

A Simple Closed-Form CAD Approach for Sensitivity Analysis and Optimization of Passive Networks Against Load Variations

*Original*

A Simple Closed-Form CAD Approach for Sensitivity Analysis and Optimization of Passive Networks Against Load Variations / Ramella, Chiara; Colantonio, Paolo; Pirola, Marco. - (2025), pp. 610-613. ( 2025 IEEE/MTT-S International Microwave Symposium, IMS 2025 San Francisco (USA) 15-20 June 2025) [10.1109/ims40360.2025.11104003].

*Availability:*

This version is available at: 11583/3004910 since: 2025-11-06T13:05:06Z

*Publisher:*

IEEE

*Published*

DOI:10.1109/ims40360.2025.11104003

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

IEEE postprint/Author's Accepted Manuscript

©2025 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

# A Simple Closed-Form CAD Approach for Sensitivity Analysis and Optimization of Passive Networks against Load Variations

Chiara Ramella<sup>#\*1</sup>, Paolo Colantonio<sup>\$\*2</sup>, Marco Pirola<sup>##3</sup>

<sup>#</sup> DET - Politecnico di Torino, Italy

<sup>\$</sup> DIE - Univeristà di Roma Tor Vergata, Italy

<sup>\*</sup> Microwave Engineering Centre for Space Applications, Italy

{<sup>1</sup>chiara.ramella, <sup>3</sup>marco.pirola}@polito.it, <sup>2</sup>paolo.colantonio@uniroma2.it

**Abstract**— This contribution presents a CAD approach based on closed-form analytic formulas that allow for a straightforward evaluation of the sensitivity of passive networks to load variations. Despite being based on very basic formulas for 2-port networks, to the author’s knowledge this approach has never been presented before in this form, nor applied to CAD optimization. Based on conformal mapping properties, the proposed technique provides the designer with simple formulas for computing and plotting sensitivity circles and maximum error due to load variations. Examples of application to biasing network optimization and matching network load-sensitivity analysis are discussed to demonstrate the effectiveness of the proposed approach.

**Keywords**— CAD optimization, load sensitivity analysis, integrated circuit design

## I. INTRODUCTION

Any microwave amplifier designer faced at least once the issue of evaluating the sensitivity of a passive network to load variations. Including the most common scenarios, in all types of amplifiers, designing a biasing network that is insensitive to external DC path load variations is crucial to its stability [1], [2]. In all applications where a circulator between the antenna and the amplifier (either in reception or transmission) cannot be used [3], the amplifier must be optimized for robustness to load variations [4]. In wideband non-linear circuits, the loading conditions of driver stages can vary significantly with frequency and power, thus requiring inter-stage matching optimization to account for these variations. [5].

Independently of the specific application, the problem to be addressed is depicted in Fig. 1: given a passive network designed to map  $Z_E$  to  $Z_M$  (subscripts E and M denote, respectively, the external impedance, that may vary, and that required to match a specific value) in a certain frequency range, a quick and reliable method to assess how a variation of  $Z_E$  from its nominal value ( $\delta_Z$ ) impacts on  $Z_M$  is desired. This is typically performed resorting to multiple simulations with different (swept) load (akin to the load-pull technique), which however become computationally intensive when a large number of frequencies and load points are required.

Since matching networks are passive linear circuits, scattering parameter theory can help finding an extremely simple, yet effective solution. In fact, as shown in Section II, the mathematical properties of conformal mapping, in conjunction with geometrical considerations, can turn textbook S-parameters formulas into a powerful tool for load-sensitivity

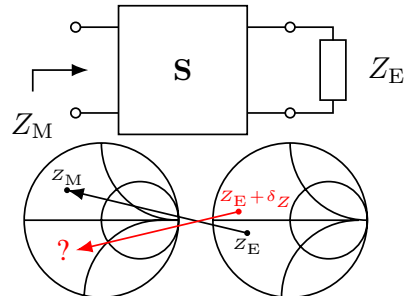


Fig. 1. Sensitivity to load variation of a generic matching network.

analysis. Although the presented theory is all-but-novel, the way we propose to apply these concepts to CAD optimization is, to our best knowledge, little known among microwave designers.

The CAD implementation of the proposed approach is straightforward in both the form of equations or user-defined macros, with the latter option enabling embedding the presented formulas as built-in functions of RF CAD tools. Two examples of application, namely the optimization of a biasing network and the load-sensitivity analysis of an ideal matching network, are considered as a proof of the usefulness of the proposed method. To prove the generality and the flexibility of the presented approach, i.e. its straightforward integration in any CAD tool, the first example was developed in Keysight Advanced Design System, while the second within Cadence AWR Design Environment.

