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AN OVERVIEW OF G. CASNATI'S MATHEMATICAL INTERESTS

Abstract. After celebrating Gianfranco Casnati's 60th birthday in September 2023 during the workshop "**G**eometry and **C**ommutative Algebra", and following Gianfranco's passing in November 2023, we decided to collect some contributions in his memory in this volume. This paper is a preface to the volume, and we use it to provide an overview of Gianfranco's mathematical interests.

1. Introduction

Professor Gianfranco Casnati passed away on November 29th, 2023, leaving his beloved wife Marina and daughter Chiara behind.

In September 2023 we organized a simple event at the Politecnico di Milano to celebrate his 60th birthday. The hardest part of the organization was convincing Gianfranco to accept the celebration. What eventually won him over was the promise that his name would not be mentioned, neither in the title "**G**eometry and **C**ommutative Algebra", nor in the announcement. We used a bolder and bigger font for the initials **G** and **C**, but just a few noticed it. Initially this volume was a follow–up of the September workshop, collecting some papers as a present for Gianfranco. After November 2023, the volume became a way to remember a friend who is no longer with us.

We want to warmly thank all the authors who contributed to the present volume, certain that Gianfranco would have read and appreciated every single paper.

Gianfranco was a landmark for geometers, kind and helpful to friends, collaborators and colleagues. He was sympathetic and humorous, and had a great intellectual honesty. The reluctance with which he accepted the celebration of his 60th birthday is an evidence of his modesty.

Gianfranco graduated in Mathematics in Ferrara under the guidance of A. T. Lascu, and received his Ph.D. in Mathematics in Pisa under the supervision of F. Catanese. His Ph.D. thesis was awarded with the "Iacopo Barsotti" prize. He began his career as a researcher in Padova, and then became Associate first, and Full Professor later, at the Politecnico di Torino. Gianfranco's mathematical interests ranged from coverings to Hilbert schemes and moduli spaces, to vector bundles on algebraic varieties, resulting in 70 publications on such topics.

Gianfranco was part of the board of many conferences, including the nineteen editions of the "Genova-Torino-Milano Seminar: Some Topics in Commutative Algebra and Algebraic Geometry", a traveling seminar. Notably, Gianfranco and T. Valla were the promoters of these events in 2006, driven by their common interests

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in Gorenstein algebras. Additionally, we recall the fifteen editions of the "School and Workshop on ...", which covered many different topics and were held in Torino first and later in Trento. The School was intended for Ph.D. students and young researchers, while the Workshop served as a platform for senior researchers to disseminate problems and results to a broader audience. Each of these schools hosted roughly 50 participants, meaning Gianfranco impacted approximately 1000 young mathematicians at a crucial stage of their careers. There are Proceedings from some editions of the "School and Workshop on ...", which include both notes from the school classes and contributions to the workshop. For example, see [39].

We believe that the best homage to a mathematician is to review his research. Therefore, in the rest of this preface, we outline Gianfranco's mathematical interests by briefly summarizing the topics of his publications.

2. Gianfranco's mathematical results

The aim of this summary is to highlight the many problems to which Gianfranco contributed, not to discuss the legacy of his results. Furthermore, even though many of the cited papers contain results obtained by Gianfranco in collaboration with various coauthors, we will only reference Gianfranco in this section. Please refer to the reference list for full attribution of the results.

