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Simulation and Identification of a Seismic Bistable Device with Hysteresis

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Abstract – Mechanical metamaterials with bistable configurations offer a promising solution for enhancing energy dissipation in existing structures subjected to dynamic excitations. This research focuses on a pre-buckled steel bistable device engineered for energy dissipation, examining the critical interplay between geometric nonlinearity and hysteretic behaviour. The classical Bouc-Wen model is here modified to incorporate the effects of bistability. The study includes parametric simulations and instantaneous identification of the proposed model parameters. Finally, an equivalent damping factor that considers both viscous and hysteretic dissipation is calculated.

I. Bistable Bouc Wen Model

Mechanical metamaterials [1] with negative stiffness (NS) allow the definition of geometries that exhibit two static stable configurations, such as tilted, curved, and arched beams [2]. When these bistable systems are subjected to dynamic excitations, they can oscillate in two distinct regimes [3]: intrawell regime and interwell regime. In the interwell regime, the snap-through phenomenon occurs, resulting in high velocities within the system and leading to significant dissipation. This behaviour suggests that a device for energy dissipation may be developed by coupling NS – specifically a steel cosine-shaped-plate (CSP) – with a vibrating mass. Bistable systems have been investigated in dynamic regimes almost exclusively in the elastic field, and their behaviour can be described by the following motion equation incorporating geometric nonlinearity [3]:

$$m\ddot{u} + c\dot{u} + k_1 u + k_3 u^3 = -m\ddot{u}_g \tag{1}$$

where m is the mass, c the linear viscous damping, k_1 the negative linear stiffness, k_3 the positive cubic stiffness, u_g the ground motion, u the displacement, and the superposed dots indicates time differentiation.

However, the effect of plasticisation on such systems still needs to be explored. Therefore, to capture hysteresis during motion, the classical Bouc-Wen model [4]-[7] is modified to include bistability, forming the bistable Bouc-Wen (BBW) model. An equivalent nonlinear restoring force, r_{BBW}, that considers both bistability and hysteresis can be defined as:

$$\begin{cases} r_{BBW} = r(u,\dot{u}) + k_3 u^3 \\ \dot{r} = f(\dot{u}, r, \boldsymbol{\theta}) \end{cases} \tag{2}$$

where r is the restoring force from the Bouc-Wen model, slightly modified to account for the negative linear stiffness of a bistable system. The derivative of this restoring force component depends on the velocity, the restoring force r, and a vector of model parameters $\boldsymbol{\theta}$. It consists of two contributions – linear and hysteretic – and can be expressed as:

$$\dot{r} = \dot{r}_L + \dot{r}_H = k_1\dot{u} + k_{BW}\dot{u} = [k_1 + k_{BW}]\dot{u} \tag{3}$$

where k_{BW} is an equivalent stiffness accounting for the hysteretic behaviour and is a function of the modal parameters $\boldsymbol{\theta}$ = [β, γ, N] the velocity $\dot{u}$, and the restoring force r. It is defined as:

$$k_{BW} = -[\beta \cdot sign(\dot{u} \cdot r) + \gamma]|r|^N \tag{4}$$

The equation of motion that governs the bistable system with hysteretic behaviour reads,

$$\begin{cases} m\ddot{u} + c\dot{u} + r_{BBW}(u,\dot{u}) = -m\ddot{u}_g \\ r_{BBW} = r(u,\dot{u}) + k_3u^3 \\ \dot{r} = \{k_1 - [\beta \cdot sign(\dot{u}\cdot r) + \gamma]|r|^N\}\dot{u} \end{cases} \quad (5)$$

corresponding to the system illustrated in Fig. 1(a).

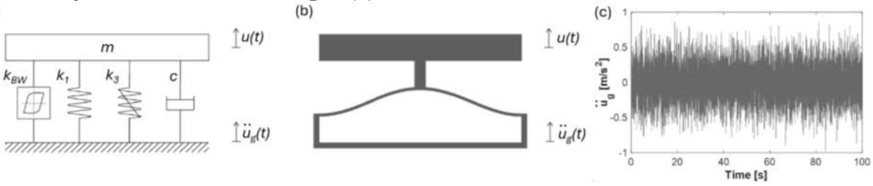

Fig. 1. (a) Bistable-Bouc-Wen (BBW) model for a SDoF, (b) seismic dissipation device unit cell, and (c) input excitation.