## II. THEORETICAL BASIS AND CAD IMPLEMENTATION

Referring again to Fig. 1 we notice that, since a passive matching network is fully described by its scattering matrix  $\mathbf{S}$ , the reflection coefficient  $\Gamma_M$  at one side is related to the reflection coefficient  $\Gamma_E$  at the other side by the well known formula [6]:

$$\Gamma_M = \frac{S_{11} - \Delta_S \Gamma_E}{1 - S_{22} \Gamma_E} \quad (1)$$

where  $\Delta_S = S_{11}S_{22} - S_{21}S_{12}$  is the determinant of  $\mathbf{S}$ , and  $\Gamma_M$  and  $\Gamma_E$  are associated to  $Z_M$  and  $Z_E$  adopting same normalization, typically  $50 \Omega$ , of the scattering parameters.

The proposed CAD tool is indeed based on recognizing that (1) is a bilinear transformation more typically written as

$$w = \frac{az + b}{cz + d} \quad (2)$$

with coefficients  $a = -\Delta_S$ ,  $b = S_{11}$ ,  $c = -S_{22}$  and  $d = 1$ , and  $ad - bc = S_{21}^2 \neq 0$ , thanks to the reciprocity of the passive networks. A bilinear transformation is a conformal transformation, preserving angles and hence mapping circles into circles from the  $\Gamma_E$  to the  $\Gamma_M$  planes, and vice versa. In particular, a circle with center  $\Gamma_{E,0}$  and radius  $r_E$  at the external port is mapped at the other port into a circle with center and radius [7]:

$$\Gamma_{M,0} = \frac{(\Gamma_{E,0}\Delta_S - S_{11})(1 - \Gamma_{E,0}S_{22})^* + \Delta_S S_{22}^* r_E^2}{|S_{22}|^2 r_E^2 - |1 - \Gamma_{E,0}S_{22}|^2}$$

$$r_M = r_E \left| \frac{S_{21}S_{12}}{|S_{22}|^2 r_E^2 - |1 - \Gamma_{E,0}S_{22}|^2} \right| \quad (3)$$

where \* denotes complex conjugate operation.

These formulas can be simply implemented as equations within any commercial CAD tools, giving the possibility to plot, at all simulation frequencies, the  $\Gamma_M$  mapping circles

$$\Gamma_{M,\text{circ}} = \Gamma_{M,0} + r_M e^{j[0, 2\pi]} \quad (4)$$

providing the designer with a straightforward graphical tool to visualize the sensitivity of the synthesized impedance to external load variations. A particular case, whose simplified formulas are already built-in function in some CAD tools for the assessment of the linear stability of active circuits, is represented by the map of the entire Smith chart at the  $\Gamma_E$  plane into the  $\Gamma_M$  plane, obtained by posing  $\Gamma_{E,0} = 0$  and  $r_E = 1$  in (3).

Besides circles, the worst  $\Gamma_M$  point, i.e., the farthest from the nominal target value  $\Gamma_{M,\text{nom}}$  can be evaluated applying the geometrical calculations illustrated in Fig. 3 providing a closed-form expression for the associated distance, representing the maximum error:

$$\mathcal{E}_{\text{max}} = r_M + |\Gamma_{M,0} - \Gamma_{M,\text{nom}}| \quad (5)$$

This equation, once implemented as a simple relationship or as a *macro* within a CAD, greatly simplifies the automatic assessment and network optimization for robustness to load variations.

### III. BIASING NETWORK EXAMPLE

The efficiency and performance of solid-state devices in microwave and RF circuits are strongly dependent on the DC biasing conditions. To ensure optimal operation, each device requires an application-specific biasing network, generically known as a decoupling network. This network significantly contributes to the circuit's overall stability, especially at low frequency, and performance. It can be considered as a 2-port network, connected from one side (hereafter port 2) to the external DC biasing supply, and to the other side (port 1) to the RF matching network, which is designed to assure the proper

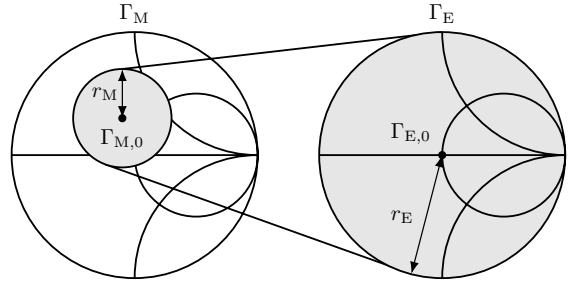


Fig. 2. Mapping the entire Smith Chart at  $\Gamma_E$ -plane into the corresponding circle at the  $\Gamma_M$ -plane.

