

Development of a Riemann variables-based technique for on-line fuel injected mass evaluation in direct injection systems

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# 1    **Development of a Riemann variables-based technique for on-line** 2    **fuel injected mass evaluation in direct injection systems**

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## 7    **Abstract**

8            On-line evaluation of injected fuel mass based on physical principles offers the potential for  
9    accurate and rapid feedback on actual injection quantities. High-accuracy pressure transducers are  
10   commonly used as primary sensors to provide inputs for model-based virtual flowmeters. While  
11   increasing the number of transducers allows for a more comprehensive analysis of elementary waves  
12   within the system, balancing measurement accuracy and economic efficiency is essential for industrial  
13   applications.

14          In this study, a physics-based technique requiring only a single pressure sensor on the injector-  
15   feeding pipe is proposed, considering two elementary waves. The algorithm employs a predefined signal  
16   that exploits quasi-constant Riemann invariants propagating from the rail to the injector, enabling  
17   accurate estimation of the flow-rate and mass entering the injector. Correlation relationships are then  
18   used to convert the entering mass into the injected mass. This approach achieves an estimation accuracy  
19   within 2 mg for single injection events.

20          The proposed method is benchmarked against a previously developed flowmeter utilizing two  
21   pressure sensors, demonstrating that comparable accuracy can be obtained with a more economical  
22   single-sensor setup. The algorithm is applicable across a range of injection sizes and working conditions,  
23   offering a practical solution for real-time injected mass evaluation and control in high-pressure fuel  
24   systems.

## 25   **Keywords**

26          Direct Injection; Injected mass evaluation; closed-loop control; Characteristic curve; Elementary  
27   wave.

28 **Nomenclature**

29	$A$	pipe cross-sectional area
30	$a$	sound speed
31	$C$	chamber
32	$C$	constant
33	$DT$	dwel time
34	$d$	pipe diameter
35	$E$	elasticity modulus
36	ECU	electronic control unit
37	$ET$	energizing time
38	FMV	fuel metering valve
39	$G$	mass flow-rate
40	$H$	vector of source terms
41	$h$	source term
42	$I$	energizing current
43	$L$	pipe length
44	$M$	fuel mass
45	PCV	pressure control valve
46	$p$	pressure
47	$T$	injection period
48	$t$	time
49	$u$	flow velocity
50	$V$	volume
51	$W$	vector of characteristic variables
52	$w$	characteristic variable
53	$x$	space coordinate
54	$\Lambda$	diagonal Jacobian matrix

55	$\rho$	density
56	$\tau$	sheer stress
57	<u>Subscripts</u>	
58	<i>avg</i>	averaged
59	<i>cc</i>	control chamber
60	<i>corr</i>	corrected
61	<i>d</i>	dynamic
62	<i>f</i>	finalize
63	<i>down</i>	transducer position positioned at the injector inlet
64	<i>EOI</i>	end of injection
65	<i>inj</i>	injected, injector
66	<i>i</i>	initial
67	<i>in</i>	inlet
68	<i>nom</i>	nominal
69	<i>one</i>	based on the innovative technique using one pressure transducer
70	<i>out</i>	fuel exiting an injector
71	<i>rail</i>	common-rail, rail side
72	<i>SOC</i>	start of current
73	<i>s</i>	static
74	<i>two</i>	based on the technique using two pressure transducers
75	<i>up</i>	transducer position close to the rail
76	<i>w</i>	wall
77	0	reference

## 78 **1. Introduction**

79 While electrification is at the forefront of research and development in the passenger car sector, the  
80 continued advancement of diesel-fueled internal combustion engines remains essential [1]. This is  
81 primarily due to their outstanding power density, which makes them particularly suitable for marine

82 propulsion and long-distance land transport applications [2]. Moreover, diesel engines are expected to  
83 contribute a significant share of CO<sub>2</sub> emissions well into the 2030s [3]. As a result, efforts to reduce  
84 emissions from diesel engines, including CO<sub>2</sub>, are driven not only by increasingly stringent regulatory  
85 requirements [4], but also by the overarching goal of achieving carbon neutrality in the coming  
86 decades [5].

87 In recent years, considerable research has been devoted to reducing carbon emissions from diesel  
88 engines. Key strategies include advancements in injection technologies [6], optimization of combustion  
89 chamber geometry [7], and the adoption of fuel blends [8]. Concerning injection systems, common-rail  
90 systems [9] have been in existence for over two decades, and they have evolved into indispensable  
91 components of contemporary diesel engines. Simultaneously, the effective implementation of specific  
92 control strategies in a common-rail system relies on accurate and stable control of the fuel dosage for  
93 each injection. Despite this, industrial applications of common-rail systems typically rely on open-loop  
94 control strategies, regulating injected fuel quantity using calibrated parameters such as nominal rail  
95 pressure ( $p_{nom}$ ) and energizing time ( $ET$ ) of the electrical current signal [10]. These parameters are  
96 mapped in the engine control unit (ECU) based on extensive pre-calibration experiments under various  
97 operating conditions. As a result, the actual injected mass during real-time engine operation remains  
98 unmeasured. Preliminary studies have shown that accurate closed-loop control of injected fuel mass can  
99 significantly reduce engine-out CO<sub>2</sub> emissions [11]. Therefore, the development of an affordable and  
100 robust technique for online evaluation of injected fuel quantity is of great importance.

101 Existing online measurement methods for fuel injections are typically classified into two main  
102 groups: those based on engine combustion models and those utilizing mechanical-hydraulic models of  
103 the injection system. The latter can be further divided into two types: one that extracts a few scalar  
104 quantities from continuous signals within a single engine cycle, and another that employs the entire  
105 acquired signal time history as model input.

106 Techniques based on combustion models acquire performance-related data for each engine cycle.  
107 The fuel injected quantity is determined through simplified and empirical models. For instance, Finesso  
108 et al. [12] derived the injected mass by measuring the in-cylinder pressure. This method involves the  
109 inversion of a predictive combustion model [13]. Initially, the heat release rate is ascertained based on

110 the measured pressure signal within the combustion chamber, and the injected fuel mass can be  
111 estimated using predefined ignition delay and the combustion rate of the heat release model. However,  
112 these methods require intricate modeling and careful calibration, and the simplified models are typically  
113 accurate only within limited operating conditions.

114 Mechanical-hydraulic model-based approaches focusing on scalar features use empirical  
115 corrections to address errors in traditional ECU calibration maps. Ferrari et al. [14] implemented a time-  
116 frequency-based virtual needle-lift sensor [15] to detect nozzle opening and closing events, correlating  
117 injected mass with nozzle opening duration. Payri et al. [16] proposed a mathematical model using  
118 inputs such as rail pressure ( $p_{rail}$ ),  $ET$  and back pressure. After injector characterization, where a set  
119 of coefficients is determined for the mathematical expressions and correlations, the model can compute  
120 both injected flow-rate and quantity. Similarly, Ma et al. [17] evaluated the injected quantity by  
121 monitoring  $p_{rail}$  reduction, introducing a parameter  $K$  (the pressure decline rate) as an indicator. Such  
122 approaches are mainly used in industry for ECU feedback. Advanced injector-integrated sensors, such  
123 as Bosch's NCS (Needle Closing Sensor) [18] and DENSO's i-ART injector [19] utilize injector internal  
124 pressure time histories to estimate injected mass through needle-lift-based models [20]. Delphi's Switch  
125 technology provides a practical means of detecting needle positions in non-ballistic injectors [21].  
126 While these approaches balance model complexity and practicality, they primarily serve as  
127 compensatory strategies and are sensitive to factors like thermal regimes, which can affect accuracy.

128 Recently, machine learning techniques have been introduced to predict injected quantity or flow-  
129 rate. Williams et al. [22] developed two machine learning algorithms to estimate the injection rate of a  
130 common-rail injection system using only a single input signal of  $p_{rail}$ , though such reliance may lead  
131 to errors in identifying needle events when pressure waves from other injectors propagate in the rail.  
132 In [23], the injected flow-rate of a piezoelectric injector was estimated using an artificial neural network  
133 algorithm, which received four different input parameters: injection pressure, measuring chamber  
134 pressure and temperature, and command signal duration. Deep learning models, such as those in [24],  
135 have been developed for multi-injection scenarios using rail pressure and current signal features as  
136 inputs. However, these approaches often function as "black-box" models and require large training  
137 datasets, which can limit interpretability [25].

138 Mechanical-hydraulic methods that process entire signal histories directly estimate representative  
139 flow-rates or nozzle outflow, and integration over time provides the total injected quantity per cycle.  
140 Yang et al. [26] extracted pilot-main injection rates from pressure signals near the injector inlet via a  
141 lumped electrical model, decoupling water hammer effects and applying needle-lift modeling, though  
142 this remains limited to non-ballistic injectors. The Bosch method enables estimation of pipe internal  
143 flow rate for small injections using a single sensor [27]. However, this is invalid when pressure waves  
144 reflected from the rail reach the transducer after nozzle opening.

