Angular velocity of Kolmogorov-scale fibers as proxy for turbulent dissipation: Supplemental material

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I. DETAILS ABOUT EXPERIMENTAL DATA PROCESSING

Fibers are illuminated by a laser, imaged at a frequency of 200 and 400 Hz for Re_τ equal with 180 and 360, respectively, using six high-speed cameras in a cross-like configuration below the channel [1], reconstructed with a tomographic algorithm, tracked with an in-house code. Their tumbling and spinning rates are measured following the methodology of Ref. [2]. The experiment was performed at three wall-normal locations: near-wall ($1 < y < 14$mm), intermediate ($17 < y < 33$mm), and center ($30 < y < 47$mm). This was achieved by displacing the carriage on which the cameras were mounted in the wall-normal direction between experiments. The achieved resolutions are 50.8, 52.7, and 55.7 vox mm^{-1} for the near-wall, intermediate, and center regions, respectively. In total, 6176 and 7040 trajectories have been used for the statistical analysis of this study for $Re_\tau = 180$ and 360, respectively. The components of the vectors of the fiber principal axes have been filtered in time using an "rlowess" filter with a kernel length, τ_f, adapted based on the distance from the wall, following Ref. [3]. The values chosen for τ_f are summarized in Tab. I. Moreover, the Kolmogorov time-scale, τ_η, for each region and each shear Reynolds number are shown in Tab. I to motivate the choice of the filtering kernel.

Moreover, using synthetic fibers, Ref. [4] shows that there is an optimum spinning angular displacement between time-steps which achieves the lowest spinning rate measurement relative total error. In the curvature range of the present experiment, $0.2 < \kappa^* < 0.6$, this optimum lies approximately between 4° and 10° between time-steps considered for estimating the rotation matrix temporal derivative, and thus for measuring the spinning rate. However, note that, Ref. [4] showed this relative total error map only for fibers with a resolution approximately double of the present experiments. The average spinning angular displacement is found to increase when approaching the wall, thus the rotation matrix temporal derivative is estimated between time-steps separated by a different δt for each region and Reynolds number. The values chosen for δt are summarized in Tab. I. With these parameters, the average measured spinning angular displacement between time-steps separated by δt lies approximately between 5° and 9° for all regions and for both Reynolds numbers.

TABLE I. Filtering kernel length τ_f, range of the Kolmogorov time-scale τ_η, and time-step separation δt used to estimate the rotational matrix derivative [4] for different wall-normal regions and friction Reynolds numbers Re_τ.

	$Re_\tau = 180$			$Re_\tau = 360$		
Region	τ_f [s]	τ_η [s]	δt [s]	τ_f [s]	τ_η [s]	δt [s]
Near-wall	0.25	0.14–0.34	0.15	0.10	0.04–0.12	0.04
Intermediate	0.50	0.34–0.62	0.20	0.18	0.14–0.22	0.08
Center	0.50	0.60–0.71	0.30	0.25	0.21–0.24	0.11

II. DETAILS ABOUT POINT-FIBER DIRECT NUMERICAL SIMULATIONS

The Navier-Stokes equations are solved in a plane channel flow of size $L_x = 4\pi h$, $L_y = 2h$, and $L_z = 2\pi h$, in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively. Across all the shear Reynolds number simulated ($Re_\tau = 180$, 360, 700) the grid resolution (in wall units) in both the streamwise and spanwise direction is $\Delta x^+ = \Delta z^+ \approx 5.9$. In the wall-normal direction, the resolution Δy^+ spans from 0.02 to 2.82 at $Re_\tau = 180$, from 0.02 to 3.77 at $Re_\tau = 360$, and from 0.02 to 5.24 at $Re_\tau = 700$. Following the well-established methodology presented in Ref. [5], the spatial discretization of the Navier-Stokes equations uses the Fourier–Galërkin method [6] in the homogeneous streamwise and spanwise directions, and the Chebyshev-tau method [6] in the wall-normal direction. Periodic bound-

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ary conditions are imposed in x and z, while no-slip conditions are enforced at the walls at $y = \pm h$. As in Ref. [5], we adopt a pressure-free formulation of the Navier–Stokes equations and advance the solution in time using the Crank–Nicolson scheme for the viscous terms and the two-step Adams–Bashforth scheme for the non-linear convective terms. The resulting integration scheme is second-order accurate in time.

