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Robust Adaptive Model Predictive Control for Tracking in Interconnected Systems via Distributed Optimization

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ABSTRACT

This study presents a novel Distributed Robust Adaptive Model Predictive Control (DRAMPC) for tracking in multi-agent systems. The framework is designed to work with dynamically coupled subsystems and limited communication, which is restricted to local neighborhoods. The proposed approach explicitly accounts for parametric uncertainties and additive disturbances by employing a tube-based formulation to bound the system response for any possible uncertainty realizations. To ensure recursive feasibility and asymptotic stability, contractivity properties for the terminal cross-section are derived alongside a structured stabilizing gain for the closed-loop dynamics. The conservativeness of the tube-based formulation is relaxed by exploiting a distributed set membership via recursive identification of the parameter uncertainty set. The control problem is formulated by leveraging the Artificial Reference method for piecewise reference signals to ensure feasibility even when the desired reference is not directly reachable. The consensus ADMM algorithm is employed to solve the distributed optimization problem efficiently while maintaining scalability as the number of agents increases. Furthermore, the artificial reference formulation is extended to trajectory tracking, allowing the controller to track time-varying references while preserving feasibility. The effectiveness of the proposed method is demonstrated through illustrative examples, highlighting its capability to achieve accurate and robust tracking in multi-agent uncertain systems.

1 | Introduction

Model Predictive Control (MPC) [1, 2] is a powerful optimization-based control strategy, widely employed in applications requiring constraint handling and optimal performance. However, in complex multi-agent systems (MAS), such as robotic systems, autonomous vehicles, or chemical plants, tracking desired setpoints while handling uncertainties and interactions among the network remains a significant challenge. In recent years, a growing interest in distributed problems for MAS has emerged [3, 4], particularly concerning networked systems where central coordination is impractical. Within the field of control, Distributed (MPC) [5, 6] has been identified as an effective approach for addressing computational burdens and limitations or privacy requirements in communication, especially when agents are coupled through constraints [7], cost functions [8], or dynamic relations [9]. Different solutions have been proposed for tackling dynamic interactions, focusing on cooperative [10] and non-cooperative systems [11]. Achieving robustness against disturbances and model uncertainties is a significant challenge in the advancement of MPC [12]. Therefore, early research on DMPC focused on meeting robustness require-

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ments against bounded disturbances in MAS [13–15], and some of them adopted a robust formulation to address non-cooperative behaviors in a network of interconnected agents [11]. However, only a few works studied how to cope with model uncertainties for DMPC using robust [16] or stochastic approaches [17]. Furthermore, only recent studies have investigated, including adaptive features in robust formulations of the distributed Optimization Control Problem (OCP) [18, 19].

On the other hand, a few studies [20–22] proposed advanced control techniques such as Robust Adaptive MPC (MPC) in centralized settings. The proposed solutions incorporate robustness by leveraging tube-based MPC [23, 24], where the tube represents a sequence of homothetic sets containing all the feasible system responses under the potential uncertainty realizations and terminating in a robust invariant set [2]. Additionally, RAMPC offers adaptability, coping with the inherent conservativeness that is often associated with robust approaches, by refining the uncertainty bounds with Set Membership (SM) identification [25]. Nonetheless, several challenges arise with RAMPC approaches, including increasing complexity in the OCP, which is influenced by the tube structures and the system's dimensionality. In the context of MAS, a viable strategy to mitigate this issue involves distributing computation across various subsystems. In addition, ensuring recursive feasibility and robustness presents clear challenges, mainly due to the effects of neighboring agents. These interactions also influence the definition of the setpoint while addressing tracking issues, thereby complicating the formulation of the OCP.

Although some studies have proposed distributed robust control with uncertainty learning, integrating distributed OCP formulations with robust adaptive methods still represents an open challenge. For instance, Parsi et al. [18] focused on regulation but noted issues in defining terminal set invariance. Conversely, Aboudonia et al. [19] suggested a different approach by treating neighboring agents as non-cooperative, which necessitates recursive updates of the terminal conditions and the open-loop strategy. Furthermore, they adopted the artificial reference method for tracking piecewise constant references. Nevertheless, the tracking of dynamically changing references within these distributed frameworks has yet to be fully investigated. This research aims to develop a tracking Distributed RAMPC (RAMPC) framework tailored for cooperative MAS characterized by coupled dynamics and local communication constraints while also addressing exogenous disturbances and parametric uncertainties, and recursively learning the uncertainty set to enhance adaptability. In particular, the formulation is derived by considering dynamic coupling within the states.

Several key advancements in MPC have contributed to the development of the proposed approach, particularly in the areas of robust, adaptive, and distributed control. Early work on RAMPC, introduced by Lorenzen et al. [20], exhibits a key limitation as the number of constraints in the optimization problem increases combinatorially with the number of vertices in the parameter uncertainty set. Lu and Cannon [21, 26] addressed this issue by enforcing fixed-complexity polytopic parameter sets, while K hler et al. [27] proposed a moving window approach, balancing computational complexity and adaptability.

While these works primarily focused on regulation, later studies extended RAMPC to tracking problems. Didier et al. [28] and Sasfi et al. [29] applied RAMPC to quadrotor control, demonstrating its effectiveness in real-world applications. Moreover, Parsi et al. [22] introduced an explicit dual control approach for constrained reference tracking, formulating an optimization that actively balances parameter estimation and tracking performance. Conversely, Peschke and Görges [30] developed a different tracking strategy for piecewise constant references based on artificial references. The adoption of the artificial reference approach, originally proposed in Reference [31], remains a crucial element in tracking MPC because it allows for feasible solutions even when the desired reference is not directly reachable, hence increasing the region of attraction. Indeed, as recently outlined in survey [32], a few studies exploited this approach to solve tracking problems with a robust framework [33–35] and for distributed cooperative MPC [36–38].

Moreover, the implementation of DMPC necessitates the use of distributed optimization algorithms [5], which can be divided into iterative and sequential methods. The former involves solving an optimization problem iteratively and in parallel, while exchanging information multiple times until the solutions converge to that of the centralized overall optimization problem. In contrast, the latter approach allows one subsystem to optimize while the other subsystems maintain fixed predicted trajectories. Among the iterative methods, one of the most popular techniques is the Alternating Direction Method of Multipliers (ADMM) [39], which is based on dual decomposition. In particular, the class of subproblems that must be solved for achieving distributed control is the consensus ADMM (ADMM), which adopts the dual variable for enabling consensus constraint on the decision variables shared among a neighborhood [40, 41].

The main contributions of this work can be summarized as follows:

- A novel framework for DRAMPC is proposed for the purposes of setpoint and trajectory tracking. The methodology utilizes the artificial reference approach to maintain feasibility and incorporates cADMM to enhance scalability with respect to the number of agents.
- The terminal set is modeled to satisfy the robustly invariant property, accounting explicitly for the neighbors' influence and leveraging the definition of a structured gain in the feedback control law to robustly stabilize the system around steady-state conditions.
- The procedure for distributed SM identification is generalized to accommodate polytopic uncertainty sets. This scheme facilitates the sharing of local identification results among agents, thus accelerating the collective learning process concerning the uncertainty set.
- The artificial reference formulation is extended beyond the conventional piecewise-constant case, enabling trajectory tracking of time-varying signals and thereby expanding the applicability of DRAMPC.

The layout of this paper is as follows. Section 2 offers an overview of the system and the control problem that is to be addressed.

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Section 3 presents essential preliminaries, including an overview of the artificial reference method, RAMPC for tracking, and distributed optimization employing cADMM. Section 4 outlines the proposed DRAMPC formulation, with a focus on the distributed SM and extension to trajectory tracking. Section 5 details numerical experiments and validation results, followed by conclusions and future research directions in Section 6.

Notation 1. The set of real numbers is denoted by $\mathbb{R}$, while the natural numbers are represented by $\mathbb{N}$. The notations $\mathbb{R}_{>}$, $\mathbb{S}_{>}$, and $\mathbb{S}_{\geq}$ signify, respectively, the collection of positive definite matrices, symmetric positive definite matrices, and symmetric positive semi-definite matrices. The symbol I_n refers to the identity matrix in $\mathbb{R}^{n\times n}$. The vectors of ones and zeros are denoted by $\mathbf{1}$ and $\mathbf{0}$, respectively. Furthermore, the notation $\mathbf{s} = \{s_k\}_0^N$ identifies a sequence of predicted variables in the interval $[0, N]$. The operator $\|x\|_Q = \sqrt{x^T Q x}$ is used to represent the weighted norm. The notation $\text{row}_i(a_i) = \begin{bmatrix} \cdots & a_i & \cdots \end{bmatrix}$ indicates the horizontal concatenation of vectors a_i, while $\text{col}_j(b_j) = \begin{bmatrix} \cdots & b_j^T & \cdots \end{bmatrix}^T$ represents the vertical concatenation of vectors b_j. Additionally, $[c]_i$ denotes the i-th element of the vector c. Furthermore, $\text{Proj}_f(\mathcal{F})$ denotes the orthogonal projection of the set $\mathcal{F}$ onto the axis defined by f, and the symbol $\oplus$ represents the Minkowski sum between two sets. $\mathcal{A} = \{1, 2, \cdots, M\}$ represents the set of $M \in \mathbb{N}$ agents, and $\mathcal{N}_i$ represents the set of in-neighbors of each agent $i \in \mathcal{A}$. The convex hull of a set S is written as $Co\{S\}$.

2 | Problem Formulation

This study considers a discrete-time linear system of the form,

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + w_k, \tag{1a}$$

$$y_k = Cx_k, \tag{1b}$$

where $x_k \in \mathbb{R}^n$ represents the states, $u_k \in \mathbb{R}^m$ the inputs, and $y_k \in \mathbb{R}^p$ the outputs. Moreover, $w_k \in \mathbb{R}^n$ is the additive disturbance, and $\theta = \theta^* \in \mathbb{R}^q$ is the unknown but constant real parameter vector. This parameter represents the parametric uncertainty as suggested in References [20, 21]. The outlined system corresponds to the global representation of a MAS consisting of $M \in \mathbb{N}$ interconnected agents. The interaction between agents is represented by an undirected graph $\mathcal{G} = (\mathcal{A}, \mathcal{E})$, where $\mathcal{A}$ is the set of agents and $\mathcal{E}$ defines the communication links. Each agent $i \in \mathcal{A}$ has an in-neighborhood $\mathcal{N}_i$, comprising all agents whose states influence its dynamics, including itself. The graph $\mathcal{G}$ is assumed to be strongly connected, ensuring that each agent can exchange information with its relevant neighbors for coordination and consensus. Specifically, this study focuses on a class of MAS in which dynamic coupling only concerns the agent's states. Hence, the global system in Equation (1a) can be expressed as,

$$x_{i,k+1} = \sum_{j\in\mathcal{N}_i} A_{ij}(\theta_i)x_{j,k} + B_i(\theta_i)u_{i,k} + w_{i,k}, \tag{2a}$$

$$y_{i,k} = C_i x_{i,k} \tag{2b}$$

where $x_{j,k} \in \mathbb{R}^{n_j}$ represents the states $\forall j \in \mathcal{N}_i$, $u_{i,k} \in \mathbb{R}^{m_i}$ the inputs, and $y_{i,k} \in \mathbb{R}^{p_i}$ the outputs, evaluated at instant k. Each subsystem matrices $A_{ii}(\theta_i) = A_i(\theta_i)$, $A_{ij}(\theta_i)$ for $j \neq i$, along with $B_i(\theta_i)$, are functions of a vector $\theta_i \in \mathbb{R}^{q_i}$ that is contained in θ. The performance output matrix is known and is defined such that $C_i \in \mathbb{R}^{p_i\times n_i}$.

The notation in Equation (2a) has been simplified below by separating the i-th agent states from the in-neighbors states. Then, the in-neighbors states have been vertically concatenated in a single vector $x_{n_i} = \text{col}_{\forall j}(x_j)$, while the interaction matrices have been horizontally concatenated such that $A_{n_i}(\theta_i) = \text{row}_{\forall j}(A_{ij}(\theta_i))$, with $j \in \mathcal{N}_i \setminus \{i\}$.

$$x_{i,k+1} = A_i(\theta_i)x_{i,k} + B_i(\theta_i)u_k + A_{n_i}(\theta_i)x_{n_i,k} + w_{i,k} \tag{3}$$

where $x_{n_i} \in \mathbb{R}^{(n_{\mathcal{N}_i} - n_i)}$ and $n_{\mathcal{N}_i} - n_i$ is the dimension of the states in the in-neighbor set excluding the agent itself.

The following Assumptions delineate the additive disturbances and multiplicative parametric uncertainties.

Assumption 1 (Additive Disturbance). The additive disturbance w_i is confined within a convex closed set such that for each subsystem,

$$\mathbb{W} = \{w_i \in \mathbb{R}^{n_i} : H_{w_i} w_i \leq b_{w_i}\}. \tag{4}$$

Assumption 2 (Parametric Uncertainty). The system matrices of each subsystem are represented as affine functions of the uncertain parameter vector θ_i such that,

$$(A_i(\theta_i), A_{n_i}(\theta_i), B_i(\theta_i)) = \left(A_i^{(0)}, A_{n_i}^{(0)}, B_i^{(0)}\right) + \sum_{g=1}^{q_i} [\theta_i]_g \left(A_i^{(g)}, A_{n_i}^{(g)} B_i^{(g)}\right) \tag{5}$$

where $A_i^{(0)}$, $A_{n_i}^{(0)}$ and $B_i^{(0)}$ denote the nominal matrices, while $A_i(\theta_i) : \mathbb{R}^{q_i} \mapsto \mathbb{R}^{n_i\times n_i}$, $A_{n_i}(\theta_i) : \mathbb{R}^{q_i} \mapsto \mathbb{R}^{n_i\times(n_{\mathcal{N}_i}-n_i)}$ and $B_i(\theta_i) : \mathbb{R}^{q_i} \mapsto \mathbb{R}^{n_i\times m_i}$. The real parameter vector θ^* is unknown and lies inside a known bounded convex polytopic set $\Theta_0 \subset \mathbb{R}^q$,

$$\Theta_0 = \{\theta | H_\Theta \theta \leq b_{\Theta_0}\} = \text{Co}\{\theta_0^{(1)} \cdots \theta_0^{(v)}\} \tag{6}$$

where $\theta_0^{(g)}$, $\forall g \in \mathbb{N}_i^v$ are the vertices of Θ_0.

According to Assumption 2, the true parameter vector θ_i^* belongs to the projection of the global set Θ_0 onto $\mathbb{R}^{q_i}$, which remains a convex polytopic set $\Theta_i = \text{Proj}_{\mathbb{R}^{q_i}}(\Theta_0)$. It is noteworthy that the polytopic one is just a particular choice of uncertainty description and that also others have been considered in robust MPC, such as linear fractional transformation (LFT) descriptions inspired by robust control [42, 43].

Assumption 3 (Shared Parameters). The vector θ_i for all $i \in \mathcal{A}$ shares at least one parameter with at least one other agent $j \in \mathcal{N}_i \setminus \{i\}$.

