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Predictive Modelling of bridge bearing displacements with Physics-Enhanced Machine Learning (PEML) environmental effects filtering

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Abstract

In bridge Structural Health Monitoring (SHM), identifying anomalies is challenging due to environmental and operational variability (EOV), such as temperature changes, traffic loads, and else. This study develops a predictive model to isolate normal structural responses, enabling the detection of damage-induced anomalies. Using displacement and temperature sensors, the model evaluates longitudinal displacements at the bridge bearings. Temperature is the primary independent variable, combined with time, to capture daily and seasonal cycles characterised by nonlinear behaviour. Regression-based Machine Learning algorithms, such as Gaussian Process Regression (GPR), are employed to predict the expected displacements. A Physics-Enhanced Machine Learning (PEML) approach, or grey-box model, integrating physical knowledge with data-driven insights is adopted, improving accuracy and interpretability. Tested on real-world data from a highway viaduct, the grey-box model demonstrates superior performance and robustness, even with limited datasets. This confirms the potential of PEML-based approaches for damage assessment with data from static monitoring, paving the way for more reliable SHM systems and enhanced bridge safety.

Keywords

Bridge bearings, displacement transducers, thermal effects, EOVs, SHM, Static Monitoring, Machine Learning, Physics-Enhanced Machine Learning, Grey-Box model

1 Introduction

Bridge bearings play a crucial role in accommodating translational and rotational movements due to factors such as thermal changes and seismic activity. However, if not properly maintained, these components can suffer from structural degradation and lead to localised malfunctioning or even catastrophic failure. While traditional visual inspections have long been the standard for assessing bearings, they often fall short in capturing time-dependent and complex behaviour due to the long time intervals between inspections. This is especially significant for early damage detection when macroscopic effects are barely visible. To overcome this limitation, Structural Health Monitoring (SHM) provides data-driven insights that complement traditional visual assessments [1]. A common goal of SHM is to establish baselines representing the "normal" behaviour of the target structure in regular (undamaged) conditions [2]. These baselines are then used to detect anomalies that may indicate damage. However, external environmental conditions—such as wind, temperature, and live loads—can significantly influence structural responses, potentially masking the effects of damage [3] [4]. This presents a challenge for effective damage detection. In long-span bridges, as in many other Civil structures and infrastructures, vibration-based SHM methods are commonly used, focusing on modal parameters. However, these methods are limited by low signal-to-noise ratios, high computational requirements, and the assumption that unknown inputs are white noise. Temperature-driven quasi-static methods have gained attention as a potential alternative, offering different perspectives on the relationships between environmental conditions and structural responses. Temperature-driven quasi-static methods have proven particularly effective in Machine Learning (ML)-based SHM, as they typically exhibit high signal-to-noise ratios [5], making them well-suited for detecting localised damage. However, the thermal behaviour of structures can be inherently nonlinear and requires advanced modelling techniques to capture its complexity.

One promising solution to this challenge is the development of Physics-Enhanced Machine Learning (PEML), which integrates domain knowledge with data-driven learning to enhance model accuracy, interpretability, and generalisability. A key example of this approach is the use of grey-box models, which combine the interpretability of physics-based white-box models with the flexibility of data-driven black-box models [6]. For instance, the black-box component can be refined by incorporating a prior mean function derived from physical principles, enabling the seamless integration of theoretical insights with measured data. Such hybrid approaches have demonstrated significant potential in predicting structural responses. Integrating hybrid modelling techniques is particularly valuable for monitoring bridge bearings, which experience both gradual movements due to thermal changes and rapid movements during traffic loads, wind, and seismic events. While physics-based models can provide insights into the underlying mechanisms of these movements, data-driven methods are better suited to handle the complex and nonlinear behaviour observed in real-world conditions, which are not accounted for in the white-box models. Combining these approaches makes it possible to develop robust baselines sensitive to damage-related changes in the system but unaffected by confounding influences.

To illustrate these concepts, a case study was conducted on a long highway viaduct (1,360 meters in length), focusing on the relationship between temperature variations and bearing longitudinal displacements. The study employed grey-box models to predict structural responses under varying environmental conditions. Models based on Gaussian Processes (GPs) [7], trained on real-world measured data, constituted the black-box component, whereas the white-box counterpart was derived from the well-established principles of thermoelasticity. The results highlight the potential of hybrid approaches to enhance the sensitivity and accuracy of SHM systems, providing a more effective strategy for long-term monitoring and in-service bridge evaluation.

