Doctoral Dissertation
Doctoral Program in Computer and Control Engineering (37th cycle)

Quantum Computing:

Analysis and design of quantum algorithms for financial applications

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Summary

Quantum computing has the potential to become a powerful tool for solving complex problems that classical computation struggles to tackle. The financial sector presents numerous such problems, and quantitative finance is considered one of the fields where the benefits of employing quantum processing units might first become evident.

The purpose of this manuscript is to analyze quantum computing as a paradigm for solving certain computational problems in the domain of finance. These are problems for which traditional classical methods may be unsatisfactory in terms of speed or accuracy. Among these, we can certainly include tasks such as portfolio optimization, risk management, and derivative pricing. These problems have intrinsic complexity due to their combinatorial or stochastic nature. Quantum approaches have shown potential in efficiently solving some of these tasks, although significant limitations in applying these approaches on scales relevant to the industry remain. This is undoubtedly partially due to the limitations of currently available quantum hardware, which is restricted by the number of qubits and low resilience to errors.

This work begins with a brief introduction to the fundamentals of quantum computing. It presents the key characteristics that distinguish quantum bits (qubits) from their classical counterparts and introduces the concept of the Bloch sphere as an effective geometric representation of the state of a single qubit. Subsequently, the main quantum gates are briefly discussed to illustrate how they act on qubit states and manipulate their phases. The discussion is then extended to multi-qubit systems, through which fundamental concepts that characterize quantum computation, such as entanglement, are introduced.

To conclude the introductory section, several fundamental and well-known quantum algorithms are presented. The Quantum Fourier Transform is introduced, highlighting its role as the quantum counterpart of the classical discrete Fourier Transform. Building on this foundational quantum subroutine, the Quantum Phase Estimation (QPE) algorithm is then illustrated, which leverages the phase kickback phenomenon to extract the eigenvalues of unitary operators. As a direct application of QPE, Shor's factorization algorithm is presented, demonstrating its ability to efficiently decompose large integers into prime factors, thereby offering an exponential advantage over classical methods. Next, Grover's search algorithm is introduced,

showcasing its quadratic speedup for searching in unsorted databases. Finally, attention is given to quantum amplitude amplification and estimation methods, which can be seen as generalizations of Grover's algorithm to arbitrary superpositions in the case of amplitude amplification and as an extension of quantum counting in the case of amplitude estimation.

Based on these premises, the text transitions to practical applications. Specifically, some of the algorithmic subroutines already mentioned, along with newer methods, are applied to address key financial challenges. Regarding portfolio optimization, the QUBO (Quadratic Unconstrained Binary Optimization) formulation is presented as an effective tool to deal with the fundamental asset selection problem. This formulation allows for the efficient exploration of an exponentially large space of asset combinations using quantum or quantum-inspired solvers.

For Credit risk analysis, which is another domain where quantum methods are expected to have significant impact, we start by introducing how financial institutions have the need to estimate the risk of debtors' default, to determine the appropriate amount of Economic Capital needed to buffer against potential (although unlikely) losses. Then, this dissertation introduces novel quantum approaches to simulate more efficiently the stochastic behavior of asset values. By modeling both systematic and idiosyncratic risks, the quantum approach aims to generate accurate loss distributions and Value at Risk (VaR) estimates with a quadratic speedup over classical Monte Carlo methods. Such advancements could ultimately lead to improved risk management strategies in banking and finance.

The final area of analysis is derivative pricing. Increasingly, Monte Carlo methods are used to simulate future market scenarios for pricing financial derivatives such as options and swaps. These methods, while flexible, require significant computational resources, especially when addressing complex payoff structures. Therefore, Quantum Monte Carlo techniques can be advantageous also in this case and are being actively explored. The objective is to assess potential enhancements for a more efficient implementation in terms of necessary quantum resources.

In general, the main contribution of this work is the proposal of algorithmic enhancements that make specific quantum approaches more scalable, and thus more suitable for currently available hardware. Recognizing the limitations of today's noisy intermediate-scale quantum (NISQ) hardware, the dissertation outlines strategies to reduce resource overhead without sacrificing the accuracy of financial models. These optimizations represent a contribution towards the realistic deployment of quantum algorithms in operational financial settings.

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