145 In such cases, Euler equations must be combined with additional pressure time data [28], with  
146 subsequent integration yielding pipe-passed mass, which is correlated to the injected mass. Nonetheless,  
147 the precise relationship between internal pipe flow and nozzle outflow remains unclear, and most  
148 physics-based methods require at least two pressure sources for large injections, making them less  
149 generalizable. Thus, a method that provides comparable accuracy while requiring only a single pressure  
150 transducer per injector would be more practical for industrial application.

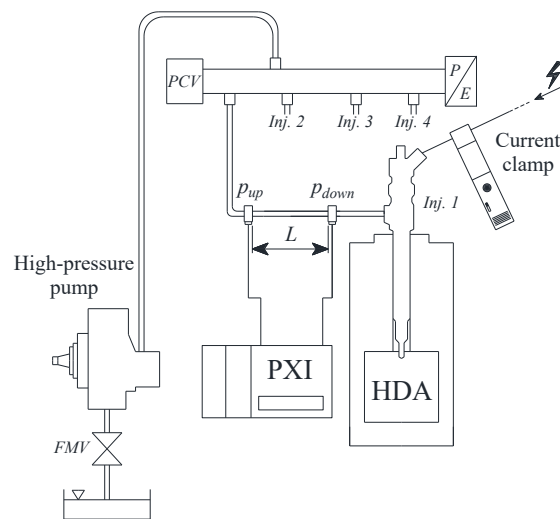
151 In this study, a previously developed flowmeter is implemented on the rail-to-injector pipe of a  
152 common-rail injection system to evaluate the space-averaged internal fuel flow-rate. We conduct a  
153 comprehensive analysis of both static and dynamic leakage under a wide range of single injection  
154 operating conditions, and further elucidate the complex relationship among internal pipe flow, leakage,  
155 and injected mass. Building on a validated one-dimensional numerical model, we propose an innovative  
156 method for determining the internal flow rate using only a single pressure sensor. This method leverages  
157 quasi-constant Riemann invariants propagating from the rail to the injector, and utilizes a predetermined  
158 signal, partially captured by the flowmeter. Finally, two algorithms are employed to evaluate the injected  
159 fuel mass, with a comparative analysis of their results, and the potential application of this approach as  
160 a control strategy is discussed.

## 161 **2. Experimental facility**

162 The experimental campaign was conducted at the Politecnico di Torino ICE Laboratory using a  
163 Moehwald-Bosch hydraulic test bench with a nominal power of 35 kW, a maximum speed of 6100 rpm,  
164 and a maximum torque of 100 Nm. Temperature and pressure levels within the high-pressure hydraulic

165 circuit were monitored using an array of thermocouples and piezoresistive pressure transducers  
 166 mounted along the investigated high-pressure hydraulic circuit. As reported in Figure 1, an HDA  
 167 flowmeter is employed to measure the injected flow-rate traces and the corresponding masses [29], and  
 168 the injector leakages were quantified with a Coriolis flowmeter. The electric command signal to the  
 169 injector was accurately measured employing a current clamp. Moreover, pressure signals near the rail  
 170 ( $p_{up}$ ) and near the injector ( $p_{down}$ ) along the injector-feeding pipe were recorded with a National  
 171 Instruments PXI system at a sampling frequency of 30 kHz.

172 Shell V-Oil 1404 (ISO 4113) was selected as the calibration fluid, accurately replicating the  
 173 properties of renewable diesel under relevant pressure and temperature conditions (up to 120 °C) [30].  
 174 For the test campaign, a Bosch CR system from the latest generation was employed as the injection  
 175 system. A high-pressure pump, with a 430 mm<sup>3</sup> displacement and a single double-acting piston, supplies  
 176 the rail and features a 1:1 transmission ratio. This rail is connected to four solenoid-actuated CRI 2.20  
 177 injectors via 320 mm long, 2.7 mm internal diameter high-pressure pipes.



178

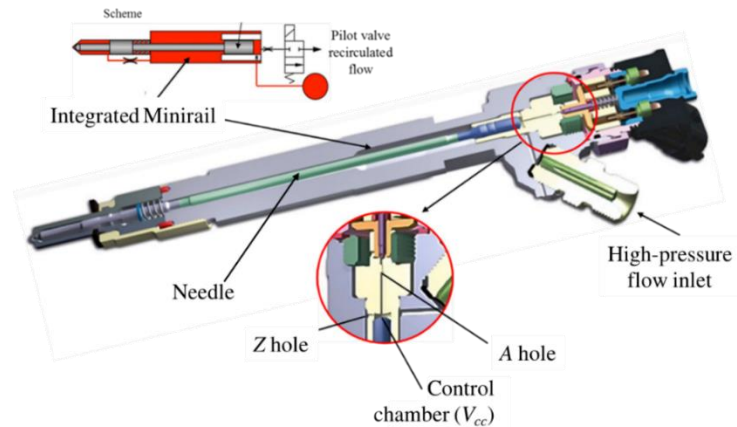
179 **Figure 1. Injection system layout.**

180

181 The ECU utilizes a PID controller to regulate  $p_{rail}$ . The PID controller input is determined by the  
 182 discrepancy between the measured pressure inside the rail and the target rail pressure value. The  
 183 pressure control valve (cf. PCV in Figure 1), located at one end of the rail, modulates excess pumped  
 184 fuel based on the duty cycle received from the PID pressure controller. Simultaneously, the duty cycle

185 is directed to the fuel metering valve (cf. FMV in Figure 1), positioned at the pump inlet, to adjust the  
186 flow-rate being drawn in, thereby preventing fuel from being throttled by the PCV.

187 The pilot stage includes a pressure-balanced pilot valve, and the injector integrates a Minirail.  
188 Figure 2 illustrates the layout of the tested injector.



189

190

**Figure 2. Injector CRI 2.20.**

191

192 In terms of the injector operating principle, a significant portion of the fuel is directed to the  
193 integrated Minirail, while the remainder enters the control chamber (volume  $V_{cc}$ ) through the Z hole.  
194 The ECU activates the pilot valve solenoid by supplying the electrical current  $I$ , initiating the injection  
195 process. Consequently, fuel is discharged from the control chamber through the A hole, facilitating the  
196 upward movement of the needle, which opens the nozzle. Following the cessation of the ECU current  
197 signal, signifying the initiation of the needle closing phase, the control chamber pressure begins to rise  
198 once more. The injection cycle concludes when the needle makes contact with the nozzle as it reaches  
199 the downstroke end.

200 For varying  $ET$  values, single injections have been taken into consideration, with nominal pressure  
201  $p_{nom}$  ranging from 800 and 1800 bar. The chosen pump speed for the entire testing campaign was set  
202 at 2000 rpm, corresponding to an engine speed of 2000 rpm.

### 203 **3. Flowmeter based on incompressible flow assumption**

204 Focusing on the rail-to-injector pipe (i.e., the section equipped with the  $p_{up}$  and  $p_{down}$  pressure  
205 transducers, as shown in Figure 1), and assuming an isothermal process, the energy equation reduces to

206 the equation of state. Accordingly, the pipe can be represented as a one-dimensional slender tube,  
 207 governed by the generalized Euler partial differential equations for mass and momentum conservation  
 208 [31]:

$$209 \quad \frac{\partial}{\partial t} [\rho u] + \frac{\partial}{\partial x} [\rho u^2 + p] = \begin{bmatrix} 0 \\ -4\tau_w/d \end{bmatrix} \quad (1)$$

210 where  $x$  and  $t$  denote the spatial and temporal coordinate.  $u$ ,  $p$ ,  $\rho$  represent the 1D flow velocity, the  
 211 pressure and the density, respectively.  $\tau_w$  signifies the wall shear stress, which is determined through  
 212 the Darcy-Weisbach equation. Assuming incompressible flow, the continuity equation simplifies to  
 213  $\frac{\partial u}{\partial x} = 0$ , and consequently, the momentum balance equation can be expressed as:

$$214 \quad \frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{4\tau_w}{\rho d} \quad (2)$$

215 where  $d$  is the pipe internal diameter. Integrating over the length  $L$  (the distance between  $p_{up}$  and  
 216  $p_{down}$  cf. Figure 1) and multiplying by  $\rho A/L$ , where  $A$  stands for the pipe cross-section area, Eq. (2)  
 217 becomes:

$$218 \quad \frac{d\bar{G}}{dt} = \frac{A}{L} \Delta p - \pi d \overline{\tau_w} \quad (3)$$

219 where  $G$  represents the mass flow-rate and  $\Delta p = p_{up} - p_{down}$ , and the overlined quantities are space-  
 220 averaged. Integrating Eq. (3) with respect to time yields the space-averaged flow-rate time history:

$$221 \quad \bar{G}(t) = \bar{G}_0 + \int_0^t \frac{A}{L} \Delta p dt - \pi d \int_0^t \overline{\tau_w} dt \quad (4)$$

222 where  $\bar{G}_0$  is the initial value of  $\bar{G}(t)$ . Integrating Eq. (4) over an entire cycle, the friction term can  
 223 be expressed in terms of the time-average of the pressure difference  $\Delta p$  as [32]:

$$224 \quad \bar{G}(t) = \bar{G}_0 + \frac{A}{L} \int_0^t [\Delta p - \langle \Delta p \rangle] dt \quad (5)$$

225 where angular brackets represent the time-averaged quantities. As inferred from Eq. (5), the  
 226 instantaneous mass flow-rate can be determined by measuring the  $\Delta p$  along the considered pipe with  
 227 an offset  $\bar{G}_0$ . Since the rail-to-injector pipe is investigated, the measured  $\bar{G}(t)$  in Eq. (5) corresponds  
 228 to the mass flow-rate  $G_{inj,in}$  at the injector inlet. The presented flowmeter has been already tested  
 229 satisfactorily on gear pumps [33], [34] and on a GDI injection system [35].

