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Doctoral Dissertation
Doctoral Program in Artificial Intelligence (37^{th} cycle)

On Some Evolutionary Game-Theoretic Vector Flows

By

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Declaration

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On Some Evolutionary Game-Theoretic Vector Flows

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In this thesis, we present our theoretical findings on a class of continuous-time dynamical systems that found application in computer vision and pattern recognition. These systems are studied in evolutionary game theory, where they are known under the name of replicator dynamics, and are related to Shahshahani gradient systems. Additionally, they provide a first-order continuous-time method for optimization subject to probability constraints, and are related to proximal gradient descent. Among these dynamical systems, two typologies were the main focus of our research. The first typology consists of the so called clique-finding replicator dynamics, which M. Pelillo and I. Bomze independently introduced to heuristically attack the maximum clique problem, and which were further generalized to address several computer vision tasks. In this thesis, we study the stationary points of these dynamics and we describe some advances related to the information encoded in these points. The second typology is related to a classical information-theoretical problem, namely computing the capacity of a discrete memoryless channel. To address this problem, we introduce the capacity-computing differential equations, and we study their theoretical properties. We establish a link between these systems and the classical Blahut-Arimoto algorithm. Moreover, by studying this information-theoretic application, we describe how the presence of singularities can be theoretically overcome for more general systems within the examined class.