Effective early-warning tool in detecting structural anomalies and preventing progressive collapse

Lorenza ABBRACCIAVENTO [1], Valerio DE BIAGI [1]
[1] DISEG, Politecnico di Torino, Torino, Italy lorenza.abbracciavento@polito.it

Addressing the challenges of structural monitoring, this research introduces a cutting-edge method for Structural Health Monitoring (SHM). The study investigates the correlation between deformation work patterns occurring during random damage scenarios. The method focuses on deformation work and related parameters as key indicators of structural health, showing promising results in the early detection of potential issues. Despite evidence indicating the robust nature of structurally complex systems in parallel arrangements, no specific study has been conducted in the case of mixed systems presenting series and parallel elements configurations. In this study, the structural response of a set of systems made of rods in series and parallel arrangements has been analyzed. Damage to the system, occurring randomly on elements, is considered, and an energy-based metric is employed. The purpose of the study is not only to explore the behaviour of rods working in the coupled mechanism, but also to identify relevant trends in the case of certain load patterns or geometrical configurations in order to design an effective targeted monitoring system.

Keywords: Structural Health Monitoring (SHM), random damage scenarios, load-paths redistribution, Energy-Based Metrics.

1. Introduction

In statically determinate structures, the way that loads travel from their point of application to the nearest support condition is uniquely defined, allowing only one possible path-way of load transmission. By the other hand, in statically indeterminates structures the matter is more complex [1]. The load paths are not uniquely defined due to the presence of additional supports and internal reactions. In these cases more sophisticated methods, including the finite element method or the integration of compatibility conditions with equilibrium equations, are required [2], [3]. The structural redundancy allows for multiple potential paths for load distribution, depending on factors like material properties, geometry, and stiffness.

This complexity not only underscores the necessity for sophisticated analysis but also introduces the wider concept of structural complexity. De Biagi and Chiaia (2013) [4] described a complex structure as a system composed of numerous components that interact in intricate ways under various loading conditions. This definition expands upon Simon's earlier work in structural mechanics from 1962, encompassing the structure's shape, stiffness, and applied loads [5]. The assessment of structural complexity is derived from the Information Content concept, initially introduced by Shannon (1948) [6]. In this context, the information content is quantified by the efficiency of load paths throughout the structure. A structure is considered simple if it has few effective load paths [8]. Conversely, a structure achieves maximum complexity when every possible load path is equally effective, as further discussed by Cennamo et al. (2014) [7] and De Biagi (2014) [9]. Recent advancements in the field of damage tolerance have embraced the integration of probabilistic models, advanced materials, and smart technologies, significantly enhancing the structural resilience and safety [10], [11].

Although numerous analytical and numerical models have been investigated into the branch of structural complexity and load redistribution [12], [13], [14], a gap exists in current literature regarding the examination of the dynamic load path variations in statically indeterminate structures under active damage conditions. This study aims to bridge this gap by extending the concepts already investigated to not only enhance understanding of structural complexity but also to provide a new and efficient tool in the field of structural monitoring.

A comprehensive mathematical model was developed to analyze changes in load paths as random structural elements undergo damage, adopting an energy-based metric for enhanced analysis. Our findings reveal significant trends in load patterns, offering deep insights into structural robustness and suggesting potential strategies for improving structural health monitoring systems, particularly regarding optimal sensor placement.

2. Methodology

The strain energy in the deformed structure was considered as a parameter that measures the ability of the system to support the loads. For a given set of loads, the smaller the internal energy the better the performance of the structure, with reduced displacements. In elastic bodies, the internal energy can be easily related to the work W of the applied forces by Clapeyron Theorem. The damage on a structural element entails an increment in the strain energy, causing an increment in the deformation of the system.

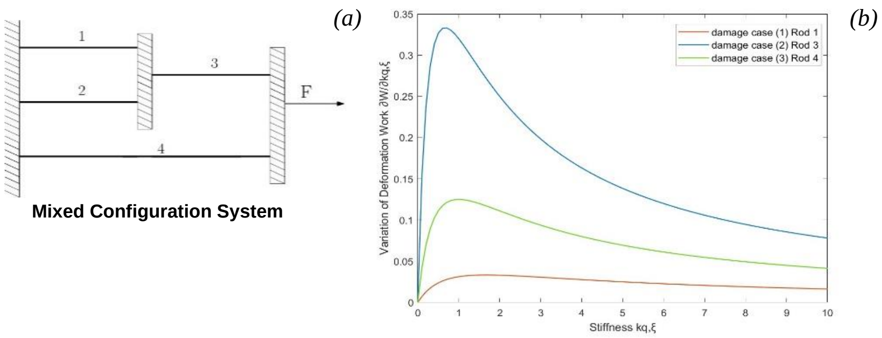
The present study focuses on a 21-bars planar statically indeterminate truss subjected to different loading conditions. The Lemaitre-Chaboche Eq. (1) model was considered to impose a damage on a structural element, say a rod of the truss. The degree of damage is controlled by a damage parameter ξ, which serves a multiplier of the axial stiffness of the element, as

