Low Computational Demand Nonlinear Correction of the Inverse Problem in Microwave Brain Imaging

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***Abstract*—This paper investigates a nonlinear correction factor for the direct inverse problem solution in medical microwave imaging (MMI), focusing on acute brain stroke monitoring. The correction factor relies on a pseudo-Rytov approximation, which employs the ratio between total and incident electric fields in the scattering model to enhance quantitative accuracy. This approach enables the direct correction of the approximate linearized imaging kernel without requiring iterative computations of the direct scattering model, significantly reducing the inversion computational effort and improving the system's robustness to numerical inaccuracies. MMI represents a promising modality for fast, potentially real-time response, delivering quantitative insights that complement gold-standard imaging techniques. This study presents a realistic numerical experiment for hemorrhagic stroke detection, demonstrating the proposed correction's impact on the accuracy of dielectric contrast reconstruction within a 3-D imaging framework and underscoring its potential benefits for clinical applications.**

I. Introduction

Medical microwave imaging (MMI) has attracted significant interest due to its potential to non-invasively examine the electrical properties of human tissues using a low-cost and safe technology. While established imaging techniques like magnetic resonance imaging (MRI) and computed tomography (CT) offer high-resolution results, they come with drawbacks such as high costs, long examination times, and, in the case of CT, exposure to ionizing radiation, limiting frequent usage. MMI has been explored in various applications requiring fast or real-time image reconstruction, such as temperature monitoring during hyperthermia cancer treatment and brain stroke monitoring to assess treatment effects [1]. Furthermore, in most implementations quantitative information is crucial for distinguishing the condition and nature of tissues, as these factors directly influence their dielectric characteristics.

Image reconstruction in MMI involves solving an ill-posed, nonlinear scattering problem. Conventional direct inversion algorithms, based on simplified scattering models like the Born approximation (BA) or Rytov approximation (RA), offer computational efficiency and qualitative results but lack quantitative precision [2]. To achieve accurate mapping of the actual permittivity values, some approaches incorporate additional physical information by characterizing the system's response to a small scatterer with known properties, effectively defining its point spread function (PSF) [3], [4]. Nonlinear iterative methods aim for greater accuracy by updating both the permittivity distribution and the electromagnetic field. However, these methods are computationally intensive and prone to convergence issues, especially in complex biomedical scenarios with significant tissue variability depending on the patient [5], [6]. Additionally, some works have explored effective rewriting of the scattering equation and alternative approximation of the internal field. They exhibit a lower "degree of non-linearity" and faster convergence. Some examples are the extended Born approximation [7], [8], the contrast source extended Born model [9], or the diagonalized contrast source approach [10]. Notably, direct methods have the potential to operate in real-time, whereas the others require additional time and computational resources, an essential consideration for monitoring applications.

This study regards an original approach to recovering the inherent nonlinearity in electromagnetic imaging neglected in direct inversion (e.g. based on BA), without the computational cost of iterative updates. We propose a direct correction factor to refine the reconstructed image. Inspired by the exponential relationships found in Rytov's approximation, we propose a correction factor enabling a one-step enhancement of the initial reconstruction. The method was first conceptualized and tested in 2-D simplified scenario in [11]. This paper extends and numerically validates the framework for a realistic scenario of brain hemorrhage monitoring with the 3-D imaging system in [12].

The paper is structured as follows: Sect. II introduces the mathematical model and the proposed correction factor for refining dielectric contrast reconstruction. Section III presents the numerical results, including the case study, method implementation, and imaging results. Finally, Sect. IV concludes the paper by summarizing the potential and effectiveness of the proposed approach for correcting contrast maps in brain stroke detection in a realistic 3-D electromagnetic scenario.

II. Mathematical model

As outlined in [11], the framework involves a direct inversion procedure to generate the "initial guess" for the dielectric contrast reconstruction, followed by the application of an approximated forward model. This model defines a correction factor that refines the reconstruction, compensating for unaccounted nonlinearities in a single step.

