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Optimal sequence of replacements in the renovation of water distribution networks

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Abstract. Water distribution networks (WDNs) are vital urban infrastructure. Aging components within WDNs pose a major concern due to their susceptibility to leakages. One possible strategy to addressing this issue -especially in severely degraded WDNs- is to replace a large part (up to 20%) of the conduit in the network (i.e., to perform a network renovation). However, the replacement of conduits presents challenges, especially in urban environments where construction must seamlessly integrate with city dynamics while minimizing disruptions to transportation infrastructure and water supply. Because of this, WDN renovation cannot be performed all at once, rather it requires a progressive and sequential replacement of conduits that can last months. Due to the nonlinear nature of the problem, it is expected that different sequences of conduits replacement will have different effects on the WDN dynamics. This study explores the existence of an optimal sequences of pipe replacement, that minimizes disturbances to users.

Keywords. Renovation, Leakages, Optimization, Genetic Algorithms, Modelling, EPANet.

1. Introduction

Water distribution networks (WDNs) are a key type of infrastructure that is deeply connected to the urban fabric. WDNs are made up of heavy equipment and components, are installed underground (usually below busy roads) and are therefore very difficult to maintain and monitor. As a result of this, WDNs worldwide are aging with little or no maintenance [1]. The main issue related to aging of WDNs is that aged components exhibit a higher likelihood to break and cause leakages [2]. This phenomenon causes -over time- an increment of leakages along the network.

Leakages are a major problem for the management of WDNs. Leakages reduce the availability of the water resources, and this is a major issue in countries affected by water scarcity [3]. Leakages are a problem also in countries where water is plentiful. One major issue is the economic damage caused by leakages: water has often to be potabilized and pumped. These operations are costly, and so leaking water means leaking money [4]. In addition, leakages may induce a reduction in the network pressure, which may end in customers dissatisfaction, and the noncompliance of the water utility with minimum supply specification defined in contracts with users. It should be finally noted that leakages in some cases can flow through the soil reaching sewer systems. This can cause an increment of wastewater to be treated in plants, but also dangerous paths of underground erosion.

To reduce leakages, a number of strategies has been adopted. Strategies span from the management of pressure in the network [5], the creation of districts, [6, 7], the research of main leakages location for localized intervention in damaged pipes [8]. In severely degraded WDNs, the complete replacement of large portions of the old conduits with new conduits is a strategy to be considered.

This last strategy is defined as "renovation" of a WDN [9]. In renovation works it is not uncommon to replace as much as 10-20% of the total length of the WDN. This is necessary to achieve a significant reduction of leakages. Nowadays, investments for renovation of WDNs that entail the replacement of vast portion of conduits in the network are widespread.

Replacement of conduits is a time-consuming work, especially in an urban setting, where the worksites for the replacement of conduits are to be integrated in the urban dynamics, and should interfere to a minimum degree with transportation infrastructures. In addition, water users should be supplied even during pipe replacement works. Total outages of water supply are to be sporadic, and can be tolerated only for few hours a day. These strict requirements entail that WDNs' renovation is to be performed through a progressive (sequential) replacement of few conduits at a time.

This sequential replacement of one conduit at a time during WDNs' renovation is called "sequence of replacement". This sequence of replacement may last weeks, affects the users, and alters the dynamics of the WDN. The scope of this study is limited to the "hydraulic" effect of renovation. The hydraulic effect encompasses all the aspects related to WDN dynamics and user satisfaction (e.g., pressure at network's junction, demand deficit). All other aspects (e.g., traffic issues) are disregarded. To go further into details, users and the WDN can be affected by the sequential replacement of conduit for a number of reasons. One first example concerns the users, that can be disconnect by the network for few hours, when the water main that feeds the building is disconnected for replacement. One second example concerns the WDN dynamics. After a conduit is replaced, some key hydraulic characteristics (e.g., roughness and diameter) can be deeply modified. These modifications can alter (to name a few): (i) the pressure dynamics in the surroundings of this conduit; (ii) the magnitude of leakages in conduits not yet renovated.

The few examples discussed in the previous paragraph show that the considered problem is strongly nonlinear. It is therefore probable that different sequences of replacements (i.e., the order in which conduits are replaced) affect to different extents users and the WDN dynamics.

In this picture, the scientific questions that is addressed in this study is "Is there an optimal sequence of replacements in the renovation of water distribution networks?". In this context, optimal means that the negative effect on users (e.g., low pressure, demand deficit).

In order to answer this question, it is necessary: (i) to assess the hydraulic effect (in terms of time-series of pressure at junctions) of a generic sequence of the replacement of conduits; (ii) quantify (with a scalar) the negative effect on users induced by the considered progressive replacement of conduits; (iii) minimize these negative effects on users finding the most appropriate (i.e., optimal) sequence of replacements.

Historically, the assessment of optimal sequence of works and operations in complex and time-consuming projects is a topic mainly studied in transportation engineering (e.g., on the progressive implementation of bus lanes [10]). In this study, the methodology is adapted and implement to the problem of renovation of WDNs.