Each i-th agent is subject to local constraints defined by bounded convex polytopic sets for the system states $x_{i,k} \in \mathcal{X}_i$ and inputs $u_{i,k} \in \mathcal{U}_i$ by the following,

$$\mathcal{X}_i = \{x_i \in \mathbb{R}^{n_i} | H_{x_i} x_i \leq b_{x_i}\}, \tag{7a}$$

$$\mathcal{U}_i = \{u_i \in \mathbb{R}^{m_i} | H_{u_i} u_i \leq b_{u_i}\}. \tag{7b}$$

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However, constraints in Equation (7) can be expressed through a general representation that allows combined constraints,

$$\mathcal{Z}_i = \{(x_i, u_i) | F_i x_i + G_i u_i \leq \mathbf{1}\} = \mathcal{X}_i \times \mathcal{U}_i. \tag{8}$$

Thus, global constraints are denoted by the cartesian product of all local constraints such that

$$\mathcal{Z} = \mathcal{Z}_1 \times \mathcal{Z}_2 \times \cdots \times \mathcal{Z}_M. \tag{9}$$

Given the subsystem dynamics from Equation (3), a known disturbance set $\mathbb{W}$ as defined in Assumption 1, and the known parameter uncertainty set Θ_0 from Assumption 2, this study aims to design a distributed solution to the following robust adaptive tracking problem.

The goal of the studied OCP is to let each agent track a reference signal $r_{i,k}$ while satisfying the constraints in Equation (8) robustly with respect to the uncertainty in the model's parameters. The true parameter θ_i^* which satisfies the Assumption 2, is estimated through Least Mean Squares (LMS) filter as $\hat{\theta}_i$. The parameter set $\Theta_{i,k}$ is iteratively refined by constructing a non-falsified parameter set from measured data and adopting SM identification. This approach enhances tracking performance while maintaining robust constraint satisfaction against all possible realizations of $\theta_i \in \Theta_{i,k}$ and disturbances $w_{i,k}$. To this end, this study will address a distributed solution of a recursive finite-horizon OCP, whose centralized formulation can be expressed by the following,

$$\min_{\hat{\mathbf{x}}_i, \mathbf{u}_i} \quad \sum_{i \in \mathcal{A}} J_{N,i}(\hat{\mathbf{x}}_i, \mathbf{u}_i, r_{i,k}, \hat{\theta}_{i,k}) \tag{10a}$$

$$\text{s.t.} \quad \hat{x}_{i,l+1|k} = A_i(\hat{\theta}_{i,k})\hat{x}_{i,l|k} + B_i(\hat{\theta}_{i,k})u_{i,l|k} + A_{n_i}(\hat{\theta}_{i,k})\hat{x}_{n_i,l|k}, \tag{10b}$$

$$\hat{y}_{i,l|k} = C_i \hat{x}_{i,l|k} \tag{10c}$$

$$\hat{x}_{i,0|k} = x_{i,k}, \quad \hat{x}_{n_i,0|k} = x_{n_i,k}, \tag{10d}$$

$$u_{i,l|k} = \kappa_i(\hat{x}_{i,l|k}, \hat{x}_{n_i,l|k}, r_{i,k}) \tag{10e}$$

$$\hat{x}_{i,l|k} \in \mathbb{X}_{i,l|k}(\theta_i), \quad \hat{x}_{i,N|k} \in \mathbb{X}_{i,N|k}(\theta_i), \quad (\hat{x}_{i,l|k}, u_{i,l|k}) \in \tilde{\mathcal{Z}}_i,$$
$$\forall \theta_i \in \Theta_{i,k}, \quad \forall l \in \mathbb{N}_0^{N-1}, \quad \forall i \in \mathcal{A}. \tag{10f}$$

where $\hat{\mathbf{x}}_i = \{\hat{x}_{i,l}\}_{l=1}^{N}$, $\mathbf{u}_i = \{u_{i,l}\}_{l=0}^{N-1}$ represent the predicted state and input trajectories, $\mathbb{X}_{i,l|k}(\Theta_k)$ the tube cross-section and $\tilde{\mathcal{Z}}_i \subseteq \mathcal{Z}_i$ denotes the tightened state-input constraints which will provide robust constraint satisfaction. The cost function J_i captures tracking performance, control effort, and other terms that will be detailed in subsequent sections. Additional decision variables, introduced later, will account for artificial references [31] and a structured evolution of tube cross-sections.

In addition to robust constraint satisfaction, ensuring the recursive feasibility of the optimization problem is fundamental for maintaining control feasibility over time. This can be achieved with the design of a terminal set $\mathbb{X}_{i,N|k}(\theta_i)$ and a control law $\kappa_i(x_{i,l|k}, x_{n_i,l|k}, r_{i,k})$ that enforce a contractive property on the closed-loop system. The formulation will later be extended to enable trajectory tracking.

3 | Preliminaries

This section outlines the methods underpinning the proposed solution to the problem formulation. It begins with an overview of the artificial reference method in centralized MPC and examines its extension into the RAMPC framework to address uncertainties and disturbances in system dynamics. The section concludes with a brief discussion on distributed optimization techniques for achieving consensus in the distributed formulation of the problem presented in Equation (10).

3.1 | Artificial Reference MPC

The objective in tracking MPC is to steer the system to a desired output reference $r_k \in \mathbb{R}^p$ through optimal setpoints while satisfying the system constraints. The artificial method introduced by [31], aims to track piecewise constant signals by adopting artificial setpoints as a proxy for the desired ones. The artificial setpoints are defined as decision variables of the tracking OCP, encompassing all the reachable steady states and setpoints of the system.

Definition 1 (Reachable Setpoints). The reachable steady states and set points belonging to the system $x^+ = Ax + Bu$ subject to constraints $\mathcal{Z} = \{(x, u) | Fx + Gu \leq \mathbf{1}\}$ are defined as the set of states and inputs such that, given $\sigma \in [0, 1)$.

$$\mathcal{Z}_s = \{(x, u) \in \sigma\mathcal{Z} : x = Ax + Bu\} \subseteq \text{int}(\mathcal{Z}), \tag{11a}$$

$$\mathcal{Y}_s = \left\{y = Cx + Du : (x, u) \in \mathcal{Z}_s\right\}. \tag{11b}$$

The resulting problem offers a relaxation of classical tracking schemes, as it provides feasible solutions even when the reference is not linked to feasible steady states. This is achieved by adding an offset cost to the OCP, which penalizes the distance of the artificial setpoints from the desired ones, as follows,

$$V_o(y_a, r_k) = \|y_a - r_k\|_S^2. \tag{12}$$

The stage cost and the terminal cost for tracking the artificial setpoints are expressed in the canonical quadratic form, respectively,

$$\ell(x_{l|k}, u_{l|k}, x_a, u_a) = \|x_{l|k} - x_a\|_Q^2 + \|u_{l|k} - u_a\|_R^2, \tag{13a}$$

$$V_f(x_{N|k}, x_a) = \|x_{N|k} - x_a\|_P^2. \tag{13b}$$

Accordingly, the total cost is given by,

$$J_N(\mathbf{x}_k, \mathbf{u}_k, x_a, u_a, r_k) = \sum_{l=0}^{N-1} \ell(x_{l|k}, u_{l|k}, x_a, u_a) + V_f(x_{N|k}, x_a) + V_o(y_a, r_k) \tag{14}$$

Thus, the artificial reference tracking problem is summarized in,

$$\min_{\mathbf{x}_k, \mathbf{u}_k, x_a, u_a} \quad J_N(\mathbf{x}_k, \mathbf{u}_k, x_a, u_a, r_k) \tag{15a}$$

$$\text{s.t.} \quad x_{l+1|k} = Ax_{l|k} + Bu_{l|k}, \quad \forall l \in \mathbb{N}_0^{N-1} \tag{15b}$$

$$(x_a, u_a) \in \mathcal{Z}_s, \quad y_a = Cx_a + Du_a \in \mathcal{Y}_s, \tag{15c}$$

$$(x_{l|k}, u_{l|k}) \in \mathcal{Z}, \quad \forall l \in \mathbb{N}_0^{N-1} \tag{15d}$$

$$(x_{N|k}, x_a, u_a) \in \mathcal{X}_a. \tag{15e}$$

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where $\mathbf{x}_k = \{x_{l|k}\}_{l=1}^{N}$ and $\mathbf{u}_k = \{u_{l|k}\}_{l=0}^{N-1}$ denote the predicted state and input trajectories, respectively. Equation (15c) guarantees the artificial variables to be reachable steady states and setpoints, while Equation (15e) denotes the terminal ingredients. The recursive feasibility of the OCP is ensured by designing the terminal set $\mathcal{X}_a$ to be the largest positively invariant set for the augmented system (x, x_a, u_a), and can be encapsulated by the following Assumptions.

Assumption 4. Given $Q \in \mathbb{S}^n_{\geq}$ and $R \in \mathbb{S}^m_{\geq}$, there exist a matrix $K \in \mathbb{R}^{n\times m}$ such that the closed-loop system $A_k = A + BK$ is asymptotically stable, and a matrix $P \in \mathbb{S}^n_{>}$ that satisfy the following,

$$A_k^T P A_k - P \leq -(Q + K^T R K). \tag{16}$$

Assumption 5. There exists $\mathcal{X}_a \in \mathbb{R}^{2n+m}$ such that $\forall (x, x_a, u_a) \in \mathcal{X}_a$ holds,

$$(x, K(x - x_a) + u_a) \in \mathcal{Z}, \tag{17a}$$

$$(Ax + B(K(x - x_a) + u_a), x_a, u_a) \in \mathcal{X}_a. \tag{17b}$$

Choosing σ in Equation (11a) arbitrarily close to 1 enlarges the region of feasible steady states that satisfy the invariance condition. Moreover, the computation of this set for large-scale systems is non-trivial. For this reason, [31] and [44] proposed to parametrize the steady states and inputs through Singular Value Decomposition (SVD).

3.2 | RAMPC and Tracking Formulation

The tracking problem in the presence of parametric uncertainties and additive disturbances in the global MAS in Equation (1a) can be addressed by exploiting methods based on the artificial reference as suggested by [22, 27, 30]. Specifically, this section describes the tracking scheme proposed by [30]. Firstly, the RAMPC is addressed by summarizing the SM, the polytopic tube structure, and the robust constraints. Subsequently, the integration of the artificial reference method is discussed, as outlined in Section 3.1 to provide the centralized solution to the OCP presented in Equation (10).

The recursive SM identification of the uncertain parameter set Θ within the control algorithm is introduced by [20]. The uncertainty set must be defined as a convex polyhedron having a fixed shape given by $H_\Theta = \left[I_q - I_q\right]^T$ such that,

$$\Theta_k = \left\{\theta | H_\Theta \theta \leq b_{\Theta,k}\right\} = \mathrm{Co}\left\{\theta_k^{(1)} \cdots \theta_k^{(v)}\right\}, \quad \forall k \geq 0 \tag{18}$$

According to Assumption 2, the initial condition for the parameter set is defined such that $\Theta \subseteq \Theta_0$. Moreover, let $D(x,u) \in \mathbb{R}^{n+m} \mapsto \mathbb{R}^{n\times q}$ and $d_k \in \mathbb{R}^n$ be defined as,

$$D(x,u) = \left[A^{(1)}x + B^{(1)}u \;\; A^{(2)}x + B^{(2)}u \;\cdots\; A^{(q)}x + B^{(q)}u\right], \tag{19a}$$

$$d_k = A^{(0)}x_{k-1} + B^{(0)}u_{k-1} - x_k. \tag{19b}$$

Recalling Assumption 1, the non-falsified parameter set can be defined as,

$$\begin{aligned}\Phi_k &= \left\{\theta : x_k - A(\theta)x_{k-1} - B(\theta)u_{k-1} \in \mathbb{W}\right\} \\ &= \left\{\theta : -H_w D(x_{k-1}, u_{k-1})\theta \leq b_w + H_w d_k\right\} \\ &= \left\{\theta : H_{\Phi,k}\theta \leq b_{\Phi,k}\right\}.\end{aligned} \tag{20}$$

The recursive identification of the parameter set by definition must be such that $\Theta_{k+1} \subseteq \Theta_k$. This condition can be achieved by computing the intersection of the sets Θ_k and Φ_k at each time step k, hence $\tilde{\Theta}_{k+1} = \Theta_k \cap \Phi_k$. In addition, the following proposition from Reference [45] determines the set inclusion condition useful for bounding the complexity of the uncertainty set.

Proposition 1. *Given two polytopes $\mathcal{P}_1 = \{x : F_1 x \leq g_1\}$ and $\mathcal{P}_2 = \{x : F_2 x \leq g_2\}$, then $\mathcal{P}_1$ is a subset of $\mathcal{P}_2$ if and only if there exists a non-negative matrix Ξ such that $\Xi F_1 = F_2$ and $\Xi g_1 \leq g_2$.*

Knowing that the target set Θ_{k+1} must be a superset of $\tilde{\Theta}_{k+1}$, the set-membership identification can be performed by solving the following Linear Program (LP) in a row-wise fashion, as proposed by [2] and [30].

$$\begin{aligned}\left[b_{\Theta,k+1}\right]_r &= \min_{b,\Xi_r} b, \\ \text{s.t.} \quad \Xi_r \begin{bmatrix} H_\Theta \\ H_{\Phi,k}\end{bmatrix} &= \left[H_\Theta\right]_r, \quad \Xi_r \begin{bmatrix} b_{\Theta,k} \\ b_{\Phi,k}\end{bmatrix} \leq b, \quad \Xi_r \geq 0.\end{aligned} \tag{21}$$

The fundamental concept behind tube-based MPC is to create a tube, a sequence of sets representing the manifold of feasible system trajectories.

Definition 2. Let $\mathbb{X}_{l|k}$ be a sequence of sets and $u_{l|k}$ a sequence of control inputs, where $l \in \mathbb{N}_0^N$. The sequence $\mathbb{X}_{l|k}$ is defined as a tube if:

$$A(\theta)\mathbb{X}_{l|k} + B(\theta)u_{l|k} \oplus \mathbb{W} \subseteq \mathbb{X}_{l+1|k}, \quad \forall l \in \mathbb{N}_0^{N-1}, \forall \theta \in \Theta. \tag{22}$$

The bounded growth of this tube must ensure that the operating constraints are robustly satisfied. Moreover, the system is pre-stabilized adopting a feedback control law $\kappa(\cdot)$ such that

$$u_{l|k} = \begin{cases} K x_{l|k} + c_{l|k}, & \text{if} \quad l \in \mathbb{N}_0^{N-1}, \\ K x_{N|k}, & \text{if} \quad l = N. \end{cases} \tag{23}$$

where $c_{l|k}$ represents the free control action, while the gain K is assumed to robustly stabilize the system. Exploiting Assumption 2, the closed-loop system dynamics can be expressed as an affine function of the uncertain parameter θ.

$$A_K(\theta) = (A^{(0)} + B^{(0)}K) + \sum_{g=1}^{q} [\theta]_g (A^{(g)} + B^{(g)}K). \tag{24}$$

The dynamics of the tube can be determined by confining its terminal cross-section within a robust λ-contractive set.

Definition 3. A set $\mathcal{P} = \{x : Vx \leq \mathbf{1}\}$ is said to be robust λ-contractive under parametric uncertainties for the system (1a),

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with the linear policy $u = Kx \in \mathcal{U}$, if it satisfies the condition $A_K(\theta)x \in \lambda P, \forall \theta \in \Theta_0$ and $\lambda \in [0, 1)$.

