Doctoral Dissertation
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Integral-Equation Modelling of Metasurfaces via the Impedance Boundary Condition Approximation

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Summary

The Impedance Boundary Condition (IBC) is a widely used tool for the electromagnetic analysis and design of metasurfaces; it is a homogenization approximation that replaces the actual sub-wavelength details with the averages of the surface electric and magnetic fields. This allows a much faster simulation, but it enables the design of metasurface-based devices of practical sizes.

The use of surface Integral Equations (IE) numerical formulations is very widely employed in the metasurface field, especially so, for design purposes. The IE for a planar metasurface then gives rise to the Electric-Field Integral Equation (EFIE), which with the IBC approximation becomes the EFIE-IBC formulation.

Despite its advantages, the EFIE-IBC formulation has been shown to suffer from instabilities for certain values of the surface impedance, most notably for inductive reactance. This results in poor conditioning, loss of accuracy, problematic convergence with iterative solvers, and, in the worst case, doubts about the existence of the solution for the EFIE-IBC approximate model.

This manuscript focuses on the above-mentioned limitations of the EFIE-IBC by proposing a systematic analysis of the numerical formulation to enlighten the source of ill-conditioning and proposes, therefore, strategies to fix the EFIE-IBC shortcomings.

This analysis has led basically to two main axes.

A first approach has attempted to avoid the problematic reactance values by representing the surface via admittance instead of impedance. This requires using the magnetic field and magnetic current instead of the electric one; the approach is not obvious for an open surface, and this has led to proposing an approximation that circumvents that difficulty.

The eigenvalue spectrum analysis of the EFIE-IBC, on the other hand, has shown that certain values of surface reactance can lead to small or even zero eigenvalues. In the latter case, akin to a "resonance" condition, doubts arise on the existence of a unique solution in the absence of a regularization. This has led to a filtering strategy that overcomes the resonances induced by the IBC addition in the worst-case scenario; using the surface shape to generate Basis Functions that allow a control of the spectrum

of the solution. The discretization is done in terms of triangular mesh and Rao-Wilton-Glisson functions, and the “spectral” basis functions are defined over this discretization. As practical metasurfaces have almost always simple contours, the cases dealt with specifically have employed waveguide modes as spectral basis functions.

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