$$k_{q,\xi} = k_q(1-\xi) \tag{1}$$

where k_q and $k_{q,\xi}$ are the stiffnesses of the q*th* element in the undamaged and damaged condition, respectively. The damage parameter varies between 1 and 0, for the undamaged to complete removal, respectively. Following the results of a previous research [15], here summarized, the curvature of the strain energy plot *vs* the damage parameter, i.e., the sign of $\partial W/\partial k_{q,\xi}$, was considered as a measure of the ability of the element undergoing the damage to be part of a main load path on the structure. Tests performed on purely parallel and purely serial arrangements revealed distinct trends in how deformation work responds to changes in structural stiffness. In serial arrangements, a reduction in stiffness leads to an increase in deformation work ($\partial W > 0$). Conversely, in parallel arrangements, a decrease in stiffness results in a reduction of deformation work ($\partial W < 0$).

2.1 Example on a basic structure

The approach was applied to the basic structural system depicted in Figure 1.a. The system, made of rods in parallel and series, is loaded by a 1N force in the right node. Damage on the elements highlights complex interaction patterns between structural elements. Figure 1.c reports the values of $\partial W/\partial k_{q,\xi}$ for the damage on rods 1, 3 and 4 (q=1,3,4). The variation of deformation work (∂W) no longer follows a simple monotonic pattern but exhibits a bell curve with a peak. This indicates a critical point in stiffness reduction k_q^* marking a significant shift in the structural response to damage. The mathematical model can determine whether the system is operating in a series or parallel regime, thus, it identifies when the element under damage is part of the preferential load pathway (series regime) or when it is part of the load path but not preferential (parallel regime) [15].



Tab.1 Stiffness values for different damage scenario in the mixed configuration system.

(c)

Rod	k_q [N/m]	$k_{q,\xi}$ [N/m]		k_q [N/m]	$k_{q,\xi}$ [N/m]		k_q [N/m]	$k_{q,\xi}$ [N/m]	
1		10	**Damage Case 1**	1,00		**Damage Case 2**	1,00		**Damage Case 3**
2	1,00			1,00			1,00		
3	2,00				10		2,00		
4	1,00			1,00				10	

Figure 1. *Mixed Configuration System layout (a); Variation of deformation work dW in the case of mixed in the case of damage on rod 1 (red curve), rod 3 (blue curve) and rod 4 (green curve) (b);Table.1 contains the values of stiffness when different damage scenarios are investigated.*

3. Case study: 21-bars planar truss

The study approach was applied to the 9 nodes, 21-bars planar truss structure depicted in Figure 2. The length of the rods forming the upper chord or lower chord, and vertical elements is equal to 2 m, while bracings are 2.82 m long. The structure exhibits redundancy with a degree of static indeterminacy of 4. All the members are characterized by an elastic axial stiffness equal to 100×10^6 N/m. The properties of the elements are kept fixed throughout the analysis.

The changes in the load paths within the structure in response to damage occurring to a random element were investigated.

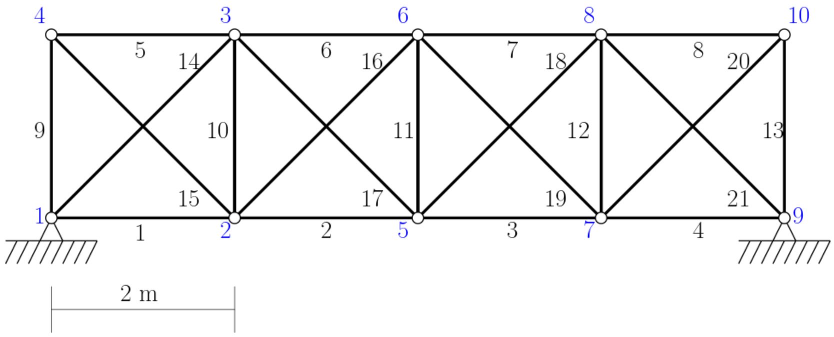


Figure 2. *Sketch of the planar truss. The nodes are on a 2 x 2 m grid.*

3.2 Loading conditions

The external forces acting on the truss are exclusively nodal loads. The structure is subjected to independent loading configurations, namely load cases n_{LC}, with a 1 kN force applied on the node. Five different loading configurations are studied, i.e. with the force acting nodes 2, 3, 4, 5, and 6, alternatively.