The resulting cross-sections will maintain a fixed structure and complexity, determined by the matrix V and varying parameter $\alpha_{l|k}$, as follows,

$$\mathbb{X}_{l|k} = \{x | Vx \leq \alpha_{l|k}\}. \tag{25}$$

By definition, the subsequent cross-sections of the tube must satisfy,

$$\mathbb{X}_{l+1|k} \supseteq A(\theta)\mathbb{X}_{l|k} \oplus B(\theta)u_{l|k} \oplus \mathbb{W} \supseteq \mathbb{X}_{l|k}, \quad \forall \theta \in \Theta. \tag{26}$$

This condition can be enforced by defining the set inclusion condition extended from Proposition 1 on linearly parametrized sets as proposed by [26].

Proposition 2. *Given two polytopes $\mathcal{P}_1 = \{x : F_1 x \leq g_1\}$ and $\mathcal{P}_2(\theta) = \{x : F_2(\theta)x \leq g_2\}$, with $F_1 \in \mathbb{R}^{n_1 \times n_x}$ and $F_2(\theta) \in \mathbb{R}^{n_2 \times n_x}$. Then $\mathcal{P}_1$ is a subset of $\mathcal{P}_2(\theta)$ if and only if there exists $\Sigma_r \in \mathbb{R}^{q \times n_1}$, such that*

$$\begin{aligned} &[\Sigma(\theta)]_r \geq 0, \quad [\Sigma(\theta)]_r F_1 = \left[F_2(\theta)\right]_r, \\ &\qquad [\Sigma(\theta)]_r g_1 \leq \left[g_2\right]_r, \quad \forall r \in \mathbb{N}_1^{n_2}, \quad \forall \theta \in \Theta \end{aligned} \tag{27}$$

where $[\Sigma(\theta)]_r = \theta^T \Sigma_r$.

According to Equation (25), the tube cross-sections $\mathbb{X}_{l+1|k}$ can be represented as follows,

$$V(A_K(\theta)\, x + B(\theta)c_{l|k}) + \overline{w} \leq \alpha_{l+1|k}, \quad \forall \theta \in \Theta_k, \quad \forall x \in \mathbb{X}_{l|k}. \tag{28}$$

where $\overline{w}$ is the maximum realization of the bounded disturbance $\mathbb{W}$ and can be computed by solving the following LP,

$$[\overline{w}]_r = \max_{w \in \mathbb{W}} [V]_r w, \quad \forall r \in \mathbb{N}_1^n. \tag{29}$$

Thus, leveraging Proposition 2, and the tube definition in Equation (26), it holds

$$\Sigma(\overline{\theta}_k^{(j)})\alpha_{l|k} \leq \alpha_{l+1|k} - VB(\theta_k^{(j)})c_{l|k} - \overline{w}, \tag{30a}$$

$$\Sigma(\overline{\theta}_k^{(j)})V = VA_K(\theta_k^{(j)}), \quad \forall j \in \mathbb{N}_1^v, \quad \forall l \in \mathbb{N}_0^{N-1}. \tag{30b}$$

where $\overline{\theta}_k^{(j)} = \text{col}(1, \theta_k^{(j)})$. Furthermore, the matrices Σ_r can be obtained by solving the LPs embedded in the following min-max optimization problem,

$$\begin{aligned} \Sigma_r &= \arg\min_{\Sigma} \max_{j \in \mathbb{N}_1^v} \theta_0^{(j)^T} \Sigma \mathbf{1}, \\ \text{s.t.} \quad &\theta_0^{(j)^T} \Sigma \geq 0, \quad \forall j \in \mathbb{N}_1^v, \quad \Sigma V = \text{col}_{t \in \mathbb{N}_0^q}([V]_r A_K^{(t)}) \end{aligned} \tag{31}$$

Robust constraint satisfaction holds by applying Proposition 1, substituting (23) into (9) as follows,

$$\begin{aligned} &Fx_{l|k} + Gu_{l|k} \leq \mathbf{1} \Leftrightarrow \\ \Omega V = F + GK, \quad &\Omega\alpha_{l|k} + Gc_{l|k} \leq \mathbf{1}, \quad \forall l \in \mathbb{N}_0^{N-1}. \end{aligned} \tag{32}$$

Equation (32) holds if there exists a non-negative matrix Ω that can be computed by row-minimization as proposed by [2] solving the following LP,

$$\begin{aligned} [\Omega]_r = \arg\min_{\omega} \quad &\omega^T \mathbf{1} \\ \text{s.t.} \quad &\omega^T V = [F + GK]_r, \quad \omega \geq 0. \end{aligned} \tag{33}$$

While the equations above establish the formulation of the RAMPC for regulation, the following elaborates on its extension to accommodate the tracking objective. The tracking of the artificial reference is achieved by substituting the feedback control policy in Equation (23) with the following,

$$u_{l|k} = \begin{cases} K(x_{l|k} - x_a) + u_a + c_{l|k}, & \text{if} \quad l \in \mathbb{N}_0^{N-1}, \\ K(x_{N|k} - x_a) + u_a, & \text{if} \quad l = N. \end{cases} \tag{34}$$

Subsequently, by adopting Equation (34) to derive the tube dynamics from Equation (30a) and the robust constraint satisfaction from Equation (32), the new set of inequalities is given by,

$$\Sigma(\overline{\theta}_k^{(j)})\alpha_{l|k} + V((I_n - A_K(\theta_k^{(j)}))x_a + B(\theta_k^{(j)})c_{l|k}) + \overline{w} \leq \alpha_{l+1|k}, \tag{35a}$$

$$\Omega\alpha_{l|k} + Gc_{l|k} - GKx_a + Gu_a \leq \mathbf{1}, \quad \forall j \in \mathbb{N}_0^v \quad \forall l \in \mathbb{N}_0^{N-1}, \tag{35b}$$

with terminal condition, respectively,

$$\Sigma(\overline{\theta}_k^{(j)})\alpha_{N|k} + V((I_n - A_K(\theta_k^{(j)}))x_a) + \overline{w} \leq \alpha_{N|k}, \tag{36a}$$

$$\Omega\alpha_{N|k} - GKx_a + Gu_a \leq \mathbf{1}. \tag{36b}$$

Assuming that the system dynamics and the artificial setpoints depend on the estimate $\hat{\theta}_k$, the decision variables for the robust adaptive problem are represented by,

$$\chi_k = \{\hat{\mathbf{x}}_k, \mathbf{c}_k, \boldsymbol{\alpha}_k\ \hat{x}_a, \hat{u}_a\}. \tag{37}$$

Thus, the RAMPC for tracking can be formulated as follows,

$$\min_{\chi_k} \ J_N(\hat{\mathbf{x}}_k, \mathbf{u}_k, \hat{x}_a, \hat{u}_a, r_k) \tag{38a}$$

$$\text{s.t.} \quad \hat{x}_{0|k} = x_k, \quad \hat{x}_{0|k} \in \mathbb{X}_{0|k}, \tag{38b}$$

$$(\hat{x}_a, \hat{u}_a, \hat{x}_{n,a}) \in \mathcal{Z}_s, \tag{38c}$$

$$\hat{x}_{l+1|k} = A_K(\hat{\theta}_k)(\hat{x}_{l|k} - \hat{x}_a) + B(\hat{\theta}_k)c_{l|k} + A(\hat{\theta}_k)\hat{x}_a + B(\hat{\theta}_k)\hat{u}_a, \tag{38d}$$

$$u_{l|k} = K(\hat{\theta}_k)(\hat{x}_{l|k} - \hat{x}_a) + \hat{u}_a + c_{l|k}, \tag{38e}$$

$$\Sigma(\overline{\theta}_k^{(j)})\alpha_{l|k} + V((I_n - A_K(\theta_k^{(j)}))x_a + B(\theta_k^{(j)})c_{l|k}) + \overline{w} \leq \alpha_{l+1|k}, \tag{38f}$$

$$\Sigma(\overline{\theta}_k^{(j)})\alpha_{N|k} + V((I_n - A_K(\theta_k^{(j)}))x_a) + \overline{w} \leq \alpha_{N|k}, \tag{38g}$$

$$\Omega\alpha_{l|k} + Gc_{l|k} - GKx_a + Gu_a \leq \mathbf{1}, \tag{38h}$$

$$\begin{aligned} &\Omega\alpha_{N|k} + Gu_a - GKx_a \leq \mathbf{1}, \\ &\forall j \in \mathbb{N}_1^v, \quad \forall l \in \mathbb{N}_0^{N-1}. \end{aligned} \tag{38i}$$

where the cost to be minimized $J_N(\cdot)$ is the same adopted for tracking in Equation, while (14) (38d) represents the system dynamics, (38b) the initial conditions, (38e) is the pre-stabilizing control policy, and (38c) represents the available artificial references. The tube dynamics and terminal tube cross-section are

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defined by (38f) and (38g) respectively, while (38h) and (38i) provide the system constraints and the terminal constraints reformulated according to the tube evolution. The recursive feasibility of the OCP can be exploited in [2, Th. 5.8].

3.3 | Distributed Consensus ADMM

This work aims to solve the OCP in Equation (10) via distributed optimization to achieve a distributed control structure.

Accordingly, the goal is to address the OCP without relying on a central coordinator C^*. Furthermore, the optimization problem cannot be fully separated, as cooperation among agents is essential for achieving consensus on the interconnected dynamics defined in Equation (2a).

Among the various algorithms for distributed optimization, this work focuses on the Alternating Direction Method of Multipliers ADMM, adapted in its consensus form cADMM to enforce agreement among neighboring agents, as proposed by [39].

The solution of the OCP relies both on shared and non-shared decision variables, respectively χ_i and γ_i. The shared variables vector represents a local copy of the global variables ζ, restricted to the subset of variables shared among the i-th agent's neighborhood. ADMM has gained considerable popularity in DMPC in recent years, thanks to its robustness against variations in algorithm parameters and its ability to handle constrained optimization problems, which are central in MPC. In comparison with non-iterative schemes for distributed optimization, iterative approaches such as ADMM are generally less conservative, since repeated communication allows neighboring agents to negotiate their local trajectories and converge toward globally consistent solutions. On the other hand, ADMM-based methods incur some computational overhead, as the arising subproblems do not necessarily admit closed-form solutions. Nevertheless, in practice is often observed that convergence is typically achieved within a few iterations, making it a standard choice in many multi-agent control applications, e.g., Shorinwa et al. [3, 41] and Stomberg et al. [4].

The distributed OCP must be solved such that consensus is reached between local copies of the decision variables and a vector of the global variables $\tilde{\zeta}_i$, as stated in Equation (39).

$$\min_{\chi_i,\gamma_i} \sum_{i=1}^{M} f_i(\chi_i,\gamma_i), \tag{39a}$$

$$\text{s.t.} \quad \chi_i = \tilde{\zeta}_i, \quad \forall i = 1,\cdots,M. \tag{39b}$$

To achieve global convergence between $\mathrm{col}_{i\in\mathcal{A}}(\chi_i)$ and ζ, a communication protocol for sharing information among neighbors in the network is necessary.

$$\|r^k_{\mathcal{N}_i}\|_2 \le \epsilon_p, \quad \|s^k_{\mathcal{N}_i}\|_2 \le \epsilon_d \tag{40a}$$

$$\|r^k_{\mathcal{N}_i}\|_2^2 = \sum_{j\in\mathcal{N}_i} \|\chi^k_j - \tilde{\zeta}^k_i\|_2^2 \tag{40b}$$

$$\|s^k_{\mathcal{N}_i}\|_2^2 = |\mathcal{N}_i|\,(\rho^k)^2 \|\tilde{\zeta}^k_i - \tilde{\zeta}^{k-1}_i\|_2^2 \tag{40c}$$

ALGORITHM 1 | Simplified cADMM.

Require: ρ, $N_{\max}$
Initialise: $k \leftarrow 0, \Lambda_i^k \leftarrow 0, \zeta^k \leftarrow 0, \forall i = 1,\dots,M$
while not (40a) **and** $k \le N_{\max}$ **do**
  for all agents i (in **parallel**) **do**
    Solve:

$$\{\chi_i^{k+1},\gamma_i^{k+1}\} \leftarrow \arg\min_{\chi_i,\gamma_i} \Big(f_i(\chi_i,\gamma_i) + (\Lambda_i^k)^\top(\chi_i - \tilde{\zeta}_i^k) + \frac{\rho}{2}\|\chi_i - \tilde{\zeta}_i^k\|_2^2 \Big)$$

    Communicate χ_i^{k+1} to neighbours $j \in \mathcal{N}_i$
    Update the global variables:

$$\tilde{\zeta}_i^{k+1} \leftarrow \frac{1}{|\mathcal{N}_i|} \sum_{j\in\mathcal{N}_i} \chi_j^{i,k+1}$$

    Communicate $\tilde{\zeta}_i^{k+1}$ to neighbours $j \in \mathcal{N}_i$
    Update the global vector:

$$\tilde{\zeta}^{k+1} \leftarrow \Big[(\tilde{\zeta}_i^{i,k+1}) \ \cdots \ (\tilde{\zeta}_i^{\mathcal{N}_i,k+1}) \Big]$$

    Update the dual variable:

$$\Lambda_i^{k+1} \leftarrow \Lambda_i^k + \rho(\chi_i^{k+1} - \tilde{\zeta}_i^{k+1})$$

    Communicate primal residual r_i^k to neighbours $j \in \mathcal{N}_i$
  end for
end while

Equation (40) defines the convergence condition for the distributed ADMM, where ϵ_p and ϵ_d are the primal and dual convergence thresholds, respectively, which can be calculated as suggested in Reference [39]. The residuals $r^k_{\mathcal{N}_i}$ and $s^k_{\mathcal{N}_i}$ are computed in the neighborhood of the i-th agent and shared with neighboring agents. While the dual residual can be computed locally, calculating the primal residual requires each agent to share its residuals with its neighbors. The complete algorithm for distributed ADMM is outlined in Algorithm 1.

The algorithm is initialized by setting the iteration counter k to zero, and the dual and global variables, Λ_i^k and ζ^k, are initially set to zero for each agent i.

In this work, warm-starting strategies are employed to accelerate convergence, which proved highly effective in reducing the number of iterations required. While more advanced acceleration techniques (e.g., adaptive ρ updates [39]) exist, a simple warm-start was sufficient to achieve efficient performance in the considered scenarios.

4 | DRAMPC for Tracking

This section presents a novel DRAMPC framework for tracking, using the artificial reference. It builds on the research by [18], the formulation for tracking by [31], using the robust adaptive framework presented by [30]. The primary objective is to design a distributed control architecture that can ensure robust tracking

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performance for dynamically interconnected agents, as described in Section 2. Adaptation in this control strategy allows reducing the conservativism of the robust control law by recursively tightening the uncertain parameter set Θ_k via SM identification. Further improvements in the learning process can be achieved by defining a distributed method for the identification, as proposed by [18].