2. System considered and formulation of the problem

2.1 Steps in a sequential renovation

The renovation of a WDN is performed with the progressive replacement of a selected number of conduits. This sequential replacement entails a well-defined sequence of many different single operations. Examples of single operations are: (i) the disconnection from the network of an old pipe;

(ii) the opening/closing of valves; (iii) the putting into service of a new pipe. These operations may affect the hydraulic of the system: key characteristics such as the topology of the network, the water demands at junctions, the conduits' resistance may be modified during these operations. The modification of these key characteristics may have in turn an effect on the hydraulic behavior of the WDN itself, that is described by the time series of flow in links, and pressure and water outflow (demand) at junctions. One additional characteristic of renovation works is that single operations require some time to be implemented, and so they begin end at different time, and therefore can interact with other time-variations of WDNs (e.g., the variation of demand over the day, tanks dynamics).

In this context, the aim of this work is to find the "optimal sequence" of the operations to be implemented. The focus is therefore on the time that spans from the beginning of the works of renovation (at the calendar time $t = t_0$) up to the end of these works, that occurs at the calendar time $t = t_0 + T_W$. As stated before, modifications of the network can induce unwanted effects (e.g., pressure reductions that cause users' dissatisfaction or technical problems). In order to quantify such negative effects, the focus will be on the analysis of quantitative data such the time-series of pressure and demand over the period $[t_0, t_0 + T_W]$. In the following, it will be shown that possible adversities registered in the time-series of pressure and flow can be aggregated to scalars that quantify the "cost" of the operations of network replacement. In this way, the problem of minimizing disturbances during network renovation becomes a problem of cost minimization.

We stress that the first issue is that the objective of the optimization is not the final design status of the network (i.e., a static configuration). Rather, the objective is the sequences of operations to be implemented to go from the initial (current) status of the network to the design configuration of the network. The objective of the optimization is therefore a dynamic sequence.

3. Modelling

3.1 Hydraulics

The first step to solve the optimization problem is to model how the hydraulic behavior of the considered WDN evolves operation after operation. To do that, we focus on a generic WDN, made up of links (pipes, pumps, valves) that connect junctions (users, reservoirs, tanks).

In order to model the hydraulics of a WDN, a standard approach based on the steady state modelling of extended-periods with spatially and temporally varying water demand is adopted. This approach is implemented by the well-known and well-documented software EPANet [11]. For this reason, most of the details about the numerical modelling are here omitted. It is of interest to re-formulate the continuity equations that holds at junction as

$$\sum_{i \in \mathbf{E}_j} Q_i - \sum_{i \in \mathbf{L}_j} Q_i - \hat{Q}_j = 0, \tag{1}$$

Where Q_i is the flow rate in the i −th link, and $\mathbf{E}_j$ is the list of links that enter the junction. This means that the downstream (DS) end of the link merges in the junction; $\mathbf{L}_j$ is the list of links that leave the junction. This means that the upstream (US) end of the link merges in the junction, and where

$$\hat{Q}_j = Q_{D,j} + \sum_{i \in \mathbf{E}_j \cup \mathbf{L}_j} Q_{L,i,j}. \tag{2}$$

$\hat{Q}_j$ is the total outflow from the junction, that is made up of the users' demand ($Q_{D,j}$) and leakages occurring along pipes. The key term here is $Q_{L,i,j}$, i.e., the leakage that occurs over the i −th pipe and that is aggregated at the j −th junction. To evaluate this term, it is assumed that the leakage depends

on the pressure occurring at the j −th junctions (see [12] for a comprehensive discussion of the validity of this approach). Under this assumption, $Q_{L,i,j} = 0.5 \cdot \xi_{L,i} \cdot L_i \cdot [P_j]^{0.5} = \chi_{L,i} \cdot [P_j]^{0.5}$, where L_i is the length of the pipe, and $\xi_{L,i}$ quantifies the leakages per unit length.