Given the measured scattering parameters, S_{nm}, the scattering problem in the frequency domain is described by:

$$S_{nm} = \frac{j\,\omega\,\varepsilon_0}{2\,a_n\,a_m} \iiint_{V_S} \Delta\varepsilon(\mathbf{r}')\,\mathbf{E}_n^{\mathrm{inc}}(\mathbf{r}',\omega)\cdot\mathbf{E}_m^{\mathrm{tot}}(\mathbf{r}',\omega)\,d^3\mathbf{r}', \tag{1}$$

where $n, m = 1, ..., N$ are the indices of the receiving and transmitting antennas, respectively, j is the imaginary unit, ε_0 is free space permittivity, $\omega = 2\pi f$ is the angular frequency, and $a_n a_m$ is the incoming root-power waves product. Moreover, $\Delta\varepsilon$ is the relative permittivity contrast with respect to a known background, $\mathbf{E}_n^{\rm inc}$ is the incident field impinging on the receiver, $\mathbf{E}_m^{\rm tot}$ is the total field due to the transmitting antenna, evaluated in the domain of interest V_S.

The correction factor $\psi_m(\mathbf{r}', \omega)$ is introduced to express the total field depending on the incident field, as follows:

$$\mathbf{E}_m^{\rm tot}(\mathbf{r}', \omega) \approx \mathbf{E}_m^{\rm inc}(\mathbf{r}', \omega)\psi_m(\mathbf{r}', \omega). \tag{2}$$

Then, we rewrite (1) substituting the new total field expression in (2), and transferring the correction factor $\psi_m(\mathbf{r}', \omega)$ to the contrast distribution. We obtain that the direct solution, $\Delta\varepsilon^{(0)}$ of a zero-order BA model (available under the approximation $E^{\rm tot} \simeq E^{\rm inc}$), is actually:

$$\Delta\varepsilon^{(0)}(\mathbf{r}') = \Delta\varepsilon(\mathbf{r}')\,\psi_m(\mathbf{r}', \omega). \tag{3}$$

Provided that we know $\psi_m(\mathbf{r}', \omega)$, the actual contrast can be simply obtained inverting (3). It is noted that, while the correction factor varies with frequency and depends on m, $\Delta\varepsilon$ itself does not. In previous analyses in 2-D models, averaging ψ_m among all transmitters has been selected an effective optimal solution for (3).

As discussed in [11], a simulation-free approach determining $\psi_0(\mathbf{r}', \omega)$ can be adopted based on simplifying assumption on the E field, only available in canonical scenarios. The experiment presented in this work, instead, considers a realistic scenario where the complexity of the reference domain of imaging (including heterogeneous tissues in the head) prevents the success of this strategy. However, a priori information on tissue distribution can be accounted via high-fidelity full-wave electromagnetic simulation, providing an accurate numerical estimate for the total electric field.

III. Numerical evaluation

A. Case Study

The experiment employs a realistic MMI system for brain scanning, consisting of a 22-transceiving probe system of monopole antennas in contact with the skin, as shown in Fig.1(a). The procedure was implemented using single-frequency data at 1 GHz, following the standard prototype approach [12]. A custom Finite Element Method (FEM)-based simulator is employed to compute the full scattering matrix and field distribution [13]. The modeled scenario is based on real human segmentations, capturing diverse tissue architectures. For simplicity, the structure is reduced to five layers: skin, fat, skull, cerebrospinal fluid (CSF), and brain, with the brain tissue comprising gray matter, white matter, and ventricles, represented by weighted average dielectric properties. In this numerical experiment, the head model is used to compute the image operator, assuming access to anthropomorphic data from prior medical scans.

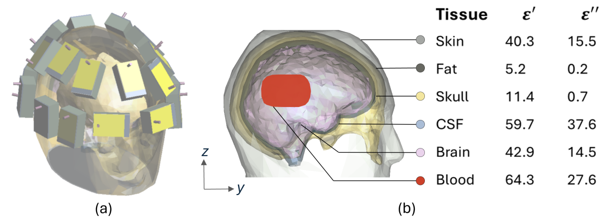

Fig. 1. Models of (a) the antenna system and (b) multitissue head. The listed real and imaginary parts of the permittivity are assigned at 1 GHz.