2 Grey-Box Modelling

2.1 The necessity to go beyond Data-Driven Black-box approaches

Machine Learning provides powerful tools for modelling nonlinear systems using observational data. However, in many engineering applications, experimental data are noisy, scarce, or incomplete. Consequently, purely data-driven models are limited by the quality and availability of training data, which can hinder their reliability in critical applications. These models often lack three key attributes: generalisation to unseen conditions, interpretability, and robustness to uncertainties or data variations [8]. Consequently, integrating additional knowledge is often essential to enhance their reliability in engineering contexts.

When applied to monitoring systems, such as those for bridges, the challenges are compounded by time-varying environmental conditions, operational variations, and degradation phenomena. Furthermore, the increasing demand for interpretability in ML models, especially in safety-critical areas like civil engineering, where decisions direct impact safety and reliability, and errors can lead to catastrophic failures, underscores the necessity of integrating physical knowledge into these systems. Hybrid approaches such as PEML have been developed to address these challenges. PEML combines physical constraints with ML techniques, offering several advantages:

- Improved generalisation and reduced overfitting problems.
- Enhanced interpretability through physically meaningful predictions.
- Reduced data requirements compared to traditional ML approaches.
- Greater resilience to environmental variability, mitigating the influence of temperature and other environmental factors.

Studies have shown PEML's effectiveness in bridge monitoring [9], notably through residual modelling techniques, where ML models learn discrepancies between measured data and physics-based predictions (using tools like Gaussian Processes or Neural Networks) [10, 11].

2.2 Grey-Box models

The rise of continuous monitoring systems has expanded the use of Machine Learning (ML) and Artificial Intelligence in Structural Health Monitoring (SHM). Techniques like Gaussian Processes, Support Vector Machines, and Neural Networks are essential for regression and classification tasks, uncovering intricate patterns in data without the need for detailed physical models. These approaches, often called "black-box" models, are data-driven and do not rely on theoretical knowledge of the system, making them especially useful in dynamic systems, where constructing accurate physics-based models can be challenging. However, on the other hand, these models are difficult or even impossible to interpret. Furthermore, their validity is limited to the range of EOV conditions included in the training observed data.

In contrast, "white-box" models rely on well-established physical principles, such as finite element analysis or differential equations. Yet, their applicability can be constrained by the system's complexity and the challenge of incorporating unmeasured variables. This research investigates a hybrid approach—"grey-box" models—that combines the adaptability of Machine Learning with the domain knowledge of physics-based models. Rather than merely correcting biases, these models enhance the explanatory power of traditional physical frameworks by leveraging Machine Learning to account for unmodelled phenomena or structural behaviours.

Specifically, this study explores the so-called residual modelling [10], where Machine Learning techniques bridge the gap between physical predictions and observed measurements. Such methods effectively mitigate biases in the underlying physics-based model or detect previously unaccounted-for anomalies. The overarching goal is to maintain simplicity in the fundamental physical assumptions while leveraging Machine Learning to improve both flexibility and predictive accuracy.

In particular, in this research, Gaussian Process (GP) regression, a highly versatile and robust regression method, is used for this purpose. GPs are ideally suited for SHM applications due to their semi-nonparametric nature, ability to perform well with limited training data, and probabilistic formulation within a Bayesian framework [12, 13, 14]. This probabilistic aspect allows GPs to generate predictions as distributions, providing not only point estimates but also confidence intervals and the ability to propagate uncertainties into subsequent analyses [15], which is particularly valuable in fields where knowing the confidence in a prediction (thus its reliability and confidence intervals) is as important as the prediction itself. As Gaussian Processes are extensively studied, their fundamental equations are omitted here. For a comprehensive reference, see [16].

2.3 Prior mean function and residual modelling

In the context of system modelling, engineers often possess valuable knowledge of the underlying physical processes, yet this knowledge is not always effectively integrated into data-driven models. Traditionally, uninformative priors are utilised in GP regression, disregarding the prior knowledge about the systems of interest. This section explores the potential of grey-box modelling to bridge this gap to improve predictive performance.

Gaussian Process regression requires the specification of a mean and covariance function, which together form the prior process. The training data then condition this prior, yielding a posterior mean and covariance that represent the model's predictions. Typically, a zero-mean prior is used, with flexible covariance functions (e.g., exponential or Matérn) adapting to data patterns. However, this approach does not leverage any domain-specific knowledge. By contrast, the grey-box model introduces prior knowledge using a physics-based model for the GP's mean function, allowing the GP to model residuals – i.e., the discrepancies between observed data and white-box predictions.