In all simulations, the motion of $N_p = 3\,000\,000$ prolate ellipsoids [7] is tracked by integrating over time the equations of rigid-body motion for each particle. The ellipsoids have an aspect ratio of $\alpha = b/a = 120$, where a and b are the semi-minor and semi-major axes of the particle, respectively. The dynamics of ellipsoids are modeled using the methodology for point-particle simulations of anisotropic particles widely adopted in the literature [8–12]. Their translational and rotational kinematics are described by:

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{v}_p, \tag{1}$$

$$\frac{\mathrm{d}\mathbf{e}}{\mathrm{d}t} = \frac{1}{2}\,\mathbf{L}\,\boldsymbol{\omega}'_p, \tag{2}$$

where $\mathbf{x}_p$ and $\mathbf{v}_p$ are respectively the position and the velocity of the ellipsoid, while $\boldsymbol{\omega}'_p$ is its angular velocity. Here, the prime denotes quantities expressed in the frame aligned with the principal axes of the particle (x', y', z'), with z' oriented as the symmetry axis of the ellipsoid. The quantity $\mathbf{e} = (e_0, e_1, e_2, e_3)$ represents the unit quaternion—also known as Euler parameters—describing the instantaneous particle orientation with respect to the channel reference frame [11], while $\mathbf{L}$ is a matrix of kinematic coefficients depending only on the Euler parameters [8], as follows:

$$\mathbf{L} = \begin{bmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_3 & e_0 & -e_1 \\ -e_2 & e_1 & e_0 \end{bmatrix}. \tag{3}$$

The translational and rotational dynamics of the ellipsoids are solved by integrating the following ordinary differential equations:

$$m_p\,\frac{\mathrm{d}\mathbf{v}_p}{\mathrm{d}t} = \mu\,\mathbf{K}\left[\mathbf{u}(t,\mathbf{x}_p) - \mathbf{v}_p\right], \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(\mathbf{I}'\cdot\boldsymbol{\omega}'_p\big) + \boldsymbol{\omega}'_p \times \big(\mathbf{I}'\cdot\boldsymbol{\omega}'_p\big) = \mathbf{M}'. \tag{5}$$

Here, m_p is the mass of the ellipsoid, μ is the fluid dynamic viscosity, $\mathbf{u}(t,\mathbf{x}_p)$ is the fluid velocity at the particle location, and $\mathbf{K}$ is the ellipsoid resistance tensor [13] defined in the channel frame (x, y, z). In the frame aligned with the principal axes of the ellipsoid, $\mathbf{K}'$ is diagonal, $\mathbf{K}' = \mathrm{diag}(K_{x'x'}, K_{y'y'}, K_{z'z'})$, and its components are given by:

$$K_{x'x'} = K_{y'y'} = \frac{16\pi a\,(\alpha^2-1)^{3/2}}{(2\alpha^2-3)\,\ln\!\big(\alpha+\sqrt{\alpha^2-1}\big)+\alpha\sqrt{\alpha^2-1}}, \tag{6}$$

$$K_{z'z'} = \frac{8\pi a\,(\alpha^2-1)^{3/2}}{(2\alpha^2-1)\,\ln\!\big(\alpha+\sqrt{\alpha^2-1}\big)-\alpha\sqrt{\alpha^2-1}}. \tag{7}$$

In Eq. (5), $\mathbf{I}'$ represents the particle inertia tensor. In the particle frame, the inertia tensor of an ellipsoid is diagonal and reads

$$\mathbf{I}' = \frac{m_p a^2}{5}\begin{pmatrix} 1+\alpha^2 & 0 & 0 \\ 0 & 1+\alpha^2 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \tag{8}$$

Following Jeffery [14], the hydrodynamic torque $\mathbf{M}'$ in the body frame can be written in terms of the strain-rate, $\mathbf{S}$, and rotation-rate tensors, $\boldsymbol{\Omega}$, both evaluated in the fiber's principal-axes frame, as follows:

$$M_{x'} = A\Big[(1-\alpha^2)\,S_{y'z'} + (1+\alpha^2)\big(\mathit{\Omega}_{x'} - \omega_{p,x'}\big)\Big], \tag{9}$$

$$M_{y'} = B\Big[(\alpha^2-1)\,S_{x'z'} + (1+\alpha^2)\big(\mathit{\Omega}_{y'} - \omega_{p,y'}\big)\Big], \tag{10}$$

$$M_{z'} = C\big(\mathit{\Omega}_{z'} - \omega_{p,z'}\big), \tag{11}$$

where A, B, and C are constants given by:

$$A = \frac{16\pi\mu a^3\alpha}{3\big(\beta_0+\alpha^2\gamma_0\big)}, \tag{12}$$

$$B = \frac{16\pi\mu a^3\alpha}{3\big(\alpha_0+\alpha^2\gamma_0\big)}, \tag{13}$$

$$C = \frac{32\pi\mu a^3\alpha}{3\big(\alpha_0+\beta_0\big)}. \tag{14}$$

Here, α_0, β_0, and γ_0 are the geometric factors for spheroids (functions of the aspect ratio α only) appearing in Jeffery's solution; explicit formulas are given in Refs. [8, 10, 14].

