, considering the investment decisions made by the previous module. This problem is solved using the SDDP algorithm. So, all aspects related to time-coupling dispatch decisions, uncertainty in the hydrology and VRE production, and time-dependent variability of the demand and VRE are considered during the expansion planning.

From the dual variables (Lagrange multipliers) of the operating constraints and the value of the objective function, Benders cuts are calculated and sent back to the master problem (investment module). Those cuts represent a linear approximation of the expected value of the operating cost due to different investment decisions. Throughout the iterations, this approximation improves, and the investment module can establish better investment decisions.

These iterations repeat until the convergence criterion is satisfied. Again, the convergence criterion consists of calculating an upper and a lower bound at each iteration, calculating the gap between them, and comparing this gap to a predefined tolerance. But now, the upper bound represents the sum of the investment cost and the operating cost calculated by the operating subproblem. The lower bound, in turn, is the sum of the investment cost and the approximation of the operating cost, assembled by the Benders cuts.

The investment subproblem can be formulated as follows:

$$\min\left\{\sum_{t=1}^{T}\sum_{j\in J_{TC}\cup,J_{RC}\cup,J_{HC}} p_{j,t}x_{j,t}+\alpha\right\} \tag{3.17}$$

s.t.

(3.15), (3.16)

$$\alpha \geq w(x^{\mu})-\sum_{t=1}^{T}\left(\sum_{j\in J_{TC}\cup J_{RC}}\left(\overline{g_j}\sum_{\tau=t}^{T}\pi^{\mu}_{j.\tau}\right)\left(x_{j,t-}x^{\mu}_{j,t}\right)+\sum_{j\in J_{HC}}\left(\overline{v_j}\sum_{\tau=t}^{T}\pi^{\mu,v}_{\tau,j}+\overline{u_j}\sum_{\tau=t}^{T}\pi^{\mu,u}_{\tau,j}\right)\left(x_{j,t-}x^{\mu}_{j,t}\right)\right)\ ;\ \forall\mu=1,\cdots\varphi \tag{3.18}$$

and the operation subproblem is formulated as follows:

$$w(x^{\varphi})=\min\left\{\sum_{t=1}^{T}\left(\sum_{j\in J_T}c_{j,t}g_{j,t}+\delta\, r_t\right)\right\} \tag{3.19}$$

s.t.

(3.2), (3.3)
(3.4), (3.5) ; $\forall j\in J_{HE}; \forall t=1,\cdots T$
(3.6) ; $\forall j\in J_{TE}; \forall t=1,\cdots T$
(3.12), (3.13), (3.14)

The constraints (3.18) represent the Benders cuts generated at each iteration μ of the operation subproblem, composing the linear piecewise function α, which represents the approximation of the operating cost function. The constraints (3.12), (3.13) and (3.14) use the candidate solution $x^{\varphi}_{j,t}$ of the expansion problem. As mentioned, this subproblem is solved by the SDDP algorithm, which can incorporate uncertainties in the operating planning by considering multiple scenarios during the optimization. The Benders cut coefficients, in that case, will be equal to the average of the coefficients calculated for each scenario defined. In that way, uncertainties are also considered in the investment decision-making process.

3.1.3. Generation expansion planning with simplified network representation (GEP&T)

It is possible to consider a simplified representation of the transmission network in the generation expansion problem [37]. In this case, some variables and constraints related to the transmission system must be added to the GEP problem, as follows:

Investment subproblem:

$$\min\left\{\sum_{t=1}^{T}\left(\sum_{j\in J_{TC}\cup,J_{RC}\cup,J_{HC}} p_{j,t}x_{j,t}+\sum_{k\in K_C}p_{k,t}x_{k,t}\right)+\alpha\right\} \tag{3.20}$$

s.t.

(3.15), (3.16)

$$\alpha \geq w(x^{\mu})-\sum_{t=1}^{T}\left(\sum_{j\in J_{TC}\cup J_{RC}}\left(\overline{g_j}\sum_{\tau=t}^{T}\pi^{\mu}_{\tau,j}\right)\left(x_{j,t-}x^{\mu}_{j,t}\right)+\sum_{j\in J_{HC}}\left(\overline{v_j}\sum_{\tau=t}^{T}\pi^{\mu,v}_{\tau,j}+\overline{u_j}\sum_{\tau=t}^{T}\pi^{\mu,u}_{\tau,j}\right)\left(x_{j,t-}x^{\mu}_{j,t}\right)+\sum_{k\in K_C}\left(-\overline{f_k}\sum_{\tau=t}^{T}\pi^{\mu,f}_{\tau,k}\right)\left(x_{k,t-}x^{\mu}_{k,t}\right)\right)\ ;\ \forall\mu=1,\cdots\varphi \tag{3.21}$$

$$x_{k,t}\in\{0,1\}\ ;\ \forall k\in K_C\ ;\ \forall t=1,\cdots T \tag{3.22}$$

$$\sum_{t=1}^{T}x_{k,t}\leq 1\ ;\ \forall k\in K_C \tag{3.23}$$

Operation subproblem:

$$w(x^{\varphi})=\min\left\{\sum_{t=1}^{T}\left(\sum_{j\in J_T}c_{j,t}g_{j,t}+\delta\sum_{i=1}^{I}\delta\, r_{i,t}\right)\right\} \tag{3.24}$$

s.t.