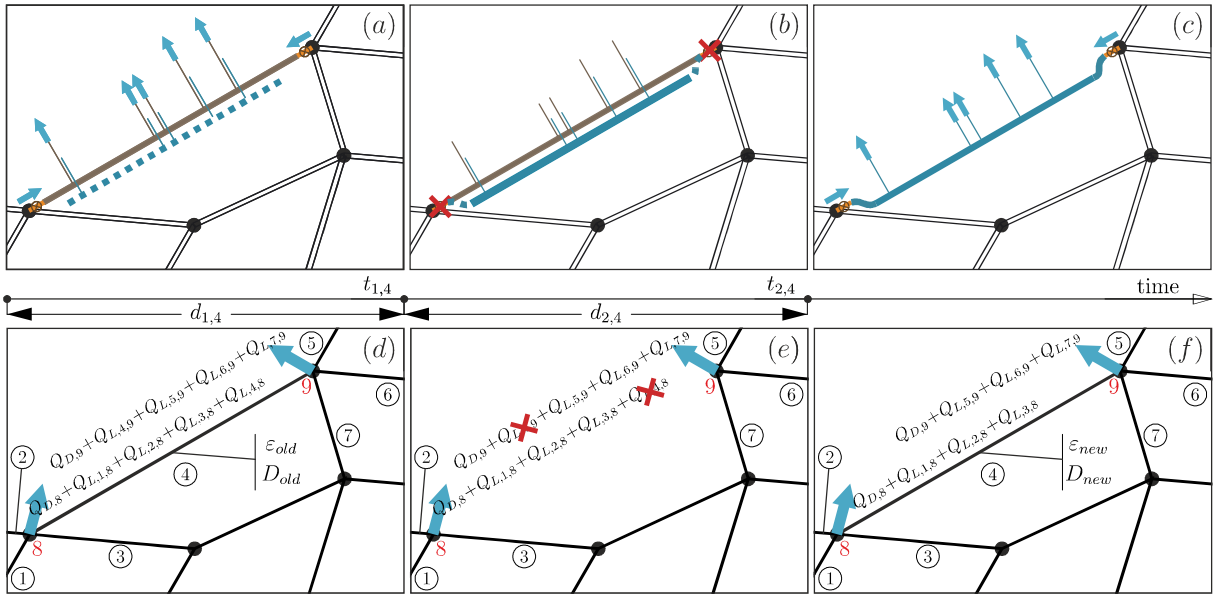

Figure 1. Panels (*a*)-(*c*) describe the phases required for the replacement of an old pipe (brown) with a new pipe (blue) Dashed lines mean that an element is under installation. Panels (*d*)-(*e*) describe the hydraulic model of the phases reported in panels (*a*)-(*c*). See the main text for description.

3.2 Sequential replacement of pipes or other links

One key step of this work is to model the variation of hydraulic characteristics and WDN topology brought about single operations of renovation. Among the different possible operations, the focus is restricted to the replacement of links, even though other operations can be accounted for with the same approach. The replacement of a link between two junctions consists of different phases. To better illustrate these phases, figures 1*a*-1*c* focus on the replacement of the pipe characterized by the link with index $i = 4$. During the first phase (panel *a*, duration $d_{1,4}$) preliminary works for the installation of the new pipe (blue) are done. The first phase ends at the calendar time $t_{1,4}$. During the second phase (panel *b*, duration $d_{2,4}$) the old pipe is disconnected from the network (closing the valves that connect the pipe with the junctions), and the connections with the new pipe are installed. The second phase ends at the calendar time $t_{2,4}$. In the third phase (panel *c*) the valves that connect the new pipe with the junctions are opened, and the new pipe is operational.

The three phases previously discussed are now interpreted in the perspective of hydraulic modelling. Figures 1*d*-1*f* will help for this purpose. The pipes (or other links) to be replaced are identified with the index k (introduced to differentiate them from the generic i −th link). **R** is the list of links to be replaced**.** In figure 1*d-f*, k =4. It should also be noted that the US end of the pipe merges in junction 8, whereas the DS end merges in junction 9 (see the red numbers in figure 1*d-f*). Phase one (generic duration $d_{1,k}$) spans until the conditions $t \leq t_{1,k}$ holds. During this phase, the generic k −th pipe to be replaced is characterized by the old values of roughness $\varepsilon_{old,k}$ and old diameter $D_{old,k}$. The demand at the nodes $\hat{Q}_8$ and $\hat{Q}_9$ is evaluated with equation (2). A key point is that all pipes merging at the junctions contribute to inducing leakages. In the sum of equation (2), all pipes are accounted for, and in particular $Q_{L,k,j} \neq 0$). In the considered case (k =4, see figure 1*d*), $Q_{L,4,8} \neq 0$ and $Q_{L,4,9} \neq 0$.

Phase two (generic duration $d_{2,k}$) spans until the conditions $t \leq t_{2,k}$ holds. During this phase, the generic k −th link to be replaced is disconnected from the network. From a modelling point of view, the pipe is removed from the network topology. Phase two entails also a modification of the water demand at junctions. During Phase 2, the leakages induced by the considered pipe to be replaced are set to zero (the pipe is disconnected from the network). This means that the terms $Q_{L,k,j}$ to be put in equation (4) is set to zero. In the example of figure 1, $Q_{L,4,8}$ =0 and $Q_{L,4,9}$ =0 (see the red crosses in figure 1*e*). Phase three begins when $t > t_{2,k}$. During this phase, the generic k −th pipe to be replaced is connected back to the network. During this phase, the generic k −th pipe has been replaced. Now is characterized by the new value of roughness $\varepsilon_{new,k}$ and new diameter $D_{new,k}$. During Phase 3, the leakages induced by the considered pipe to be replaced are set to zero (the pipe is new and with no damage that causes leakages). This means that the term $Q_{L,k,j}$ to be put in equation (4) are kept to zero. In the example of figure 1, $Q_{L,4,8}$ and $Q_{L,4,9}$ are not listed among the outflow at junctions.

All the operations to be performed and described in the previous paragraphs were scheduled in calendar time (i.e, with a precise day-hours-minutes format, see [13]). This was necessary because the operations interfere with a dynamically evolving system (water demand, tank level, pump status change over time). Hence, the same operation occurring over different weekdays or over different part of the day can affect to a different extent the WDN hydraulics.

3.4 Estimation of costs

During the process of WDN renovation, pipes (or other link elements) will be progressively replaced. This sequence of replacements will induce changes of pressure within the network. The changes of pressure induce two effects that are particularly relevant in the context here considered. The first effect (the focus of this paper) stems from the fact that during the disconnection period (from $t_{1,k}$ to $t_{2,k}$) at the nodes where the $k-$ th pipe is connected to the network, a reduction of pressure is experienced. This can lead to an increment of the demand deficit $\Delta Q_j(t)$. The second fact stems from the fact that after the disconnection period leakages are reduced. As a result, at the junctions of the $k-$ th pipe, an increment of pressure is experienced. This again can lead to a reduction of the demand deficit $\Delta Q_j(t)$, but an increase of leakages in pipes that are not yet replaced.