In the grey-box GP model, the white-box mean function models the expected trend in the data, while the GP's covariance function models residuals, capturing deviations from white-box predictions. This approach focuses on modelling differences, rather than the entire dataset, by training the GP on the residuals between observed data and white-box predictions. This method offers several advantages:

- Incorporation of Prior Knowledge: The white-box mean function provides a physically interpretable baseline.
- Improved Predictive Capability: The GP models finer details and non-linearities not captured by the white-box model.
- Reduced Data Requirements: Leveraging prior knowledge improves performance with limited training data.

2.4 Problem Formulation

The grey-box hybrid model proposed is based on residual modelling, introducing two main components, namely the black-box and the white-box contributions. The total displacement is formulated as:

$$y(x) = f(x) + \delta(x) + \sigma \qquad (1)$$

Where:

- $y(x)$ is the total observed displacement.
- $f(x)$ is the white-box component based on physical knowledge, expressed by the analytical formula of thermoelasticity:

$$\delta_T = \alpha \cdot \Delta T \cdot L \qquad (2)$$

  where δ_T is the thermal bearing displacement, α is the coefficient of thermal expansion of steel, and L is the unrestrained length of the bridge deck in the considered span. This linear dependency correlates displacements with temperature variations. This formulation provides a straightforward and physically interpretable baseline model for the system.

- $\delta(x) + \sigma$ are the residual component (or discrepancy), modelled by the GP model.

In this application, the variable of interest, *x,* is the temperature variation measured by the monitoring system's sensors. In addition, the models are trained using a time input to incorporate temporal information into the training phase. Temporal information was represented by two vectors: one for the time of day (in decimal hours) and one for the day of the year (1 to 365). This allows the model to capture both daily and seasonal displacement cycles.

3 Case study of the highway viaduct

3.1 Introduction to the case study

This section presents a case study that illustrates the application of the proposed methodology. A Gaussian Process model is implemented to predict bridge deck displacements while experiencing varying environmental conditions. Monitoring the horizontal displacements of the bearings is critical as they serve as significant indicators of structural performance. For steel girder bridges, such as the one examined here, temperature variations are a primary factor influencing the temporal changes observed in deck displacement records.

The dataset used in this study originates from a continuous monitoring system installed on a long highway viaduct, 1,360 meters in length, within the Italian highway network. Constructed in the mid-1970s, the viaduct consists of two adjacent decks—one per traffic lane—spanning a total length of 1360 meters. Each deck comprises 18 spans, with lengths ranging from 52 to 92 meters. The structure features a single hollow steel box girder per carriageway, with a variable height between 2.75 and 3.55 meters and a base width of 6.00 meters. A reinforced concrete slab, pre-stressed transversely, is placed atop the box girder, with an average thickness of 30 cm.

A key focus of this study is the configuration of the bearings at the viaduct's abutments. Each abutment features four unidirectional sliding pot bearings, which decouple deck and abutment movements in the longitudinal direction, thereby reducing transmitted stresses. These bearings allow longitudinal displacements of up to ±200 mm, facilitated by the reciprocal sliding of two flat contact surfaces.

3.2 Data Availability and Methodology

The viaduct was monitored using a system of displacement sensors and thermal probes, with data acquired at a frequency of one reading per second. However, given the large volume of data collected and the focus of this study on static monitoring, the analysis was conducted using data downsampled to one reading per minute. Approximately 1,440 data points were recorded daily for each sensor, totalling around 44,500 data points per month and approximately 516,000 data points over nearly one full year of monitoring. The dataset covers the period from September 7, 2023, to September 1, 2024. The data analysed in this study originate from eight displacement transducers installed on the bearings, with one sensor per bearing and four sensors on each viaduct abutment. Additionally, temperature sensors installed on the bridge deck were considered, selecting those most representative of deck temperatures near the abutments. For the analysis of the viaduct measured data, given the extensive size of the available dataset, covering nearly a year of continuous monitoring, only a random 2% of data samples were utilised, selected after a random shuffling, to prevent the model from identifying patterns related to data order. From this subset, 70% (training ratio) of the data were allocated to training, while the remaining 30% was reserved for testing.

The models were trained using a full year of data. Selecting an appropriate training period is crucial to mitigate environmental and operational influences. When training a model for damage detection, it is essential to account for the full range of environmental and operational conditions before applying damage identification [17]. Failing to do so may lead to novel environmental or operational conditions mistakenly identified as damage. Therefore, to ensure the highest possible accuracy, this case study employed a training dataset encompassing nearly an entire year of data [18].

Sensor identifiers include both a number (1 or 2) and a letter (A, B, C, D). The number designates the abutment: abutment 1 on the West side and abutment 2 on the East. Each letter corresponds to a distinct sensor associated with a specific bearing device. As a reference, only the results from sensor 2A, which is positioned at the northernmost bearing of the west abutment, are reported. The findings for all other sensors are qualitatively similar. Therefore, the conclusions drawn apply to the entire sensor set.