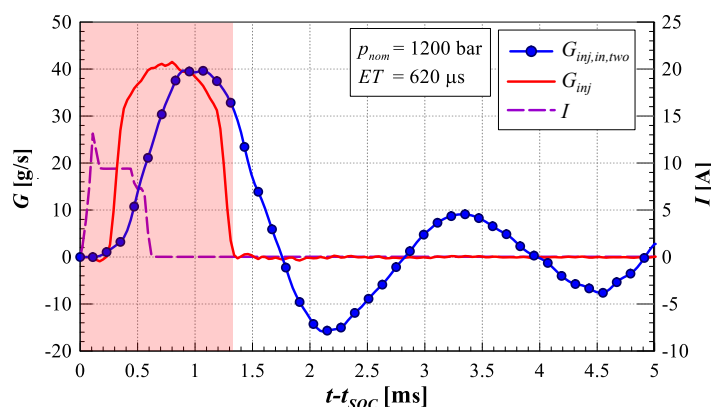
230 **4. Correlation between fuel injected mass and injector inlet/leakage mass**

231 By applying Eq. (5) to evaluate  $G_{inj,in}$ , it is crucial to first establish its initial value  $\overline{G}_0$ . This can  
 232 be assumed to be the flow-rate entering the injector, which is the portion throttled by the closed pilot  
 233 stage (known as static leakage, denoted as  $G_{leak,s}$ ). The measurement of static leakages was performed  
 234 using the Coriolis flowmeter, with the  $ET$  set to zero while the hydraulic system was pressurized. The  
 235 results for  $G_{leak,s}$  at different  $p_{nom}$  are presented Table 1.

236 **Table 1. Static leakages for different nominal pressure levels.**

$p_{nom}$ [bar]	400	600	800	1000	1200	1400	1600	1800
$G_{leak,s}$ [g/min]	1.95	2.10	2.3	3.3	4.5	6.6	9.8	13.3

237  
 238 Figure 3 illustrates the space-averaged flow-rate along the rail-to-injector pipe entering the injector  
 239 (line with symbols), the injected flow-rate (continuous red line), and the current signal (dashed line) for  
 240 a single injection with  $p_{nom} = 1200$  bar and  $ET = 620 \mu s$ . The injection event generates pressure  
 241 waves that propagate along the rail-to-injector pipe and can locally induce negative flow-rates. These  
 242 reverse flows, typically observed after the end of injection, result from fluid dynamic interactions within  
 243 the high-pressure pipe and are induced by water hammer effects caused by the rapid closure of the  
 244 injector's needle.



245  
 246 **Figure 3. Flow-rate entering the injector, injected flow-rate and current signal.**

247 Regarding the mass exiting the injector, two distinct contributions are considered: the mass injected  
 248 through the nozzle holes and the mass flow throttled by the pilot stage. The Coriolis flowmeter, installed

249 at the pilot stage outlet, measures the latter during injection cycles but cannot provide instantaneous  
 250 flow-rate data. Therefore, it only yields a time-averaged flow-rate, denoted as  $G_{leak,Coriolis}$ .

251 Considering the instantaneous flow-rate going through the pilot stage, it can be approximated that  
 252 over an entire injection cycle,  $G_{leak,Coriolis}$  aligns with the static value measured when  $ET = 0 \mu s$   
 253 (as indicated in Table 1). However, it increases when the ECU provides a current signal to the injector  
 254 solenoid. Figure 4 illustrates the values of  $G_{leak,Coriolis}$  for different  $ET$  values at varying  $p_{nom}$ .

255 If one considers an injection period  $T$ , the instantaneous flow-rate through the pilot stage (denoted  
 256 as  $G_{leak,real}$ ) and the time-averaged flow-rate  $G_{leak,coriolis}$  measured by the Coriolis flowmeter are  
 257 linked by the following relation

$$258 \quad \int_0^T G_{leak,real} dt = \int_0^T G_{leak,Coriolis} dt \quad (6)$$

259 Therefore, the dynamic leakage mass, denoted as  $M_{leak,d}$  and corresponding to the flow throttled by  
 260 the pilot stage when it is open, can be calculated for each operating condition as:

$$261 \quad M_{leak,d} \equiv \int_0^T (G_{leak,real} - G_{leak,s}) dt = \int_0^T (G_{leak,Coriolis} - G_{leak,s}) dt \quad (7)$$

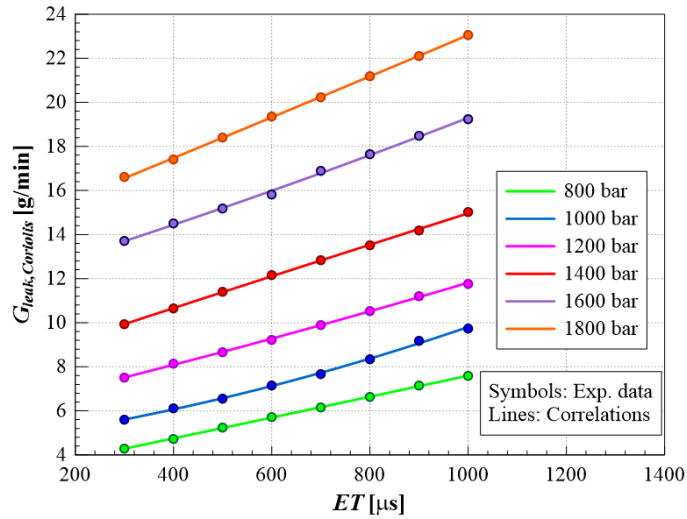
262 With regard to the mass entering the injector, this quantity can be obtained by integrating  $G_{inj,in}$  over  
 263 the relevant time interval, and is expressed as:

$$264 \quad M_{inj,in} \equiv \int_{t_i}^{t_f} G_{inj,in} dt \quad (8)$$

265 where  $t_i$  corresponds to the time instant  $t_{SOC}$ , at which the energizing current starts to rise (i.e.,  $t -$   
 266  $t_{SOC} = 0$  ms in Figure 3), while  $t_f$  is defined as follows. Assuming the injector can be modeled as a  
 267 0D chamber with a given pressure time history  $p_{inj}$ , and with different flow-rates entering and leaving  
 268 the domain, the continuity equation can be written as:

$$269 \quad \frac{V_{inj}}{a^2} \int_{t_i}^{t_f} dp_{inj} = \int_{t_i}^{t_f} (G_{inj,in} - G_{inj} - G_{leak}) dt = M_{inj,in} - M_{inj} - M_{leak} = M_{inj,in} - M_{out} \quad (9)$$

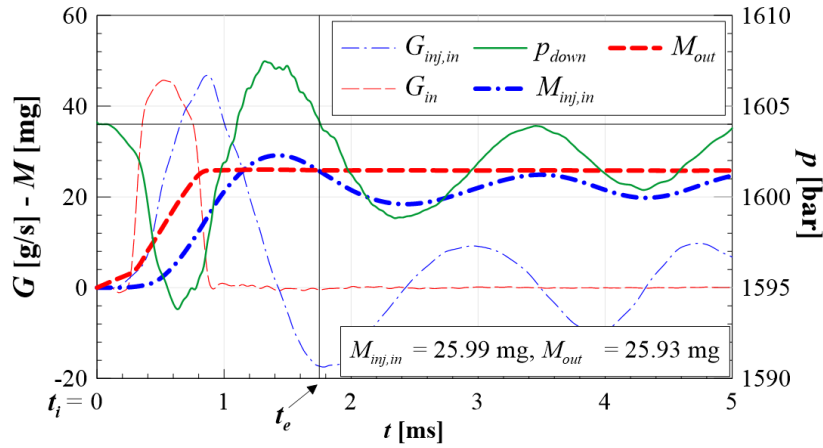
270 where  $a$  represents the fuel sound speed, defined as  $a = \sqrt{dp/d\rho}$ ,  $V_{inj}$  denotes the volume of the  
 271 0D chamber modeling the injector, and  $M_{out}$  represents the mass of fuel exiting the injector. Eq. (9)  
 272 reveals that a balance between the inlet and outlet masses  $M_{inj,in} = M_{inj} + M_{leak} = M_{out}$  is achieved  
 273 when the integration interval is chosen such that  $p_{inj}(t_i) = p_{inj}(t_f)$ , implying  $\int_{t_i}^{t_f} dp_{inj} = 0$ .