This study addresses new critical aspects for reformulating the OCP accounting for the neighbors' influence. These aspects include computing a pre-stabilizing gain with an appropriate structure, the design of the robust contractive set, and the reformulation of the tube dynamics and terminal conditions designed in Section 3.2. This work contributes to the formulation of the distributed tracking OCP while guaranteeing recursive feasibility and robust constraint satisfaction.

The problem of tracking a constant signal over the entire prediction horizon has been tackled in Section 4.2 by defining the artificial variables $(x_a, u_a, x_{n,a})$ that allow the construction of an admissible steady state for tracking the reference. Through the redefinition of these variables, it became possible to establish a control policy along with the polytopic structure and the dynamics of the tube, such that the terminal states belong to a robust λ-contractive set. The algorithm for tracking the particular class of constant references is synthesized in Section 4.3, and later the proposed architecture has been extended in Section 4.5 to the problem of trajectory tracking (i.e., time-varying reference signals). This challenge is addressed by redefining the artificial variables to represent the whole artificial trajectory capable of reaching a stationary condition at the end of the prediction horizon. This novel architecture significantly enhances tracking performance, especially for more complex applications relying on a trajectory planner, as shown later through numerical examples in Section 5.

In both cases for constant set points and reference trajectory tracking, the reachable steady-states and set points in Definition 1 must be redefined to consider the influence of the neighboring agents. Each i-th agent is subject to constraints defined by compact polytopic sets as in Equation (8). Moreover, the neighboring states are bounded within a compact polytopic set $\mathcal{X}_{n_i}$, formally expressed as follows,

$$\mathcal{X}_{n_i} = \{x_{n_i} \in \mathbb{R}^{(n_{\mathcal{N}_i} - n_i)} : H_{x_{n_i}} x_{n_i} \le b_{x_{n_i}}\} = \prod_{j \in \mathcal{N}_i \setminus \{i\}} \mathcal{X}_j. \tag{41}$$

Thus, $\mathcal{X}_{n_i}$ can be interpreted as the Cartesian product of an indexed family of sets, belonging to the neighborhood of the i-th agent. The steady-state condition for the i-th agent subject to the dynamics (3) can be expressed as,

$$\{(x_i, u_i, x_{n_i}) : x_i = A_i(\hat{\theta}_{i,k})x_i + B_i(\hat{\theta}_{i,k})u_i + A_{n_i}(\hat{\theta}_{i,k})x_{n_i}\}. \tag{42}$$

The asymptotic stability of the system around the steady state condition can be achieved if there exists a reachable set of steady states and inputs satisfying the following definition.

Definition 4. The reachable steady-states and set points for the i-th agent subject to dynamics (3) and polytopic constraints (8), are defined as the set of states, inputs, and neighbors' states such that, given $\sigma \in [0, 1)$ it holds,

$$\mathcal{Z}_{\mathcal{N}_i,s} = \{(x_i, u_i, x_{n_i}) \in \sigma(\mathcal{Z}_i \times \mathcal{X}_{n_i}) : (42)\}, \tag{43a}$$

$$\mathcal{Y}_{i,s} = \{y_i = C_i x_i : (x_i) \in \mathrm{Proj}_{(x_i)}(\mathcal{Z}_{\mathcal{N}_i,s})\}. \tag{43b}$$

The above definition implies that obtaining the reachable steady-states and set points requires each agent to know the constraint set of its neighbors, i.e., the polytopic set $\mathcal{X}_{n_i}$ defined in Equation (41). In the considered cooperative MAS with a static communication topology, this information can be exchanged once during the initialization phase, which is consistent with the cooperative paradigm. To design the tracking OCP using the artificial reference method proposed in Section 3.1 for the system in Equation (3), the artificial variables $x_{i,a}$, $u_{i,a}$ and $x_{n_i,a}$ must be defined such that $(x_{i,a}, u_{i,a}, x_{n_i,a}) \in \mathcal{Z}_{\mathcal{N}_i,s}$. The coefficient σ is designed arbitrarily close to 1 to guarantee the existence of an admissible invariant set, as terminal conditions of the augmented system depending on $(x_i, x_{i,a}, u_{i,a}, x_{n_i,a})$.

4.1 | Pre-Stabilizing Gain Computation

In Reference [18], a method was proposed to compute the pre-stabilizing gain for each i-th agent, relying on a structured Linear Matrix Inequalities (LMI)s formulation. This approach is similar to the one presented by [46], where the problem is to find a structured stabilizing gain and terminal ingredients within a distributed MPC framework. To advance the construction of the feedback gain, this work leverages the definition of some lifting matrices that map the overall MAS states and inputs into smaller subsets of states and inputs relative to each agent or to its in-neighborhood $\mathcal{N}_i$. Thus, there can be defined the matrices $T_i \in \{0,1\}^{n_i \times n}$, $W_i \in \{0,1\}^{n_{\mathcal{N}_i} \times n}$, $U_i \in \{0,1\}^{m_i \times m}$, such that $x_i = T_i\, x$, $u_i = U_i\, u$ and $x_{\mathcal{N}_i} = W_i\, x$. The objective here is to obtain a structured feedback gain such that the following Assumption holds.

Assumption 6. There exists a linear feedback law $k(x)$ of the form

$$k(x) = Kx = \mathrm{col}_{i \in \mathcal{A}}(K_{\mathcal{N}_i}\, x_{\mathcal{N}_i}). \tag{44}$$

that asymptotically stabilizes the system $x^+ = Ax + B\, k(x)$, where $K \in \mathbb{R}^{m \times n}$ and $K_{\mathcal{N}_i} = U_i\ K\ W_i^T \in \mathbb{R}^{m_i \times n_{\mathcal{N}_i}}$.

Furthermore, the sufficient conditions for the asymptotic stability of the cooperative system are provided in [46, Th. 6]. These conditions are achieved by defining a Lyapunov function for each subsystem $V_{f,i}(x_i) = \|x_i\|^2_{P_i}$ that may increase locally while ensuring a global decrease of the function $V_f(x) = \|x\|^2_P$. This stability result is summarized in the following Lemma,

Lemma 1. *The interconnected system $x^+ = Ax + Bk(x)$ with the feedback control law in Equation (44) is asymptotically stable if there exists a block-diagonal matrix $P = \mathrm{diag}(P_1, \dots, P_M) \in \mathbb{R}^{p \times p}_{>}$, where each block is denoted as $P_i\ \forall i \in \mathcal{A}$, such that,*

$$(A_{\mathcal{N}_i} + B_i K_{\mathcal{N}_i})^T P_i (A_{\mathcal{N}_i} + B_i K_{\mathcal{N}_i}) - \overline{P_i} \preceq -(Q_i + K^T_{\mathcal{N}_i} R_i K_{\mathcal{N}_i}) + \Gamma_{\mathcal{N}_i} \tag{45a}$$

$$\sum_{i \in \mathcal{A}} W_i^T \Gamma_{\mathcal{N}_i} W_i \preceq 0, \quad \forall i \in \mathcal{A}. \tag{45b}$$

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where $\overline{P}_i = W_i T_i^T P_i T_i W_i^T$ *and* $A_{\mathcal{N}_i} = T_i A W_i^T$, *given* $Q_i \in \mathbb{S}_{\succeq}^{n_i}$ *and* $R_i \in \mathbb{S}_{\succeq}^{m_i}$, $\forall i \in \mathcal{A}$.

In Reference [46], the authors suggest to select, among the solutions to the LMIs (45), the stabilizing gain K and the matrix P maximizing the volume of the 1-level set ellipsoid $\varepsilon = \{x \in \mathbb{R}^n | x^T P x \leq 1\}$, effectively identifying the largest invariant ellipsoidal set.

However, as noted by [47], a variety of metrics can be selected to fulfil particular criteria. In this study, no additional specific requirements have been incorporated; indeed, the problem is similar to Linear Quadratic Regulator (LQR) design but with the added complexity because of the system uncertainty. The objective of this subsection is to determine a structured gain that satisfies the following Assumption,

Assumption 7. There exists a linear feedback law $\kappa(x)$ that robustly stabilizes the system $x^+ = A(\theta)x + B(\theta)\,\kappa(x)\ \forall \theta \in \Theta$, of the same form of Equation (44).

Assumption 7 can be guaranteed by evaluating the LMIs at edges of the parameter uncertainty set, in accordance with the approach proposed by [48]. The sufficient conditions for quadratic stability of the uncertain system are provided in [45, Th. 6.21] and summarized in the following Lemma,

Lemma 2. *The uncertain system* $x^+ = A(\theta)x + B(\theta)\,\kappa(x)$ *with* $\kappa(x) = Kx$ *is stable if exists* $P \in \mathbb{S}_{\succ}^n$ *such that,*

$$A_K(\theta^{(j)})^T P A_K(\theta^{(j)}) - P \preceq -(Q + K^T R K), \quad \forall j \in \mathbb{N}_1^v. \tag{46}$$

The condition stated in Lemma 2 ensures robust quadratic stability by requiring the existence of a common quadratic Lyapunov function for all systems corresponding to the vertices of the uncertainty set. In other words, stability is guaranteed if a single positive definite matrix P satisfies the inequality for each vertex realization of the system, as expressed in Equation (46). The calculations performed offline necessitate that the LMIs in Equation (46) are defined at the vertices of initial set Θ_0. As a result, the derived solution will have some conservatism, which can be reduced by adapting the gain online as proposed by [19]. However, this approach is based on the recursive solution of Semi-Definite Program (SDP)s, which would be computationally expensive.

This work extends the results of Lemmas 1 and 2 to formulate a robust stability condition for the interconnected uncertain systems.

Theorem 1. *Consider an uncertain interconnected system of* M *agents, where each agent follows the dynamics* $x_i^+ = A_{\mathcal{N}_i}(\theta)\, x_{\mathcal{N}_i} + B(\theta)\,\kappa(x_{\mathcal{N}_i})$ *and is controlled by a feedback law* $\kappa(x_{\mathcal{N}_i}) = K_{\mathcal{N}_i} x_{\mathcal{N}_i}$. *Assume that the uncertainty set is polytopic with* v *vertices, and let* $\theta^{(j)}$, $j \in \mathbb{N}_1^v$, *denote each vertex of the uncertainty set. The overall system is robustly quadratically stable if there exists a block-diagonal matrix* $P = \mathrm{diag}(P_1, \ldots, P_M) \in \mathbb{R}_{\succ}^{p\times p}$, *such that for each block* P_i *the following LMIs hold for all vertices* $\theta^{(j)}$, $j \in \mathbb{N}_1^v$:

$$A_{K,\mathcal{N}_i}(\theta^{(j)})^T P_i A_{K,\mathcal{N}_i}(\theta^{(j)}) - \overline{P}_i \preceq -(Q_i + K_{\mathcal{N}_i}^T R_i K_{\mathcal{N}_i}) + \Gamma_{\mathcal{N}_i}, \quad \forall i \in \mathcal{A}, \tag{47a}$$

$$\sum_{i\in\mathcal{A}} W_i^T \Gamma_{\mathcal{N}_i} W_i \preceq 0, \tag{47b}$$

where $A_{K,\mathcal{N}_i}(\theta^{(j)}) = A_{\mathcal{N}_i}(\theta^{(j)}) + B_i(\theta^{(j)}) K_{\mathcal{N}_i}$, $A_{\mathcal{N}_i}(\theta^{(j)}) = W_i A(\theta^{(j)}) W_i^T$ *and* $\overline{P}_i = W_i T_i^T P_i T_i W_i^T$, *given* $Q_i \in \mathbb{S}_{\succeq}^{n_i}$ *and* $R_i \in \mathbb{S}_{\succeq}^{m_i}$.

Proof. The result follows by extending the stability condition in Equation (45) to the case of polytopic uncertainty. Specifically, the condition

$$A_{K,\mathcal{N}_i}(\theta)^T P_i A_{K,\mathcal{N}_i}(\theta) - \overline{P}_i \preceq -(Q_i + K_{\mathcal{N}_i}^T R_i K_{\mathcal{N}_i}) + \Gamma_{\mathcal{N}_i}, \quad \forall i \in \mathcal{A}, \tag{48a}$$

$$\sum_{i\in\mathcal{A}} W_i^T \Gamma_{\mathcal{N}_i} W_i \preceq 0, \tag{48b}$$

must hold for all admissible parameter values $\theta \in \Theta$. To guarantee robust satisfaction of this condition over the entire uncertainty set Θ, it is sufficient to enforce it at all vertices $\theta^{(j)}$, $j \in \mathbb{N}_1^v$, of the polytopic set, following the argument used in Lemma 2. Therefore, the robust stability condition is obtained by evaluating the LMIs in Equation (45) at each vertex of Θ. □

Following the approach in Reference [46], the stability conditions stated in Lemma 1 can be equivalently reformulated using a structured representation. The resulting formulation preserves the original stability guarantees while enabling a distributed implementation. In particular, this structure leverages the lifting matrices to express the neighbor-to-neighbor couplings, and facilitates scalable synthesis of local controllers via distributed SDP.

Lemma 3. *The stability conditions in Equation* (47) *are equivalent to the following set of coupled LMIs,*

$$\begin{bmatrix} \bar{E}_i + F_{\mathcal{N}_i} & * & * & * \\ A_{\mathcal{N}_i}(\theta^{(j)}) E_{\mathcal{N}_i} + B_i(\theta^{(j)}) Y_{\mathcal{N}_i} & E_i & * & * \\ Q_i^{1/2} E_{\mathcal{N}_i} & 0 & I_{n_{\mathcal{N}_i}} & * \\ R_i^{1/2} Y_{\mathcal{N}_i} & 0 & 0 & I_{m_i} \end{bmatrix} \succeq 0,$$
$$F_{\mathcal{N}_i} \preceq S_{\mathcal{N}_i}, \quad \sum_{t\in\mathcal{N}_i} T_t W_t^T S_{\mathcal{N}_i} W_t T_t^T \preceq 0, \quad \forall j \in \mathbb{N}_1^v, \quad \forall i \in \mathcal{A} \tag{49}$$

where $E_i = P_i^{-1}$, $Y_{\mathcal{N}_i} = K_{\mathcal{N}_i} E_{\mathcal{N}_i}$, $\bar{E}_i = T_i W_i^T E_i W_i T_i^T$, $A_{\mathcal{N}_i}(\theta^{(j)}) = W_i A(\theta^{(j)}) W_i^T$, $F_{\mathcal{N}_i} = E_{\mathcal{N}_i} \Gamma_{\mathcal{N}_i} E_{\mathcal{N}_i}$ *and* $E_{\mathcal{N}_i} = W_i E W_i^T$.

Proof. The equivalence follows by applying the Schur complement to the condition in Equation (47a), using the variable substitutions $E_i = P_i^{-1}$ and $Y_{\mathcal{N}_i} = K_{\mathcal{N}_i} E_{\mathcal{N}_i}$, as in [46, Lem. 10]. The constraint in Equation (47b) is then reformulated to reflect neighbor-to-neighbor coupling by introducing block-diagonal matrices $S_{\mathcal{N}_i}$ that upper bound $F_{\mathcal{N}_i}$, as discussed in [46, Sec. 4.1.2] □

Therefore, if the SDP defined by Equation (49) is feasible and $E_i \succ 0\ \forall i \in \mathcal{A}$, its solution yields the local matrices $K_{\mathcal{N}_i}$ and P_i that compose the overall structured feedback gain K and the block-diagonal matrix P, respectively.