(3.3)
(3.4), (3.5) ; $\forall j\in J_{HE}; \forall t=1,\cdots T$
(3.6) ; $\forall j\in J_{TE}; \forall t=1,\cdots T$
(3.8) ; $\forall t=1,\cdots T$
(3.10) ; $\forall k\in K_E; \forall t=1,\cdots T$
(3.12), (3.13), (3.14)

$$-\overline{f_k}\sum_{\tau=1}^{t}x_{k,\tau}\leq f_{k,t}\leq\overline{f_k}\sum_{\tau=1}^{t}x_{k,\tau}\ ;\ \forall k\in K_C\ ;\ \forall t=1,\cdots T \tag{3.25}$$

The introduction of the investment variable $x_{k,t}$ in the constraint (3.9) for candidate circuits would cause a non-linearity. This problem could be bypassed using the disjunctive formulation, as detailed in the TEP problem (Section 3.1.4). However, method M2 proposes disregarding this constraint at this stage to avoid using the disjunctive formulation, which can make the algorithm ill-conditioned, given the huge values of the disjunctive constants. Then, a TEP must be performed with full representation of the network afterward to ensure that the final expansion plan meets all network constraints.

3.1.4. Transmission expansion planning (TEP)

TEP aims to calculate the optimal transmission expansion plan, considering that the generation investment decisions have already been taken.

The non-linearity caused by the investment variable $x_{k,t}$ included in the constraint (3.9) is bypassed by the disjunctive formulation. As mentioned, the presence of the disjunctive constants (M_k) can make the iterative process of the Benders decomposition ill-conditioned, where from a certain iteration onward, the Benders cuts generated by the operating subproblem no longer provide relevant information to guide the investment problem toward optimal decisions, causing the algorithm to fall into a trial-and-error process. To mitigate this effect, Binato et al. [38] propose to calculate the smallest value for the disjunctive constant based on the reactance of the shortest path between the terminal buses of the circuit.

Moreover, the operating subproblem can be seen as an optimal power flow (OPF) problem, aiming to minimize the total deficit of the system. In that case, the dispatch of each generator is an input data ($g^{*}_{j,t}$), both for hydro and thermal, determined by the previous stage of the generation expansion.

Applying the same decomposition scheme described in the previous section, the TEP problem can be formulated as follows:

Investment subproblem:

$$\min\left\{\sum_{t=1}^{T}\sum_{k\in K_C}p_{k,t}x_{k,t}+\alpha\right\} \tag{3.26}$$

s.t.

(3.22), (3.23)

$$\alpha \geq w(x^{\mu}) - \sum_{t=1}^{T} \left(\sum_{k \in K_C} \left(M_k \sum_{\tau=t}^{T} \pi_{\tau,k}^{\mu,\gamma} - \overline{f_k} \sum_{\tau=t}^{T} \pi_{\tau,k}^{\mu,f} \right) \left(x_{k,t-} x_{k,t}^{\mu} \right) \right) \; ; \; \forall \mu = 1, \cdots \varphi \tag{3.27}$$

Operation subproblem:

$$\min \left\{ \sum_{i=1}^{I} \sum_{t=1}^{T} \delta \, r_{i,t} \right\} \tag{3.28}$$

s.t.

(3.9), (3.10) ; $\forall k \in K_E; \forall t = 1, \cdots T$

$$\sum_{j \in \Phi_i} g_{j,t}^{*} + \sum_{k \in \Omega_i^{+}} f_{k,t} - \sum_{k \in \Omega_i^{-}} f_{k,t} + \delta \, r_{i,t} = d_{i,t} \; ; \; \forall i \in I \; ; \forall t = 1, \cdots T \tag{3.29}$$

$$-M_k \left(1 - \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \right) \leq f_{k,t} - \gamma_k \Delta\theta_{k,t} \leq M_k \left(1 - \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \right) \; ; \; \forall k \in K_C \; ; \forall t = 1, \cdots T \tag{3.30}$$

$$-\overline{f_k} \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \leq f_{k,t} \leq \overline{f_k} \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \; ; \; \forall k \in K_C \; ; \forall t = 1, \cdots T \tag{3.31}$$

Regarding the number of periods, considering many periods in the same optimization problem may make the problem computationally heavy. So, the problem described above can be formulated for a single period (usually, this represents an entire year). If the expansion horizon is longer than one period (year), this strategy is performed period by period, going forward in time and considering that the investment decisions of previous periods are fixed.