274

275

Figure 4. Injector pilot stage throttled flow-rates for different  $ET$  and  $p_{nom}$  values.



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Figure 5. Definition of the end of integration to calculate the mass entering the injector.

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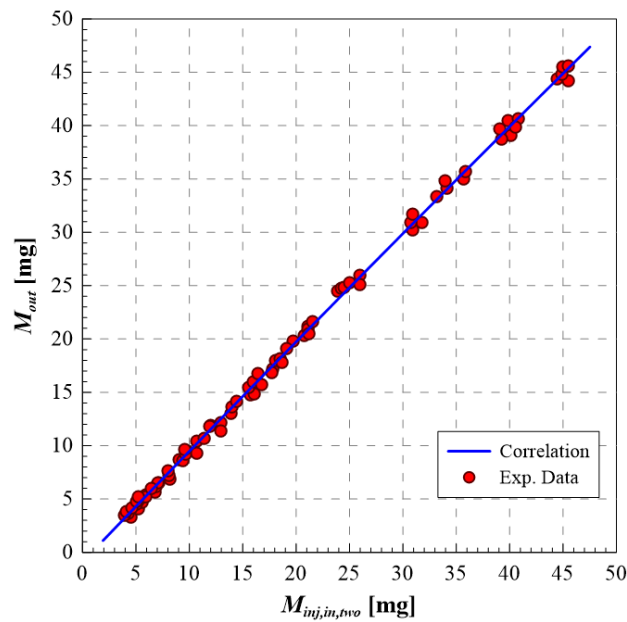
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Although real injectors have more complex hydraulic circuits than a 0D chamber, the pressure signal  $p_{down}$ , measured near the injector inlet, can serve as a representative pressure for the internal hydraulic circuit. For an injection with  $p_{nom} = 1600$  bar and  $ET = 400$   $\mu$ s, Figure 5 shows the injector inlet mass flow-rate  $G_{inj,in,two}$  (thin blue dash-dot curve), the injected mass flow-rate  $G_{inj}$  (thin red dashed curve), and the pressure time history  $p_{down}$  (solid green curve). The figure also displays the cumulative values of  $G_{inj,in,two}$ , denoted as  $M_{inj,in,two}$  (thick blue dash-dot curve), and  $M_{out}$ , which is calculated as the sum of the injected flow-rate and the flow-rate throttled by the pilot stage (thick dashed red curve). It is noteworthy that the data related to variables with subscript *two* were evaluated using the aforementioned technique, which applies the incompressible flow assumption and utilizes two pressure time histories.

By selecting  $p_{down}(t_f) = p_{down}(t_{SOC})$ , the instant  $t_f$  marks the moment when the injector inlet

289 pressure returns to its initial value at  $t_{SOC}$ . Although the wave dynamics in the rail-to-injector pipe are  
 290 still active, this condition provides a state that most closely resembles the pressure state at  $t_{SOC}$ . In  
 291 Figure 5, this moment corresponds to  $t_f \approx 1.75$  ms, as indicated by the intersecting black solid line.  
 292 At this point, the curves of  $M_{inj,in,two}$  and  $M_{out}$  intersect, satisfying the mass balance described in  
 293 Eq. (9).

294 As the values of  $M_{inj,in,two}(t_f)$  are calculated over a wide range of working points (82 in total), a  
 295 correlation between  $M_{inj,in,two}(t_f)$  vs  $M_{out}(t_f)$ , as shown in Figure 6, can be determined. This  
 296 correlation closely follows the bisector line, confirming the validity of the flowmeter concept according  
 297 to Eq. (9).



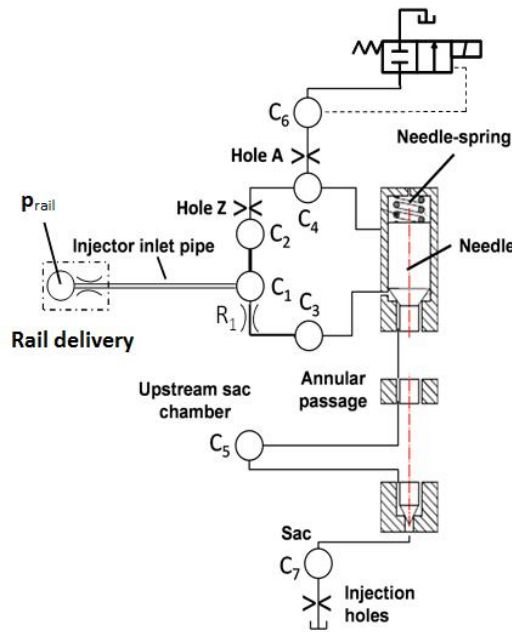
298  
 299 **Figure 6. Correlation between the mass entering the injection and the outlet one.**  
 300

301 The maximum and average errors stand at 1.12 mg and 0.39 mg, respectively. This close agreement  
 302 between  $M_{inj,in,two}$  and  $M_{out}$  further demonstrates the viability of the flowmeter concept. The  
 303 proposed approach provides a reliable benchmark for high-pressure transient mass flow-rate assessment,  
 304 paving the way for the development of an innovative on-line flowmeter that relies on a single pressure  
 305 transducer.

306 **5. Elementary wave analyses of rail-to-injector pipe internal flow**

307 The barotropic numerical model of the injection system, previously developed and validated, is  
 308 shown in Figure 7. This model comprehensively incorporates the vital attributes of the hydraulic circuit,  
 309 mechanical configuration, and electromagnetic elements. It takes inputs of  $p_{rail}$  and  $I$  trace acting on  
 310 the solenoid.

311 The hydraulic circuit is composed of interconnected 0D chambers linked by 1D pipes, with some  
 312 pipes featuring gauged orifices at their extremities. The initial conditions are specified as uniform  
 313 pressure and zero velocity for the fluid states. Within the model, the pipes are describe using Eq. (1).  
 314 The influencing forces are a result of hydraulic, mechanical, and electromagnetic processes, with the  
 315 moving components following ordinary differential forms of Newton’s laws of motion.



316

317 **Figure 7. Schematic of the 1D hydraulic-mechanical- numerical model.**

318

319 Given that the sound speed remains below 0.02 Mach number across all operating conditions, Eq. (1)  
 320 reduces to a set of quasi-linear hyperbolic partial differential equations [36]. Consequently, the system  
 321 can be decoupled into a series of advection equations with source terms, expressed as:

322 
$$\frac{\partial W}{\partial t} + \mathbf{A} \frac{\partial W}{\partial x} = \mathbf{H} \quad (10)$$

323 where  $\mathbf{A}$  is the diagonalized Jacobian matrix of the system, defined as:

324 
$$\mathbf{A} = \begin{bmatrix} u - a & 0 \\ 0 & u + a \end{bmatrix} \quad (11)$$

325  $\mathbf{W}$  represents the characteristic variables, and its differential form is written as:

326 
$$\delta\mathbf{W} = \begin{bmatrix} \delta w_1 \\ \delta w_2 \end{bmatrix} = \begin{bmatrix} \delta u - (a/\rho)\delta\rho \\ \delta u + (a/\rho)\delta\rho \end{bmatrix} \quad (12)$$

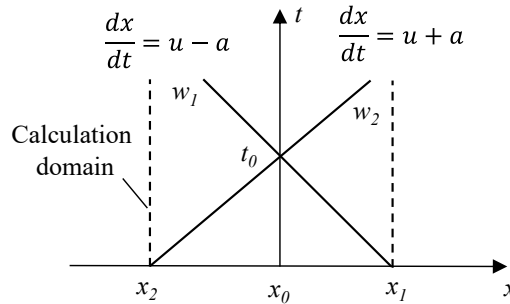
327 and  $\mathbf{H}$  is the source term:

328 
$$\mathbf{H} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 4\tau_w/\rho d \\ -4\tau_w/\rho d \end{bmatrix} \quad (13)$$

329 As the fuel properties, in accordance with the ISO 4113 standard, are described using a fitting  
 330 method, it has been previously demonstrated in [37] that the characteristic variables can be expressed  
 331 as follows:

332 
$$\mathbf{W} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} u - 2 \cdot \rho_0^{-\frac{1}{2}} \cdot E_0^{\frac{1}{2}} \cdot \frac{(E_0 + \chi p)^{\frac{1}{2}} \cdot \frac{1}{2\chi}}{\chi - 1} + C_1 \\ u + 2 \cdot \rho_0^{-\frac{1}{2}} \cdot E_0^{\frac{1}{2}} \cdot \frac{(E_0 + \chi p)^{\frac{1}{2}} \cdot \frac{1}{2\chi}}{\chi - 1} + C_2 \end{bmatrix} \quad (14)$$

333 where  $\rho_0$  and  $E_0$  indicate the density and elasticity modulus at a reference pressure, and  $\chi$  is an  
 334 intermediate variable dependent on temperature. Since the model assumes an isothermal process,  $\rho_0$ ,  
 335  $E_0$  and  $\chi$  remain constants in this model.