To quantify with scalars these complex dynamics, a number of costs can be defined. Here the focus is on the so called “user cost”, where “user” indicates that the focus is on the effect of pipe replacement on users’ satisfaction. In this perspective, the emphasis is thus on minimizing demand deficit. We recall that the demand deficit in each junction is $\Delta Q_j(t) = \max[0, Q_{DU,j} - Q_{D,j}(t)]$, so the “user cost” is the sum of the demand deficit in all the junctions of the network over T_W, namely

$$C_Q = \sum_{j=1}^{N_j} \int_0^{T_W} \Delta Q_j(t)\, \mathrm{d}t\,. \tag{4}$$

4. **Algorithm for the optimization of the sequence of replacement**

In order to optimize the sequence of replacement, the so called “Population Based Incremental Learning (PBIL) algorithm” is used.

5. Case study and results

In the previous section, a technique that can be used to optimize the sequence of replacement of pipes in an aging WDN was discussed. We now apply such technique to a realistic case study in order to show that: (i) the discussed approach can be applied to a real world problem; (ii) different sequences

of replacement of pipes may lead to different costs, and so the idea of finding an "optimal" sequence is meaningful

To do that, we make use of a realistic case study that has been widely used in the lieterature about analysis of WDN. This case study is referred in the lierature (see [14]) as "C-Town" (see figure 3). The WDN of the entire C-Town is organized in 5 districts (see see figure 3). For the purposes of this analysis, the replacement of 53 conduits is considered. Those 53 conduits are identified as the most problematic and cause the most severe leakages. It is supposed that all conduits are in district 5. It is also supposed that the new pipes will be characterized by the same diameter of the old pipes ($D_{new,i} = D_{old,i}$) and by the new roughness $\varepsilon_{new,i} = 0.01$ mm (typical value for HDPE pipes). The duration of the replacement operations was related to the importance of the pipe within the district. Three levels of importance (estimated with engineering judgment) were considered: (i) high (marked with red), (ii) medium (yellow marked), and (iii) low (green marked). For highly important pipes the disconnection time was one hour; medium-importance pipes required disconnection time of two hours; low-importance pipes were disconnected for 6 hours (thanks to the networked structure of the system that allowed for redundancy of flow paths). For what concerns the duration of preliminary works, the range $d_{1,k}$ =[32, 80] hours was considered. With this strategy, the total time for complete replacement was $T_W = 216$ days.

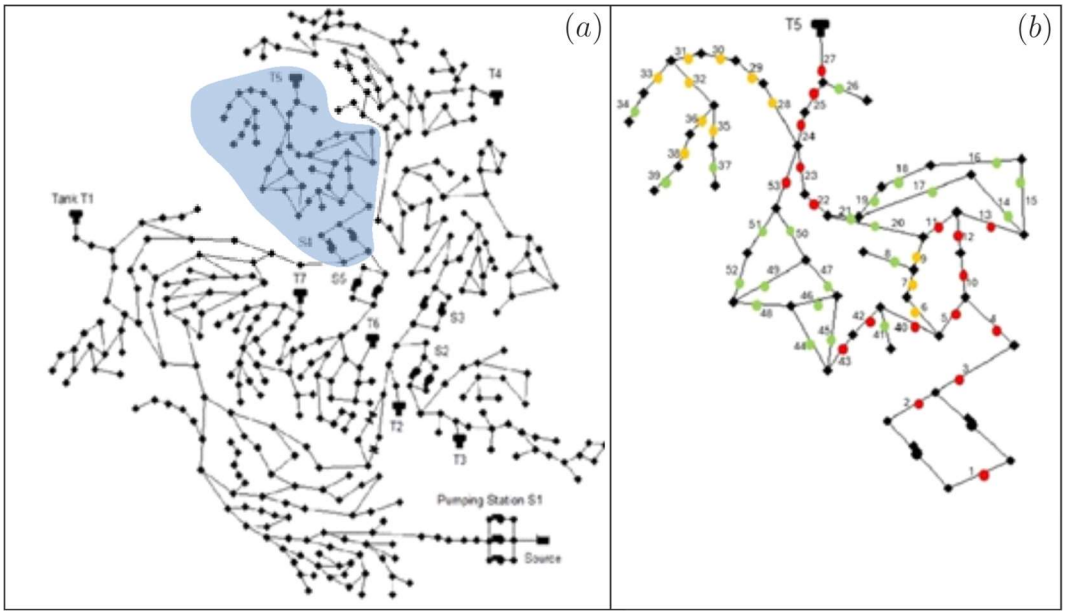

Figure 2. (a) Network considered in this study. (b) Magnified view of district 5. The number near links is the corresponding index i. Different colors represent different importance of the pipe.

The algorithm was applied with the optimization parameters LR =0.5, f =0.35, and Δf =0.1. The cost reported in (3) was considered. This led to the optimized sequence of Figure 3. The horizontal axis reports the number of iterations NI, the vertical axis is the cost. The curve show a decrease of the cost while the number of iterations increases. The algorithm successfully achieved an optimization of the sequence of replacement, effectively mitigating deficits in cumulative demand. In the user-oriented scenario, the demand deficit was reduced from approximately 240 m^3 to145 m^3.