3.1.5. *Integrated generation and transmission planning (G&T) and operation (G&T&O)*

Joining the full representation of the network to the GEP&T formulation results in the following integrated generation and transmission expansion planning problem:

Investment subproblem: the formulation is the same as in GEP&T, just replacing the constraint (3.21) by constraint:

$$\alpha \geq w(x^{\mu}) - \sum_{t=1}^{T} \left(\sum_{j \in J_{TC} \cup J_{RC}} \left(\overline{g_j} \sum_{\tau=t}^{T} \pi_{\tau,j}^{\mu} \right) \left(x_{j,t-} x_{j,t}^{\mu} \right) + \sum_{j \in J_{HC}} \left(\overline{v_j} \sum_{\tau=t}^{T} \pi_{\tau,j}^{\mu,v} + \overline{u_j} \sum_{\tau=t}^{T} \pi_{\tau,j}^{\mu,u} \right) \left(x_{j,t-} x_{j,t}^{\mu} \right) + \sum_{k \in K_C} \left(M_k \sum_{\tau=t}^{T} \pi_{\tau,k}^{\mu,\gamma} - \overline{f_k} \sum_{\tau=t}^{T} \pi_{\tau,k}^{\mu,f} \right) \left(x_{k,t-} x_{k,t}^{\mu} \right) \right) \; ; \; \forall \mu = 1, \cdots \varphi \tag{3.32}$$

Operating subproblem: the formulation is the same as in GEP&T, adding the constraints (3.9) and (3.30).

It is also recalled that the operating subproblem is solved using the SDDP methodology, which decomposes the multi-period hydrothermal dispatch problem into multiple one-period problems. To maintain the consistency of the reservoir operating along the time steps, a FCF for each one-time step problem is approximated. This approximation is named as "*policy*" calculation.

However, the construction of new hydroelectric plants with big reservoirs is becoming more unlikely, given the current global concern about the environmental impact that this undertaking causes. So, depending on the type of candidates considered in the expansion planning (if there is no hydro plant project on the list, for instance), it may be reasonable to assume that the operating policy of the reservoirs will remain mostly the same in future years.

Therefore, the method M3 proposes to avoid recalculating the FCFs at each iteration of the Benders decomposition in the first stage (referred to as G&T in Fig. 1). In this case, the FCF must be given at the beginning of the optimization and remain fixed throughout the whole convergence process. Therefore, this strategy saves CPU time in the iterations of the decomposition scheme. However, to evaluate the final expansion plan, an OP execution must be carried out at the end, disregarding the input FCF and recalculating it based on the final plan.

On the other hand, the G&T&O strategy adopted in M4 does not present any simplification in the algorithm or the problem formulated in this section, avoiding the need to run an OP model afterward.

3.2. *Inclusion of the N-1 security constraints*

The N-1 criterion is a well-known security constraint that imposes that the solution for the optimal dispatch must not violate the operating constraints whether any single contingency occurs in the system. In terms of formulation, every single contingency configures a new network topology. Thus, to represent this constraint in the problem, the network variables and constraints considering all single contingencies in the network must be added to the operating subproblem. These constraints can be formulated as follows:

$$-M_k \left(1 - \sum_{\tau=1}^{t} x_{k,\tau} \right) \leq f_{k,t,n} - \gamma_k \Delta\theta_{k,t,n} \leq M_k \left(1 - \sum_{\tau=1}^{t} x_{k,\tau} \right) \; ; \; \forall k \in K_C \; ; \forall t = 1, \cdots T \; ; \forall n \in N \tag{3.33}$$

$$\sum_{j \in \Phi_i \cap (J_T \cup J_R)} g_{j,t} + \sum_{i \in \Phi_i \cap J_H} \rho_j u_{j,t} + \sum_{k \in \Omega_i^{+}} f_{k,t,n} - \sum_{k \in \Omega_i^{-}} f_{k,t,n} + \delta \, r_{i,t} = d_{i,t} \; ; \forall i \in I \; ; \forall n \in N \tag{3.34}$$

$$f_{k,t,n} = \gamma_k \Delta\theta_{k,t,n} \; ; \; \forall k \in K_E \; ; \forall n \in N \tag{3.35}$$

$$-\overline{f_{ke}} \leq f_{k,t,n} \leq \overline{f_{ke}} \; ; \; \forall k \in K_E \; ; \forall n \in N \tag{3.36}$$

$$-\overline{f_{ke}} \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \leq f_{k,t,n} \leq \overline{f_{ke}} \sum_{\tau=1}^{t} x_{k,\tau}^{\varphi} \; ; \; \forall k \in K_C \; ; \forall t = 1, \cdots T \; ; \forall n \in N \tag{3.37}$$

The representation of the N-1 criterion notably increases the number of constraints in the problem, affecting the computation performance of the algorithm. This paper explores the strategy called *Complementary expansion planning* to improve the algorithm performance when considering the N-1 criterion. In this strategy, given a G&T expansion plan calculated without considering the N-1 criterion, the optimization can be started again, considering the given G&T plan fixed and contemplating the N-1 security criterion in the optimization. The benefits of this strategy will be shown in the next section. In contrast, the strategy called *Complete expansion planning* solves the optimization problem by considering the N-1 constraints from the beginning. The N-1 criterion is introduced in M3 and M4.

4. Application to a National power system

The abovementioned methods have been applied in a case representing the Chilean power system. The system data is summarized in the following section. The detailed data is available at [37]. The study horizon considered in the simulations is 2025 to 2030 (6 years). So, a fixed G&T expansion plan until 2024 is considered as input.