336

337 **Figure 8. Characteristic curves for the tube model.**

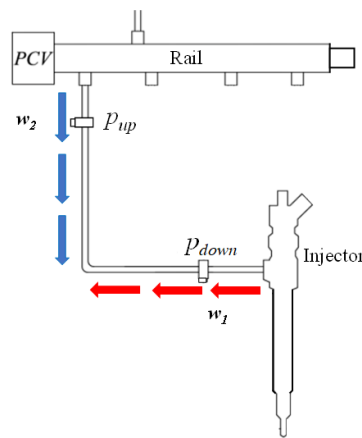
338 Figure 8 provides a schematic of the propagation of characteristic variables in the space-time  
 339 domain. The two solid lines represent the characteristic curves of the system governed by Eqs. (10)-  
 340 (13). Their slopes ( $dx/dt = x \mp a$ ) correspond to the eigenvalues of Eq. (10), as defined in Eq. (11),  
 341 and indicate the wave propagation speeds. Since the Mach number remains below 0.02 in the high-  
 342 pressure circuit for all operating conditions, the eigenvalues are always one negative and one positive.  
 343 The variables  $w_1$  and  $w_2$  evolve temporally at rates specified by Eq. (11), propagating along their

344 respective characteristics. Additionally, the absence of shockwaves under these conditions ensures that  
 345 numerical errors from the finite difference solution of the decoupled system are negligible. By  
 346 transitioning from an Eulerian to a Lagrangian framework, Eq. (10) can be reformulated as a set of  
 347 ordinary differential equations, given by:

$$348 \quad \frac{d\mathbf{W}}{dt} = \mathbf{H} \quad (15)$$

349 where  $\mathbf{W}$  and  $\mathbf{H}$  represent the corresponding variables in Eq. (12)-(14), while each elementary wave  
 350 front of the characteristic variable is analyzed individually. For instance, a wavefront of  $w_1$  (or  $w_2$ )  
 351 traveling at a speed of  $u - a$  (or  $u + a$ ) is tracked over time, and its state is described by Eq. (15).

352 A single hydraulic injection represents a highly transient hydrodynamic process. Elementary waves  
 353 are generated within the high-pressure circuit due to pressure fluctuations from the high-pressure pump  
 354 or internal hydraulic-mechanical effects in the injector [38]. Figure 9 schematically illustrates the  
 355 propagation of these waves along the injector-feeding pipe:  $w_1$  propagates from the injector toward  
 356 the rail, while  $w_2$  travels in the opposite direction, from the rail to the injector.

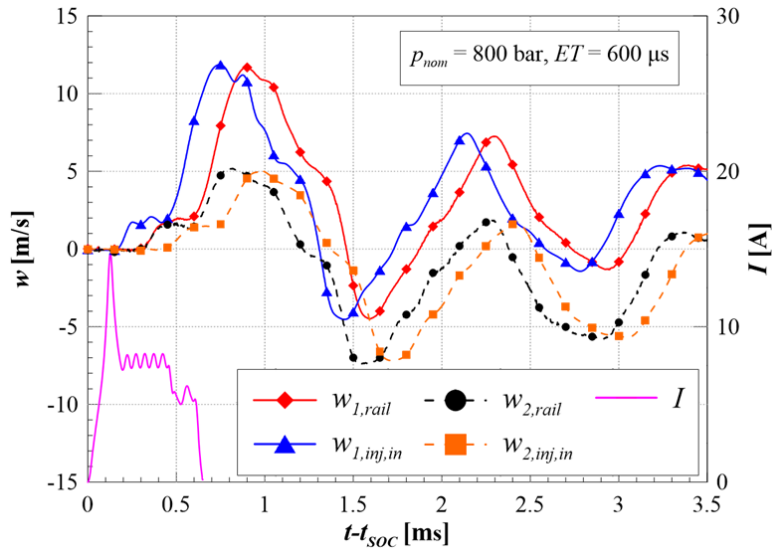


357

358 **Figure 9. Schematic of elementary waves.**

359 Figure 10 presents simulated time histories of the two waves at both ends of the pipe when  $p_{nom} =$   
 360 800 bar and  $ET = 600 \mu s$ . In this figure,  $w_{1,inj,in}$  and  $w_{2,inj,in}$  represent the time distributions of  
 361  $w_1$  and  $w_2$  at the injector inlet, while  $w_{1,rail}$  and  $w_{2,rail}$  denote the same variables at the rail-pipe  
 362 interface. As established in Figure 9, the waves  $w_{1,inj,in}$  and  $w_{2,rail}$ , propagating at the injector inlet  
 363 side and at the rail side, respectively, are of particular interest, since they reflect within the considered  
 364 domain (the rail-to-injector pipe). Meanwhile, the constants  $C_1$  and  $C_2$  in Eq. (14) can be arbitrarily

365 defined, as only the variation of  $w_1$  and  $w_2$  along the pipe are relevant. Therefore,  $C_1$  and  $C_2$  are set  
 366 such that  $w_{1,inj,in}$  and  $w_{2,rail}$  are zero at  $t_{SOC}$ . The initial evolution of  $w_{1,rail}$  and  $w_{2,inj,in}$   
 367 depends on the pre- $t_{SOC}$  values of  $w_{1,inj,in}$  and  $w_{2,rail}$ , during which  $p$  and  $u$  changes are  
 368 negligible. Thus, identical values of  $C_1$  and  $C_2$  can be assigned throughout the injection phase.  
 369 Specifically,  $C_1 = 2 \cdot \rho_0^{-\frac{1}{2}} \cdot E_0^{\frac{1}{2\chi}} \cdot (E_0 + \chi p(t_{SOC}))^{\frac{1}{2} - \frac{1}{2\chi}} / (\chi - 1)$  and  $C_2 = -2 \cdot \rho_0^{-\frac{1}{2}} \cdot E_0^{\frac{1}{2\chi}} \cdot (E_0 +$   
 370  $\chi p(t_{SOC}))^{\frac{1}{2} - \frac{1}{2\chi}} / (\chi - 1)$  are used consistently.



371  
 372 **Figure 10. Elementary waves traveling from the rail-to-end pipe extremities.**  
 373

374 It can be observed that when a wave propagates from one side to the other, a time lag appears  
 375 between the wavefronts at these two locations due to the finite wave speeds, as given in Eq. (11).  
 376 Additionally, both  $w_{1,rail}$  and  $w_{2,rail}$  deviate from zero simultaneously, triggered by the arrival of  
 377 the none-zero wavefront  $w_{1,inj,in}$  at the rail side. The variable  $w_2$  contains all information about the  
 378 wave reflected at the connector, and traveling toward the injector. Notably, as  $w_{1,inj,in}$  propagates to  
 379  $w_{1,rail}$ , the peak amplitude decreases, a phenomenon also observed as  $w_{2,rail}$  transforms into  $w_{2,inj,in}$ .  
 380 This reduction in peak values indicates energy dissipation due to friction, as described by the source  
 381 term in Eq. (13) and the right-hand side of Eq. (15).

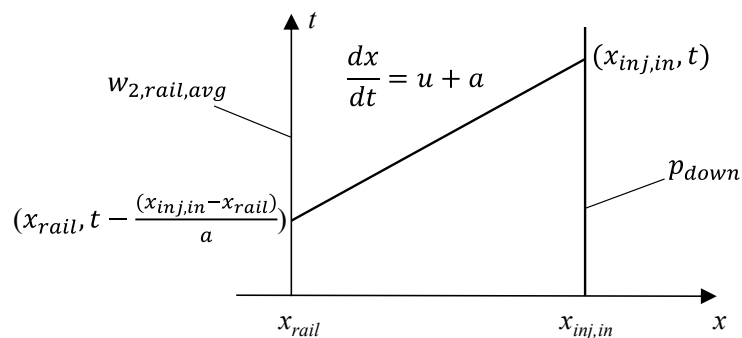
## 382 6. Injected mass evaluation based on elementary wave analyses

383 As reported in [37], the time profiles of the reflected wave  $w_2$  at the rail-pipe connector are nearly

384 identical within a specific range of operating conditions from  $t_{SOC}$  to a certain point after hydraulic  
 385 injection. This interval, highlighted in red in Figure 3, indicates that  $w_{2,rail}$  can be treated as a constant  
 386 time history across a various rail pressures during this period, from  $t_{SOC}$  to  $t_{EOI}$  (end of the injection).  
 387 Since the injector-feeding pipe hydrodynamics are governed by Eq. (10), a system of two hyperbolic  
 388 partial differential equations, knowledge of the time evolution of  $w_2$  at a fixed spatial location enables  
 389 the system to be simplified to a single advection equation with  $w_1$  as the only unknown. This allows  
 390 the internal mass flow rate in the pipe to be determined using just a single pressure signal and a pre-  
 391 determined  $w_{2,rail}$  profile. The injected mass can then be calculated analytically.

