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Machine-learning-based constraint handling for particle swarm optimization within structural optimization

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Abstract. Despite their historical roots and formally rigorous mathematical framework, mathematical programming is often not sufficient to effectively deal with real-world complex optimization problems. For instance, focusing on the structural optimization field, heuristic and meta-heuristic computational intelligence methods represented a promising solution for addressing many real-world challenges since their early conception. Several algorithms have been formulated inspired by mimicking natural phenomena, such as Genetic Algorithms or Simulated Annealing, among others. The lack of a strong mathematical basis motivated the idea of always simultaneously implementing different soft-computing techniques both for comparisons and mutual validation, and also because due to the No-Free Lunch, which demonstrated that the perfect algorithm able to solve any optimization problem does not exist. The study of the animal's world behavior, i.e. bird flocking or fish schooling, is the main idea behind one of the nowadays still most widely adopted meta-heuristic algorithms, i.e. the particle swarm optimization (PSO) technique. In the beginning, PSO was formulated to solve unconstrained optimization problems only, and later numerous scholars attempted to introduce some new constraint handling mechanisms in order to exploit the PSO's powerful optimization capabilities even with most likely real-world constrained problems. In this study, the authors focused on the constraint handling problem in PSO for solving structural optimization tasks, by leveraging the nowadays new compelling Machine Learning tools offered by the digital revolution currently in progress. Specifically, the authors formulated a new constraint-handling method based on the support vector machine (SVM) classifier to progressively update the feasible search region while the swarm explores the search space. This novel technique has been tested on a structural optimization conceptual design problem of a Warren truss steel bridge.

Keywords: Structural optimization, Machine Learning, Meta-heuristic Algorithm, Support Vector Machine.

1 Introduction

Mathematically speaking, optimization problems (OP) are generally referred to as the procedure of finding the absolute minimum (if it exists) of a function called objective function (OF) $f(\boldsymbol{x})$ by varying some governing variables denoted as design parameters $\boldsymbol{x}$ [1]. The same definition encompasses also maximization problems, if considering the opposite of the OF. The optimization problems can be classified in several ways. If there is one OF only, the problem is denoted as single-objective OP (SOOP), whereas if there are different OFs collected in a vectorial notation, the problem is a multi-objective problem (MOOP) [1,2]. Typically, MOOPs are characterized by opposing OFs, and the optimum is therefore characterized by a series of trade-off solutions among the various considered objectives denoted as Pareto Front. Other possible classification criteria of OPs depend on the presence or not of constraints functions which may limit or not the admissible values assumed by the design variables. This results in characterizing those tuples associated with feasible or infeasible regions of the OF, after projecting these tentative solutions x in the OF space.
In this study, the SOOPs are considered, specifically limited to the structural optimization problems (SOP) field [1,2,3,4,5,6,7,8,9,10,11]. SOPs are generally dealing with solving three main subproblems. The size optimization (SiSOP) task deals with the definition of the optimal cross-section properties associated with every structural element in the domain of analysis, thus governing their resisting and stiffness properties. The shape optimization (ShOP) task instead refers to optimizing those variables governing the shape of the domain of interest, without modifying the interconnection layout of the placed elements. Lastly, the topology optimization task (TSOP) deals with rearranging the structural elements in the domain of analysis to find an optimized configuration under certain loading conditions. This latter is also a more complex task, which often requires special dedicated methods.
In the present study, the authors mainly focused on solving SiSOP and ShOP leveraging the potentials offered by modern approaches based on computational intelligence [1,2,3]. Indeed, the digital revolution fostered by the progress of artificial intelligence (AI), provided new machine learning (ML) based tools that can be combined with existing approaches for solving complex OP. Indeed, despite the well-established maths branch called operational research, gradient-based algorithms have been demonstrated to suffer from strong limitations with discontinuous and constrained complex real-world SOP. Conversely, meta-heuristic algorithms provided a new powerful framework for addressing those kinds of problems and providing some sub-optimum estimates conceivably close to the actual real optimum. The meta-heuristic algorithms lack of strong mathematical basis, but they were developed by mimicking some natural phenomena and readapted to address numerical OP. Genetic algorithms, Simulated Annealing, and Particle Swarm Optimization can be cited among the others as some of the most popular and nowadays widespread meta-heuristic techniques [12].
In the current study, the ML classification algorithm denoted as Support Vector Machine (SVM), has been integrated with the PSO algorithm as a different constraint handling solution, leveraging the surrogate model provided by SVM for guiding the swarm