The expansion plans calculated by the proposed alternatives listed in Section 2.4 were compared regarding the solution quality, i.e., the total G&T expansion costs and the computational performance assessed through the CPU computation time.

4.1. System details

4.1.1. Demand projection

The electricity demand projection presents a yearly growth of 4 %, from 95 TWh in 2025 to 115 TWh in 2030. The demand input for the model has hourly resolution for the entire study horizon. However, during the simulations, the demand is clustered into 13 blocks at each time window (month), using the K-means clustering method. Thereby, the model considers 13 demand levels at each time window, with different durations according to the clustering result.

4.1.2. Generation data

Table 3 illustrates the capacity mix in 2024. The system presents a varied mix of generation sources (hydro, thermal, and renewable energy), with an estimated average operating cost of 1,051 M$ in 2024 [37].

The candidate projects listed for the expansion simulations comprise thermal plants (gas-fired) and VRE (wind and solar). For the thermal plants, 20 projects are considered [37], where two types of gas-fired generation are represented:

(i) 10 projects with open cycle gas turbines (OCGT) with a capacity of 100 MW each.
(ii) 10 projects with combined cycle gas turbines (CCGT) with relatively higher investment costs (33 %) to the OCGT, higher capacity (250 MW each), and lower operation costs (73 %).

The VRE candidates were created based on the projects that participated in past power supply auctions in Chile. The total capacity of VRE candidates considered is 9.6 GW of solar (182 projects), primarily located in the country's northern region, and 7.1 GW of wind (81 projects), mainly in the center and south regions [37]. Fig. 2 shows the geographical location of these candidates. Wind projects have an investment cost of 48 % higher than solar projects. On the other hand, the capacity factors of wind projects range between 26–46 %, and for solar projects, 20–37 %, depending on the location.

4.1.2.1. Definition of the probabilistic scenarios. The historical water inflow data, used as an input to the model, is 54 years long with monthly values. SDDP manages this information to forecast future inflow scenarios used in the expansion planning optimization.

In the case of wind and solar production, for both existing and candidate plants, a historical record was created through the following procedure:

(i) From the geographical position of each plant, the wind and irradiation resource data were obtained considering an hourly historical window of 30 years (from a public Chilean database);
(ii) These values were transformed into energy production values through a simulation model with some assumptions regarding the characteristics of the wind generator/PV panel;
(iii) For the existing plants, using the actual measurement data of the equipment, a "scaling" of the energy production values was made; and
(iv) Synthetic future scenarios of VRE production, correlated to the water inflow scenarios, were generated using a statistical model and Bayesian Network, which capture the most significant correlations existing in the historical data between renewable production and hydrological inflows [39,40].

Table 3
Generation system in 2024.

Technology	*Installed capacity (GW)*	*Average Operating Cost ($/MWh)*	*Technology*	*Installed capacity (GW)*	*Average Operating Cost ($/MWh)*
Hydro	7.4 (23 %)	0	Coal	5.1 (16 %)	33
Solar	5.9 (19 %)	0	Diesel	5.7 (18 %)	194
Wind	3.4 (11 %)	0	Others	0.9 (3 %)	93
Natural Gas	3.2 (10 %)	67	**Total**	**31.6 (100 %)**	–

In the end, 30 scenarios [37] of VRE production and water inflow for all plants were produced and considered in the simulations. For the VRE production, the scenarios are generated in hourly resolution (resulting in 8760 × 30 values of capacity factors of each VRE plant). As the demand was aggregated into 13 blocks, the VRE production followed the same aggregation, resulting in 13 × 12 × 30 capacity factors per year represented during the expansion planning optimization for each VRE plant (existing and candidate).

These VRE production scenarios are fixed and used as input to the optimization (regardless of moments of surplus VRE generation when the model can curtail it), reducing the demand to be met by the other sources (hydro and thermal power plants). The variability is illustrated in Fig. 3. Particularly, there is a significant variation of approximately 2 TWh for hydro and 0.6 TWh for wind, depending on the month. Solar does not present relevant variability between the scenarios. This uncertainty underscores the crucial role of the SDDP methodology in managing such variability. However, as the time resolution within a month is represented by 13 aggregated blocks, the hourly intermittency is not captured by the model.

4.1.3. Transmission data

The Chilean transmission system is composed of 500 kV, 220 kV, and under 154 kV lines. In total, the 2024 system has 471 existing circuits, of which 388 are transmission lines and 83 are transformers. The utilization factor of the transmission network is 28 % in 2024. Regarding the candidates, 128 projects [37] are considered, with 104 transmission lines and 24 transformers. The investment costs vary depending on the voltage level and the circuit length for transmission lines.

4.2. Results

4.2.1. Assessment of the methods

The comparison among the results obtained from the different methods is presented by first considering the solutions without incorporating the N-1 criterion, which is then included to draw further conclusions.

Fig. 4 and Fig. 5 present the resulting generation (in MW) and transmission (in km) expansion plans, respectively, calculated by applying the four methods. The convergence tolerance gap defined in the simulations is 1 %. There are differences in the total generation expansion plan, but the model insists on investing only in VRE projects due to their better competitiveness compared to thermal plants.