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Remark 1. The structured feedback gain K enable the overall MAS to satisfy Assumption 7, such that the agent dynamics subject to the local controller $K_{\mathcal{N}_i}$ can be expressed by the following,

$$x_{i,k+1} = A_{K,\mathcal{N}_i}(\theta)x_{\mathcal{N}_i,k} = A_{K,i}(\theta)x_{i,k} + A_{K,n_i}(\theta)x_{n_i,k}, \tag{50}$$

where $A_{K,\mathcal{N}_i}(\theta) = A_{\mathcal{N}_i}(\theta) + B_i(\theta)K_{\mathcal{N}_i}$, $A_{K,i}(\theta) = A_i(\theta) + B_i(\theta)K_i$, $A_{K,n_i}(\theta) = A_{n_i}(\theta) + B_i(\theta)K_{n_i}$.

Although a distributed optimization may be adopted, this work solves the SDP offline by exploiting a central coordinator and then shares $K_{\mathcal{N}_i}$ and P_i with the agents.

According to the tube-based MPC approach, the gain $K_{\mathcal{N}_i}$ can be employed to pre-stabilize the uncertain system. In addition, for enhancing the tracking of the artificial set points, the feedback control is expressed as follows,

$$u_{i,l|k} = \begin{cases} K_i(x_{i,l|k} - x_{i,a}) + K_{n_i}(x_{n_i,l|k} - x_{n_i,a}) \\ \quad +u_{i,a} + c_{i,l|k}, & \text{if } l \in \mathbb{N}_0^{N-1} \\ K_i(x_{i,l|k} - x_{i,a}) + K_{n_i}(x_{n_i,l|k} \\ \quad - x_{n_i,a}) + u_{i,a}, & \text{if } l = N \end{cases} \tag{51}$$

where $K_i = U_i\ K\ T_i^T$ and $K_{n_i} = \text{row}_{j\in\mathcal{N}_i/\{i\}}(U_i\ K\ T_j^T)$, while $c_{i,l|k}$ $\forall l \in \mathbb{N}_0^{N-1}$ and $i \in \mathcal{A}$ are the new decision variables expressing the free control action to be computed by solving the OCP which will be defined in Section 4.3.

4.2 | Structured Tube Dynamics for Interconnected Systems

The bounded growth of the tube can be characterized using a robust λ-contractive set. As noted by [18], computing a robustly contractive set for a MAS is challenging. In interconnected systems, the set design must consider both agent dynamics and the influence of neighboring entities. However, ensuring the contraction of the tube is crucial for achieving recursive feasibility of the OCP.

The strategy proposed in this work builds on the approach by [49], which computes controlled contractive sets. In Section 3.2, the matrix V defining the tube structure ensures that the set $\mathcal{P}$ is contractive, under the assumption that the closed-loop dynamics are fully autonomous and given by $x_{k+1} = A_K x_k$.

In the distributed setting, each subsystem is influenced by its neighbors. The closed-loop dynamics of subsystem i can be written as in Equation (50), where the term involving x_{n_i} captures the interconnection with neighboring subsystems. Under the assumption that the states of the neighbors remain within a known compact set $\mathcal{X}_{n_i}$, the dynamics of subsystem i can be interpreted as a controlled system subject to structured interaction from other subsystems. This structure enables the application of invariance arguments developed in Reference [50] to establish robust contractivity with respect to the parametric uncertainty $\theta_i \in \Theta_{i,0}$.

Definition 5. A set $\mathcal{P}_i = \{x_i : V_i x_i \leq \mathbf{1}\}$ is said to be robust λ-contractive under parametric uncertainties for the system (3), given the linear feedback policy $u_i = K_i x_i + K_{n_i} x_{n_i} \in \mathcal{U}_i$ and $x_{n_i} \in \mathcal{X}_{n_i}$, if it satisfies

$$\begin{aligned} &(A_{K,i}(\theta_i)x_i + A_{K,n_i}(\theta_i)x_{n_i})\mathcal{P}_i \in \lambda\mathcal{P}_i, \\ &\forall x_{n_i} \in \mathcal{X}_{n_i} \quad \forall \theta_i \in \Theta_{i,0}, \quad \lambda \in [0,1). \end{aligned} \tag{52}$$

ALGORITHM 2 | Robust λ-contractive set for interconnected systems.

Require: λ, K_i, K_{n_i}, $A_{K,i}(\theta_i^{(j)})$, $A_{K,n_i}(\theta_i^{(j)})$ $\mathcal{Z}_i$, $\mathcal{X}_{n_i}$ for each $j \in \mathbb{N}_1^v$ and $i \in \mathcal{A}$
$t \leftarrow 0$
$\mathcal{O}_t^a \leftarrow \{(x_i, x_{n_i}) : (x_i, K_i x_i + K_{n_i} x_{n_i}) \in \mathcal{Z}_i\}$
while not contractive **do**
  $t \leftarrow t + 1$
  $\bar{\mathcal{O}}_t^a \leftarrow (\mathcal{O}_t^a \times \mathcal{X}_{n_i}) \bigcap_{j=1}^{v} \{(x_i, x_{n_i}) : (A_{K,i}(\theta_i^{(j)})x_i + A_{K,n_i}(\theta_i^{(j)})x_{n_i}) \in \lambda\mathcal{O}_t^a\}$
  $\mathcal{O}_{t+1}^a \leftarrow \text{Proj}_x\left(\bar{\mathcal{O}}_t^a\right)$
  if $\mathcal{O}_{t+1}^a = \mathcal{O}_t^a$ **then**
    break ▷ is contractive
  else
    $\mathcal{O}_t^a = \mathcal{O}_{t+1}^a$
  end if
end while

The iterative method for computing such robust λ-contractive sets is described in Algorithm 2.

The results of the algorithm can be summarized in the following lemma.

Lemma 4 (Robust λ-Contractive Set). *The set $\mathcal{O}_t^a$ computed by Algorithm 2 is robustly λ-contractive for the system (50) under the feedback law $\kappa_i(x_i, x_{n_i}) = K_i x_i + K_{n_i} x_{n_i}$, for all $x_{n_i} \in \mathcal{X}_{n_i}$ and $\theta_i \in \Theta_{i,0}$.*

Proof. The closed-loop dynamics of i-th agent can be written as $x_{i,k+1} = A_{K,i}x_i + A_{K,n_i}x_{n_i}$, where $x_{n_i} \in \mathcal{X}_{n_i}$. This structure corresponds to a controlled system with bounded, structured input. The iterative procedure in Algorithm 2 constructs a sequence of sets that remain invariant under the closed-loop dynamics for all $\theta_i \in \Theta_{i,0}$ and all admissible neighbor states $x_{n_i} \in \mathcal{X}_{n_i}$. This process is analogous to the computation of the supremal (A, B) λ-contractive set as detailed in [50, Prop. 3.1], based on the contractivity condition defined in [50, Def. 2.4]. Hence, upon convergence, the set $\mathcal{O}_t^a$ satisfies the condition in Definition 5. □

Similarly to Definition 2, which defines polytopic tubes for the overall MAS, the following definition introduces tubes for each local subsystem, capturing the set of possible state trajectories under uncertainty.

Definition 6. Let $\mathbb{X}_{i,l|k}$ be a sequence of sets, $u_{i,l|k}$ a sequence of control inputs and $x_{n_i,l|k}$ a sequence of neighbors' states, where $l \in \mathbb{N}_0^N$. The sequence $\mathbb{X}_{i,l|k}$ is defined as a tube if:

$$\begin{aligned} &A_i(\theta_i)\mathbb{X}_{i,l|k} \oplus B_i(\theta_i)u_{i,l|k} \oplus A_{n_i}(\theta_i)x_{n_i,l|k} \\ &\oplus \mathbb{W}_i \subseteq \mathbb{X}_{i,l+1|k}, \quad \forall l \in \mathbb{N}_0^{N-1} \quad \forall \theta_i \in \Theta_i. \end{aligned} \tag{53}$$

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The tube dynamics evolve independently for each agent with a polytopic structure defined using a fixed matrix V_i and the decision variables $\boldsymbol{\alpha}_{i,k} = \{\alpha_{i,l|k}\}_{l=0}^{N}$,

$$\mathbb{X}_{i,l|k} = \{x_i \mid V_i x_i \leq \alpha_{i,l|k}\} \tag{54}$$

This parametrization allows the tube cross-sections to preserve the contractive property in the terminal region by embedding the λ-contractive set obtained in Lemma 4. In particular, the robust contractive property offers stronger guarantees than robust invariance and provides the condition for establishing recursive feasibility of the predictive scheme. Consequently, the tube construction can be decentralized, enabling each agent to compute its own cross-sections while accounting for the influence of neighboring trajectories. The linear inequalities defining the cross-section $\mathbb{X}_{i,l+1|k}$ can be rewritten exploiting the system dynamics from Equation (3) and the input policy from Equation (51) as follows,

$$\begin{aligned}
&V_i x_i^+ \leq \alpha_{i,l+1|k}, \Leftrightarrow\\
&V_i(A_i(\theta_i)x_i + B_i(\theta_i)u_{i,l|k} + A_{n_i}(\theta_i)x_{n_i,l|k} + w_{i,k}) \leq \alpha_{i,l+1|k}, \Leftrightarrow\\
&V_i(A_{K,i}(\theta_i)(x_i - x_{i,a}) + A_{K,n_i}(\theta_i)(x_{n_i,l|k} - x_{n_i,a}) + x_{i,a}\\
&\quad + B_i(\theta_i)c_{i,l|k} + w_{i,k}) \leq \alpha_{i,l+1|k}, \Leftrightarrow\\
&\quad V_i(A_{K,i}(\theta_i)x + (I_{n_i} - A_{K,i})x_{i,a} + A_{K,n_i}(\theta_i)(x_{n_i,l|k} - x_{n_i,a})\\
&\quad + B_i(\theta_i)c_{i,l|k} + w_{i,k}) \leq \alpha_{i,l+1|k}.\\
&\quad \forall \theta_i \in \Theta_{i,k}, \quad \forall x_i \in \mathbb{X}_{i,l|k}, \quad \forall i \in \mathcal{A}.
\end{aligned} \tag{55}$$

Thus, by applying Proposition 2, the polytopic set inclusion in Equation (53) can be rewritten in algebraic form if there exists $\Sigma_i(\theta) \geq 0$ such that,

$$\begin{aligned}
&\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,l|k} \leq \alpha_{i,l+1|k} - V_i\Big((I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)}))x_{i,a} + A_{K,n_i}\\
&\quad \big(x_{n_i,l|k} - x_{n_i,a}\big) + B_i c_{i,l|k}\Big) - \overline{w}_i,\\
&\Sigma_i(\overline{\theta}_{i,k}^{(j)})V_i = V_i A_{K,i}(\theta_{i,k}^{(j)}) \quad \forall j \in \mathbb{N}_1^{v_i}.
\end{aligned} \tag{56}$$

The matrices $\Sigma_i(\theta)$ must be computed for each $i \in \mathcal{A}$ agent defining LPs similar to Equation (31), while $\overline{w}_i$ exploits the worst-case disturbance affecting the i-th agent. In analogy, Proposition 1 and Equation (51) can be used to rewrite the polytopic constraints in Equation (8), if there exists $\Omega_i \geq 0$ such that,

$$\begin{aligned}
&F_i x_{i,l|k} + G_i u_{i,l|k} \leq \mathbf{1} \Leftrightarrow\\
&(F_i + G_i K_i)x_{i,l|k} + G_i K_{n_i}(x_{n_i,l|k} - x_{n_i,a})\\
&\quad + G_i c_{i,l|k} - G_i K_i x_{i,a} + G u_{i,a} \leq \mathbf{1} \Leftrightarrow\\
&\Omega_i V_i = F_i + G_i K_i, \quad \Omega_i \alpha_{i,l|k} + G_i K_{n_i}(x_{n_i,l|k} - x_{n_i,a})\\
&\quad + G_i c_{li,|k} + G_i u_{i,a} - G_i K_i x_{i,a} \leq \mathbf{1}.
\end{aligned} \tag{57}$$

where Ω_i are obtained for each $i \in \mathcal{A}$ agent by solving LPs similar to the one in Equation (33). Finally, imposing that $c_{i,N|k} = 0$, the terminal conditions can be derived such that,

$$\begin{aligned}
&\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,N|k} + V_i\Big((I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)}))x_{i,a} + A_{K,n_i}\big(x_{n_i,N|k} - x_{n_i,a}\big)\Big)\\
&\quad + \overline{w}_i \leq \alpha_{i,N|k},
\end{aligned} \tag{58a}$$

$$\Omega_i \alpha_{i,N|k} + G_i K_{n_i}\big(x_{n_i,N|k} - x_{n_i,a}\big) - G_i K_i x_{i,a} + G_i u_{i,a} \leq \mathbf{1}. \tag{58b}$$

4.3 | Control Algorithm

To enable distributed implementation, the centralized OCP in Equation (10) is reformulated as a local subproblem for each subsystem $i \in \mathcal{A}$. This reformulation introduces two classes of decision variables: The shared variables $\chi_{i,k}$ and the non-shared variables $\gamma_{i,k}$, defined as,

$$\chi_{i,k} = \{\hat{\mathbf{x}}_{i,k}, \hat{x}_{i,a}, \hat{\mathbf{x}}_{n_i,k}, \hat{x}_{n_i,a}\}, \quad \gamma_{i,k} = \{\mathbf{c}_{i,k}, \boldsymbol{\alpha}_{i,k}\ \hat{u}_{i,a}\}, \tag{59}$$

where $\hat{\mathbf{x}}_{i,k} = \{\hat{x}_{i,l|k}\}_{l=1}^{N}$, $\hat{\mathbf{x}}_{n_i,k} = \{\hat{x}_{n_i,l|k}\}_{l=1}^{N}$, $\mathbf{c}_{i,k} = \{c_{i,l|k}\}_{l=0}^{N-1}$, and $\boldsymbol{\alpha}_{i,k} = \{\alpha_{i,l|k}\}_{l=0}^{N-1}$ denote, respectively, the predicted trajectories of the agent states, neighboring agents states, control actions, and tube parameters. The cost function for the tracking problem is defined in Equation (14). The overall distributed optimization problem is solved using a cADMM scheme, wherein each subsystem solves the following local OCP at each time k,

$$\min_{\chi_{i,k},\gamma_{i,k}} \sum_{l=0}^{N-1} \ell_i(\hat{x}_{i,l|k}, u_{i,l|k}, \hat{x}_{i,a}, \hat{u}_{i,a}) + V_{f,i}(\hat{x}_{i,N|k}, \hat{x}_{i,a}) + V_{o,i}(\hat{y}_{i,a}, r_{i,k}), \tag{60a}$$

$$\text{s.t.} \quad \hat{x}_{i,0|k} = x_{i,k}, \quad \hat{x}_{n_i,0|k} = x_{n_i,k}, \quad \hat{x}_{i,0|k} \in \mathbb{X}_{i,0|k}, \tag{60b}$$

$$\hat{x}_{i,l+1|k} = A_i(\hat{\theta}_{i,k})\hat{x}_{i,l|k} + B_i(\hat{\theta}_{i,k})u_{i,l|k} + A_{n_i}(\hat{\theta}_{i,k})\hat{x}_{n_i,l|k}, \tag{60c}$$