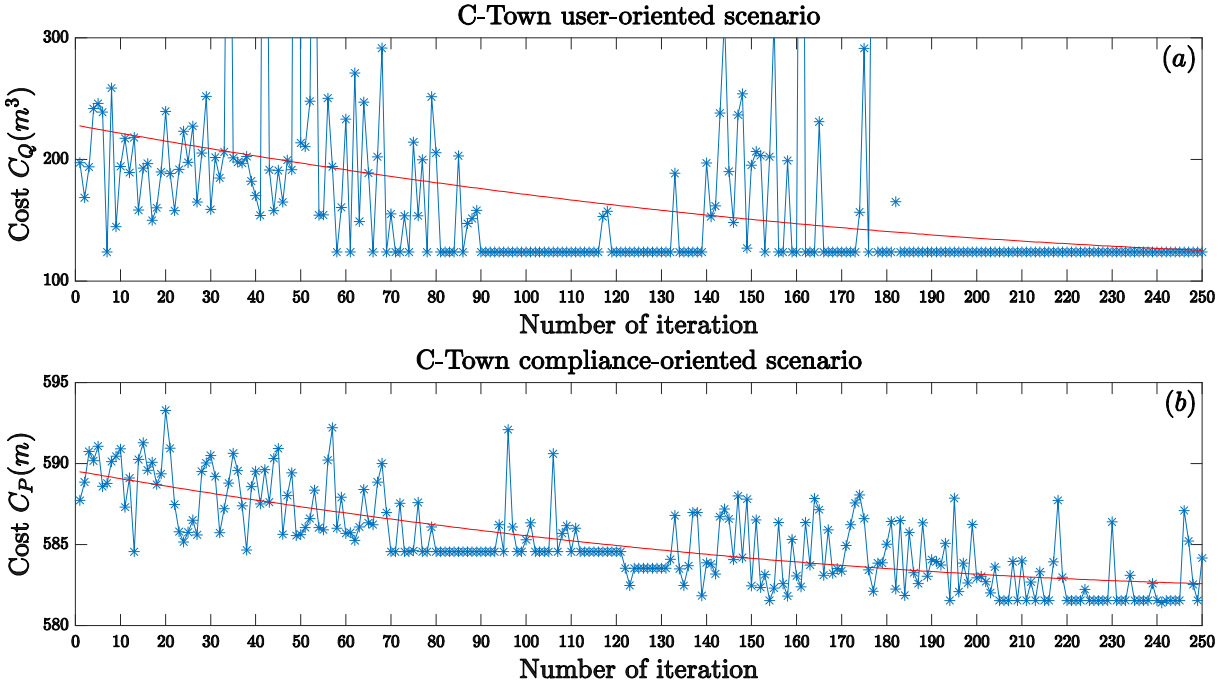

Figure 4. Optimization of the sequence of pipe replacement in the user-oriented scenario (*a*) e in the compliance-oriented scenario (*b*).

5. Concluding remarks

In this paper, a computational strategy for modelling the replacement of pipes in the context of network renovation has been presented. The strategy is based on the MATLAB's EPANet toolkit, that allows for the automatic modelling of a WDN with a time-varying topology, time-varying leakages, time-varying hydraulic characteristics of conduits.

Such modelling strategy was adopted to find the optimal sequences of conduit replacement that can be implemented during renovation works of WDNs. In the context of this paper, optimal means that the sequence of replacement minimizes a "cost". The costs that were considered in this work are scalar metrics that were derived from pressure time-series or junctions' demand time-series. Different costs can be defined. In this paper the focus was on: (i) a "user-oriented" cost, that measures possible problems related to customers satisfaction; and (ii) a "compliance-oriented" cost, that measures possible problems related to the compliance of the water utility with minimum service standards.

It was demonstrated that different sequences of pipe replacement impact to very different extents the users. It was also demonstrated that the optimization process led to significant and meaningful optimizations. To name some numbers, some sequences of pipe replacement led to a demand deficit as high as 1000 m^3, while the optimal sequences of pipe replacement led to a demand deficit of 150 m^3 (almost one order of magnitude lower).

In the case study here considered (the well-known C-TOWN), the best optimization was achieved in terms of minimization of the demand deficit. The optimization aimed at keeping the pressure above a prescribed (compliance) value was less successful. This partial failure was probability due to the characteristics of the considered district.

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Characteristics of the elements considered in the hydraulic modelling

Links (conduits, pumps valves) are identified by the index i. N_C conduits are installed in the WDN. The hydraulic resistance depends on the length L_i, the diameter D_i, and on the equivalent roughness ε_i (according to the Colebrook-White's approach). In pipes the flow velocity is V_i and the flow rate is Q_i. Along the i −th pipe, distributed leakages occur. Leakages depend on the diameter, age of material of the pipe, and on the pressure. The leakage per unit length is $q_{L,i} = C_{L,i} \cdot [P_i(x_i)]^{0.5}$, where $P_i(x_i)$ is the pressure at the location x_i in the i −th pipe, whereas $C_{L,i}$ is a leakage coefficient that takes into account the diameter, age, material and condition of the conduit.

Pumps are characterized by the pump-curves $C_{P,i}$ $(\Delta H_{P,i}, Q_{P,i})$, where $\Delta H_{P.i}$ is the total head and $Q_{P,i}$ is the flow rate. N_P pumps are installed in the WDN. The (constant) efficiency of pumps is η_i, whereas the cost of energy is c_E. Pumps are dynamic elements that can be switched on/off by controls, so the additional variable $s_{P,i}$ is defined to monitor the status of the machine. For what concerns valves, here we only consider throttle control valves with headloss coefficient $C_{V,i}$ that satisfies the equation $\Delta H_{V,i} = C_{V,i}\left[2Q_{V.i}/(g\pi D_{V,i}^2)\right]^2$ where $Q_{V,i}$ is the flow through valve, $\Delta H_{V,i}$ is the head drop across the valve, $D_{V,i}$ is the nominal diameter of the valve. N_V valves are installed in the WDN. Similar to pumps, valves are dynamic elements that can be switched on/off by controls, so the additional variable $s_{V,i}$ (that monitor the status of the valve) is defined.