in the next iterations before updating the model again based on the actual OF evaluation. The idea of hybridizing PSO with SVM is not completely new in scientific literature. Indeed, the first attempts can be found in [13], using PSO and SVM to improve feature selection procedures in data mining and classification tasks. Another example can be found in [14] using PSO-SVM for optimizing the design and control of thermal energy storage systems. Mainly focusing on structural optimization tasks, the adoption of PSO-SVM for reliability-based design optimization can be found in [15] for minimizing a cost function while limiting the failure probability within a fixed threshold. In [16], PSO-SVM has been used for estimating geo-mechanical parameters of rock masses in complex geological conditions.

The current document is organized as follows. In the next section, the main governing principles of PSO and SVM are reviewed, and the proposed PSO-SVM integration is described in detail. Section 3 illustrates a case study example of SOP in which PSO-SVM revealed its potential, i.e. referred to the size and shape optimization of the conceptual design of a truss warren beam structure, which can represent a prototype of an arch bridge structure. The obtained results are in agreement with the existing literature [17,18], and demonstrated the effectiveness of adopting this PSO-SVM tool for solving real-world structural optimization problems.

2 Particle Swarm Optimization integrated with Support Vector Machine

The particle Swarm Optimization (PSO) algorithm is a meta-heuristic optimization method formalized by Kennedy and Eberhart [12] inspired by mimicking the biological behavior of bird flocking and fish schooling during the process of getting food in natural environments. It is a population-based method, in which intelligent agents called individuals are exploring the design variable search space, sampling their values, and getting the objective function (OF) evaluation for every position assumed by every agent. In PSO nomenclature, the population of intelligent agents that delivers the tentative solutions is called a swarm. To simulate a real-world swarm, the agents are able to interact with each other transmitting information on the search space to the entire population, according to the neighborhood concept. In this way, at every iteration, it is possible to define a global attractor (global best, Gb) which represents the so far best-found solution in the entire space. Likewise in a real-world biological environment, every individual is attracted not only to the global optimum, but thanks to their own intelligence and cognitive memory, it reminds also their individual so far best-visited position, which is denoted as local attractor (personal best, Pb). Therefore, considering that the particles of the swarm are moving in the search space following a Newtonian dynamics scheme, each particle's movements among the iterations are thus fully characterized by knowing its actual position and its velocity vector, which will determine the next iteration position. Despite the presence of local and global attractors, the movement paths of the individuals in the swarm are not completely deterministic, moving along a trajectory that would be somehow influenced by both an inertial term due to the previous step velocity, and also on a random combination of local and global attractiveness. Therefore, the next-step (k+1) velocity and displacement updating rule of the Newtonian-dynamics-based PSO version formalized by Kennedy and Eberhart [12] can be thus expressed as follows:

$$ {}^{(k+1)}v_i = {}^{k}v_i + c_1^{(k+1)} r_{1i} \cdot \left[{}^{k}x_i^{Pb} - {}^{k}x_i \right] + c_2^{(k+1)} r_{2i} \cdot \left[{}^{k}x_i^{Gb} - {}^{k}x_i \right] \quad (1) $$

$$ {}^{(k+1)}x_i = {}^{k}x_i + T \cdot {}^{(k+1)}v_i \quad (2) $$

In previous Eqs. (1) and (2), the term xi represents the position of any intelligent agent of the swarm in the search space, i.e. being equal to a specific realization of the design variables vector, whereas vi denotes the velocity term of the *i*-th particle, *T=1* is a pseudo-time variable equal to unity merely defined to ensure a physical-based meaning to Eq. (2), r_{1i} and r_{2i} are two random weights sampled by a continuous uniform probability distribution spanning between 0 and 1, c_1 and c_2 are denoted as cognitive and social terms respectively, which are priorly and deterministically weighting the relative importance of the cognitive and social terms related to the local Pb attractor and the global Gb one.