$$(\hat{x}_{i,a}, \hat{u}_{i,a}, \hat{x}_{n_i,a}) \in \mathcal{Z}_{s,\mathcal{N}_i}, \qquad \hat{y}_{i,a} = C_i\ \hat{x}_{i,a} \in \mathcal{Y}_{i,s} \tag{60d}$$

$$u_{i,l|k} = K_i(\hat{x}_{i,l|k} - \hat{x}_{i,a}) + K_{n_i}(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a}) + \hat{u}_{i,a} + c_{i,l|k}, \tag{60e}$$

$$u_{i,N|k} = K_i(\hat{x}_{i,N|k} - \hat{x}_{i,a}) + K_{n_i}(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a}) + \hat{u}_{i,a}, \tag{60f}$$

$$\begin{aligned}
&\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,l|k} + V_i((I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)}))\hat{x}_{i,a} + A_{K,n_i}(\theta_{i,k}^{(j)})(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a})\\
&\quad + Bc_{i,l|k}) + \overline{w}_i \leq \alpha_{i,l|k+1},
\end{aligned} \tag{60g}$$

$$\begin{aligned}
&\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,N|k} + V_i((I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)}))\hat{x}_{i,a}\\
&\quad + A_{K,n_i}(\theta_{i,k}^{(j)})(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a})) + \overline{w}_i \leq \alpha_{i,N|k},
\end{aligned} \tag{60h}$$

$$\begin{aligned}
&\Omega_i \alpha_{i,l|k} + G_i K_{n_i}(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a}) + G_i c_{i,l|k} - G_i K_i \hat{x}_{i,a} + G_i \hat{u}_{i,a} \leq \mathbf{1},\\
&\Omega_i \alpha_{i,N|k} + G_i K_{n_i}(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a}) - G_i K_i \hat{x}_{i,a} + G_i \hat{u}_{i,a} \leq \mathbf{1},
\end{aligned} \tag{60i}$$

$$\forall j \in \mathbb{N}_1^{v_i}, \quad \forall l \in \mathbb{N}_0^{N-1}. \tag{60j}$$

where (60b) represents the initial states and the tube cross-section, while (60d) constraints the artificial variables that belong to the set of reachable steady-states and set points. Then, (60c) exploits the system dynamics and (60e) the control policy robustly stabilizes the error dynamics. The tube evolution within the cross-sections is defined by (60g) and (60h), which characterize the terminal cross-section. In the end, the system constraints are defined by (60i) and (60j).

Theorem 2 (Recursive Feasibility and Robust Constraint Satisfaction). *Let the terminal tube cross-section defined through parameter $\alpha_{i,N|k}$ to be robustly λ-contractive according to Definition 5. Then, the problem in Equation (60) is recursively feasible and guarantees robust satisfaction of state and input constraints for all $\theta_i \in \Theta_{i,k}$ and all admissible neighbor trajectories.*

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Proof. Recursive feasibility follows from the structure of the tube-based formulation and the λ-contractive construction of the terminal region, following the argument in [2, Th. 5.8]. Following standard arguments for tube-based MPC, robust constraint satisfaction is ensured by tightening the constraints such that the tube cross-sections remain within sets admissible for all $\theta_i \in \Theta_{i,k}$ and all admissible neighbor trajectories. As the true system trajectories evolve within these tubes, the original state and input constraints are satisfied by construction. □

As proposed by [30] and [26], the λ-contractivity condition in Equation (52) ensures bounded tube growth, which in turns guarantees the existence of feasible $\alpha_{i,N|k}$.

The estimate of the uncertain parameter θ_i is provided through a LMS filter, which has been reformulated as follows to account for the influence of neighboring agents,

$$\hat{\theta}_{i,k} = \arg\min_{\theta_i \in \Theta_{i,k}} \|\theta_i - (\hat{\theta}_{i,k-1} + \mu D_{\mathcal{N}_i}(x_{i,k-1}, u_{i,k-1}, x_{n_i,k-1})^T e_{i,k})\|, \tag{61}$$

where $e_{i,k}$ and $D_{\mathcal{N}_i}(x_i, u_i, x_{n_i})$ are given by,

$$e_{i,k} = x_{i,k} - A_i(\hat{\theta}_{i,k-1})x_{i,k-1} - B_i(\hat{\theta}_{i,k-1})u_{i,k-1} - A_{n_i}(\hat{\theta}_{i,k-1})x_{n_i,k-1}, \tag{62a}$$

$$D_{\mathcal{N}_i}(x_i, u_i, x_{n_i}) = \mathrm{row}_{h \in \mathbb{N}_1^{q_i}}(A_i^{(h)}x + B_i^{(h)}u + A_{n_i}^{(h)}x_{n_i}). \tag{62b}$$

The constraints in Equation (41) are omitted in the OCP in Equation (60). This omission arises because these states are inherently constrained within the optimization problems of neighboring agents. Explicitly enforcing such constraints would be redundant and would unnecessarily increase computational complexity, given that the convergence of the consensus algorithm ensures their consistency.

The workflow of the proposed control strategy is summarized in Algorithm 3.

ALGORITHM 3 | Distributed Robust Adaptive MPC Algorithm.

```
Require: Θ_{i,0}, (A_i(θ), B_i(θ), A_{N_i}), Q_i, R_i, N, Z_i, X_{N_i}, r_{i,k}
   for each i ∈ A
1: Offline:
2: Initialise k ← 0
3: for each i ∈ A do
4:    Initialise Θ_{i,k} ← Θ_{i,0} ⊆ Θ_i
5:    Compute pre-stabilising gain K_i and K_{n_i}
6:    Compute robust λ-contractive set S using
      Alg. 2
7:    Solve LPs in (29),(31),(33) to obtain
      w̄_i, Σ_i, and Ω_i
8: end for
9: Online:
10: for each time step k do
11:    if k > 0 then
12:       Solve SM problem in (21), to tighten Θ_{i,k}
13:    end if
14:    for each i ∈ A in parallel do
15:       Solve (61), to retrieve LMS estimate
          of θ̂_{i,k}
16:       Update constraints (60d),(60c),(60e),
          (60f),(60g),(60h),(60i),(60j) with θ̂_{i,k}
          and vertices of Θ_{i,k}
17:       Execute distributed cADMM in Alg. 1
          with f(χ_i, γ_i):(60)
18:       Apply u_{i,k} ← u_{i,0|k}
19:    end for
20: end for
```

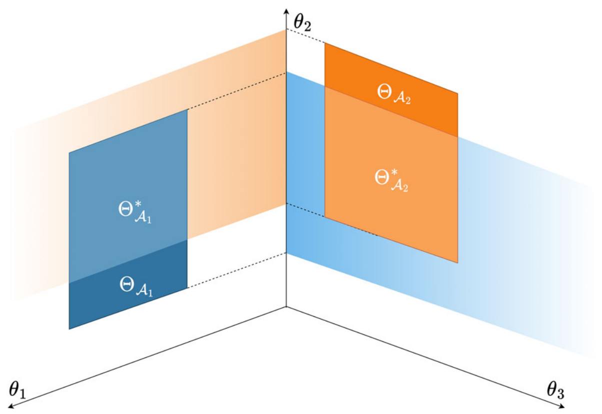

FIGURE 1 | Distributed parameter SM with box uncertainties, for a system of two agents sharing a single parameter.

Although Algorithm 3 adopts a centralized SM to update $\Theta_{i,k}$, a fully distributed identification strategy is proposed in the following section. A comparison of the two approaches is further discussed in Section 5.

4.4 | Distributed Identification

This subsection presents the distributed parameter set identification method. As pointed out by [18], this method is particularly suitable for agents sharing uncertain parameters. The identification process of the set bounds for the shared parameters may differ among the agents, depending on the available data constructing the non-falsified parameter set. Hence, to enhance the learning process, each agent collects supplementary data from the in-neighbors set $\mathcal{N}_i$. The neighbors' parameter set is projected onto the axis of the shared parameters, and the resulting set is augmented to match the dimension of the actual agent parameter subspace. The intersection between the sets may produce a tighter set, providing a mutual improvement of the identification algorithm performance, as shown in the illustrative example in Figure 1. Moreover, by employing box uncertainties that can be represented through hyperrectangles, the operation in Step 13 maintains the fixed complexity of the updated set.

Algorithm 4 outlines the distributed parameter set identification.

The non-falsified parameter set in Step 5 is constructed using the data from the agents in $\mathcal{N}_i$.

$$\begin{aligned}\Phi_{\mathcal{N}_i,k} &= \{\theta_i : x_{i,k} - A_i(\theta_i)x_{i,k-1} \\ &\quad - B_i(\theta_i)u_{i,k-1} - A_{n_i}(\theta_i)x_{n_i,k-1} \in \mathbb{W}_i\} \\ &= \{\theta_i : -H_{w_i}D_{\mathcal{N}_i}(x_{i,k-1}, u_{i,k-1}, x_{n_i,k-1})\theta_i \\ &\quad \leq b_{w_i} + H_{w_i}d_{\mathcal{N}_i,k}\} \\ &= \{\theta_i : H_{\Phi_{\mathcal{N}_i},k}\theta_i \leq b_{\Phi_{\mathcal{N}_i},k}\}.\end{aligned} \tag{63}$$

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ALGORITHM 4 | Distributed Parameter Set Identification.

Require: $\Theta_{i,0}$, $\mathbb{W}_i$, $(A_i(\theta_i), B_i(\theta_i), A_{\mathcal{N}_i}(\theta_i))$, $(x_{i,k}, x_{i,k-1}, u_{i,k-1}, x_{\mathcal{N}_i,k-1})$, for each $i \in \mathcal{A}$
1: **Initialise** $k \leftarrow 0$
2: **for** each agent $i \in \mathcal{A}$ **do**
3: **Initialise** $\Theta_{i,k} \leftarrow \Theta_{i,0} \subseteq \Theta_i$
4: **for** each time step k **do**
5: **Compute** local non-falsified parameter set $\Phi_{\mathcal{N}_i k}$
6: **Update** local parameter set $\tilde{\Theta}_{i,k+1} \leftarrow \Theta_{i,k} \cap \Phi_{\mathcal{N}_i k}$
7: **Solve** LP in Equation (65) to find fixed complexity $\Theta_{i,k+1} \supseteq \tilde{\Theta}_{i,k+1}$
8: **Get** $\Theta_{\mathcal{N}_i,k+1}$ from neighbours
9: **for** each $j \in \{\mathcal{N}_i \setminus \{i\}\}$ **do**
10: **Project** $\Theta_{j,k+1}$ on the axis of shared parameters
11: **Lift** $\Theta_{j,k+1}$ to $\tilde{\Theta}_{j,k+1}$ to fit $\dim(\Theta_{i,k+1})$
12: **end for**
13: **Update** local parameter set $\Theta_{i,k} \leftarrow \Theta_{i,k} \bigcap_{j\in\{\mathcal{N}_i\setminus\{i\}\}} \tilde{\Theta}_{j,k+1}$
14: **end for**
15: **end for**

where $D_{\mathcal{N}_i}$ has been defined in Equation (62), while $d_{\mathcal{N}_i k}$ is given by,

$$d_{\mathcal{N}_i k} = A_i^{(0)} x_{i,k-1} + B_i^{(0)} u_{i,k-1} + A_{n_i}^{(0)} x_{n_i,k-1} - x_{i,k}. \tag{64}$$

The fixed complexity on $\Theta_{i,k+1}$ is enabled by Step 7 solving the following LP,

$$\begin{aligned} \left[b_{\Theta,k+1}\right]_i &= \min_{b,\Xi_i} b \\ \text{s.t.} \quad \Xi_i H_{\tilde{\Theta}} &= \left[H_\Theta\right]_i, \quad \Xi_i b_{\tilde{\Theta},k} \le b, \quad \Xi_i \ge 0. \end{aligned} \tag{65}$$

where $H_{\tilde{\Theta}}$ and $b_{\tilde{\Theta},k}$ are the matrix and vector defining the set $\tilde{\Theta}_{i,k+1}$ obtained in Step 6. This distributed approach makes the identification more effective, thereby relaxing the conservatism of the OCP.

4.5 | Extension for Trajectory Tracking

This subsection extends the method in Section 4.3 for piece-wise constant signals to trajectory tracking by exploiting the properties of the artificial reference method to guarantee the feasibility of all the set points along the trajectories.

Analogously to what was reviewed in Section 3.1 for the constant reference tracking problem, the trajectory tracking problem is reformulated through the introduction of the *artificial trajectories*.

Definition 7 (Artificial Trajectories). The artificial trajectories of system $x^+ = Ax + Bu$, which is subject to $\mathcal{Z} = \{(x,u) | Fx + Gu \le \mathbf{1}\}$, encompass all N-long sequences of feasible states and inputs (trajectories) that culminate in the set of reachable steady-states (Definition 1). These trajectories can be defined as follows,

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_l \\ u_l \end{bmatrix} = \begin{bmatrix} x_{l+1} \\ r_l \end{bmatrix}, \quad \forall (x_l, u_l) \in \mathcal{Z}, \quad \forall l \in \mathbb{N}_0^{N-1}, \tag{66a}$$

$$\begin{bmatrix} A - I_n & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_N \\ u_N \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ y_N \end{bmatrix}, \quad \forall (x_N, u_N) \in \mathcal{Z}_s, \quad \forall y_N \in \mathcal{Y}_s \tag{66b}$$

The sequence of artificial variables to be adopted in the OCP is defined by $(\mathbf{x}_a, \mathbf{u}_a)$ satisfying Equation (66). Consequently, the OCP should be reformulated for tracking the artificial trajectories adopting the stage cost $\ell(x_{l|k}, u_{l|k}, x_{a,l|k}, u_{a,l|k})$ and the terminal cost $V_f(x_{N|k}, u_{N|k}, x_{a,N|k}, u_{a,N|k})$, depending on the new variables. In addition, the offset cost must be updated to penalize the deviation between the artificial and the desired trajectory,

$$V_o(\mathbf{r}_k, \mathbf{y}_{a,k}) = \sum_{l=0}^{N-1} \|y_{a,l|k} - r_{l+k}\|_T^2 + \|y_{a,N|k} - r_{N+k}\|_S^2, \tag{67}$$