The results from different configurations (white-, black-, and grey-box) are compared, including all performance metrics for each predictive model. These metrics assess the moment-by-moment discrepancy between predicted and actual values. In regression tasks, several metrics can be used to provide a comprehensive evaluation of model performance and prediction accuracy. The evaluation of predictive models relies on the following performance metrics:

- Mean Absolute Error (MAE)
- Mean Squared Error (MSE)
- Root Mean Squared Error (RMSE)
- Normalised Mean Squared Error (NMSE)
- R-squared (R^2)

When comparing regression models to simpler regression techniques such as linear interpolation, the metric used to evaluate the quality of a linear fitting is the Pearson correlation coefficient (r). All these metrics were computed between the real and forecasted signals.

Displacement time series plots (observed and predicted) and prediction error time series are presented to provide a visual understanding of model behaviour over time. Additionally, a temperature-domain representation illustrates the accuracy in capturing daily cyclic patterns in Temperature-Displacement plots, highlighting each model's capability to replicate temperature-dependent bridge displacement behaviour seen in the actual data.

3.3 Results

To discuss the intraday effects of temperature on bridge displacement observed in the dataset, a specific day (August 8, 2024) has been chosen as a reference. Throughout the day, temperature variations cause the bridge deck to expand and contract. As temperatures rise in the morning, the bridge deck expands, leading to longitudinal displacements at bearings. Conversely, the bridge deck contracts at night and during cooler periods, resulting in movement in the opposite direction. In a scenario where the behaviour is linear, following the laws of thermoelasticity, the expected displacement would follow Eq. (2).

However, a significant non-linearity emerges when examining the Temperature-Displacement plot (Figure 1), showcasing cyclic daily patterns. While temperature-induced displacement in bridges is a well-documented issue, no definitive interpretation of this phenomenon is available, as the underlying mechanism responsible for the hysteresis phenomenon observed in diurnal cycles remains a subject of debate [19].

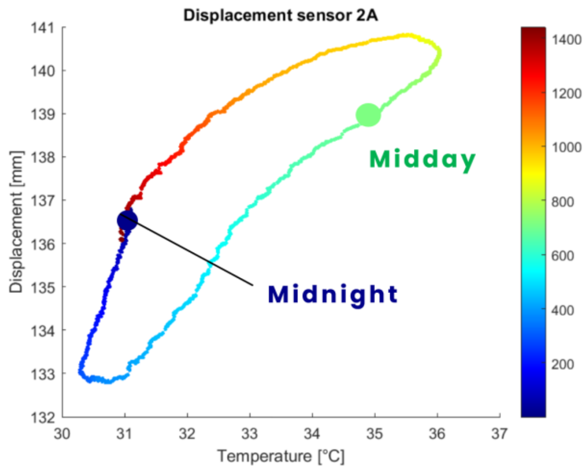

Figure 1 Daily cyclic behaviour of the temperature-displacement correlation (sensor 2A). The colour bar is time in minutes, starting from midnight. August 8, 2024 is shown as an example for illustrative purposes.

Some possible explanations for this non-linear correlation can be proposed by focusing on complex factors related to the bridge's structural and material behaviour. The observed cyclical pattern during the heating and cooling phases can be attributed to a variety of reasons. Very likely, the primary cause of this non-linearity can be caused by bearing friction: even though bearings are designed to allow for controlled movement due to thermal expansion and contraction, these devices exhibit some degree of friction, which can resist movement up to a certain threshold. During heating or cooling phases, this friction introduces a lag in displacement until the thermal force overcomes it. This phenomenon is commonly referred to in the literature as "stick-slip" behaviour [20]. As shown in Figure 1, when the structure begins to heat up in the morning, the bearing remains stationary until the temperature reaches a point where the horizontal force overcomes the friction. Once this threshold is surpassed, it begins to move. This movement continues as the temperature rises. Conversely, during the cooling phase, the process works similarly: as the temperature decreases, the bearing will only move when the static friction is overcome in the opposite direction. This behaviour results in the observed cyclic hysteresis effect, where the displacement is delayed and exhibits a non-linear response. The increasing wear condition of sliding bearings could cause friction in these components [21]. Overall, friction at the bearings is a crucial aspect, especially if the bearing joints are ageing or not functioning optimally. This could lead to noticeable hysteresis in displacement during daily thermal cycles. This relationship's non-linear and cyclic nature underscores the importance of considering time effects when attempting to build a predictive model.

