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***Abstract*—The study of electromagnetic diffraction in arbitrary linear media holds substantial significance for many practical applications. These types of problems present major challenges in computational electromagnetics, regardless of whether they are tackled using numerical, analytical, semi-analytical, or asymptotic techniques. In this paper, we introduce a new mathematical framework in the spectral domain, utilizing the Wiener-Hopf technique, which effectively addresses the underlying physical phenomena through semi-analytical and asymptotic methods.**

Keywords—arbitrary linear media, scattering, diffraction, propagation, asymptotics, Wiener-Hopf method

I. Introduction

Spectral methods in electromagnetics have gathered a fundamental role to investigate the physical phenomenon of diffraction.

The state of the art provides several effective and consolidated spectral methods to handle in particular the scattering by wedges immersed in isotropic media. Among them, we identify the Sommerfeld–Malyuzhinets (SM) method [1-5], the Kontorovich–Lebedev (KL) transform method [6-8], and the Generalized Wiener–Hopf technique (GWHT) [9-10].

However, we consider the spectral methods SM and KL capable to effectively handle wedge problems where only one propagation constant is present. In fact, they rely on the definition of the complex angular variable w, which is related to Sommerfeld integrals. This definition simplifies the investigation, in particular effectively handling the physical branch cut of diffraction problems immersed in an isotropic medium.

Similar advantage is present in the GWHT while dealing with one propagation constant problems involving wedges where it is possible and effective to use a spectral mapping that transforms the Generalized Wiener-Hopf equations (GWHEs) into Classical Wiener-Hopf equations (CWHEs) amenable of solution via classical methods or approximated methods such as the Fredholm factorization technique [9-11].

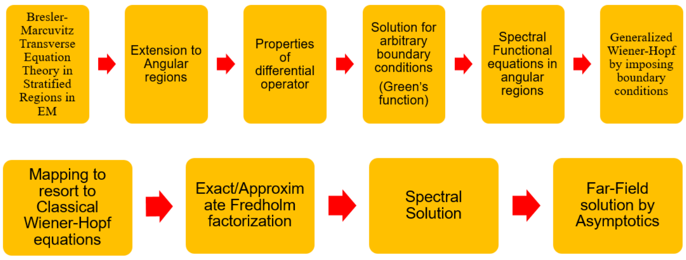

Fig. 1. Flowchart of GWHT used to deal with wedge diffraction problems immersed in isotropic media [9-10].

Moreover, the three reported spectral methods show connections among each other, and one takes advantage from the others in their synergetic development [12].

However, generally, in wedge problems immersed in (or made of) arbitrary linear media, where multiple propagation constants are present, it is not possible to define a unique complex angular plane w for SM and KL, and at the same time it is not possible to define a unique spectral mapping to transform the GWHEs into CWHEs.

To the best of our knowledge, no spectral method has been developed to address the diffraction problems by wedges in arbitrary linear media (i.e., bianisotropic media) characterized by multiple propagation constants to date.

In this work we propose a revisited GWHT capable to handle wedge scattering in arbitrary linear media based on a novel and generalized version of Fredholm Factorization Method directly applied to GWHEs without using spectral mappings [13-14]. Indeed, applying this new methodology, the spectral mapping is not necessary anymore because of the application of Cauchy representation of generalized Wiener-Hopf unknowns. It is important to highlight that the applicability of the proposed WH technique to arbitrary linear media resides in its formulation directly and uniquely in the Laplace domain avoiding other complex planes.

This novel approach provides accurate semi-analytical solutions to new and intricate scattering problems, preserving the robust features of analytical solutions found in spectral methods, including the asymptotic estimation of fields and the physical interpretation of field components through spectral singularities. Indeed, the same methodology can be applied also to the analysis of complex diffraction problems in other physics, like acoustics and elasticity [15].

In the following, we will report some preliminary results to validate the proposed technique.

II. The Method

In Fig. 1 we report in summary the GWHT methodology used to deal with wedge diffraction problems immersed in isotropic media [9-10], that is based on:

1) spectral functional equations obtained applying transversalization to Maxwell equations in layered media, Laplace transform and Green's function procedure

2) generalization of point (1) from layered media to angular stratified media

3) application of boundary conditions to get GWHEs

4) spectral mapping for isotropic regions to transform GWHEs into CWHEs

5) exact factorization or Fredholm factorization to get respectively analytical spectral solution or semi-analytical ones

6) inverse Laplace transform and asymptotic evaluation at far field.

Points 1 to 3 can be implemented also in case of arbitrary linear media with more complex mathematical results, while it is not possible to use point 4 since we have multiple propagation constants and therefore Wiener-Hopf unknowns are defined into more than two complex "Laplace" planes. We highlight that the complex planes in the GWHEs are related together through mathematical properties that take origin from the spectral properties of the operator of transverse Maxwell's equations defined in the angular domain [14].