At its origins, the PSO was formalized for working with unconstrained optimization problems, and further constraint-handling methodologies were developed to extend the possibility of a widespread use of PSO, such as penalty-based methods among the others [1,2]. In the present work, the PSO has been integrated with a machine learning (ML) methodology denoted as support vector machine (SVM) for providing a data-driven constraint handling that constructs a surrogate model of the feasible region in the search space. SVM is a supervised classification ML model based on statistical learning theory and was originally formalized by Vapnik in 1995 [19]. The SVM constructs a sort of Bayesian probability model of the search space while it is attempting to optimally separate the infeasible particle positions from the feasible ones. The ML method is based on the maximization of the margin, i.e. the optimal separation hyperplane, which can be conceived as the constraint position in the current scenario. However, since for structural optimization problems, the inequality constraints' boundaries are often represented by nonlinear functions, the optimal hyperplane can provide a poorly representative separating plane. However, the Kernel trick allows to map of data to a higher dimensional space, in which the apparently non-separable labeled data finally become linearly separable, and the hyperplane margin can be efficiently located with significantly reduced misclassification errors. If the mapping rule to higher dimensional space is invertible, the optimal found hyperplane could be restored in the initial design space, thus becoming a nonlinear boundary which this time better approximates the original constraint boundaries. The powerfulness of the kernel trick allows for the simulation of this process without properly and explicitly mapping data to higher dimensional space, therefore providing a computational effective tool. In this work, the radial basis function (RBF) kernel has been adopted. After a certain number of k of PSO iterations, the SVM can be trained based on all the explored positions of the swarm in the previous iterations, which have been labeled as feasible or infeasible. Once the

SVM has been trained, the surrogate model can provide an estimate of the feasible region of the search space, and new PSO positions can be predicted to be feasible or infeasible based on the SVM scores. Periodically, this process can be repeated because the new PSO sampled position represents new data for the SVM algorithm, which can be retrained and improve the estimate of the feasible/infeasible region, especially closer to the real optimum region. It is worth noting that the SVM method can be further tuned to improve the surrogate model estimation, e.g. varying its regularization coefficient C. For a more comprehensive description of the proposed PSO-SVM method, the interested reader can refer to [17].

3 Structural optimization example: conceptual design of a Warren truss beam

The effectiveness of current PSO-SVM implementation on numerical benchmark case studies has been already demonstrated in [17]. The real-world case study herein considered is a planar Warren truss-type beam structure depicted in Fig. 1, carrying a fixed uniform static load of 100 kN/m, acting as point loads in the nodes of the lower chord. A similar geometry has been presented also in [17,18]. In conceptual design optimization problems, the OF steel truss beams are normally conceived to minimize the material consumption usage, which is expressed directly as the total weight of the structure. Indeed, knowing the steel density ρ_i=7850 kg/m^3 of each *i*-th steel member, the weight is the summation of the products of steel density and the relative steel member volume, expressed in turn as the product of its length L_i and its cross-section area A_i. In order to limit the number of design variables, the geometry of the truss has been schematized using four different types of square hollow core (SHC) section beams for the lower chord, the upper chord, the internal diagonals, and the external diagonals. To address also the shape optimization problem, *Hmin* and *Hmax* govern the geometric height and layout of the beam truss. Therefore, the problem statement becomes finding $x=[s_i,B_i,Hmin,Hmax]^T \in \Omega$ such that minimizes the OF *f(x)* such that:

$$\min f(x) = W(x) = \sum_{i=1}^{m} \rho_i L_i A_i \text{ subjected to}$$

$$\frac{N_{Ed,i}}{N_{tRd,i}} = \frac{N_{Ed,i}}{\frac{f_{yk} \cdot A_i}{\gamma_{M0}}} \leq 1 \tag{3}$$

$$\frac{N_{Ed,i}}{N_{bcRd,i}} = \frac{N_{Ed,i}}{\chi \cdot \frac{f_{yk} \cdot A_i}{\gamma_{M1}}} \leq 1$$