To investigate the correlation between temperature and longitudinal displacement, the most common approach would be to just apply the simple and well-known white-box model of Eq. (2), represented in Figure 2 by the red dashed line. This model is based on a linear interpolation applied to the entire available dataset. The best fit was determined to maximise the correlation coefficient. The result shows a strong linear correlation between temperature and displacement, as the Pearson correlation coefficient for sensor 2A is r = 0.954.

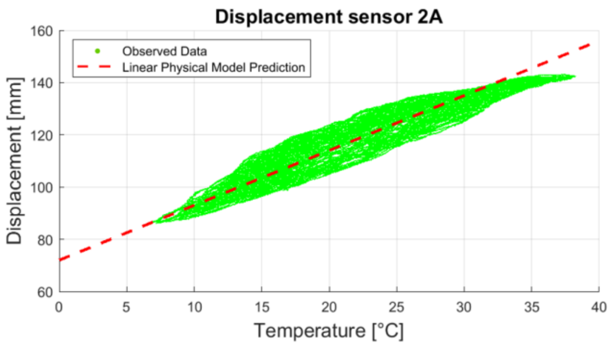

Figure 2 Linear correlation between temperature and abutment bearing displacements (sensor 2A)

The predicted displacement values are then compared with the observed displacement using the white-box model. The prediction results and errors in the time domain representation are shown in Figures 3 and 4. As can be inferred from this last one, not only is the error quite noticeable, but it is also not constant, fluctuating with temperature variations over time. This points out the necessity of correcting the white-box formulation with an additional, non-constant term.

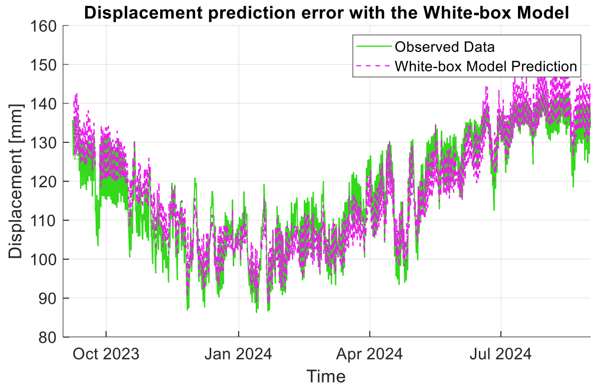

Figure 3 Observed and predicted displacements using the white-box model (sensor 2A)

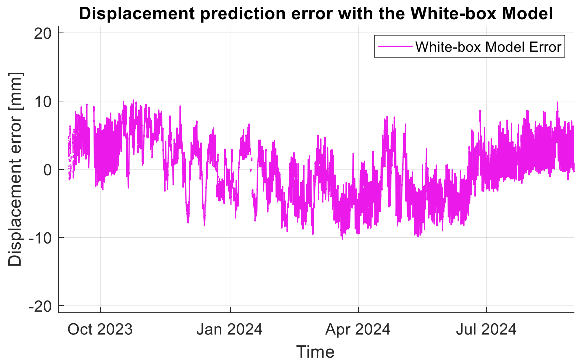

Figure 4 Displacement prediction errors using the white-box model (sensor 2A)

Table 1 reports the performance metrics of the white-box model. The estimation errors of the model are relevant, demonstrating how, as mentioned before, a linear temperature-displacement correlation fails to fully capture the structural response at the bearings.

Table 1 Performance metrics of the white-box model predictions

White-box Model		
Performance Metrics	*sensor 1A*	*sensor 2A*
MAE	2.699	3.582
standard deviation (σ)	3.368	4.283
MSE	11.346	18.365
RMSE	3.369	4.285
NMSE (%)	2.911	9.350
R^2	0.971	0.907

To account for the non-linear behaviour of the bearing displacements, data-driven components were introduced, using both black-box and grey-box approaches. Regarding Gaussian Process models, the chosen kernel function is the squared exponential. The model hyperparameters were not manually set but were determined autonomously to optimise the Negative Log Marginal Likelihood (NLML). The results of the predictions are shown in Figure 5, 6, 7, and 8.