Junctions are identified by the index j. N_J junctions are installed in the WDN. Junctions are characterized by the users' demand $Q_{D,j}$, by the leakage $Q_{L,j}$ and by the elevation z_j. Users require the time-varying demand $Q_{DU,j}(t)$, but the actual outflow that can be provided by the WDN is constrained by pressure. In particular, the parameter $P_{\min,j}$ is the minimum pressure at the j −junction so that the outflow provided by the network can be equal or larger than the demand required by users. On the other hand, P_j is the (possibly time-varying) pressure at the j −junction. When $P_j \geq P_{\min,j}$, the outflow from the j −junction is $Q_{D,j} = Q_{DU,j}(t)$ (i.e., the users' demand is fully satisfied). When $P_j < P_{\min,j}$, the outflow is $Q_{D,j} < Q_{DU,j}(t)$. The users' demand cannot be fully satisfied, and the demand deficit is $\Delta Q_j(t) = Q_{DU,j} - Q_{D,j}(t)$. It should be finally remarked that the time-varying demand can be written as $Q_{DU,j}(t) = \overline{Q}_{DU,j}\Pi_j(t)$, where $\overline{Q}_{DU,j}$ is the (usually daily-) average demand, whereas $\Pi_j(t)$ is a time pattern that describes the temporal variations. In this study, time variations are limited to a weekly typical pattern with hourly time-steps.

Additional elements of a WDN are tanks, identified by the index τ. N_T tanks are installed in the WDN. Tanks are characterized by the level $H_{T,\tau}(t)$, maximum level $H_{\max,\tau}$, minimum level $H_{\min,\tau}$, elevation of bottom $z_{T,\tau}$ and by the radius $R_{T,\tau}$. Similar to pumps and valves, tanks are dynamic elements, and to fully describe a hydraulic problem it is necessary to know the tanks' initial level $H_{0,\tau}$. The last considered physical elements of WDN are reservoirs, identified by the index r. N_{Res} reservoirs are installed in the WDN, and are characterized by the head $H_{Res,r}$.

Additional (intangible) elements that deeply affect the dynamics of a WDN are controls. Controls are a set of logical rules that modify the status of valves and pumps according to a set of state variables. State variables can be: (i) the pressure in prescribed nodes; (ii) the level of tanks; (iii) the time of the day; (iv) the flow rate in some prescribed link.

For the purpose of this study, it necessary to consider the compliance of the service provided by the Water Utility with the contracts signed between the Water Utility and customers. The key parameter in this case is the minimum pressure that the Water Utility should guarantee to customers, namely P_{cmp}.

Implementation of modelling and algorithms

Figure 2 reports the diagram of the algorithm adopted to model the sequence of replacement of all links listed in **R**. The core of the algorithm is to simulate a WDN with evolving topology and hydraulic characteristics. To do that, a cycle of simulations of the WDN are done over the time t (yellow rectangles in figure 2). The flow of time is sometimes interrupted to perform the required modifications of topology or hydraulic characteristics. These interruptions are done with the help of two parameters: $t_{\max}$ and "type". The parameter $t_{\max}$ set the time when simulations on the WDN are to be stopped, and its characteristics are to be updated (orange rhombus in figure 2). The parameter "type" set the type of modification to be performed: closure of the pipe or opening of the pipe with updated diameter, roughness, leakages (green rhombus in figure 2). To better understand the algorithm, a typical cycle (see arrows in figure 2) is followed and discussed.

The first step is to load all data of the WDN prior to renovation works. With this data, an input file for a WDN modelling software can be produced. In this work, EPANet was used, and so an ".inp" file was written. An additional task of this first step is to load the calendar time of each operation for each pipe to be replaced. We recall that: (i) $t_{1,k}$ is the calendar time when the $k-$th pipe is disconnected from the network; (ii) $t_{2,k}$ is the calendar time the $k-$th pipe is connected back to the network, with updated values of roughness and diameter; (iii) the renovation works begin at $t = t_0$ and end at $t = t_0 + T_W$.

The second step is to initialize the time at the value $t = t_0$, and set $t_{\max} = t_{1,k}$ and type=close. The first setting defines the next calendar time in which a modification of the network is to be performed. The second setting defines the type of the next modification to be performed.

The third step is to simulate the network behavior until $t \leq t_{\max}$ (yellow rectangles in figure 2). For each time step Δt the values of pressure and demand at junctions are saved in the time series $P_j(t)$ and $Q_j(t)$. Other data that are saved are the level of tanks $H_{T,\tau}(t)$; the status (open/close; on/off) of other dynamical elements (e.g., valves/pumps).

When t exceeds $t_{\max}$, the cyclic modelling of the WDN is interrupted (orange rhombus in figure 2). The check $t < t_{end}$ with $t_{end} = t_0 + T_W$ is performed (gray rhombus in figure 2). If negative (i.e., renovation works haven't yet been completed) an operation on pipes is performed. To do that, the saved levels of tanks $H_{T,\tau}(t)$ and the status of all dynamical elements is loaded. Then, two paths can be followed, depending on the operations to be performed (green rhombus in figure 2).