Solar projects stand out in the expansion due to their lower cost than wind, with added capacity ranging from 5 to 7 GW depending on the method. Even so, some wind projects are included in the expansion, representing an added capacity of 2.5 to 3.5 GW. In terms of transmission, it is notable that methods that do not initially represent (M1) or simplify the network representation (M2) result in a more substantial expansion of lines in the end, increasing costs. In general, strategies M3 and M4 saved on investment in transmission, with around 4,100–4,600 km of lines less than for method M1, implying a generation expansion closer to the load center.

This divergence in the investment in each technology and each year contributes to distinct total costs, as described in Fig. 6. Method M2, which allows the reinforcement of the transmission system at *Stage 1* but with some simplification in its representation, reached a reduction of 3.4 % compared to the fully hierarchical approach used in method M1.

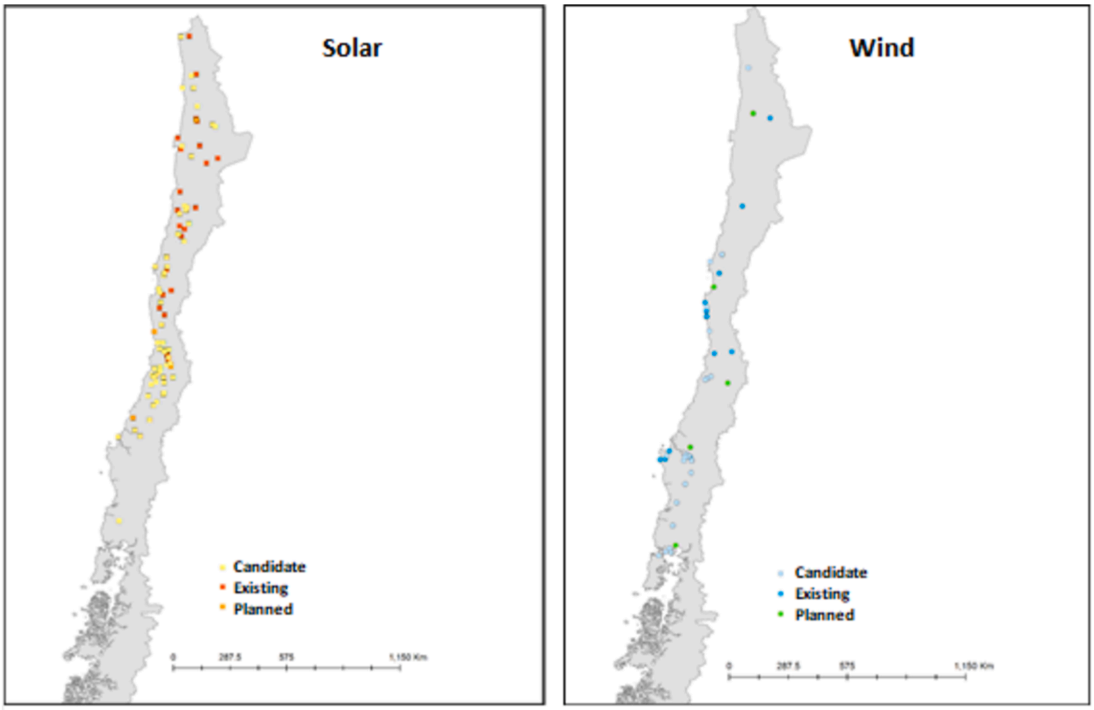

Fig. 2. VRE location.

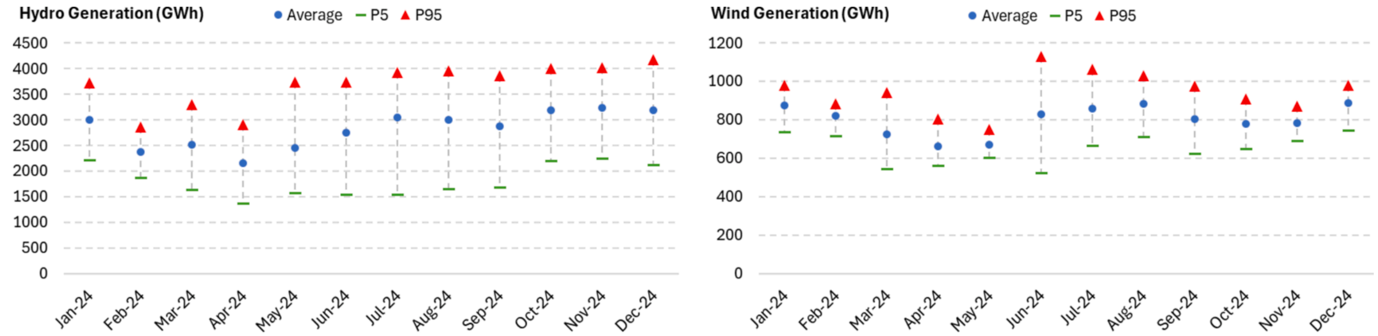

Fig. 3. Generation uncertainty.