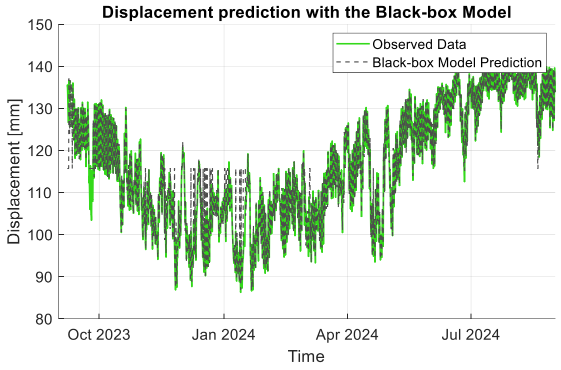

Figure 5 Prediction of the black-box model (sensor 2A)

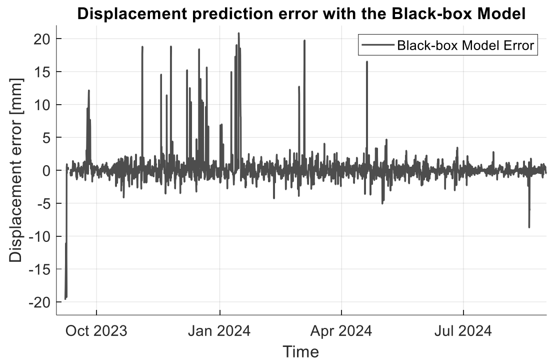

Figure 6 Prediction error of the black-box model (sensor 2A)

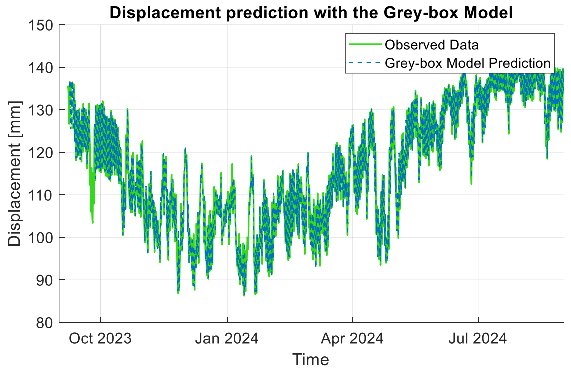

Figure 7 Prediction of the grey-box model (sensor 2A)

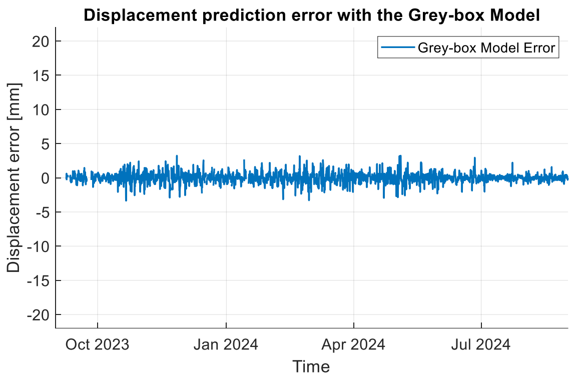

Figure 8 Prediction error of the grey-box model (sensor 2A)

The results appear to be very accurate, especially when considering the performance of the grey-box model. The NMSE value is very low, indicating an almost perfect prediction of the bearings' response. This model provides a near-perfect fit to the observed bearing behaviour, as evidenced by very low error metrics (Table 2). The grey-box model better captures daily and seasonal cycles, enabling to replicate more closely the actual displacement patterns, as shown in Figure 9 and Figure 10.

Table 2 Performance metrics comparison of the GP models (sensor 2A)

Metrics Comparison (sensor 2A)			
Performance Metrics	*White-box*	*Black-box*	*Grey-box*
MAE	3.582	0.902	0.505
standard deviation (σ)	4.283	2.220	0.706
MSE	18.365	4.966	0.499
RMSE	4.285	2.228	0.706
NMSE (%)	9.350	2.556	0.252
R^2	0.9065	0.9745	0.9975

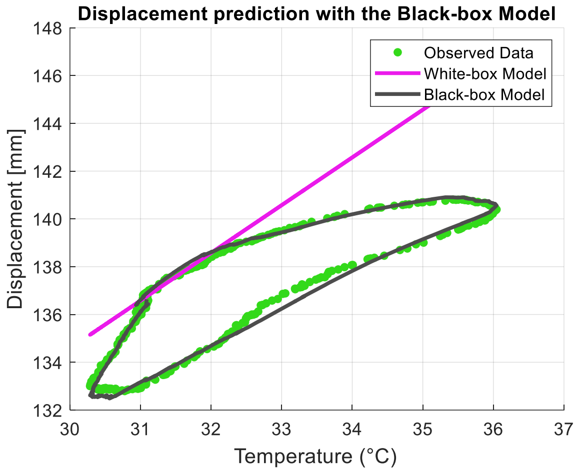

Figure 9 Prediction of a daily cycle with the black-box model (sensor 2A - August 8, 2024)