Moreover, some deformability constraints have been also considered in the optimization process, i.e. limiting the maximum deflection at a limited prescribed value of L/500. The constraints have been considered according to Eurocode design prescriptions EN-1993-2 2006, in terms of Ultimate Limit State (ULS) assessment in terms of maximum tensile strength $N_{tRd,i}$ and maximum buckling compression strength $N_{bcRd,i}$, being f_{yk}=*275* MPa the design value of structural steel S275, γ_{M0} is the partial safety factor equal to 1.05, $\chi \leq 1$ is the compressive strength reduction factor due to buckling issues, and γ_{M1} is the partial safety factor equal to 1.10 for bridge design. The Warren truss beam total length is L=20.00 m and the lower chord has been dived into 20 equal length ΔL intervals. The OP in Eq. (3) has been formalized as continuous optimization, therefore PSO explores design variables which are assumed to vary as continuous variables within an admissible range of interest. Indeed, *Hmax* has been limited to (0.05,20.00) m, and consequently, *Hmin* has been dynamically set to be a positive quantity but lower than *Hmax*. Moreover, an admissible interval of (60;360) mm was set for SHC profiles' width B_i, whilst setting (4;30) mm for SHC profiles' thickness s_i.To evaluate both deflection and stresses in steel members, at every iteration of the PSO-SVM algorithm, the Matlab CALFEM numerical finite element solver has been adopted. The swarm size has been chosen with 100 particles, and the stopping criterion has been set when the threshold number of iterations equal to 200 has been reached. Since the achievement of the actual optimal solution is not guaranteed with meta-heuristic algorithms, it is advisable to perform a series of independent runs of the algorithm, e.g. 50 runs, and then evaluate the statistics of optimal founded solutions.

The optimal results obtained from the 50 independent runs have been reported in Fig. 2. Inspecting the graph, it is worth noting that the dispersion of OF values is quite scattered, demonstrating the meta-heuristic nature of the utilized algorithm, which did not always reach the optimal value of around 3.10 tons (red dashed line). Therefore, to evaluate the significant statistics, it is necessary to consider solely those similar solutions. Thus, the outlier results far from 3.10 tons have been excluded, retaining only 21 successful runs.

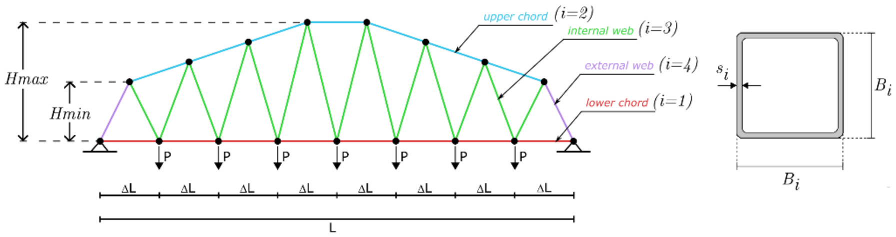

Fig. 1. Warren truss beam structure case study illustration.

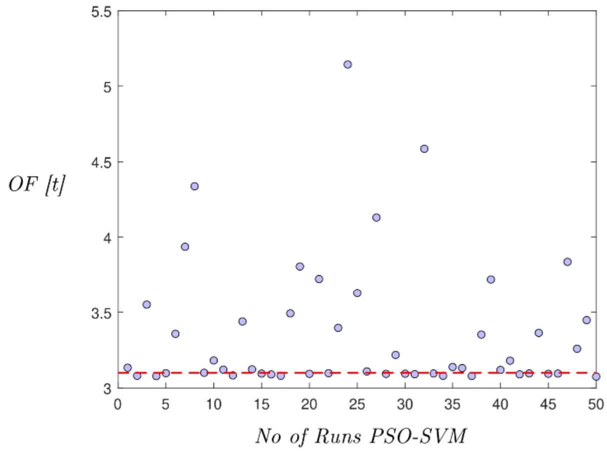

Fig. 2. Best optimal results in terms of OF evaluation for 50 independent runs of the algorithm.