392 According to Eq. (14),  $w_1$  contains the variables of  $p$  and  $u$ , both dependent on time and space,  
 393 along with other state variables. At the injector side of the pipe,  $p$  is measured directly by a pressure  
 394 transducer ( $p_{down}$ ) at  $x_{inj,in}$ , while  $u_{inj,in}$  cannot be measured directly.

395 Figure 11 provides an ad-hoc scheme for evaluating  $u_{inj,in}$ . The spatial domain, illustrated in  
 396 Figure 11, extends from the rail-pipe joint ( $x_{rail}$ ) to the location of the pressure transducer  $x_{inj,in}$ .  
 397  $w_{2,rail,avg}$  denotes the average time evolution of  $w_{2,rail}$  at the rail-pipe joint. Since the time profile  
 398 of  $w_{2,rail}$  under different operating conditions are similar between  $t_{SOC}$  and  $t_{EOI}$ , an average time  
 399 history  $w_{2,rail,avg}$  can be constructed and effectively represents  $w_{2,rail}$  for a broad range of working  
 400 conditions.



401  
 402 **Figure 11. Flow velocity evaluation based on elementary wave analyses.**  
 403

404 At any given instant  $t$ , the value of  $w_{2,rail,avg}(t)$  propagates downstream at a speed of  $u + a$ ,  
 405 with its evolution described by Eq. (15). Given that the Mach number remains consistently below 0.02  
 406 across all operating conditions, substituting  $u + a$  with  $a$  introduces a negligible error in the wave

407 transmission speed estimation.

408 During the initial period,  $t - t_{SOC} \leq (x_{inj,in} - x_{rail})/a$ ,  $w_{2,inj,in}$ , which corresponds to  $w_2$  at  
 409 the spatial coordinate of  $x_{inj,in}$ , remains constant, unaffected by the elementary wave transmitted from  
 410 the rail side. Thus, variation in  $u_{inj,in}$  are directly related to changes in  $p_{inj,in}$ , in accordance with the  
 411 Bosch method [27]. Immediately after  $t - t_{SOC} = (x_{inj,in} - x_{rail})/a$ , the property  $w_{2,inj,in}(t)$  can be  
 412 determined from  $w_{2,rail,avg}$ , and  $u_{inj,in}$  for both two phases can be calculated using the following  
 413 equation:

$$414 \quad u_{inj,in}(t) = \begin{cases} P(t_{SOC}) - P(t) & \text{if } t - t_{SOC} \leq (x_{inj,in} - x_{rail})/a \\ w_{2,rail,avg}(t - \frac{x_{inj,in} - x_{rail}}{a}) + P(t_{SOC}) - P(t) & \text{if } t - t_{SOC} > (x_{inj,in} - x_{rail})/a \end{cases} \quad (16)$$

415 where  $P(t)$  is a function of  $p_{down}$  expressed as

$$416 \quad P(t) = 2 \cdot \rho_0^{-\frac{1}{2}} \cdot E_0^{2\chi} \cdot \frac{(E_0 + \chi p_{down}(t))^{\frac{1}{2}}}{\chi^{-1}} \quad (17)$$

417 It is crucial to highlight that  $P(t_{SOC})$  plays a key role in setting the initial value of  $w_{2,rail,avg}$  when an  
 418 injection initiates, ensuring that  $w_{2,rail,avg}(t_{SOC}) = 0$  through the appropriate selection of  $C_2$ . Eqs.  
 419 (16) and (17) link the measurable signal  $p_{inj,in}$  to the target variable  $u_{inj,in}$ . As  $w_2$  propagates from  
 420 the rail to the injector inlet, it is affected by the wall shear stress, as described in Eq. (15). Consequently,  
 421 the values of  $w_{2,rail,avg}$  are iteratively updated as the wavefront travels toward  $x_{inj,in}$ , with the initial  
 422 value of  $u_{inj,in}$  from Eq. (16). This iterative adjustment is performed at each sample point using:

$$423 \quad w_{2,inj,in}^{n+1} = w_{2,rail,avg}^n - \frac{f}{2} \frac{|u_{inj,in}^n| u_{inj,in}^n}{d} \cdot \frac{x_{inj,in} - x_{rail}}{a} \quad (18)$$

424 where  $n$  represents the number of iterations, and  $f$  stands for the friction factor. Furthermore,  $u_{inj,in}^n$ ,  
 425 which is involved in the next iteration calculation, is determined based on Eq. (14) using  $w_{2,inj,in}^n$ ,  
 426  $p_{down}$ , and the determined  $C_2$ .

427 The in-pipe mass flow rate is then determined as:

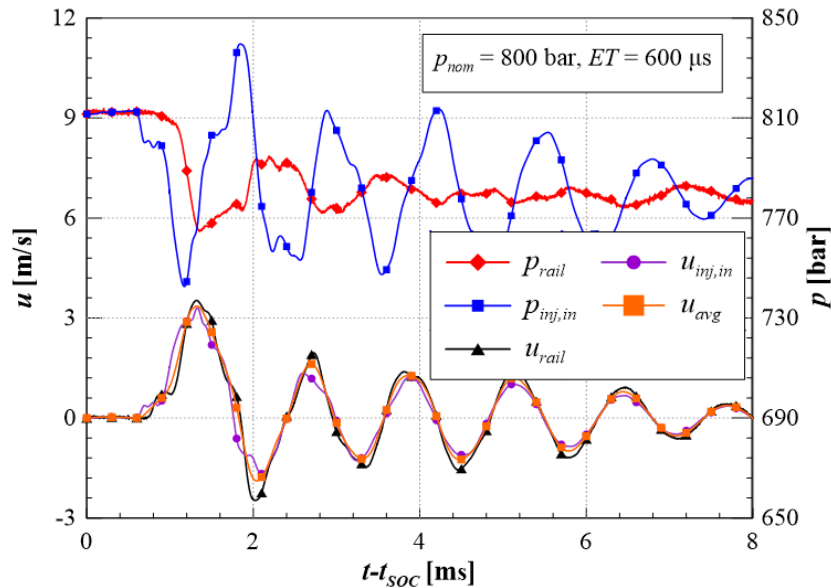
$$428 \quad G_{inj,in} = \rho A u_{inj,in} \quad (19)$$

429 The injected mass can be estimated using integration and correlation methods similar to those employed  
 430 in the aforementioned flowmeter shown in Section 4, under the assumption of incompressible flow.

431 **7. Results of the hydraulic tests and discussion**

432 In the application of the evaluation method based on elementary wave analyses,  $w_{2,rail,avg}$  serves  
 433 as a predefined signal, formed by combining the pressure and velocity data at the rail-pipe connector  
 434 across various working conditions. While the pressure signal can be directly obtained through a sensor  
 435 (cf.  $p_{up}$  in Figure 1) during the algorithm setup phase, the velocity time history cannot be directly  
 436 captured by any transducer.

437 Figure 12 presents simulated flow velocity and pressure time histories at both ends of the pipe that  
 438 feeds the injector for a typical operating condition ( $p_{nom} = 800$  bar and  $ET = 600$   $\mu$ s).  $u_{rail}$  and  
 439  $u_{inj,in}$  exhibit similar trends, particularly during the initial rise and fall phases, though they intersect  
 440 occasionally. This similarity suggests that the velocity component ( $u_{rail}$ ) of the predefined signal  
 441  $w_{2,rail}$  can be approximated using the space-averaged flow rate between the pressure time histories  
 442  $p_{up}$  and  $p_{down}$  (see Figure 1), as calculated by the incompressible flow assumption-based flowmeter  
 443 described in Eq. (5). In Figure 12,  $u_{avg}$  is calculated from simulated  $p_{rail}$  and  $p_{inj,in}$ .



444 **Figure 12. Time histories of the flow velocities at the extremities of pipe feeding the injector.**

447 Notably,  $u_{rail}$  lags slightly behind  $u_{inj,in}$  for  $t - t_{SOC} < 2$  ms. Since the space-averaged  
 448 velocity  $u_{avg}$  mainly represents the pipe center fluid property, and  $w_{2,rail,avg}$  necessitates  $u_{rail}$  at  
 449 the rail-pipe connector, a time-shift of  $u_{avg}$  in time is necessary. Therefore, a delay of 90  $\mu$ s is

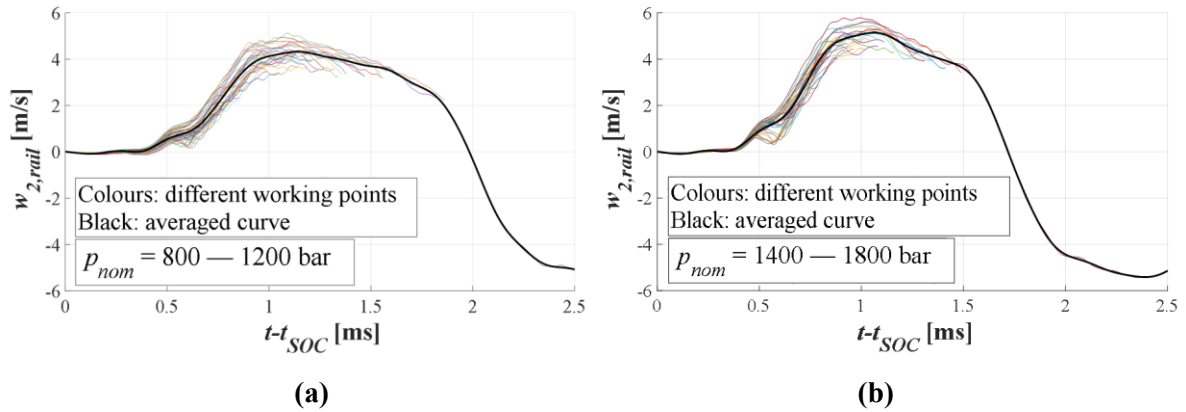
450 introduced to  $u_{avg}$ , corresponding to a propagation distance of approximately 15 cm, based on the  
451 speed of sound in ISO 4113 oil. The final  $w_{2,rail}$  signal is then derived using Eq. (14), with this delayed  
452  $u_{avg}$  and the  $p_{up}$  signal, which approximates  $p_{rail}$  at the rail-pipe joint (located about 5 cm from the  
453  $p_{up}$  sensor).