3.1 Introduction of damage

The truss is subjected to independent damaging scenarios. The simulation of damage progression in the structure has been modeled by discretizing the process of the stiffness reduction into 10 equal steps, ranging from its original, undamaged state to a fully damaged or completely removed state. Considering the Lemaitre-Chaboche model, the damage is represented by the variable d and directly decreases the stiffness of the rod according to th equation $k_{q,d} = d \cdot k_q$. As a sake of example, in the case d is equal to 0.5 means that the stiffness of the element is 50% of its original value in the undamaged configuration. The damage is alternatively applied on 7 different elements. Considering 5 load cases, 7 damaged elements and 10 damage steps, the analysis includes a total of 350 different configurations.

4. Results and discussions

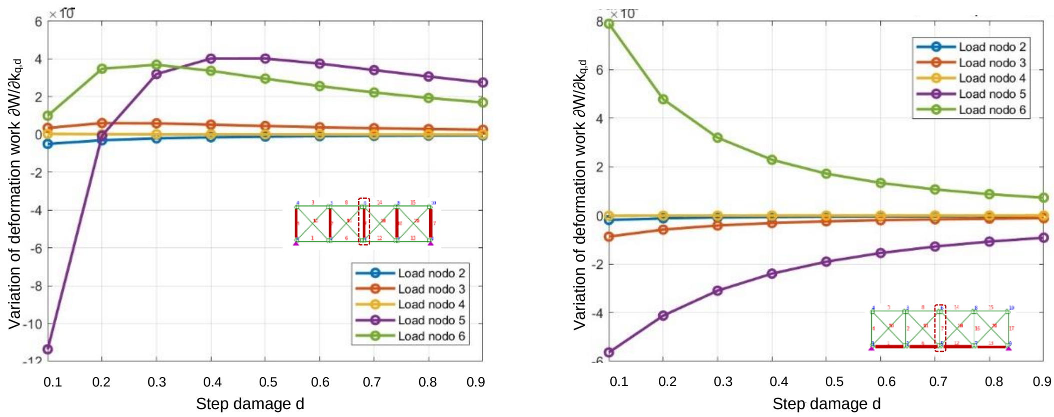
In this section, the results of the 21-bars planar truss structure analysis for independent load configurations under the damaging process affecting random elements of the structure is presented. The authors opted to present the results grouped by categories of elements rather than for individual elements separately. The basic idea is that elements of the same group perform load distribution in a similar manner, therefore, it is considered more efficient to investigate the general trend across the groups. The groups of elements are: the vertical members (#9, #10, #11, #12 and #13), the lower chord (#1, #2, #3 and #4), the upper chord (#5, #6, #7 and #8), the diagonal members pyramidal in relation to the midline (#14, #16, #19, #21), and the diagonal members flaring in relation to the midline (#15, #17, #18, #20). This dissertation does not include all the results obtained in the study but will focus on those more crucial to understand the concepts introduced and the proposed methodology.

In the first set of results, a set of plots in which the damage variable d is reported on the X-axis and the variation of elastic deformation work ∂W on the Y-axis is proposed in Figures 3-5. Each curve represents the evolution of the deformation work ∂W, from the undamaged condition to the totally removed element condition, in the case of a specific loading configuration. Each graph represents a different elements group. The graphs in Figure 3, in which the results of the variation of the deformation work for the vertical elements (Figure

3.a) and for the lower chord (Figure 3.b) are presented in the case of the rod #7 under damage. As evident, in the case of the load being applied in node 4, the change in the load paths during the damage process is zero, since the presence or removal of element #7 has no influence on the load path for this load configuration. Even in the case of load applied at node 2 and node 3, the change in the load path is neglegible, for both vertical elements (Figure 3.a) and lower chord (Figure 3.b). On the other hand, if the load is applied at node 6 or node 5 the vertical elements are part of the preferential load path. This is because, if the stiffness of element #7 is perturbed then a significant change in the system's energy is obtained. In the first case (Figure 3.a): as damage increases, positive ∂W increments are observed, thus, load in vertical elements increase; when element #7 reaches 60-70% of damaging the ∂W increments become negative, thus, the unloading of the vertical elements occur. In the second case (Figure 3.b): for the load configurations of node 5 and node 6, the lower chord is preferential part of the load path. Specifically, for load in node 5 the lower chord as the damage progresses it experiences a gradual load release while for load in node 6 the lower chord as the damage progresses it experiences a gradual loading.