The BBW model can describe the response of an energy dissipation device consisting of pre-buckled NS device coupled with a vibrating mass (Fig. 1(b)), where the device consists of a corrugated plate with specific geometrical and mechanical characteristics designed to satisfy the onset buckling condition. Among the stiffness terms, k_1 and k_3 are determined by the geometry of the CSP [3]. By fixing its span to 0.5 m, the height to 0.03 m, and the thickness to 0.001 m, k_1 is equal to -693.08 N/m, while k_3 is 6.34e05 N/m^3. Whilst k_{BW} is a function of the model parameters $\boldsymbol{\theta}$, which are not directly known. Therefore, an investigation into the influence of these parameters on the device response is necessary. For what concerns the hysteretic model parameters, the parameter N is fixed equal to 2, according to [8], whilst the constraints of $\beta > 0$ and $-\beta < \gamma < \beta$ were imposed [9]. Six combinations of the model parameters $\boldsymbol{\theta}$ are considered: *Model 1* – $\boldsymbol{\theta_1}$ = [0.5, -0.4, 2], *Model 2* – $\boldsymbol{\theta_2}$ = [0.5, 0.4, 2], *Model 3* – $\boldsymbol{\theta_3}$ = [10, -9, 2], *Model 4* – $\boldsymbol{\theta_4}$ = [10, 9, 2], *Model 5* – $\boldsymbol{\theta_5}$ = [100, -99, 2], and *Model 6* – $\boldsymbol{\theta_6}$ = [100, 99, 2]. The equation of motion (5) is solved considering a mass of 58.4 kg, a damping ratio ζ of 3%, and a Gaussian noise base acceleration of 100 s. The sampling time is 0.01 s, and with peak value is set to 0.1g (see Fig. 1(c)). Fig. 2(a) shows the displacement time-history of the vibrating mass for each model, whilst Fig. 2(b) presents the relative nonlinear restoring force r_{BBW}. Fig. 2(c) presents the two contributions to the dissipated energy: the viscous dissipated energy, $E_{d,C} = c\int_0^{t_{end}} \dot{u}^2 dt$, and the hysteretic dissipated energy, $E_{d,H} = \oint r_{BBW} du$.

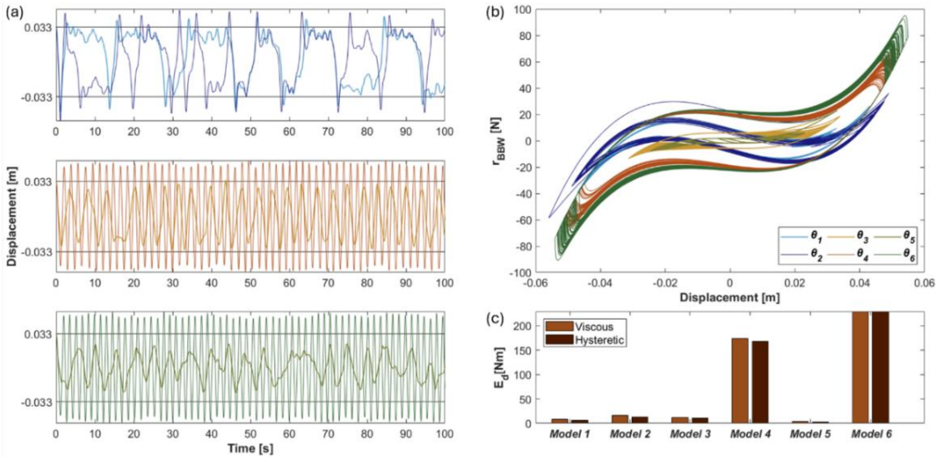

Fig. 2 (a) Displacement time-histories, (b) total restoring force r_{BBW}, and (c) viscous and hysteretic dissipated energy.

To validate the proposed model, an instantaneous identification procedure [10] of the nonlinear parameters involved in the behaviour description is proposed here. The procedure is set in a time-frequency framework, exploiting a quadratic time-frequency transform, with all the advantages that follow. The procedure is based on instantaneously minimizing the distance between the Wigner-Ville transform [11] of a generic hysteretic bistable displacement of the device and the transform of the displacement calculated using the proposed model. The results of the procedure for the Bouc-Wen parameters set $\boldsymbol{\theta_2}$ under a unitary white Gaussian noise are reported in Fig. 3.

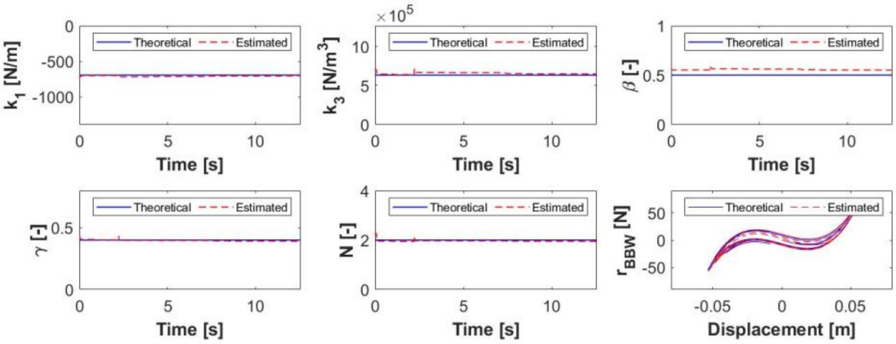

Fig. 3 Bistable and hysteretic parameters estimation results for the proposed model.

II. CONCLUSION

This study explores bistability-hysteresis interplay in NS metamaterials for energy dissipation. The results allow the determination of an equivalent damping ratio ζ_{eq} for each of the six proposed models, namely $\zeta_{eq} \approx$ *[5.3%, 5.5%, 5.7%, 5.9%, 4.7%, 6.0%]*, demonstrating that the hysteresis nearly doubles the viscous dissipation. However, degradation must be investigated to assess the device re-usability under repeated excitations. This will be investigated through an experimental campaign aimed at identifying the model parameters that optimise bistable device for energy dissipation. Finally, the introduction of parallel multi-cell configurations combined with resonators will further enhance dissipation performance in the perspective of a seismic bistable finite lattice.

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