Differing from common practice in the literature, we compute the torques in Eqs. (9)–(11) following the numerical approach of Ref. [15] which used the strain- and rotation-rate tensors obtained from the fluid velocity gradient filtered at the length scale ℓ. Specifically, denoting the filtered gradient by $\overline{A}^{\ell}_{ij} = \partial\overline{u}^{\ell}_i/\partial x_j$ [16], we define its symmetric and skew-symmetric components as $\overline{S}^{\ell}_{ij} = 1/2(\overline{A}^{\ell}_{ij} + \overline{A}^{\ell}_{ji})$ and $\overline{\mathit{\Omega}}^{\ell}_{ij} = 1/2(\overline{A}^{\ell}_{ij} - \overline{A}^{\ell}_{ji})$, respectively. In the simulations, ℓ equals the length of the experimental fibers, $\ell/h = 3\times10^{-2}$.

The ellipsoids considered in the simulations have negligible inertia. Their response time τ_p, as defined by Ref. [17], is 0.35 times the viscous time scale ($\tau_\nu = \nu/u_\tau^2$) for all Re_τ examined. Consequently, the particles behave as tracers and Eqs. (4)–(5) reduce to the imposition of zero net hydrodynamic force and torque at each instant.

This implies that the ellipsoids' slip velocity vanishes, $\mathbf{v}_p(t) = \mathbf{u}(t, \mathbf{x}_p(t))$, and their angular velocity is determined by the local, filtered velocity gradient components as follows

$$\boldsymbol{\omega}_p = \frac{1}{2}\overline{\boldsymbol{\varpi}}^\ell + \lambda \mathbf{p} \times \overline{\mathbf{S}}^\ell \mathbf{p} \qquad (15)$$

where $\mathbf{p}$ is the particle's symmetry-axis unit vector, $\lambda = (\alpha^2 - 1)/(\alpha^2 + 1)$, while $\overline{\boldsymbol{\varpi}}^\ell$ and $\overline{\mathbf{S}}^\ell$ are respectively the filtered vorticity vector and strain-rate tensor evaluated at the location of the particle.

To ensure consistency with the flow solver, the particle ordinary differential equations (Eqs. (1)–(2) and (4)–(5)) are integrated with the same explicit Adams–Bashforth scheme used for the convective terms of the Navier–Stokes equations.

III. DETAILS ABOUT THE FILTERED VELOCITY GRADIENT

To obtain the fluid velocity gradient tensor filtered at the length of the experimental fibers ($\ell/h = 3\times10^{-2}$), we apply a three-dimensional isotropic Gaussian filter to the velocity field before evaluating spatial derivatives. Following Ref. [15], the filtered velocity components $\overline{u}_i^\ell(\mathbf{x})$ are obtained by convolution with the normalized Gaussian kernel $G_\ell(\mathbf{r})$, as follows

$$\overline{u}_i^\ell(\mathbf{x}) = \int_\Omega G_\ell(\mathbf{r})\, u_i(\mathbf{x} - \mathbf{r})\, d\mathbf{r}, \qquad (16)$$

where Ω denotes the channel domain and the Gaussian kernel function is given by:

$$G_\ell(\mathbf{r}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{|\mathbf{r}|^2}{2\sigma^2}\right). \qquad (17)$$

Here, σ is the filter width and is related to the filter scale ℓ; specifically, we set $\sigma = \ell/(2\sqrt{5}) \approx 0.224\,\ell$, which ensures that the isotropic three-dimensional Gaussian has the same variance as a spherical three-dimensional top-hat kernel. This choice closely matches the value of $\sigma = 0.23\,\ell$ adopted in Refs. [15, 18].

Exploiting the separability of the Gaussian filter, we apply the filter in sequence along y, x, and z. In the wall-normal direction y, the convolution is evaluated in physical space using trapezoidal quadrature on Chebyshev–Gauss–Lobatto points [6], while in the homogeneous directions (x, z) it is computed in spectral space by multiplying the Fourier-transformed fields by the Gaussian transfer function.

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