The key tool to develop a spectral semi-analytical solution is the representation of all generalized Wiener-Hopf unknowns by means of Cauchy representation (1), yielding integral equations in spectral domain defined only in one complex plane:

$$F_+(-m_i(\eta)) = \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{F_+(t)}{t+m_i(\eta)}dt + F_+^{ns}(-m_i(\eta)) \qquad (1)$$

In (1) we have applied the representation to an arbitrary plus function $F_+(\eta)$. We then apply to the set of integral equations a modified version of Fredholm factorization to eliminate one kind of unknowns (usually the minus one), yielding a set of Fredholm integral equations (FIEs) of second kind in terms of the other type of unknowns (usually the plus unknowns). As the FIEs are with fast convergent kernel, they are amenable of semi-analytical solution with sample and hold discretization and reconstruction formula.

As usual, Fredholm factorization with this discretization process preserves exactly the spectral properties of the physical problems in terms of poles and branches allowing asymptotic evaluation as for closed form spectral solutions.

III. . Preliminary Numerical Results

In order to validate the entire procedure synthetically outlined in the previous section we consider the following simplified test problem: an isotropic angular region defined in cylindrical coordinates by $0< \varphi <\gamma=0.7\pi$, with a propagation constant $k=k_1\text{-}jk_2=1-j0.1$, terminated by PEC boundary conditions and illuminated by an Hz polarized plane wave with an incoming direction $\varphi o=0.1\pi$ and intensity $Ho=1A/m$, while the edge of the wedge is along z. Fig. 2 reports the GTD diffraction coefficient obtained from the exact solution versus the one obtained from the method outlined in section II using discretization parameters for the FIEs $A = 20$, $h = 0.05$.

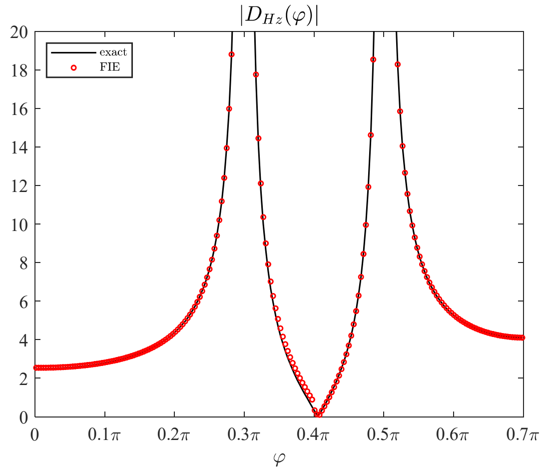

Fig. 2. GTD diffraction coefficient versus the observation angle φ in rad for the PEC wedge test problem reported in section III obtained from the exact solution versus the one obtained from the method outlined in section II.

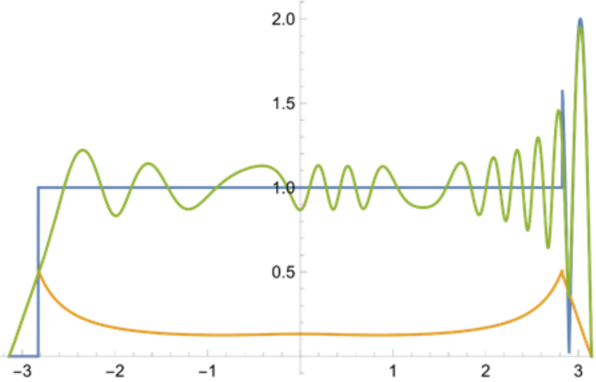

Fig. 3. GO field (blue line), UTD field (orange line) and total field (green line) in V/m versus the observation angle φ in rad at $k_1\rho=10$ for the PEC half-plane immersed in biaxial media diffraction problem described in section III.

To validate the capability of the method to analyze diffraction problem in arbitrary linear media, as a second test problem, we consider a PEC half-plane (with edge along z) immersed in a biaxial media (characterized by the relevant $\mu_{rxx}=5$, $\mu_{ryy}=1$), illuminated by an Ez polarized plane wave with an incoming direction $\varphi o=0.1\pi$ and intensity $E_o=1V/m$, and with the Ez polarization x-propagation constant $k=k_1\text{-}jk_2=1-j0.001$ (the second propagation constant is linked to Hz polarization). For this problem in Fig. 3 we show the numerical results at far field.

The promising results reported in Figs. 2 and 3 enable wedge scattering studies in arbitrary linear media with the theoretical package briefly outlined in section II.

IV. Conclusions

The work introduces a comprehensive self-consistent theory that allows the analysis of diffraction problems constituted by impenetrable/penetrable wedges immersed in or made of arbitrary linear media. The research activity proposes for the first time in literature a spectral method that analyzes problems in presence of angular regions filled by complex media with multiple propagation constants starting from the analysis of half-plane diffraction. Preliminary results demonstrate efficacy of the method providing accurate semi-analytical solutions, preserving the robust features of analytical solutions found in spectral methods, including the asymptotic estimation of fields and the physical interpretation of field components through spectral singularities.

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