The imaging target is a 60 ml intracerebral hemorrhage, a volume threshold associated with high mortality according to the literature [14]. It is modeled as a capsule within the brain, exhibiting blood electrical properties, with an actual permittivity contrast with respect to average brain tissue properties equal to $\Delta\varepsilon = 21.4 - \mathrm{i}\,13.1$ at 1 GHz. From the perspective of real-time microwave imaging, this example represents a challenging problem, as larger size and higher contrast values move further away from the conditions where the BA is applicable [2]. This means that the solution to the forward problem will be more significantly affected by the unrepresented nonlinearities, resulting in quantitative inaccuracies and image artifacts, particularly in areas where the differences between the total and incident fields are significant.

B. Method implementation

Here, the direct solution, $\Delta\epsilon_0$, is obtained via zero-order BA coupled with the Truncated Singular Value Decomposition (TSVD) inversion algorithm, which relies on a regularizing threshold applied to the image operator singular values [15]. This scheme has been thoroughly validated for brain stroke monitoring and implemented in the actual prototype system [12].

As discussed in Sect. II, for complex background scenarios, a feasible way to pursue optimal ψ solution is through a single forward model iteration providing the total field numerical estimate, $\mathbf{E}^{\rm tot}$ in the selected area of imaging. Then, one can compute:

$$\psi = \frac{\mathbf{E}^{\rm tot}}{|\mathbf{E}^{\rm inc}|} \cdot \hat{\mathbf{p}}^*_{\rm inc}, \tag{4}$$

having define the polarization vector of the incident field $\hat{\mathbf{p}}^*_{\rm inc}$.

The main steps required for achieving the correcting factor estimate are summarized in Fig. 2. In the proposed scheme, the stroke region is selected to create a dielectric contrast mask, focusing the reconstruction process on relevant regions and minimizing the impact of artifacts.

C. Imaging Results

To assess the capability of recovering quantitative information, we can observe the absolute value of the dielectric contrast, which is equal to $|\Delta\varepsilon| = 25.1$ in the stroke region of the actual model. The images in Fig. 3 present sagittal and

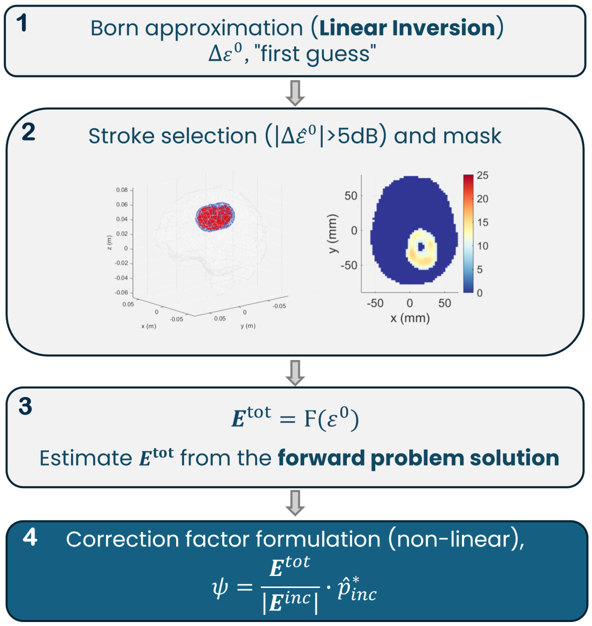

Fig. 2. Implemented steps in 3-D brain imaging scenario.