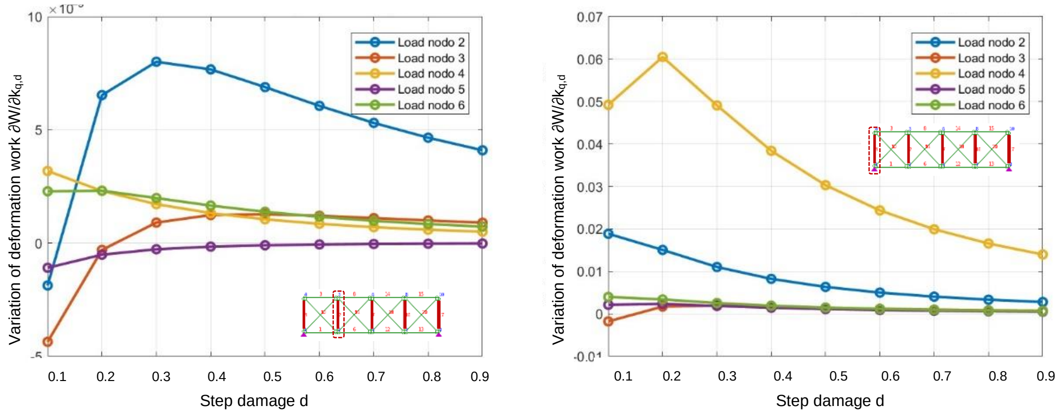
Consider now the damaging of vertical element #2 and vertical element #4 (Figure 4). It is evident that for the rod #2 under damage and load acting in node 2 and node 3, the vertical elements are a preferential part of the load path. This situation corresponds, in fact, at the load applied at the top and at the bottom of the damaged element (Figure 4.a). On the other hand, for the rod #3 under damage, the vertical elements are a preferential part of the load path only in the case load acting in node 4 (Figure 4.b).

The final graphs shown in Figure 5 highlight the outcomes of damaging diagonal element #10. the change in load paths during the damage process is minimal. This is because the path used by the loads to transfer from the point of force application to the nearest support condition does not involve the damaged element. However, the application of load in node 2 and node 3 and the progressive removal of rod #10 cause the loading of lower chord (Figure 5.a) and the discharge of upper chord (Figure 5.b). This means that, in the absence of rod #10, the structure redistributes the loads in such a way that it loads the lower chord and unloads the upper chord.



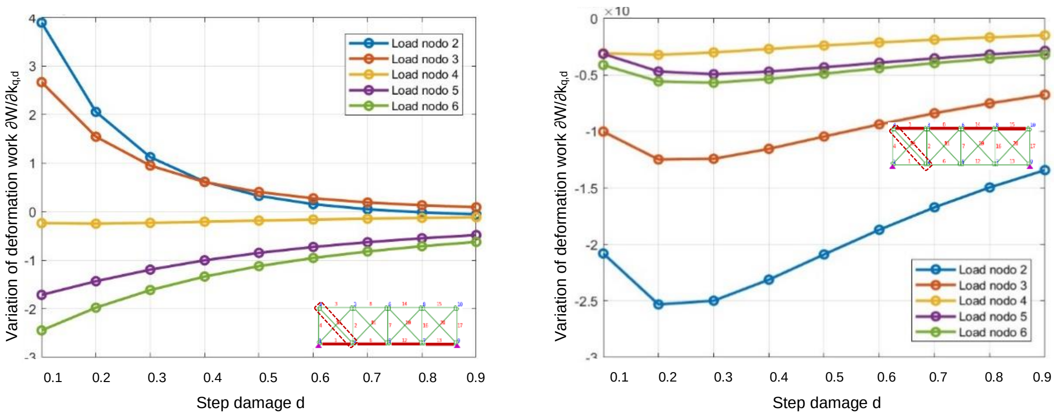
(a) *(b)*

Figure 3. *Damage to rod #7: Variation in deformation work (∂W) with respect to increasing damage, in the case of vertical elements (a) and the lower chord (b).*



(a) *(b)*

Figure 4. *Damage to rods #2 and #4: Variation of deformation work (∂W) in vertical elements, corresponding to increasing damage of rod #2 (a) and rod #4 (b).*



(a) *(b)*

Figure 5. *Damage to rod #10: Variation in deformation work (∂W) with respect to increasing damage, in the case of the lower chord (a) and the upper chord (b).*

In the second set of results, a comprehensive analysis is reported. Each curve represents the evolution of the deformation work ∂W for a specific elements group, from the undamaged condition to the totally removed element condition. The total ∂W is reported as well. Each graph represents a different load configuration.