The first main topic that initially attracted Gianfranco's interest, and that is recurrent in his studies, are covers. Let X, Y be Gorenstein varieties, and let $\rho : X \to Y$ be a finite degree d map. Then, $\mathbb{P} = \mathbb{P}(\mathscr{E})$ is a projective bundle over Y, where $\mathscr{E}$ is the cokernel of the natural map $\mathscr{O}_Y \to \rho_*(\mathscr{O}_X)$. From its definition, $\mathscr{E}$ is locally free of rank $d-1$. In [68], the authors prove that there is a natural closed embedding of X, as a Y–scheme, into $\mathbb{P}$, and that its ideal sheaf $\mathscr{J}$ has a minimal locally free resolution of the form

$$0 \to \mathscr{N}_{d-2}(-d) \to \cdots \to \mathscr{N}_1(-2) \to \mathscr{J} \to 0,$$

that satisfies $\mathrm{Hom}_{\mathscr{O}_{\mathbb{P}}}(\mathscr{N}_1, \mathscr{N}_{d-2}(-d)) \simeq \mathscr{N}_1$. Furthermore $\rho^*\rho_*\mathscr{N}_1 \simeq \mathscr{N}_1$. This result was used to classify degree 3 and 4 covers, and to construct the coarse moduli space of Enriques surfaces with polarization of degree 4. Other than in [68], Gianfranco's results on covers are summarized in [67], [64], [63], [61], [58], [52], [47], and in [34], where many different cases are considered.

A second main topic that attracted Gianfranco's interest is the rationality of moduli spaces of curves, or of some loci in them. We recall that a variety X over an algebraically closed field $\mathbb{K}$ is rational if X is birationally equivalent to a projective space $\mathbb{P}^n_{\mathbb{K}}$ for a suitable n. In general, it is a very difficult problem to show that a variety is rational. A possible approach, often used by Gianfranco, is to show that there is a birational correspondence between the variety X and a suitable variety Y, later shown to be rational. When X is a moduli space of curves, the variety Y is constructed in terms of the geometric properties of the canonical model of the parameterized curves. Even though the general methodology remains the same, the geometrical properties to be considered change as X varies, making the proofs of

the results considerably different from case to case. Some of the papers on this topic benefit of the results on coverings. Gianfranco's contributions to this problem can be found in [66], [60], [59], [57], [55], [54], [51], [50], [49], [45], [44], [42], [36], [33], [32], and [31].

In a few joint papers with F. Catanese, Gianfranco addressed some classical problems. In [65], the authors considered a set $\Delta \subseteq \mathbb{P}^3$ of points, and looked for conditions on Δ to guarantee the existence of a surface having Δ as its singular set. Their main result states that Δ occurs as the degeneracy locus of a symmetric map between a vector bundle and a suitable twist of its dual. While the surface is the locus where the rank of the map drops by 1, Δ is the set where the rank drops by 2. In [53], the same authors consider the classical problem of classifying locally Cohen–Macaulay curves in $\mathbb{P}^3$ that are self–linked, that is to say, whose ideal sheaf $\mathscr{J}_C$ satisfies $\mathscr{H}om_{\mathscr{O}_{\mathbb{P}^3}}(\mathscr{O}_C, \mathscr{O}_X) \simeq \mathscr{J}_C/\mathscr{J}_X$ where X is a complete intersection that contains C. In $\mathbb{P}^n$, the class of schemes to consider is the one of codimension 2, equidimensional and without embedded components. The main result of [53] is that C is self–linked if and only if there exists a resolution of the form

$$0 \to \mathscr{E}^\vee(-d-m) \to \mathscr{O}_{\mathbb{P}^n}(-m) \oplus \mathscr{E} \to \mathscr{J}_C \to 0,$$

where the map $\mathscr{E}^\vee(-d-m) \to \mathscr{E}$ is symmetric according to a definition in [65], and the linking scheme is a complete intersection of hypersurfaces of degrees d, m.