One possible operation is the closure of the pipe (pink rectangles). This entails to: (i) set to zero the leakages induced by the $k-$th pipe at the junctions where the US and DS merge (i.e., the terms $Q_{L,k,j}$); and (ii) disconnect the pipe (in EPANet: set the pipe's status to "close"). In addition, it is set the new value $t_{\max} = t_{2,k}$ (the calendar time when the new pipe will be put in service) and type=replace (i.e., the next operation will be to put in service the new pipe).

The other possible operation is to put in service the new pipe (light blue rectangles). This entails to: (i) modify the roughness and diameter of the $k-$th pipe to the new values; (ii) connect the pipe (in EPANet: set the pipe's status to "open"); and (iii) keep the leakages induced by the $k-$th pipe at the junctions where the US and DS merge to zero (i.e., the terms $Q_{L,k,j}$). In addition, it is set the new value $t_{\max} = t_{1,k+1}$ (the calendar time when the next pipe to be replaced $(k+1)$ will be closed and disconnected); and type=close (i.e., the next operation will be to close the $(k+1)-$th pipe).

In order to implement these modifications, a new input file for a WDN modelling software (an EPANet ".inp" file in this specific case) is written. After this point, new simulations with the updated network can be performed (see the red arrow in figure 2). It should be noted that we choose the MATLAB toolkit for EPANet (see [14]) to perform this long sequence of operations.

To sum up, the typical results of one simulation of WDN renovations (see the rectangle below the gray rhombus in figure 2) are: (i) the time series of pressure in each junctions $P_j(t)$; the time series of the demand in each junction $Q_j(t)$.

These time series can be analyzed, and can be used to evaluate additional time-series useful for determining the effect of the replacement works on users, such as: (i) the demand deficit, evaluated as the difference between the flow rate wanted by users and the flowrate provided by the network, i.e., $\Delta Q_j(t) = Q_{DU,j} - Q_{D,j}(t)$; (ii) the time series of leakages in all junction $Q_{L,j}$; (iii) the time series of pressure $P_j(t)$. It should be noted that the time series span in the time interval $[t_0, t_0 + T_W]$ with the timestep Δt.

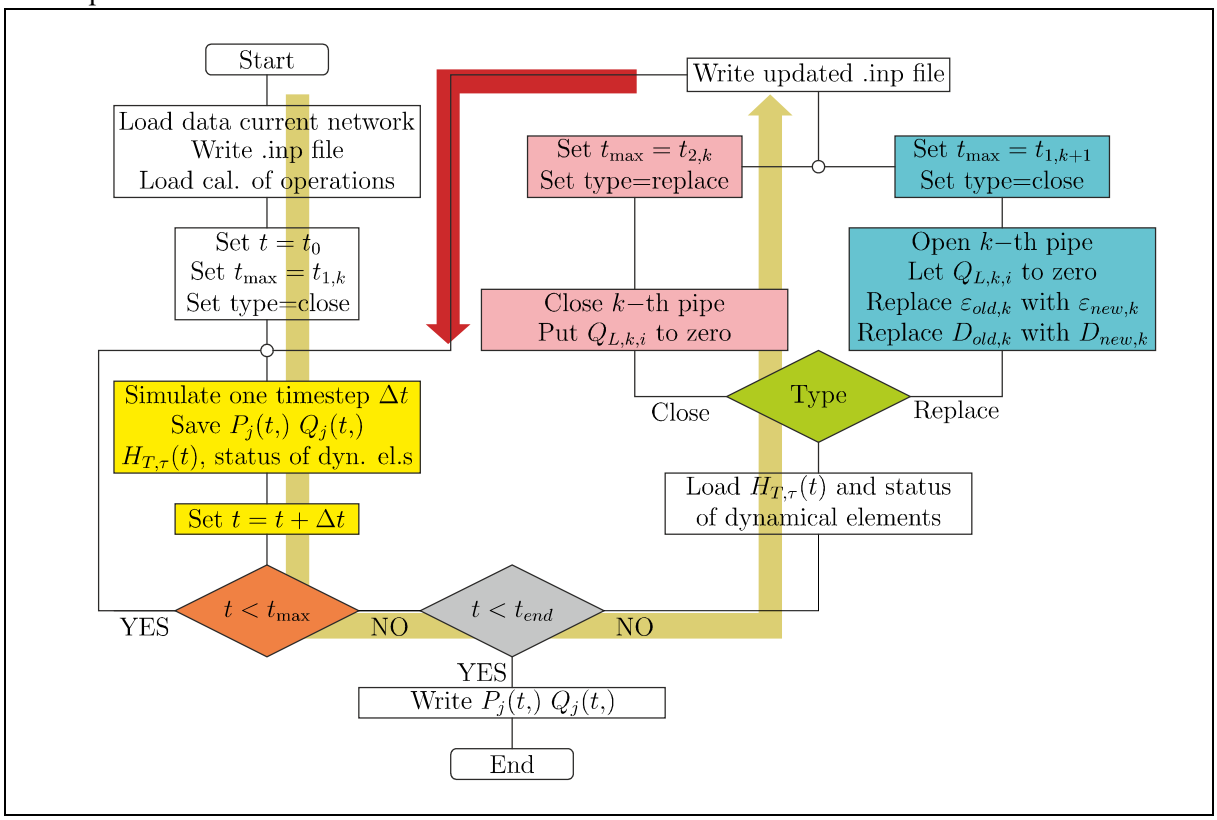

Figure 3. Algorithm adopted to model a sequence of operations for the renovation of water distribution networks. Colors are used to guide the reader in the algorithm description provided in the main text.