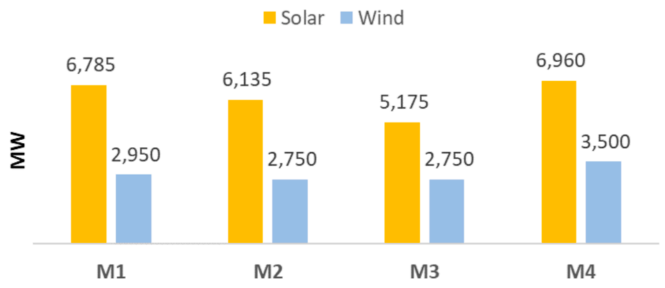

Fig. 4. Generation expansion plan of each method.

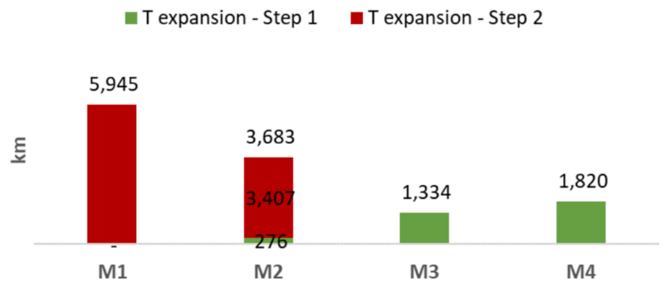

Fig. 5. Transmission expansion plan of each method.

Moreover, applying the strategies with full network representation resulted in a total reduction of 7.9 % (M3) and 8 % (M4). Although it seems like a slight percentage reduction, in absolute terms, this result represents a savings of 700 million dollars for this case study. The similarity in the final costs for M3 and M4 indicates that skipping the recalculation of FCFs in the iterative process did not impact the final result. This can be attributed to the low share of hydropower in Chile's existing capacity mix and the absence of hydropower candidates for expansion.

Fig. 7 presents the computational performance. All *Stage 1* simulations were performed in cloud computing using a cluster with 64 processes and 128 GB of RAM (c7i.16xlarge Amazon instance). Regarding *Stage 2*, the simulations were performed on a local computer with four processors (11th Gen Intell CoITM) i7-1165G7) and 16 GB of RAM.

Method M1 took a few iterations (4) to reach the convergence tolerance, followed by method M2 (53 iterations), M3 (204 iterations),

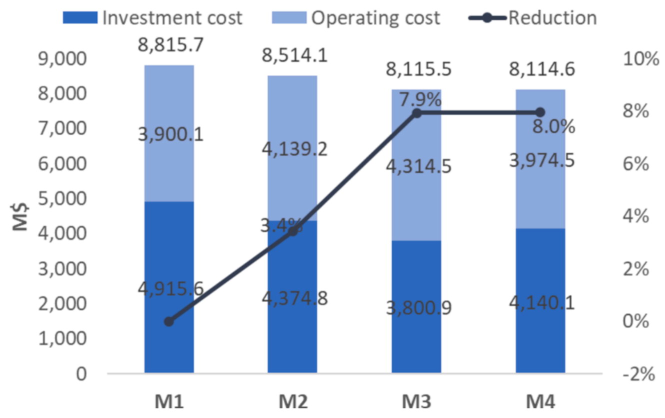

Fig. 6. Total costs of each method.

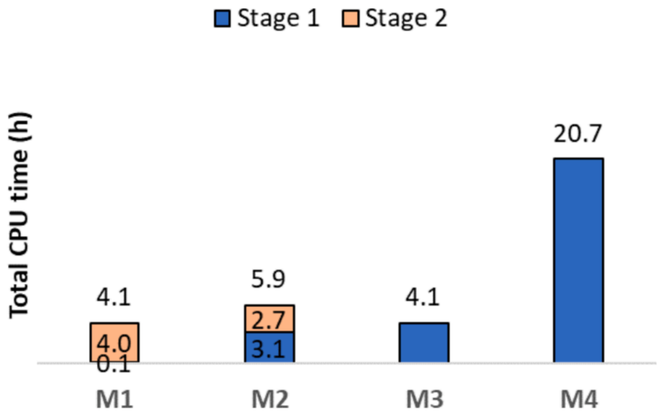

Fig. 7. Total CPU time of each method.

and M4 (315 iterations). Summing up the total CPU time of *Stage 1* and *Stage 2*, method M4 took 20 h of execution, compared to values lower than 6 h in the remaining methods. These results show how skipping FCF recalculations during algorithm iterations speeds up the convergence process without affecting the outcome for that case. On average, each iteration of M3 took 1.2 min, compared to 3.9 min for M4, with both achieving similar total costs.

Evaluating the CPU performance and the expansion results, the method M3 stands out against the others, reaching a very close cost reduction to M4, representing the method with lower total cost and better computation performance (16.6 fewer hours of execution time).

Regarding *Stage 2*, method M2 took less CPU time than M1, as this alternative already invested in some circuits at Stage 1, reducing 33 % of the CPU time in *Stage 2*.