Another research topic that has attracted Gianfranco's interest is the locus parameterizing Gorenstein schemes in the punctual Hilbert scheme. Part of his interest was motivated by the result by A.V. Iarrobino and J. Emsalem on the reducibility of the Gorenstein locus when $d = 14$. In a series of papers, Gianfranco proved that the Gorenstein locus in the punctual Hilbert scheme is irreducible for $d \leq 13$. Up to degree 10, there is a study of the singular locus of the Gorenstein locus. Unfortunately, for degree 10, the proof includes an arugment that is not well-motivated. As a result, the singularity of the Gorenstein locus has to be considered up to degree 9. The irreducibility is obtained through a detailed classification of Gorenstein, Artinian algebras: since the isomorphism classes can be both finite and infinite, different techniques have been used, according to the degree. The results are contained in [46], [43], [40], [38], [28], [26], and [23]. A byproduct of these studies is the proof that the Poicaré series of Gorenstein, Artinian algebras satisfying some conditions is rational (see [41], [29], and [24]).

Let C be a non–hyperelliptic, smooth, connected, projective curve of genus g over an algebraically closed field $\mathbb{K}$. The canonical model of C is the image $\phi(C)$ of the map $\phi : C \to \mathbb{P}^{g-1}_{\mathbb{K}}$ induced by the canonical divisor on C. Since the Artinian reduction of the coordinate ring of $\phi(C)$ in $\mathbb{K}[x_0, \ldots, x_{g-1}]$ is Gorenstein, it is possible to associate a (cubic) form to it by Macaulay's inverse system, called the apolar form. In [37], Gianfranco classified the curves whose apolarity is $g-1$, where the apolarity is the minimum number t of linear forms needed to write the apolar form of C as sum of cubes. The geometry of canonical curves was considered by Gianfranco also in [35] and [30], where he studied conditions to force the canonical model to be contained in surfaces of small degree. The case of surfaces containing curves

satisfying $\omega_C \simeq \mathcal{O}_C(\alpha)$, that is to say, α–subcanonical curves, was considered in [69], at least for surfaces of degree at most 4, or containing a line. Curves on a quadric surface were considered in the very first paper by Gianfranco, [70], written under the supervision of A.T. Lascu: he considered the ruled surface $\mathbb{P}(\mathcal{N}^{\vee}_{X|\mathbb{P}^3}) \to X$, where $X \subset Q \subset \mathbb{P}^3$ is a curve on a quadric Q, not a complete intersection, and computed bounds for the self–intersection number of a cross section C. The study of moduli spaces of bielliptic curves, in the case when the curve supports more bielliptic structures, was the topic of [48]. The moduli space of pluriregular threefolds of general type was considered in [56].

In algebraic geometry, vector bundles are a very powerful tool to study properties of supporting varieties. The existence and the properties of the moduli spaces of some classes of vector bundles, namely rank 2 arithmetically Cohen–Macaulay (aCM, for breif), and Ulrich bundles have attracted the interest of Gianfranco. Let X be a smooth, connected, projective variety over an arithmetically closed ground field $\mathbb{K}$, of characteristic 0. A vector bundle $\mathcal{E}$ on X is aCM if its intermediate cohomology vanishes, that is to say, $H^i(X, \mathcal{E} \otimes \mathcal{O}_X(t)) = 0$ for $0 < i < \dim(X)$ and for every $t \in \mathbb{Z}$. An aCM bundle $\mathcal{E}$ on X is Ulrich if the corresponding module has the maximal number of generators. From a representation theory perspective, a variety is of finite, tame, or wild representation type if, for every rank, the indecomposable aCM bundles are finitely many, up to twist, or there are finitely many families of dimension at most 1, or there are families of arbitrarily large dimension, respectively. Gianfranco's interest in this kind of problems is witnessed by the many papers on the subject: for the existence and classification of rank 2, aCM or Ulrich bundles on varieties with special properties, see [27], [22], [19], [18], [17], [16], [15], [14], [13], [12], [11], [10], [9], [4], [1]; for the geometry of the moduli space of (semi)stable rank 2, aCM or Ulrich vector bundles, see [25]; for the representation type, see [21], [20], [7]. Another class of vector bundles that attracted Gianfranco's interest are instanton bundles, and their generalizations: e.g., for their existence on varieties with special properties and the geometry of their moduli spaces, see [8], [6], [5], [3], [2].

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