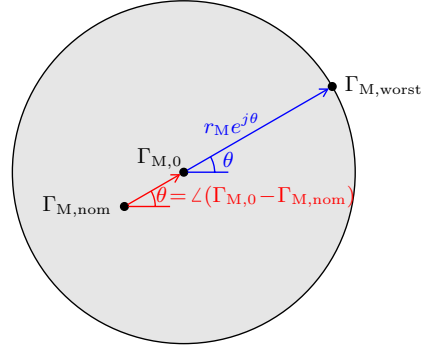


Fig. 3. Geometrical derivation of  $\Gamma_{M,\text{worst}}$  as the farthest point from the nominal (target) value.

loading conditions in the operating frequency band. The typical approach is to implement a biasing network that synthesizes at port 1 a short circuit, to avoid external feedback loops with the DC supply that may induce low frequency, out of bandwidth, oscillations. In general, it is difficult to predict the load at port 2: for a same circuit the external biasing chain may change (e.g. from the laboratory measurements phase to the system integration one), but even if it were known, low-frequency components' models are hardly integrable in RF CAD tools. The design target for the biasing network is thus to have a good RF short independently of the load at port 2. A widely adopted approach is to assume a mainly-reactive load and tune the network to optimize  $S_{11}$  parameter when considering a swept load at port 2 covering with a reasonable number of points an annular region close to the edge of the Smith Chart, as shown, for example, in Fig. 4 (left). Clearly, it is also possible to perform a sweep over the entire Smith chart, but this would require a large number of points, each to be evaluated at several frequencies, and thus longer simulation time.

By using the proposed CAD approach, instead of running multiple S-parameters simulations, it is possible to obtain all information needed with a single S-parameter simulation of the network terminated on standard  $50\Omega$  ports at both sides, as shown in Fig. 4 (right). In fact, from the scattering matrix it is possible to compute the sensitivity circles with respect to variations of the load on the entire Smith chart using (4) or the built-in full-Chart mapping function (`map1_circle()` for ADS, `SMAP` for AWR). Moreover, given the target value  $\Gamma_{M,\text{nom}}$ , it is then possible to compute the maximum error

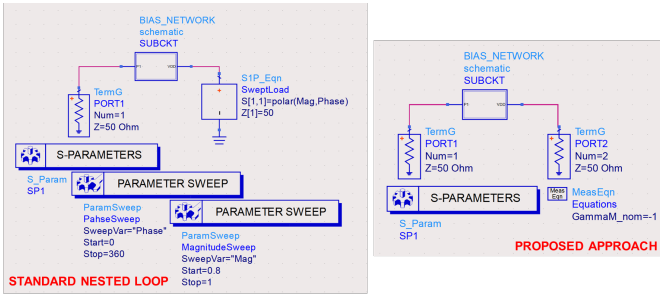


Fig. 4. Simulation setups for the design of the biasing network.

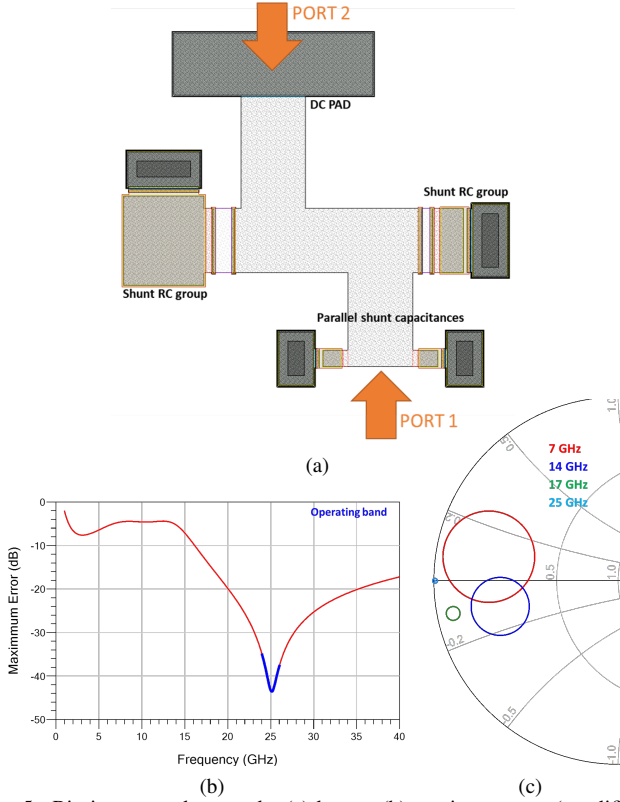


Fig. 5. Biasing network example: (a) layout, (b) maximum error (amplifier's bandwidth highlighted in blue) and (c) load-sensitivity circles at different frequencies (before final low-frequency optimization).