4.2.2. *N-1 criterion*

The simulations with N-1 security constraints in the transmission system are assessed in two executions, one considering the expansion plan calculated in the previous section fixed, so the simulation complements the initial plan with new generators/circuits to meet the N-1 constraints (*Complementary*), and the other with no fixed expansion plan (*Complete*).

Circuits with voltage levels greater or equal to 154 kV were selected for the contingency lists, representing 480 circuits, 80 % of the sum of existing and candidate circuits. Also, the emergency capacity was considered 20 % higher than the nominal capacity for all circuits. As the N-1 criterion is implemented inside the G&T methodology, only the methods M3 and M4 are simulated, and the tolerance gap was increased to 4 %. The results are shown in the sequel.

The resulting G&T expansion plan for generation and transmission is illustrated in Fig. 8 and Fig. 9 for generation and transmission, respectively. In addition to the M3 expansion plan, the *Complementary* strategy invested in 1,300 MW of solar power, 1,500 MW of wind power, and 560 km of circuits to meet the N-1 criterion. For method M4, the further investment is 575 MW of solar energy and 1,777 km of circuits. These additions in the expansion plan culminate in an increase of 5.4 % and 6.6 % in the total cost, respectively, as illustrated in Fig. 10. Besides, the *Complementary* executions present slightly higher final costs (0.4 % and 0.5 %, respectively) than the *Complete* executions. For the convergence process, Fig. 11 shows the result for both strategies for M3 and M4.

The *Complementary* executions have fewer iterations, with method M3 taking 270 iterations and method M4 stopping after 147 iterations. The *Complete* executions, on the other hand, took 328 and 399 iterations, respectively. The increase in the complexity of optimization problems is notable, as *Complete* approaches require more iterations than simulations without introducing the N-1 criterion.

Looking at the CPU time, the *Complementary* simulations required significantly less time, with M3 spending 12 h and M4 14 h. *Complete* executions required 44.1 and 73.7 h, respectively. To have a fair overall comparison, the *Complementary* strategýs total CPU time is considered the sum of the simulation time without and with the N-1 criterion. For M3, there were 4.1 + 12 = 16.1 h of execution, while M4 spent 20.7 + 14 = 34.7 h. Still, the CPU time applying the *Complementary* strategy can reach a 65 % reduction compared to the *Complete* execution, as seen in M3.

Regarding total execution times, the methods, particularly M3, demonstrated an efficient representation of the N-1 security criterion, effectively handling an extensive list of single-circuit contingencies (80 % of the entire network) in a model of a real-world transmission network. Naturally, as the size of the system and the number of contingencies to be considered increase, the computational time will tend to increase.

From the cases analyzed, if the N-1 criterion is not applied, the hierarchical method M1 obtains the best computational performance in *Stage 1*. However, the final expansion plan is about 8 % more expensive than the best solution found. Method M2 presents more attractive results than the previous ones because it has better costs with a slight increase in total CPU time (1.8 h). Method M3, compared to the fully integrated method M4, has very similar total costs and the lowest CPU burden. When applying the N-1 criterion, both M3 and M4 present similar results in terms of total cost. Moreover, using the solution obtained without applying the N-1 criterion as a starting point for the optimization, as in the *Complementary* strategy, shows reduced computation time and similar costs to the *Complete* strategy. However, comparing the methods, M3 spent 50 % less CPU time than M4. Therefore, after looking at all sets of simulations, the proposed method M3, applying the complementary strategy to consider the N-1 criterion, stands out with the best trade-off between solution quality and computation time.

5. Conclusions

This work has proposed and explored alternative methods for integrated G&T expansion planning with N-1 security constraints. Some

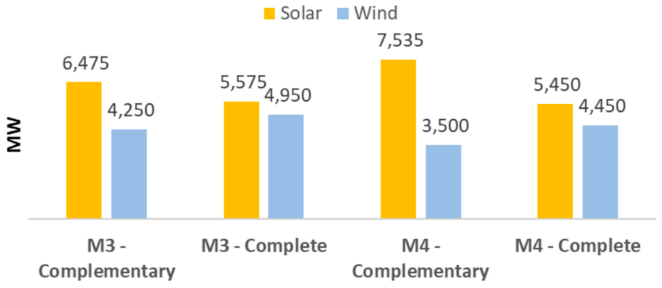

Fig. 8. N-1 Generation expansion plan.

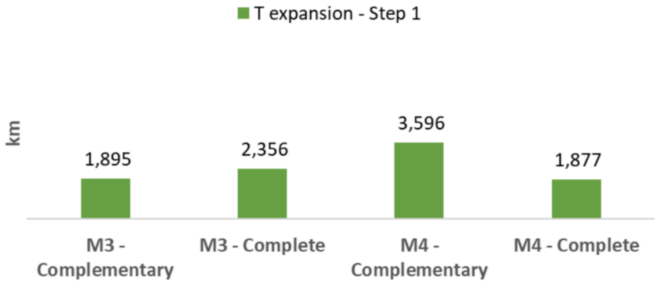

Fig. 9. N-1 Transmission expansion plan.