Definition 7 can be extended to the artificial trajectories of each agent in an interconnected MAS, which are given by $(\mathbf{x}_{i,a}, \mathbf{u}_{i,a}, \mathbf{x}_{n_i,a})$. For the system in Equation (3) subject to constraints in Equations (8) and (41), the artificial trajectories are estimated along the horizon as follows,

$$\begin{bmatrix} A_i(\hat{\theta}_{i,k}) & B_i(\hat{\theta}_{i,k}) & A_{n_i}(\hat{\theta}_{i,k}) \\ C_i & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_{i,a,l|k} \\ \hat{u}_{i,a,l|k} \\ \hat{x}_{n_i,a,l|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{i,a,l+1|k} \\ \hat{y}_{i,a,l|k} \end{bmatrix},$$
$$\forall\, (\hat{x}_{i,a,l|k}, \hat{u}_{i,a,l|k}) \in \mathcal{Z}_i \quad \forall\, \hat{x}_{n_i,a,l|k} \in \mathcal{X}_{n_i} \quad \forall\, l \in \mathbb{N}_0^{N-1}, \tag{68a}$$

$$\begin{bmatrix} A_i(\hat{\theta}_{i,k}) - I_{n_i} & B_i(\hat{\theta}_{i,k}) & A_{n_i}(\hat{\theta}_{i,k}) \\ C_i & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_{i,a,N|k} \\ \hat{u}_{i,a,N|k} \\ \hat{x}_{n_i,a,N|k} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hat{y}_{i,a,N|k} \end{bmatrix},$$
$$\forall\, (\hat{x}_{i,a,N|k}, \hat{u}_{i,a,N|k}, \hat{x}_{n_i,a,N|k}) \in \mathcal{Z}_{\mathcal{N}_i,s}, \quad \forall\, \hat{y}_{i,a,N|k} \in \mathcal{Y}_{i,s} \tag{68b}$$

Accordingly, the OCP in Equation (60) can be reformulated by updating constraints in Equations (60e) to (60j) as follows

$$u_{i,l|k} = K_i(\hat{x}_{i,l|k} - \hat{x}_{i,a,l|k}) + K_{n_i}(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a,l|k}) + \hat{u}_{i,a,l|k} + c_{i,l|k}, \tag{69a}$$

$$u_{i,N|k} = K_i(\hat{x}_{i,N|k} - \hat{x}_{i,a,N|k}) + K_{n_i}(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a,N|k}) + \hat{u}_{i,a,N|k}, \tag{69b}$$

$$\begin{aligned} &\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,l|k} + V_i(I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)})\hat{x}_{i,a,l|k} + A_{K,n_i}(\theta_{i,k}^{(j)})(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a,l|k}) \\ &\quad + Bc_{i,l|k}) + \overline{w}_i \le \alpha_{i,l|k+1}, \end{aligned} \tag{69c}$$

$$\begin{aligned} &\Sigma_i(\overline{\theta}_{i,k}^{(j)})\alpha_{i,N|k} + V_i((I_{n_i} - A_{K,i}(\theta_{i,k}^{(j)}))\hat{x}_{i,a,N|k} \\ &\quad + A_{K,n_i}(\theta_{i,k}^{(j)})(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a,N|k})) + \overline{w}_i \le \alpha_{i,N|k}, \end{aligned} \tag{69d}$$

$$\begin{aligned} &\Omega_i\alpha_{i,l|k} + G_iK_{n_i}(\hat{x}_{n_i,l|k} - \hat{x}_{n_i,a,l|k}) + G_ic_{i,l|k} \\ &\quad + G_i\hat{u}_{i,a,l|k} - G_iK_i\hat{x}_{i,a,l|k} \le \mathbf{1}, \end{aligned} \tag{69e}$$

$$\Omega_i\alpha_{i,N|k} + G_iK_{n_i}(\hat{x}_{n_i,N|k} - \hat{x}_{n_i,a,N|k}) + G_i\hat{u}_{i,a,N|k} - G_iK_i\hat{x}_{i,a,N|k} \le \mathbf{1}. \tag{69f}$$

Moreover, the artificial variables definition in Equation (60d) must be replaced by (66), while the cost function for the trajectory tracking problem becomes,

$$\begin{aligned} &J_i(\hat{\mathbf{x}}_{i,k}, \mathbf{u}_{i,k}, \hat{\mathbf{x}}_{i,a,k}, \hat{\mathbf{u}}_{i,a,k}, \hat{\mathbf{y}}_{i,a,k}, \mathbf{r}_{i,k}) \\ &\quad = \sum_{l=0}^{N-1} \ell_i(\hat{x}_{i,l|k}, u_{i,l|k}, \hat{x}_{i,a,l|k}, \hat{u}_{i,a,l|k}) + V_{f,i}(\hat{x}_{i,N|k}, \hat{x}_{i,a,N|k}) \\ &\quad + V_{o,i}(\hat{\mathbf{y}}_{i,a,k}, \mathbf{r}_{i,k}). \end{aligned} \tag{70}$$

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As in the Section 4, the following problem represents the local subproblem solved by each subsystem $i \in \mathcal{A}$ within the distributed cADMM framework. Given the set of decision variables,

$$\chi_{i,k} = \{\hat{\mathbf{x}}_{i,k}, \hat{\mathbf{x}}_{i,a,k}, \hat{\mathbf{x}}_{n_i,k}, \hat{\mathbf{x}}_{n_i,a,k}\}, \quad \gamma_{i,k} = \{\mathbf{c}_k, \boldsymbol{\alpha}_k\ \hat{\mathbf{u}}_{i,a,k}\}, \tag{71}$$

the trajectory tracking OCP can be formulated as

$$\min_{\chi_{i,k},\gamma_{i,k}} (70), \ \text{s.t.}\ (60b), (60c), (68), (69), \forall j \in \mathbb{N}_1^{v_i}, \ \forall l \in \mathbb{N}_0^{N-1}. \tag{72}$$

Remark 2. Recursive Feasibility of the problem in Equation (72) along with robust constraint satisfaction, follows from the contractive properties of the terminal tube cross-section and constraint tightening strategy, as outlined in Lemma (2).

5 | Illustrative Examples

This section presents an illustrative example to demonstrate the effectiveness of the proposed tracking control strategies. It provides a comprehensive comparison of tracking performance in both centralized and distributed settings, relative to the nominal and robust adaptive MPCs for tracking. The outcomes show that the proposed methods can achieve the desired goal while maintaining the scalability of the OCP. Moreover, the artificial reference methodology is shown to perform well for the problem of tracking a reference trajectory. The example focuses on interconnected mass-spring-damper systems consisting of M agents, similar to the examples proposed by [18, 42] and [46], depicted in Figure 2.

For the sake of simplicity, the multiplicative uncertainty has been considered only for the springs' stiffnesses, assuming the same mass for each agent but different stiffness and damping coefficients. Multiple simulations were conducted with 3,5,7, and 9 agents connected in a graph with line topology using the parameters outlined in Table 1. In the first set of simulations, the reference signals are supposed to be constant for the whole prediction horizon, while the second set considers a reference trajectory along the prediction horizon.

The system's states include the position and velocity of each mass, while the input consists of the applied forces and the output consists of the agent's position. The states x_i are constrained within $\pm\left[2.22\ \text{m}\ \ 6\ \text{m/s}\right]^T$, and the inputs are bounded between ± 25 N, while the disturbance is assumed to be bounded by ± 0.05 N. In addition, the dynamic model adopts Euler discretization at a sampling time of $T_s = 0.25$ s. The weight matrices for the cost function are $Q_i = 10 I_2$ and $R_i = 5$ for all agents. The terminal weight P_i and the gains K_i and $K_{n,i}$ can be obtained by the procedure defined in Section 4.1. The centralized algorithms employ the same weights arranged in block-diagonal matrices, and the prediction horizon has been set to $N = 10$. The cADMM presented in Section 3.3 was implemented with $\rho = 50$ and $N_{\max} = 500$. Simulations were conducted using MATLAB R2022b [51] on a 2.6 GHz Intel Core i7 with 32 GB of RAM. The YALMIP [52] toolbox was utilized to formulate the OCP, while MPT3 [53] to manage operations involving polyhedral sets. The MOSEK [54] solver was employed for solving LPs and SDPs, while OSQP [55] was used for Quadratic Program (QP)s.

The simulation code and implementation of the proposed DRAMPC framework are available at: https://github.com/Falier0s/IJRNC_material.

5.1 | Constant Reference Signal

The following simulations show the results of the controller proposed in Section 4.3 to extend the DRAMPC formulation

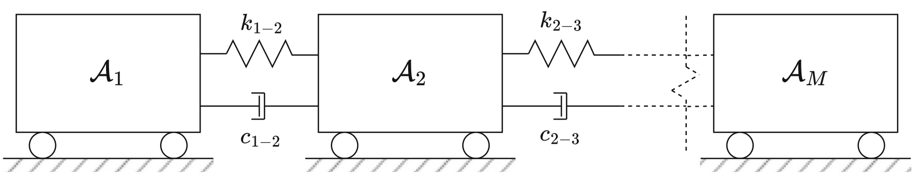

FIGURE 2 | Interconnection scheme of M masses with springs and dampers.

TABLE 1 | System nominal parameters, uncertainties, and real parameters.

Agent	m [kg]	c [Ns/m]	k [N/m]	Δk [N/m]	k_r [N/m]
1	1	2	1.4	0.18	1.26
2	1	2	2.6	0.23	2.39
3	1	1.65	1.8	0.25	2.0
4	1	1.8	2.2	0.15	2.34
5	1	2.6	1.5	0.11	1.4
6	1	1.9	2	0.2	1.85
7	1	2.4	1.7	0.17	1.77
8	1	1.7	1.9	0.2	1.75
9	1	—	—	—	—

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for tracking using artificial references. Thus, the reference signals are treated as constant for the whole prediction horizon. Hence, the artificial variables $(x_a, u_a, x_{n,a})$ identify the artificial steady states to be tracked. Accordingly, these examples adopt a piecewise-constant function with a step change every 12.5 s, and the offset cost weight is defined by $S_i = sI_{p_i}$, where s is an arbitrarily large scalar, set to $s = 1000$.

Figure 3 presents the tracking results for both nominal and robust adaptive MPC solved with both a centralized and distributed approach. Specifically, the Figure shows the output of the first agent in the configuration having five interconnected agents.

The overall tracking performance has been evaluated by computing the RMS error at steady condition, as reported in Table 2.

This table highlights the higher accuracy of the robust adaptive MPC concerning the nominal formulation, which is consistent in centralized and distributed configurations. These findings are achieved thanks to the adaptive estimation of the system parameters. Additionally, Figure 3 underlines the control algorithm's robustness, as the RAMPC and DRAMPC provide an admissible output even if the reference signal approaches the boundaries. Conversely, the MPC and the DMPC violate the feasible region. To show this violation, the MPC and DMPC constraints have been relaxed using a slack variable.

Further insights can be gained by comparing the RAMPC and the DRAMPC to assess the benefits of distributed optimization and identification.

An increasing M leads to a higher complexity of the OCP because of the growing number of constraints and decision variables. Therefore, using a distributed strategy becomes highly beneficial, as shown in Figure 4, which compares the computational time of RAMPC and DRAMPC over an increasing number of agents. This comparison shows how the computational burden of the distributed OCP does not vary significantly, while the centralized solution shows an exponential trend.

The distributed SM allows for a more precise estimation of the uncertain parameters, with a faster tightening of the uncertainty set Θ_k, as shown in Figure 5.

Regardless of M, the distributed identification turns out to be more effective in terms of reduction of the uncertainty set, as shown in Table 3. The Reduction Factor (RF) has been evaluated for each parameter by projecting Θ_k on each θ_j axis, and then

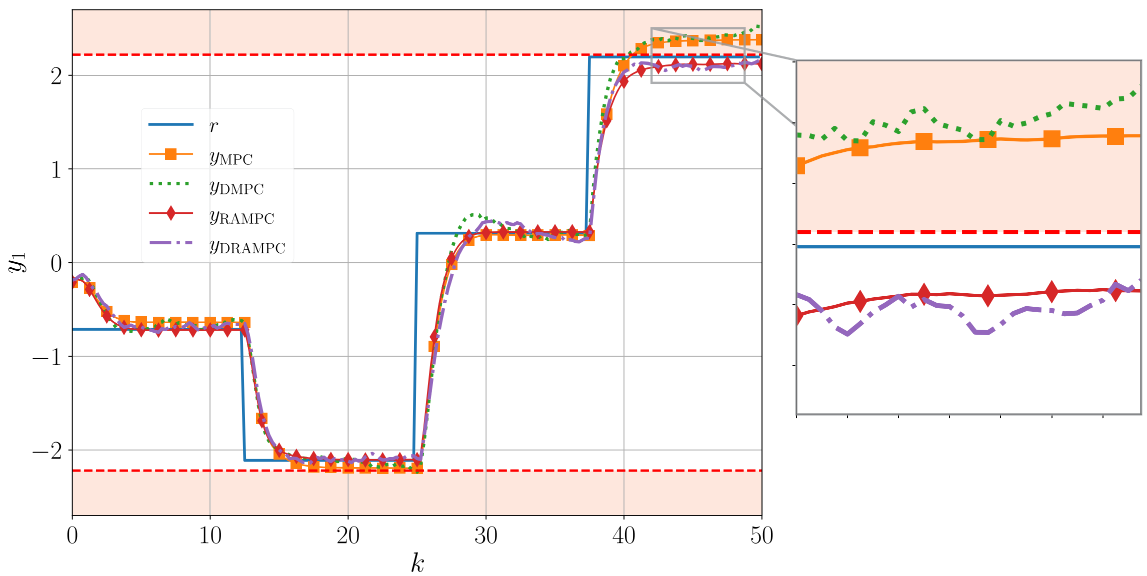

FIGURE 3 | On the left: Evolution over the discrete time steps k of the output y_1 for agent $\mathcal{A}_1$ in the five-agent simulation under different MPC methods. On the right: Detailed view highlighting constraint violations of non-robust controllers. The solid blue line denotes the reference: Centralized MPC (yellow, solid with squares), distributed MPC (green, dotted), centralized Robust Adaptive MPC (red, solid with diamonds), and distributed Robust Adaptive MPC (purple, dash-dot). Output constraints are shown as light red bands with dashed red boundaries.

TABLE 2 | Tracking errors for different MPC methods.

	3 agents	5 agents	7 agents	9 agents
MPC	1.111E-1	8.300E-2	9.988E-2	1.097E-1
DMPC	1.304E-1	9.612E-2	1.022E-1	1.029E-1
RAMPC	1.659E-2	2.707E-2	2.234E-2	3.174E-2
DRAMPC	3.509E-2	4.825E-2	4.263E-2	2.978E-2

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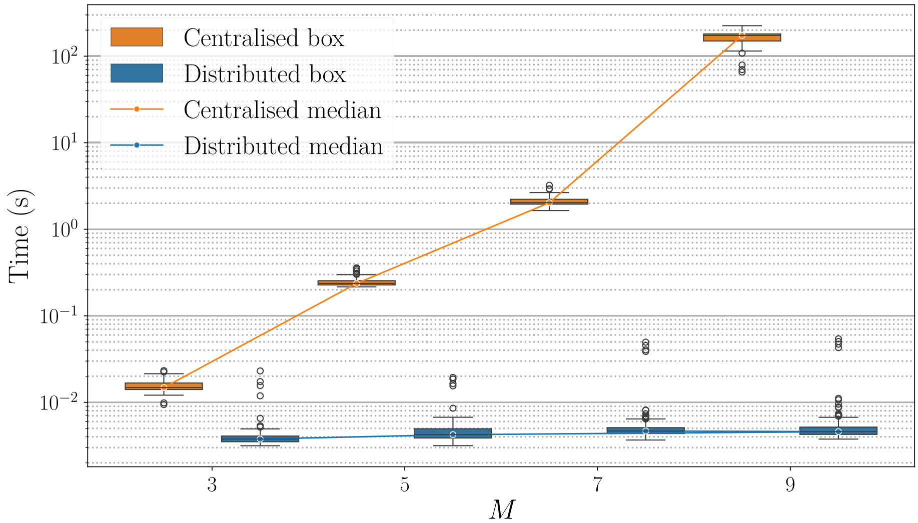

FIGURE 4 | Boxplot of computational time in log-scale (y-axis) for different numbers of agents M (x-axis, $M = \{3, 5, 7, 9\}$). Orange denotes the centralized RAMPC solution and blue the distributed DRAMPC solution. For each box, values were obtained by collecting the solution times at each control step over an entire simulation; in the distributed case, per-step values correspond to the average computation time across agents. Median values across simulations are connected with solid lines of the respective color.