with (5) to be used as optimizer goal.

As a practical example, Fig. 5 reports the drain biasing network of a GaN MMIC power amplifier operating in the frequency range 24–26 GHz designed adopting the proposed tool. As is often the case, to optimize chip compactness, the output network of this amplifier exploits the bias line as a matching element (RF-shorted stub), thus the insensitivity to external variations of the biasing network, shown in Fig. 6a, is crucial to amplifier performance. The maximum error, computed with (5) versus frequency is shown in Fig. 6b: as can be noticed, it represents at any effect a simulation result usable for defining a target in an automatic network optimization. Fig. 5c reports instead the circles obtained with (4) for 4 sample frequencies. From Fig. 5c it can be immediately noticed that below 17 GHz the load synthesized at port 1 is no

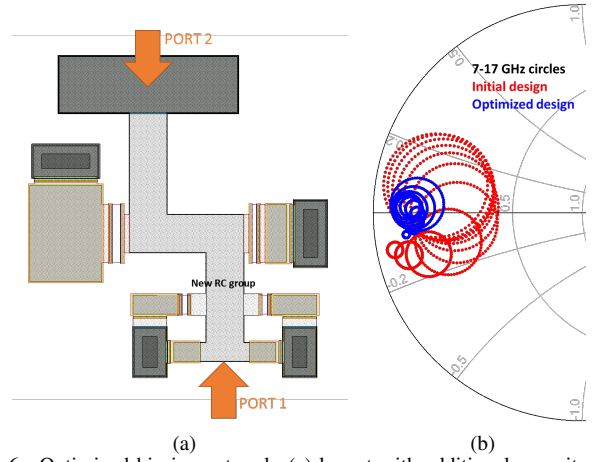


Fig. 6. Optimized biasing network: (a) layout with additional capacitance(s) and (b) sensitivity circles in the medium-low frequency range.

longer a good short circuit since it can vary in a significant range. An automatic optimization based on the reduction of the maximum error, defined according to (5), has been hence set up adopting a mixed circuit-/electromagnetic-level simulation approach. In particular, we added a third capacitor to the network, and we re-optimized all values so as to keep the maximum error below -10 dB in the intermediate frequency range 7–17 GHz. The results of the optimization are shown in Fig. 6.

#### IV. MATCHING NETWORK EXAMPLE

In massive MIMO systems, the use of a circulator between the power amplifier (PA) and the antenna is typically avoided to achieve high integration, scalability, and cost-efficiency. However, this implies that the PA suffers from large load impedance variation, with VSWR values up to 2:1 or more. If the PA is not optimized to be robust against these variations, efficiency and linearity degradation can be unacceptable [8]. In this very basic example we show how the proposed CAD approach can be exploited in this scenario. The simple ideal-element matching network shown in Fig. 7a is designed to power match, at a single frequency, a microwave transistor ( $Z_{L,opt} = [10 + j35] \Omega$ ) to a  $75 \Omega$  antenna and we want to assess the sensitivity of the synthesized impedance at the drain of the device to a VSWR of 2:1.

Built-in functions to map circles of arbitrary center and radius are not available, but implementing equations (3) and (4) the load-sensitivity circles with respect to an arbitrary sub-region of the Smith chart can be easily obtained. In this example, the center of the region of external load variation is  $\Gamma_{E,0} = 1/5$ , while the radius is  $r_E = 1/3$ . The corresponding circle at the device drain terminal is shown in Fig. 7b, where it is also compared against load pull contours for power and efficiency: a highly effective way to immediately assess performance degradation due to VSWR. The approach can be straightforwardly extended to wideband and/or parameterized analyses, while, adopting again the maximum error formula (5), an automatic CAD