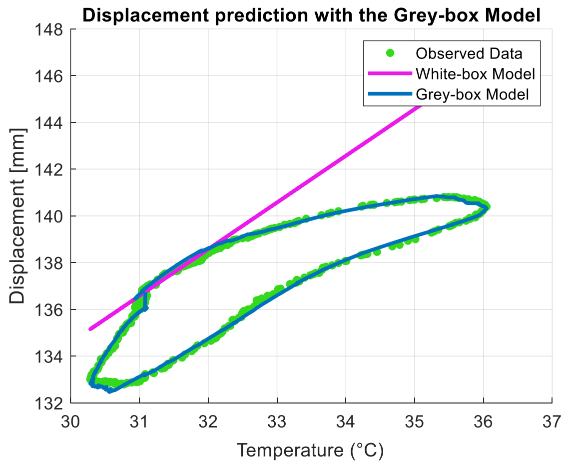

Figure 10 Prediction of a daily cycle with the grey-box model (sensor 2A - August 8, 2024)

For completeness, results from sensors on the other abutment (sensor 1A) have also been included in Table 3. These findings exhibit a similar trend to those obtained for abutment 2, reinforcing the consistency of the observed behaviours across both abutments. This parallel in results underscores the reliability of the modelling approach, further validating the generalisability of the proposed predictive framework.

Table 3 Performance metrics comparison of the models (sensor 1A)

Metrics Comparison (sensor 1A)			
Performance Metrics	*White-box*	*Black-box*	*Grey-box*
MAE	2.699	0.957	0.532
standard deviation (σ)	3.368	2.232	0.714
MSE	11.346	4.979	0.510
RMSE	3.369	2.231	0.714
NMSE (%)	2.911	1.280	0.131
R^2	0.9709	0.9872	0.9987

3.4 Discussion of the results

This section summarises the key points drawn from the analysis's main findings. The behaviour of the bearings on the viaduct is characterised by strong non-linearity. Observing how displacements vary in the temperature domain reveals cyclic patterns, indicating that a simple linear physical law is insufficient to explain the relationship between temperature variations and the longitudinal displacements observed at the bearings. To better capture this complex behaviour, temperature measurements are combined with multiple levels of temporal information. Since cyclic phenomena depend on the specific time at which the system is observed, temporal data are essential for achieving a complete understanding of structural behaviour. The inclusion of time as an additional input allowed the model to attain excellent predictive performance, particularly in addressing the non-linear correlation between temperature variations and bearing displacements.

The implementation of the hybrid grey-box model, integrating physical laws describing structural behaviour (white-box) with data-driven models (black-box), produced highly satisfactory results. The findings were consistently observed across all examined bearings, demonstrating the developed model's ability to adapt well to different conditions. The accuracy in predicting displacement values was significantly better than that of the white-box model and even outperformed the black-box model, demonstrating that the residual-based modelling strategy within the grey-box framework further enhanced accuracy relative to black-box approaches. The ability to provide explanations grounded in physical laws makes grey-box models more suitable for decision-making applications, particularly in high-risk scenarios, where relying solely on an unexplainable black-box model may be considered insufficiently reliable.

4 Conclusions

This study explored the development of a predictive framework for estimating bridge displacement at bearings, using a grey-box modelling strategy within the Physics-Enhanced Machine Learning (PEML) paradigm. The aim was to overcome the limitations of purely physics-based (white-box) models and conventional black-box Machine Learning approaches by integrating both methodologies. By combining domain knowledge with data-driven refinement, the grey-box model effectively captured the bridge bearings' complex, time-dependent behaviour under varying environmental and operational conditions, offering a balance between interpretability and predictive accuracy. These results demonstrate that grey-box modelling provides a robust framework for structural assessment, effectively addressing the challenges posed by environmental and operational variations.

However, further investigation is needed, particularly in testing the model’s ability to generalise to extreme or unseen scenarios, such as unusual temperature fluctuations. Enhancing the model's responsiveness to non-linear variations and incorporating more extensive, long-term environmental data will be objectives for future work. Also, further enhancement of the physics-based component could improve the model’s generalisation, applicability, and transferability of the proposed method across diverse structural configurations and conditions.

In conclusion, this study establishes the groundwork for the predictive analysis of bridge-bearing displacements using grey-box modelling, highlighting the potential of PEML in static SHM. The findings emphasise the value of hybrid modelling frameworks in bridging the gap between physics-based knowledge and data-driven adaptability, paving the way for more advanced and resilient SHM methodologies.

5 References

[1] Yarnold, M. T.; Dubbs, D. C. (2015) Bearing Assessment with Periodic Temperature-Based Measurements. Transportation Research Record Journal of the Transportation Research Board 2481, pp. 115–123.