Considering only these 21 latter solutions, the descriptive statistics have been computed and reported in Tab 1. The descriptive statistics have been reported in terms of average value, standard deviation, and coefficient of variation (COV), which is the normalized ratio of the standard deviation and the average value. It represents a normalized measure of dispersion. The table also reports the absolute optimal founded solutions among the selected 21 as a reference set of design parameters, which have been rounded considering actual constructability tolerances issues and technical-commercially available steel profiles solutions. It is evident that despite selected optimal results are characterized by the same OF, different combinations of design variables are possible. This fact is evidenced by the relatively high COV of some variables such as the thickness of the lateral outer diagonals s_4, or the thickness of the upper chord s_2, and the corresponding width of the SHC section profiles. Indeed, considering the buckling resistance of steel members, the combination of width and thickness of the profile determines the class of the steel section profiles, permitting the full usage of its plastic resources (classes 1 and 2) or limiting the resistance to the elastic stage (class 3), or even exhibiting local sectional buckling in elastic stage (class 4). Therefore, it is evident that despite the absolute optimal solution is sometimes different from the average result, in this special case, the PSO-SVM has finally found an absolute optimal solution characterized by the minimum usage of steel material whilst preferring thicker profiles for the top chord and lateral outer diagonals. Further refinements could be performed in future studies, such as reducing the degree of freedom of the optimization search process, in order to better focus the algorithm to find similar structural solutions, and even including some clustering approaches for gathering and analyzing different optimal solutions, critically analyzing the different technical aspects for possibly preferring a solution rather than another, despite characterized by similar OF.

Table 1. OF and design variables' optimal results, and their statistics over the 21 selected independent runs characterized by similar OF around 3.10 tons.

Result	Rounded absolute optimal solution (mean; std.dev.; COV)
OF [ton]	3.09 (3.09; 0.007; 0.23%)
s_1 [mm]	4.00 (6.00; 1.500; 25.00%)
B_1 [mm]	95.00 (72.50; 15.700; 21.66%)
s_2 [mm]	26.00 (14.70; 7.400; 50.34%)
B_2 [mm]	105.00 (190.20; 70.200; 36.91%)
s_3 [mm]	4.00 (4.00; 0.030; 0.75%)
B_3 [mm]	130 (128.90; 1.400; 1.09%)
s_4 [mm]	19.00 (14.00; 8.400; 60.00%)
B_4 [mm]	110.00 (211.70; 107.90; 50.97%)
$H\ min$ [m]	0.41 (399.40; 21.50; 5.38%)
$H\ max$ [m]	4.15 (4064.7; 113.500; 2.79%)

4 Conclusions

The nowadays complexity of structural optimization problems (SOP) is demanding new powerful algorithms able to overcome limitations of operational research and mathematical gradient-based existing methods. Indeed, in the current study, a hybrid optimization algorithm has been presented. This was developed by integrating the meta-heuristic particle swarm optimization (PSO) algorithm with the machine learning (ML) support vector machine (SVM) classifier. The method attempts to combine the powerful search capabilities of swarm intelligence with a surrogate model of the search space provided by the SVM technique, acting as a non-penalty constraint handling method. The training labeled data for the SVM directly comes from iterations of the PSO algorithm which explores both feasible and infeasible regions. Periodically, the SVM is retrained, in order to progressively refine the feasible region boundaries. Specifically, when approaching the stopping criterion, the algorithm should migrate from an exploration phase to an exploitation phase to refine the optimal solution. Therefore, the SVM in these latter iterations should accurately represent the constraint boundaries in the optimum nearby. The PSO-SVM algorithm has been effectively employed for studying a real-world structural optimization problem of the conceptual design of a planar Warren truss beam problem. The Warren beam optimal design results are in agreement with the existing literature [18], and provided some interesting insights and remarks for future research improvements.

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