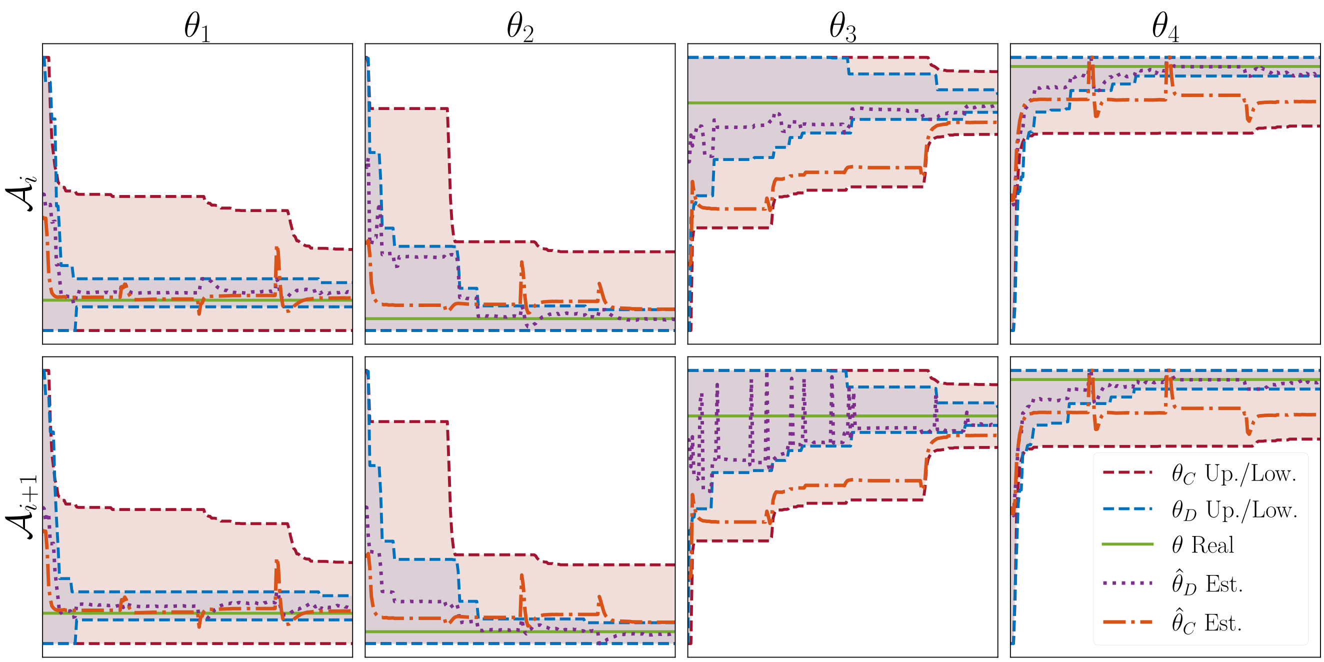

FIGURE 5 | Comparison of identified uncertainty sets Θ_k for the 5-agent case. Each column corresponds to one parameter θ_i ($i = 1, \ldots, 4$), and each row shows the projection for the two agents sharing that parameter ($\mathcal{A}_i$ above, $\mathcal{A}_{i+1}$ below). Light red shading with dashed red bounds indicates the projection of the centralized (C) identified set over time; light blue shading with dashed blue bounds indicates the projection of the distributed (D) identified set over time. The true parameter value is shown in solid green. Parameter estimates are reported as dotted purple (distributed) and dash-dotted orange (centralized).

TABLE 3 | RFs of the uncertainty set for centralized and distributed SM.

	3 agents	5 agents	7 agents	9 agents
Distributed	93.638%	92.547%	84.772%	88.167%
Centralized	79.253%	73.343%	52.880%	65.754%

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computing the interval size at the beginning and at the end of the simulation as follows,

$$\mathrm{RF}_j = \left(1 - \frac{\max \mathrm{Proj}_{\theta_j}\left(\Theta_{T_f}\right) - \min \mathrm{Proj}_{\theta_j}\left(\Theta_{T_f}\right)}{\max \mathrm{Proj}_{\theta_j}\left(\Theta_0\right) - \min \mathrm{Proj}_{\theta_j}\left(\Theta_0\right)}\right), \quad \forall j \in \mathbb{N}_1^q. \tag{73}$$

For centralized identification, the reported value is the average over all parameters, whereas for distributed identification, it is first averaged among the agents sharing the same parameter and then across all parameters.

The results are consistent with the fact that sharing information among agents during the identification allows different non-falsified parameter sets, hence different solutions of Step 7 in Algorithm 4. The conservativeness in this process arises from the fixed complexity requirement. However, the intersection of solutions within the neighborhood proves to be less conservative than addressing the problem in a centralized manner.

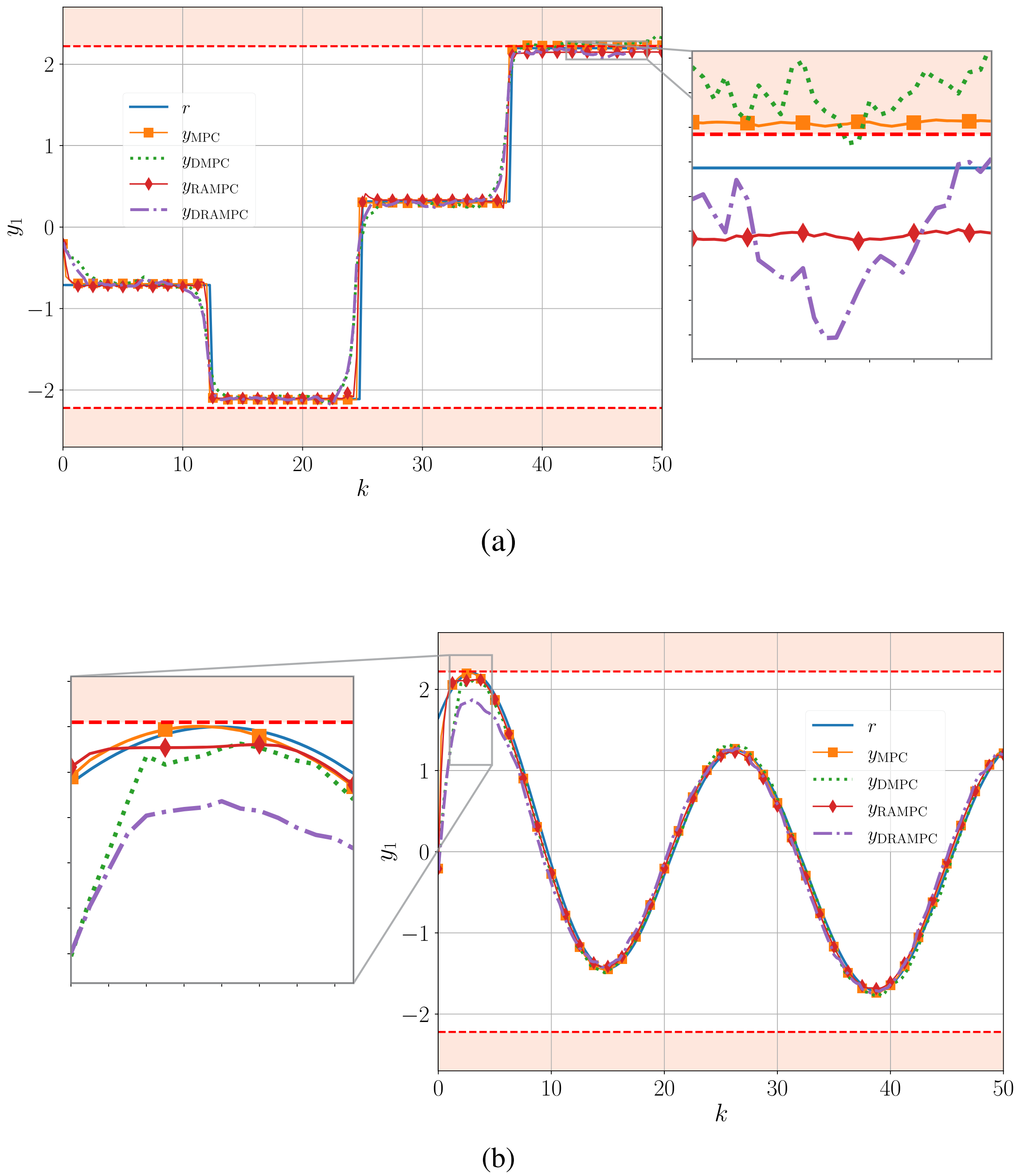

FIGURE 6 | Output evolution of y_1 over the discrete time steps k for agent $\mathcal{A}_1$ under trajectory tracking (a) piecewise-constant reference and (b) harmonic reference. In (a), the right subplot provides a detailed view of constraint violation for non-robust controllers. In (b), the left subplot zoom highlights the conservative behavior close to the boundary. The solid blue line denotes the reference: Centralized MPC (yellow, solid with squares), distributed MPC (green, dotted), centralized Robust Adaptive MPC (red, solid with diamonds), and distributed Robust Adaptive MPC (purple, dash-dot). Output constraints are shown as light red bands with dashed red boundaries.

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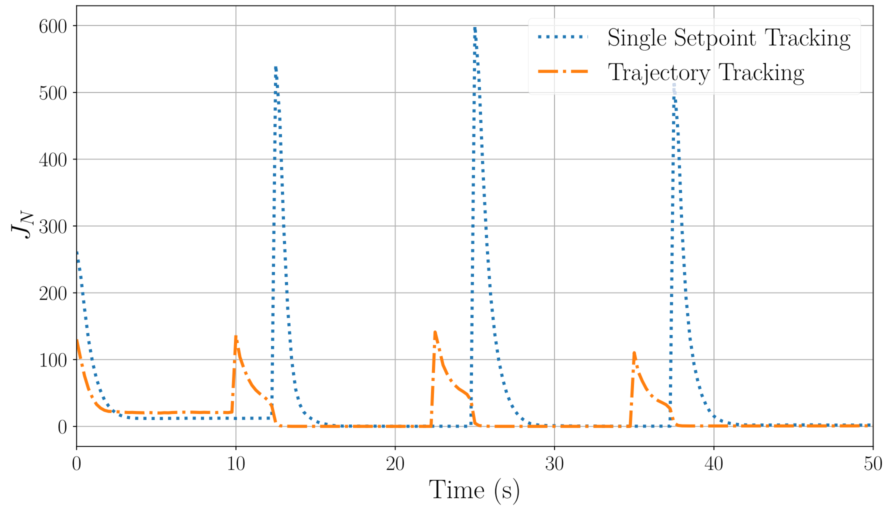

FIGURE 7 | Evolution of the cost function over time under the proposed DRAMPC. The plot compares single setpoint tracking (dotted blue) and trajectory tracking (dash-dotted orange) using a piecewise-constant reference.

Indeed, in the formulation of the set inclusion $\Theta_{k+1} \supseteq \tilde{\Theta}_{k+1}$ with a fixed complexity, the number of constraints grows with the number of hyperplanes. The distributed SM operates for each agent on projections of Θ_k along the axis of the corresponding parameters. Accordingly, the number of constraints in the distributed SM is significantly reduced, still guaranteeing a successful identification.

5.2 | Reference Trajectory

The following simulations show the results of the controller proposed in Section 4.5, which modifies the original formulation of the artificial reference for effectively tracking trajectories. The reference is defined as a sequence of desired positions along the prediction horizon, and the strengths are demonstrated by simulating two reference trajectories: A piecewise-constant function and a harmonic function. In particular, the piecewise-constant function is the same adopted in Section 5.1.

The simulations adopt the same parameters as the previous examples, with the newly defined offset cost weights $T_i = 10I_{p_i}$ and $S_i = 50I_{p_i}$ for boosting the artificial trajectories whose terminal output $y_{a,N}$ is closer to the desired r_{k+N}. The results are presented in Figure 6.

It can be observed that the proposed extension of the artificial reference approach for trajectory tracking is effective in both centralized and distributed configurations. A more responsive control behavior is achieved, as evidenced by the faster convergence to the desired reference trajectory in Figure 6a with respect to the results depicted in Figure 3. This improvement arises from the controller's ability to anticipate future reference changes, thus enabling proactive control action. This behavior is further illustrated in Figure 7, which compares the cost function evolution of the proposed DRAMPC when operating under a reference treated as constant for the whole horizon vs. a sequence of references.

Notably, the cost function associated with trajectory tracking exhibits shifted and generally lower peaks. This shift reflects that the upcoming reference steps are available N steps in advance within the prediction horizon. This anticipative behavior can be modulated by appropriately tuning the weighting matrices and the prediction horizon, allowing the controller to start adjusting its output earlier. Furthermore, the proposed approach supports the tracking of more complex reference profiles, as shown in Figure 6b.

In all scenarios, the robust constraint satisfaction properties of both the RAMPC and DRAMPC formulations are confirmed, consistently ensuring constraint adherence across varying tracking tasks. However, the conservative nature of the proposed DRAMPC becomes more evident in the scenario of Figure 6b. As shown in the zoomed view on the left of the figure, the DRAMPC is not able to follow reference signals that approach the constraint boundary too closely. This behavior may be interpreted as a structural limitation of the formulation, whereby no admissible artificial trajectory exists that simultaneously ensures constraint satisfaction and convergence to the desired reference.

Although this approach requires an increased number of decision variables in the OCP, the computational effort of the distributed configuration remains limited. For instance, the tracking of the harmonic function for the 5-agent system requires a median of 2 iterations of the cADMM and a computational time of 0.00368 s for the DRAMPC and 0.29801 s for the RAMPC.

6 | Conclusions

The current study proposed a novel DRAMPC strategy to perform tracking in MAS. This formulation faces the effects of dynamic coupling and limited communication while guaranteeing the control robustness to additive disturbances and parametric uncertainties. The neighbors' influence is tackled by adopting a strategy based on controlled robust invariant sets. In addition, the feedback control law includes a structured gain to robustly stabilize the system around steady-state conditions. The definition of the above solutions enables recursive feasibility and

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asymptotic stability of the resulting scheme. The recursive feasibility is guaranteed in piecewise reference tracking by adopting the artificial reference and extending the consensus constraint to the artificial variables. Finally, the tracking formulation has been extended beyond piecewise constant references to encompass trajectory tracking, thereby facilitating robust control of time-varying signals. The effectiveness of this approach has been validated via numerical simulations of MAS with an increasing number of agents, demonstrating its computational efficiency, scalability, and improved tracking performance. Further studies will extend the distributed approach to offline computations and improve the structured feedback gain formulation to explicitly attenuate disturbances.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The authors have nothing to report.

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