[2] Cross, E. J.; Worden, K.; Koo, K. Y.; Brownjohn, J. M. (2012) Filtering Environmental Load Effects to Enhance Novelty Detection on Cable-Supported Bridge Performance. In Bridge Maintenance, Safety, Management, Resilience and Sustainability, pp. 745–752.

[3] Worden, K.; Cross, E. (2018) On Switching Response Surface Models with Applications to the Structural Health Monitoring of Bridges. Mechanical Systems and Signal Processing 98, pp. 139–156.

[4] Figueiredo, E.; Cross, E. (2013) Linear Approaches to Modelling Nonlinearities in Long-Term Monitoring of Bridges. Journal of Civil Structural Health Monitoring 3, no. 3, pp. 187–194.

[5] Yarnold, M. T.; Moon, F. L. (2015) Temperature-Based Structural Health Monitoring Baseline for Long-Span Bridges. Engineering Structures 86, pp. 157–167.

[6] Cross, E. J.; Rogers, T. J.; Pitchforth, D. J.; Gibson, S. J.; Zhang, S.; Jones, M. R. (2023) A Spectrum of Physics-Informed Gaussian Processes for Regression in Engineering. Data-Centric Engineering 5, no. 11.

[7] Williams, C.; Rasmussen, C. (1995) Gaussian Process for Regression. Advances in Neural Information Processing Systems 8.

[8] Cicirello, A. (2024) Physics-Enhanced Machine Learning: A Position Paper for Dynamical Systems Investigations. arXiv preprint arXiv:2405.05987.

[9] Haywood-Alexander, M.; Liu, W.; Bacsa, K.; Lai, Z.; Chatzi, E. (2024) Discussing the Spectrum of Physics-Enhanced Machine Learning: a Survey on Structural Mechanics Applications. Data-Centric Engineering 5, p. e31

[10] Cross, E. J.; Gibson, S. J.; Jones, M. R.; Pitchforth, D. J.; Zhang, S.; Rogers, T. J. (2022) Physics-Informed Machine Learning for Structural Health Monitoring. Structural Integrity 21, pp. 347–367.

[11] Behkamal, B.; Entezami, A.; De Michele, C.; Arslan, A. (2023) Elimination of Thermal Effects from Limited Structural Displacements Based on Remote Sensing by Machine. Remote Sensing 15, no. 12, p. 3095.

[12] Sacks, J.; Welch, W.; Mitchell, T.; Wynn, H. (1989) Design and Analysis of Computer Experiments. In Statistical Science, pp. 409–423.

[13] Holmes, G.; Sartor, P.; Reed, S.; Southern, P.; Worden, K.; Cross, E. (2016) Prediction of Landing Gear Loads Using Machine Learning Techniques. Structural Health Monitoring 15, no. 5, pp. 568–582.

[14] Moravej, H.; Chan, T.; Nguyen, K.; Jesus, A. (2019) Vibration-Based Bayesian Model Updating of Civil Engineering Structures Applying Gaussian Process Metamodel. Advances in Structural Engineering 22, no. 16, pp. 3487–3502.

[15] Gibson, S.; Rogers, T.; Cross, E. (2023) Distributions of Fatigue Damage from Data-Driven Strain Prediction Using Gaussian Process Regression. Advances in Structural Engineering 22, no. 5, pp. 3065–3076.

[16] Rasmussen, C.; Williams, C. (2006) Gaussian Processes for Machine Learning. MIT Press, Cambridge, MA, USA.

[17] Kullaa, J. (2009) Eliminating Environmental or Operational Influences in Structural Health Monitoring Using the Missing Data Analysis. Journal of Intelligent Material Systems and Structures 20, no. 11, pp. 1381–1390.

[18] Sousa Tomé, E.; Pimentel, M.; Figueiras, J. (2020) Damage Detection Under Environmental and Operational Effects Using Cointegration Analysis – Application to Experimental Data from a Cable-Stayed Bridge. Mechanical Systems and Signal Processing 135, p. 106386.

[19] Yang, X.; Zhou, Y.; Li, X.; Zhang, J. (2024) Effect of Temperature Changes on Bearing Motion of Long-Span Steel Truss Continuous Girder Bridge. Construction and Building Materials 430, p. 136440.

[20] Alexander, J.; Yarnold, M. (2020) Quasi-Static Bearing Evaluation and Monitoring – A Case Study. Frontiers in Built Environment 6, no. 69.

[21] Warren, M. E.; Dubbs, N. C. (2017) Periodic Temperature-Based Bearing Assessment of a Steel Multigirder Bridge. Transportation Research Record 2642, no. 1, pp. 26–34.