454 Figure 13 presents all  $w_{2,rail}$  time histories (thin colored curves) involved in the averaging  
455 calculation for a wide range of operating conditions, along with the derived  $w_{2,rail,avg}$  (thick black  
456 curve) used for injected mass evaluation. To minimize the error between  $w_{2,rail,avg}$  and each  $w_{2,rail}$   
457 over the valid temporal length, the working conditions are segregated into two categories, respectively  
458 for  $p_{nom}$  ranging from 800 to 1200 bar and from 1400 to 1800 bar. This separation is necessary  
459 because the value of  $a$  in Eq. (16) is predefined for each pressure range, and the resulting error in  $a$   
460 within a single category can be as large as 100 m/s. Grouping the working conditions in this way reduces  
461 the error in  $a$  thereby improving the accuracy of injected mass evaluation. It is worth noting that only  
462 the segments between  $t_{SOC}$  and  $t_{EOI}$  of  $w_{2,rail}$  for the considered operating conditions are included  
463 in the averaging, and only one curve tail of  $w_{2,rail}$  for a large injection after its  $t_{EOI}$ , positioned at the  
464 center of the curve family, is not truncated in order to smoothen out the final averaged signal. Within  
465 each category, all curves of  $w_{2,rail}$  are averaged across different operating conditions over the valid  
466 time range. Additionally, a Butterworth filter is applied to smooth the averaged signal  $w_{2,rail,avg}$  while  
467 retaining its primary physical information. Consequently, it is evident that  $w_{2,rail,avg}$  is situated at the  
468 center of  $w_{2,rail}$  and closely correlates with these  $w_{2,rail}$  in each category.

469 Figure 14 presents the estimated injector inlet fuel mass flow-rates  $G_{inj,in,one}$  (blue squares) for  
470 two typical cases following Eqs. (16)–(19) based on  $w_{2,rail,avg}$  plotted in Figure 13. The estimated  
471 mass flow-rate  $G_{inj,in,two}$  (red curves), evaluated based on the method described in Eq. (5) and  
472 previously validated in the literature [11], is used as a reference to assess the feasibility of the new  
473 approach.

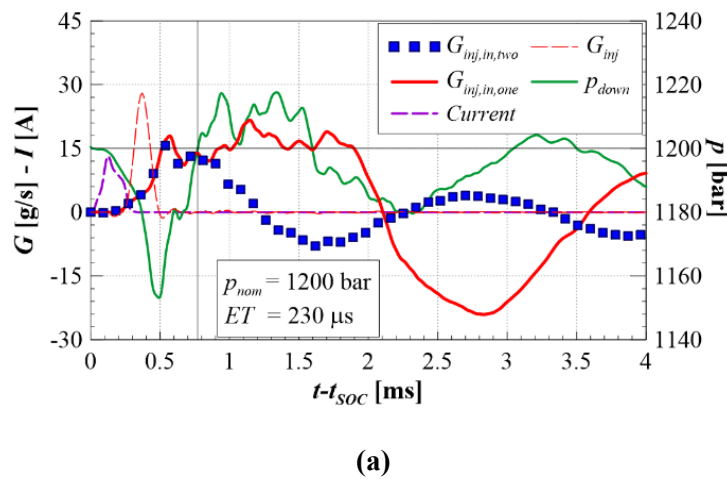
474 Figure 14(a), representing a small injection,  $G_{inj,in,one}$  closely follows  $G_{inj,in,two}$  until  $t -$   
475  $t_{SOC} \approx 0.8$  ms, beyond which the two curves begin to diverge. This divergence occurs because  $w_{2,rail}$

476 also deviates from  $w_{2,rail,avg}$  under these conditions. However, during the interval from  $t_{SOC}$  to the  
 477 instant when  $p_{down}$  first return to its value at ( $p_{down}(t_f) = p_{down}(t_{SOC})$ ), the agreement remains  
 478 strong, validating the use of the single-sensor estimate up to this point.

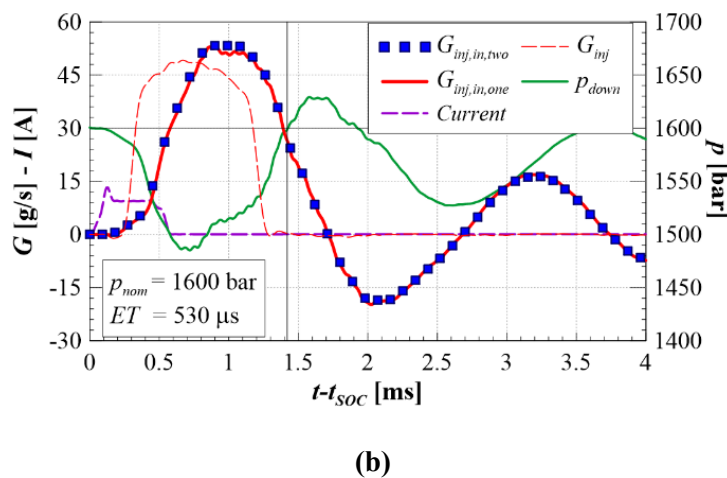


479  
 480  
 481 **Figure 13.  $w_{2,rail}$  time histories when  $p_{nom}$  between 800 bar and 1800 bar:**

482 **(a)  $p_{nom} = 800 - 1200$  bar; (b)  $p_{nom} = 1400 - 1800$  bar.**



483  
 484 **(a)**



485  
 486 **(b)**

487 **Figure 14. Comparison between the methods applying one or two pressure sensors:**  
 488 **(a)  $p_{nom} = 1200$  bar and  $ET = 230 \mu s$ ; (b)  $p_{nom} = 1600$  bar and  $ET = 530 \mu s$ .**

489

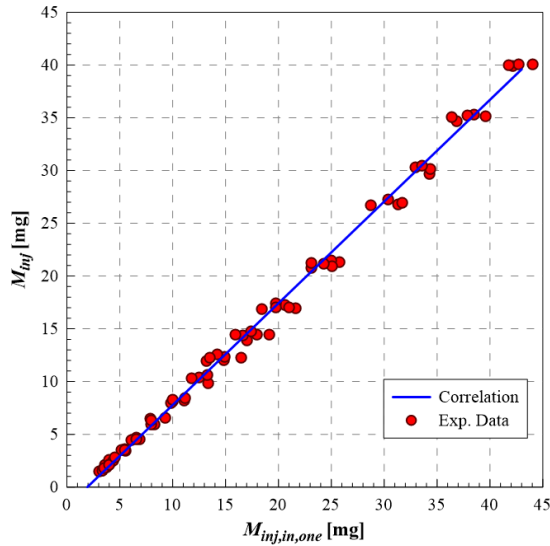
490 Figure 14(b) presents a similar graph for a large injection. As the  $w_{2,rail,avg}$  presents the average  
491 of  $w_{2,rail}$  across a range of working conditions, and large injections exhibit longer valid  $w_{2,rail}$  phase  
492 during the evaluation of  $w_{2,rail,avg}$ . As a result,  $G_{inj,in,one}$  and  $G_{inj,in,two}$  are nearly overlapped. This  
493 signifies that the estimated injector inlet flow-rate remains valid until a slightly later instant after  $t_{EOI}$ .  
494

495 Given the lack of an obvious integration endpoint with the single-sensor approach,  $t_f$  is defined as  
496 the time when  $p_{down}$  first returns to its initial value ( $p_{down}(t_f) = p_{down}(t_{SOC})$ ), which is  
497 approximately  $t - t_{SOC} \approx 0.7$  ms in Figure 14(a) and  $t - t_{SOC} \approx 1.4$  ms in Figure 14(b) The total  
498 mass passing through the pipe is then computed by integrating  $G_{inj,in,one}$  over its valid time interval:

499 
$$M_{inj,in,one} = \int_{t_{SOC}}^{t_f} G_{inj,in,one} dt \quad (20)$$

500 Figure 15 plots the HDA-measured hydraulic injected fuel mass  $M_{inj}$  versus  $M_{inj,in,one}$ . Although  
501 the operating conditions are categorized into two families based on  $p_{nom}$  (cf. Figure 13), Figure 15  
502 unmistakably reveals a significant correlation between  $M_{inj}$  and  $M_{inj,in,one}$ . The correlations (blue  
503 curves) in Figure 15 follows linear trend with a maximum observed error of 1.7 mg.