Algorithm for the optimization of the sequence of replacement

In order to optimize the sequence of replacement, the so called "Population Based Incremental Learning (PBIL) algorithm" is used. In order to understand how to implement this algorithm, the definition of a number of variables and parameters is first required. Vector **I** lists all links in the network. Vector **R** includes all links to be replaced. $\mathbf{R} \in \mathbf{I}$ and the number of elements of **R** is N_R. The number of phases of link replacement is N_m. In principle, N_m can be different from N_R, as more links can be replaced during one single phase. Here we focus on the case $N_m = N_R$.

With the definitions previously provided, it becomes clear that the optimization algorithm is aimed at determining the best sequence of the elements listed in the vector **R**. In this regard, "best sequence" means that the cost associated with the sequence of **R** is minimum.

The steps implemented to apply the PBIL algorithm to the specific are as follows.

- Step 1– Definition of the probability matrix. Firstly, the probability matrix $\mathbf{P}^s$with size $N_R \times N_R$ is defined. Each element of the matrix $\mathbf{P}^s(k,m)$ is the probability that the pipe k will be replaced during the phase m. The superscript s refers to the current iteration in the optimization algorithm. At the beginning of the optimization ($s = 1$) all the elements of $\mathbf{P}^s$ are equal to $1/N_R$ (same probability).
- Step 2 – Generation of a random sequence of pipe replacement (called sequence vector). For each pipe k^*, the phase m during which the pipe will be replaced is determined. This is done generating sample values (i.e., random extraction) from the probability distributions $\mathbf{P}^s(k^*,m)$. One example of sequence vector is $J = [i_5, i_3, i_2, i_1, i_4, \dots\dots, i_{N_R}]$, where i_m is the index (i) of the pipe to be replaced during phase m.
- Step 3 – Generation of the population. In this step, a population of N_{rdm} sequence vectors J_η with $\eta = 1{:}\,N_{rdm}$ is created. The sequence vectors are obtained according to Step 2. The population vector is thus $\mathbf{J} = [J_1, J_2, \dots, J_{N_{rdm}}]$.
- Step 4 – Reparation of the sequence vectors. The elements i_m of the η −th sequence vector are randomly chosen from the vector **R.** These elements are supposed to be unique in each sequence vectors $J_\eta$of the population **J**, as each pipe will be changed once in each sequence of operation. The algorithm checks if there are repetitive elements, and replace them with new randomly chosen elements. The procedure is repeated until each sequence vectors $J_\eta$of the population **J** includes all the elements of the matrix **R**.
- Step 5 – Cost evaluation for each sequence vector. The cost incurred during the replacement works is calculated for each sequence vector that make up the population. The sequence vector that provides the lowest cost among the entire population is J^*.
- Step 6 -Update of the probability matrix. Each row (index k) of the probability matrix in the algorithm at the iteration $s+1$ will be updated as follows. At first, the condition $J^*(k) = i_m$ is checked for each element of $J^*(k)$, i.e., $k = 1{:}\,N_R$. The condition identifies when –in the most convenient sequence found so far– the k −th pipe is replaced during the m −th phase. When the condition holds true, the probability of such an event is increased. This is done modifying the probability matrix as $\mathbf{P}^{s+1}(k,m) = (1 - LR) \times P^s(k,m) + LR$, where LR is the learning rate. When the condition doesn't hold true, the probability of such an event occurring is reduced. Again, this is done modifying the probability matrix as $\mathbf{P}^{s+1}(k,m) = (1 - LR) \times P^s(k,m)$.
- Step 7 -Probability matrix mutation. During optimization, it is possible for algorithms to got stuck in local optimal solution. Mutations are thus used to escape from these local minimums. In each row k of the probability matrix $\mathbf{P}^{s+1}$, one element is randomly selected. The selection is done with mutation probability f. The so called "direction of mutation" is then randomly determined as $d_f =$ (−1 or 1). Mutations are then applied on the row k as follows. For the location (k,m), $\mathbf{P}^{s+1}(k,m) = \left[\mathbf{P}^{s+1}(k,m) \times \left(1 - \left(d_f \times \Delta f\right)\right)\right] + \left(d_f \times \Delta f\right)$; for locations

other than (k, m)in row k: $\mathbf{P}^{s+1}(k, m) = \left[\mathbf{P}^{s+1}(k, m) \times \left(1 - \left(d_f \times \Delta f\right)\right)\right]$, where Δf is the value of the mutation shift. If $\mathbf{P}^{s+1}(k, m) > 1$ the condition $\mathbf{P}^{s+1}(k, m) = 1$ is set, whereas if $\mathbf{P}^{s+1}(k, m) < 0$ the condition $\mathbf{P}^{s+1}(k, m) = 0$ is set.

- Step 8 - Check the algorithm stop criteria. If the number of prescribed iterations of the algorithms s has reached the pre-set number of iterations NI to be performed, the algorithm is stopped, otherwise another iteration begins from Step 2.