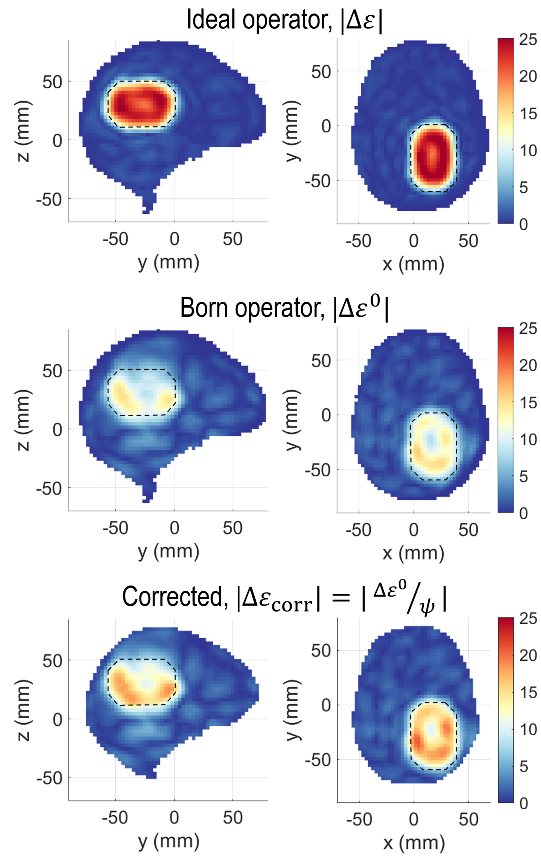

Fig. 3. Absolute value of the contrast obtained with the ideal operator through the TSVD procedure (upper row), the initial estimate generated by the TSVD algorithm using the Born approximation (middle row), and the reconstruction achieved after applying the ψ_0 correction (bottom row).

transverse 2-D slices of the reconstructed maps, centered on the stroke region. The upper row reports the ideal inversion solution (using the TSVD), which results from utilizing the exact integral kernel $\mathbf{E}_n^{\text{inc}} \cdot \mathbf{E}_m^{\text{tot}}$ and considering all relevant singular values, specifically those with normalized values above -90 dB relative to the maximum. This reconstruction represents the best achievable result with this algorithm, serving as a point of comparison. The reconstruction obtained from the traditional BA approximation, $\Delta\varepsilon_0$, in the middle row, serves as the initial guess of the implemented method. However, it lacks quantitative accuracy, particularly in the target region. Finally, the bottom row of Fig.3 shows the corrected estimate $\Delta\varepsilon_{\text{corr}}$, derived after a single forward problem implementation clearly refining the initial result.

IV. Conclusion and Perspectives

This work has investigated an innovative approach to direct electromagnetic inverse scattering, addressing the challenge of recovering quantitative information lost during direct inversion through the introduction of a correction variable that accounts for nonlinearities. For the first time this study has implemented and validated the strategy in a complex 3-D electromagnetic scenario, emulating a realistic medical application.

The numerical results have demonstrated the potential of the proposed approach to recover significant quantitative information, even in scenarios where the limitations of the Born approximation are strongly violated. This validation in a realistic context underscores the practical feasibility of the method for advanced imaging tasks. For example, in brain stroke monitoring during clinical treatment, the accuracy of the dielectric contrast value is crucial for distinguishing between different conditions, such as hemorrhagic and ischemic stroke, as each requires distinct medical management.

In the future, validation with experimental data will also be pursued to confirm the applicability and robustness of the technique in real-world configurations.

Acknowledgement

This research was supported in part by the project PON Research and Innovation "Microwave Imaging and Detection powered by Artificial Intelligence for Medical and Industrial Applications" (DM 1062/21), funded by the Italian Ministry of University and Research (MUR), and it was carried out partially within the Agritech National Research Center, funded by the European Union Next-Generation EU (Piano Nazionale di Ripresa e Resilienza (PNRR) – MISSIONE 4 COMPONENTE 2, IN- VESTIMENTO 1.4 – D.D. 1032 17/06/2022, CN00000022). It was also supported by the research projects "MedWaveImage", funded by Interreg Central Europe (CE0200670), and "3BAtwin - Bone, Brain, Breast and Axillary Medical Microwave Imaging Twinning", funded by Horizon Europe Framework Programme (GA 101159623).