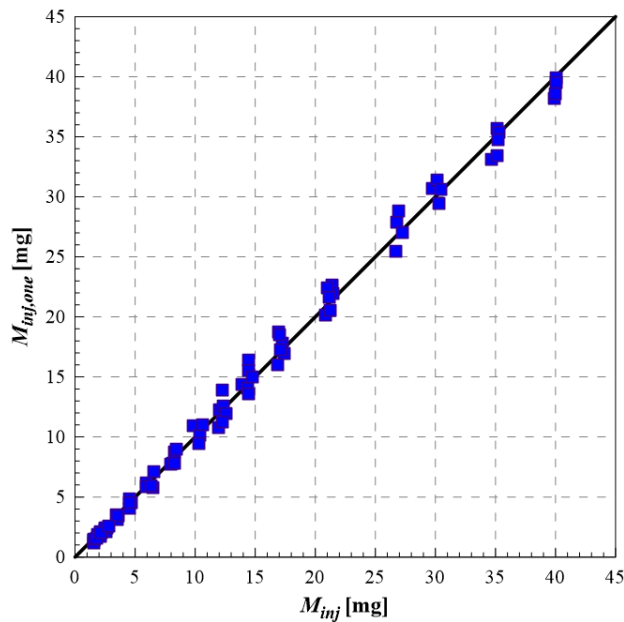
504 The predictive performance is further illustrated in Figure 16, where the estimated injected mass  
505  $M_{inj,one}$  calculated using the fitted function  $M_{inj,one} = f(M_{inj,in,one})$ , is plotted against  $M_{inj}$ . The  
506 data closely follow the bisector line, with a maximum deviation of 2 mg and an average error of 0.6 mg,  
507 indicating that the method is reliable for control applications due to its minimal margin of error.



508  
 509 **Figure 15. Correlation between  $M_{inj}$  and  $M_{inj,in,one}$  for evaluation method employing one**  
 510 **sensor.**

511

512 Turning to double injections, Figure 17 shows  $w_{2,rail}$  curves corresponding to different dwell time  
 513 ( $DT$ ), with identical  $ET$ s for both injections. It is evident that  $w_{2,rail}$  fails to converge throughout the  
 514 entire hydraulic injection phase for multiple injections, as all curves diverge immediately at the onset  
 515 of the second injection.



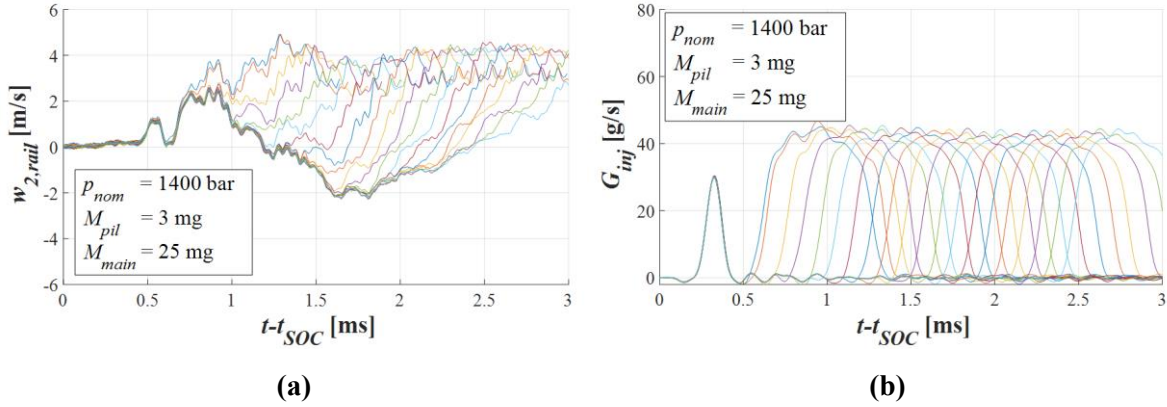
516  
 517 **Figure 16. Prediction performance of the  $M_{inj}$  vs  $M_{inj,in,one}$  correlation.**

518

519 Consequently, while the proposed technique is not applicable for measuring injected mass in

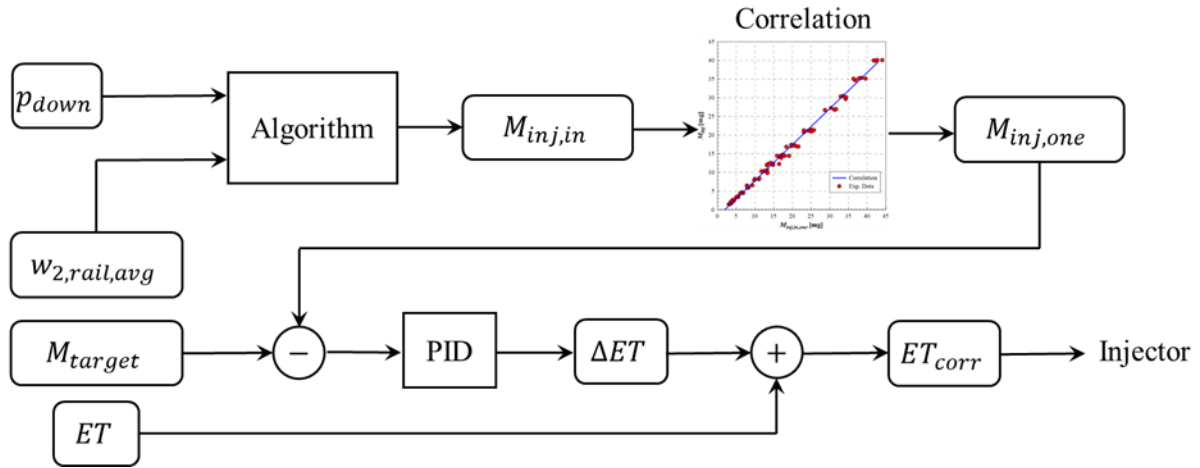
520 multiple injection scenarios, it provides accurate and robust estimates for the first injection, independent  
 521 of its size or the presence of reflected waves during the event.

522 It is worth highlighting that the methodology presented here is broadly applicable to all liquid fuels,  
 523 including OME<sub>x</sub> and methanol, because it relies solely on the assumption of incompressible flow.



524  
 525 **(a)** **(b)**  
 526 **Figure 17. Time distributions of  $w_{2,rail}$  and  $G_{inj}$  for pilot-main injections with different  $DT$**   
 527 **values: (a)  $w_{2,rail}$ ; (b)  $G_{inj}$ .**

528 As already presented in the literature [11, 35], the mass entering the injector can be used to set up a  
 529 closed-loop strategy for the injected mass. A possible scheme for this closed-loop control is presented  
 530 in Figure 18. During an injection event, the acquired  $p_{down}$  signal is used together with the stored  
 531  $w_{2,rail,avg}$  pattern to work-out the entering flow-rate. This flow-rate is integrated to calculate the  
 532 entering mass and, by means of the correlation presented in Figure 18, the injected mass can be  
 533 estimated ( $M_{inj,one}$ ). Such an estimation is compared with the target value of fuel quantity ( $M_{target}$ )  
 534 and the difference  $\Delta M_{inj} = M_{target} - M_{inj,one}$  is provided as an input value of a PID controller: the  
 535 output of this PID controller is an energizing time correction ( $\Delta ET$ ), that is employed to provide a new  
 536  $ET$  value ( $ET_{corr}$ ) to the injector, that will compensate the injected mass inaccuracy.



537

538 **Figure 18. Example of a possible closed-loop control strategy for the  $ET$  correction.**

539 **8. Conclusions**

- 540
- A strong bisector correlation was observed between the fuel mass passing through the injector-
  - 541 feeding pipe and the total outflow, comprising both injected and leaked mass.
  - 542
  - The injected mass was estimated using a hyperbolic PDE system characterized by two
  - 543 elementary waves; one wave is obtained from the measured pressure, while the other is
  - 544 predefined based on the convergence behavior of first injections.
  - 545
  - The proposed method accurately estimates the fuel flow-rate from  $t_{SOC}$  to the point when
  - 546  $p_{down}$  returns to its initial pressure level at  $t_{SOC}$ , using only a single pressure transducer.
  - 547
  - The estimated injected mass achieves an error within 2 mg, with an average deviation of 0.6 mg.
  - 548
  - The injected mass estimation can be employed to set-up a closed loop control for the injected
  - 549 mass to compensate inaccuracy of the fuel quantity.

550 **Acknowledgements**

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553

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