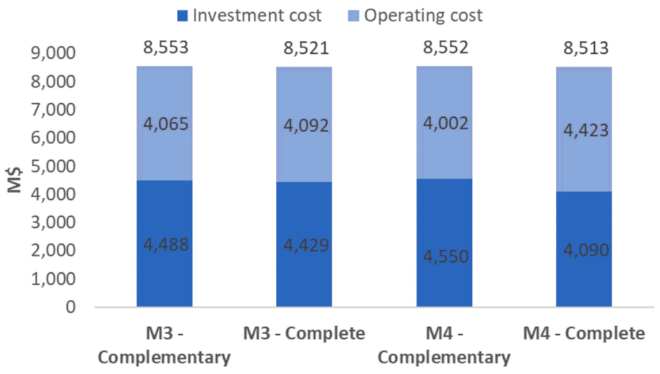

Fig. 10. Total cost of each method with the N-1 criterion.

strategies have been presented that involve a chain execution of optimization methods. For the sake of comparison among the structure of the methods, an overall scheme of multi-stage optimization has been created. The methods have been applied to a representation of a real system (the Chilean system) to evaluate the trade-off between the quality of the solution (in terms of the total cost of the expansion planning) and the computation time. First, the G&T expansion plan has been calculated without N-1 security constraints for all the methods. Then, considering the expansion plan calculated as a starting point, the additional expansion has been determined, ensuring that the system meets the N-1 criterion. Finally, with the final G&T expansion plan in hand, the hydrothermal dispatch optimization has been simulated based on the SDDP method to calculate the system's actual operation costs.

From the results obtained, the best trade-off between the total cost and the computation time has been found by applying the proposed method M3, in which the future cost function is determined and remains fixed during the G&T optimization process and is recalculated only in the final operation stage. This approach, in an execution without N-1 constraints, enables a significant reduction (over 80 %) in the computation time, whereas the final solution differs only about 1 % from the solution obtained from the fully integrated G&T planning and operation approach in which the future cost function is continuously recalculated. Besides, in the case analyzed, this approach represented savings of 8 % (around 700 million dollars) compared to the usual hierarchical approach M1.

When including the N-1 constraints, the reduction in CPU time is still significant (over 50 %) compared to the fully integrated method with similar solutions. The preliminary calculation of an optimal solution without applying the N-1 security constraint as a starting point, followed by re-optimization with active N-1 security constraints, also contributes to a 65 % reduction of the computation time without significantly impacting the quality of the solution.

The proposed methods were also influential in addressing uncertainties in hydroelectric and VRE production, incorporating security criteria, and are applicable to large-scale systems with relevant VRE penetration, such as in Chile. Regarding applying the method in real markets, although the expansion plan produced by this integrated approach may not be fully realized in unbundled markets, it offers a significant advantage over hierarchical methods. It better supports market participants, such as transmission planners, in coordinating and anticipating their investment decisions. It also provides the government with more cost-effective tools for making strategic decisions for energy planning, especially in the context of energy transition.

This study is limited to analyzing the Chilean power system, so its conclusions cannot be generalized to other power systems with different scales, generation mixes, or network topologies. For example, applying the proposed methods to larger systems, such as the Brazilian power system, could make computation time a critical factor, highlighting the value of alternatives with better computational efficiency over those that offer more detailed network representations but at a higher computational cost. Additionally, M3 may not perform as well in power systems with a higher dependence on hydropower, such as those in Brazil and Colombia, where skipping FCF recalculations during the iterative process could lead to suboptimal results. Future work will explore applying these methods to power systems with varying characteristics to assess broader applicability and performance.

CRediT authorship contribution statement

Lucas Y. Okamura: Writing – original draft, Software, Methodology, Investigation, Data curation. **Carmen L.T. Borges:** Conceptualization, Investigation, Supervision, Formal analysis, Writing – review & editing. **Gianfranco Chicco:** Writing – review & editing, Validation, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence

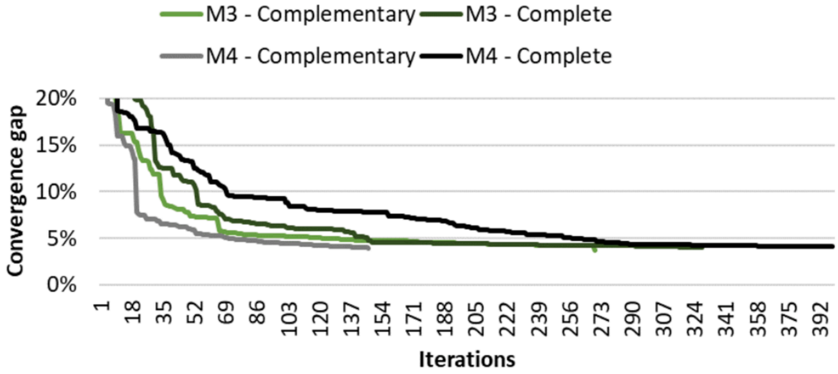

Fig. 11. Convergence for G&T methodology N-1 criterion.

the work reported in this paper.

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Data availability

Data will be made available on request.

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