REFERENCES

[1] C. Origlia, D. O. Rodriguez-Duarte, J. A. Tobon Vasquez, J.-C. Bolomey, and F. Vipiana, “Review of microwave near-field sensing and imaging devices in medical applications,” *Sensors*, vol. 24, no. 14, 2024.

[2] N. K. Nikolova, *Introduction to Microwave Imaging*. EuMA High Frequency Technologies Series, Cambridge University Press, 2017.

[3] D. Tajik, R. Kazemivala, and N. K. Nikolova, “Real-time imaging with simultaneous use of Born and Rytov approximations in quantitative microwave holography,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 70, no. 3, pp. 1896–1909, 2022.

[4] D. S. Shumakov and N. K. Nikolova, “Fast quantitative microwave imaging with scattered-power maps,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 66, no. 1, pp. 439–449, 2018.

[5] A. Zakaria, C. Gilmore, and J. LoVetri, “Finite-element contrast source inversion method for microwave imaging,” *Inverse Problems*, vol. 26, p. 115010, sep 2010.

[6] V. Mariano, J. A. Tobon Vasquez, and F. Vipiana, “A novel discretization procedure in the CSI-FEM algorithm for brain stroke microwave imaging,” *Sensors*, vol. 23, no. 1, 2023.

[7] T. M. Habashy, R. W. Groom, and B. R. Spies, “Beyond the Born and Rytov approximations: A nonlinear approach to electromagnetic scattering,” *Journal of Geophysical Research: Solid Earth*, vol. 98, no. B2, pp. 1759–1775, 1993.

[8] C. Torres-Verdin and T. Habashy, “A two-step linear inversion of two-dimensional electrical conductivity,” *IEEE Transactions on Antennas and Propagation*, vol. 43, no. 4, pp. 405–415, 1995.

[9] T. Isernia, L. Crocco, and M. D’Urso, “New tools and series for forward and inverse scattering problems in lossy media,” *IEEE Geoscience and Remote Sensing Letters*, vol. 1, no. 4, pp. 327–331, 2004.

[10] A. Abubakar, T. M. Habashy, P. M. van den Berg, and D. Gisolf, “The diagonalized contrast source approach: an inversion method beyond the Born approximation,” *Inverse Problems*, vol. 21, p. 685, feb 2005.

[11] C. Origlia, D. Rodriguez-Duarte, J. T. Vasquez, N. Nikolova, and F. Vipiana, “Nonlinear correction of the direct inverse problem solution in real-time imaging,” in *2024 18th European Conference on Antennas and Propagation (EuCAP)*, pp. 1–4, 2024.

[12] D. O. Rodriguez-Duarte, C. Origlia, J. A. T. Vasquez, R. Scapaticci, L. Crocco, and F. Vipiana, “Experimental assessment of real-time brain stroke monitoring via a microwave imaging scanner,” *IEEE Open Journal of Antennas and Propagation*, vol. 3, pp. 824–835, 2022.

[13] D. O. Rodriguez-Duarte, J. A. T. Vasquez, R. Scapaticci, L. Crocco, and F. Vipiana, “Assessing a microwave imaging system for brain stroke monitoring via high fidelity numerical modelling,” *IEEE Journal of Electromagnetics, RF and Microwaves in Medicine and Biology*, vol. 5, no. 3, pp. 238–245, 2021.

[14] K.-C. Teo, S.-M. Fong, W. C. Leung, I. Y. Leung, Y.-K. Wong, O. M. Choi, K.-K. Yam, R. C. Lo, R. T. Cheung, S.-L. Ho, A. C. Tsang, G. K. Leung, K.-H. Chan, and K.-K. Lau, “Location-specific hematoma volume cutoff and clinical outcomes in intracerebral hemorrhage,” *Stroke*, vol. 54, no. 6, pp. 1548–1557, 2023.

[15] M. Bertero and P. Boccacci, *Introduction to inverse problems in imaging*. CRC press, 2020.