## REFERENCES

- [1] G. Gonzalez, *Microwave transistor amplifiers (2nd ed.): analysis and design*. USA: Prentice-Hall, Inc., 1996.
- [2] N. Ayllon, J.-M. Collantes, A. Anakabe, I. Lizarraga, G. Soubercaze-Pun, and S. Forestier, "Systematic Approach to the Stabilization of Multitransistor Circuits," *IEEE Trans. Microw. Theory Techn.*, vol. 59, no. 8, pp. 2073–2082, 2011.
- [3] D. Yoo, J. Hwang, and B.-W. Min, "An Ultra-Wideband and Compact Active Quasi-Circulator With Phase Alternated Differential Amplifier," in *IEEE Radio Freq. Integrated Circ. Symp., RFIC 2023*, 2023, pp. 285–288.
- [4] A. Berthier, A. Ghiotto, E. Kerherve, and L. Vogt, "Active VSWR Robustness Comparison for Different Phase Combining Topologies," in *IEEE Int. New Circ. Syst. Conf., NEWCAS 2021*, 2021, pp. 1–4.
- [5] P. Neining, L. John, F. Thome, C. Friesicke, P. Brückner, R. Quay, and T. Zwick, "Limitations and Implementation Strategies of Interstage Matching in a 6-W, 28–38-GHz GaN Power Amplifier MMIC," *IEEE Trans. Microw. Theory Techn.*, vol. 69, no. 5, pp. 2541–2553, 2021.
- [6] R. E. Collin, *Foundations for microwave engineering*, 2nd ed. New York: McGraw-Hill.
- [7] H. Kober, *Dictionary of Conformal Representations*. Dover Publications, 1952.
- [8] A. Prata, S. C. Pires, M. Acar, A. S. R. Oliveira, and N. B. Carvalho, "Towards circulator-free multi antenna transmitters for 5G," in *IEEE MTT-S Int. Microw. Symp., IMS 2017*, 2017, pp. 677–680.

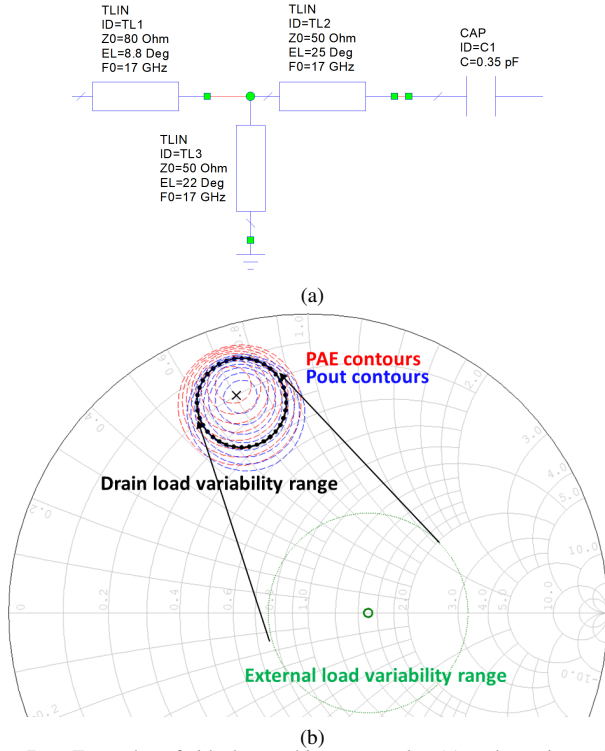


Fig. 7. Example of ideal matching network: (a) schematic and (b) load-sensitivity circle against device's load pull contours.

optimization can be set up to desensitize the network with respect to load variation. Notice that, thanks to the simplicity of the closed-form approach, which does not require sweeps, load-sensitivity can be included as a main optimization constraint since the initial network design phase rather than being considered just as a final refinement.

## V. CONCLUSION

In this work, we presented an effective, yet simple, method for load-sensitivity analysis in passive network design based on closed-form analytic formulas. The approach leverages conformal mapping and applies well-established principles in a way that has not been fully explored in CAD optimization before, providing the designer with straightforward graphical tools for immediate evaluation of network's robustness against load variations. The proposed technique is applied to specific real-world cases to demonstrate its practical utility.

## ACKNOWLEDGMENT

This work was carried out within the "Stacked power architectures for next generation mobile and space communications in European gallium arsenide technology (STARGATE)" project – funded by European Union – Next Generation EU within the PRIN 2022 PNRR program (D.D.1409 del 14/09/2022 Ministero dell'Università e della Ricerca). This manuscript reflects only the authors' views and opinions and the Ministry cannot be considered responsible for them.