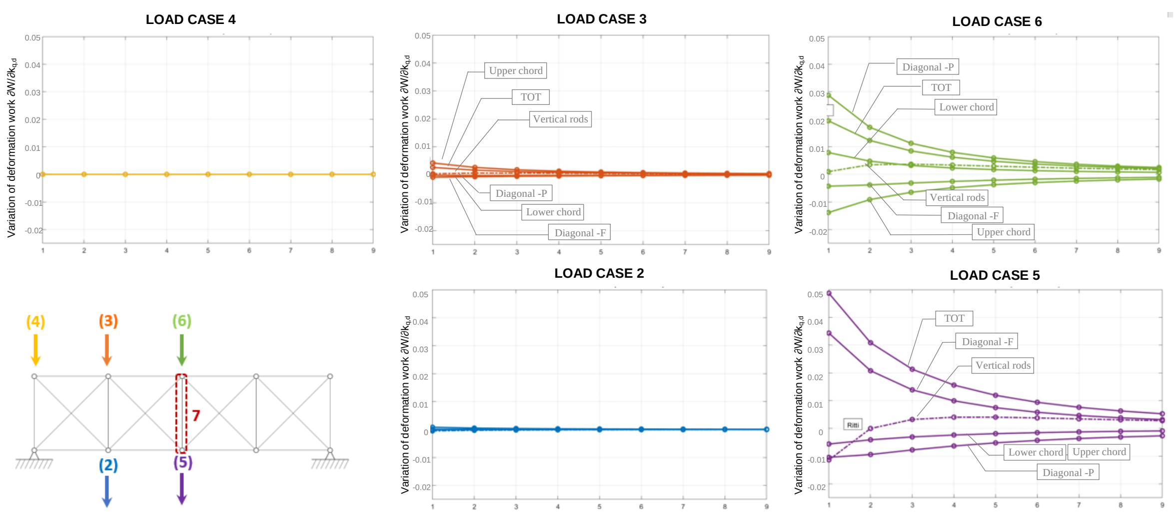


__Figure 6.__ Damage to rod #7: Variation in deformation work (∂W) in the case of all the load cases, each curve represents the evolution of the deformation work ∂W for a specific elements group. (a) Load case node 4. (b) Load case node 3. (c) Load case node 6. (d) Load case node 2. (e) Load case node 5.

In a representative manner, the following paragraph exclusively presents the results pertaining to the damage of rod #7. This presentation of results facilitates an even clearer visualization of the outcome. The load configurations at nodes 2, 3, and 4 do not lead to significant changes in the load paths. This is determined by the fact that the presence or absence of the element under damage, in this case rod #7, is not decisive in the pathways of loads because the structure redistributes the force applied, from the point of application to the nearest support condition, without including critically the passage through the element under damage.

On the other hand, the load configurations at nodes 5 and 6 cause a significant change in the system's energy. As can be observed across all groups of elements—vertical, diagonal, lower chord, and upper chord—there is a significant variation in the system's energy, consistently resulting in a balance between positive and negative changes. In the graphs of Figure 5, the dashed line is used to indicate the group of elements to which the damaged element belongs, in this case, rod #7 and thus the group of vertical elements. Furthermore, it is possible to observe that from the application of the load at node 6, as the damage to rod #7 increases, compromising his structural function, then the diagonal pyramidal elements and the lower chord are increasingly loaded, while the vertical elements and the upper chord unloaded. A similar situation applies to the load at node 5.

The analysis conducted reveals that the variation of deformation work ∂W contains key information about the structural system. Precisely, two concepts can be pointed out: i) the variation of deformation work ∂W indicates which element is part of the preferential load path for a specific load configuration; ii) the variation of deformation work ∂W describes how the load redistribution occurs within the structure when damage to an element is ongoing.

With the help of the value mapping tool in Matlab (Figure 7) [16], it is possible to graphically represents the results in a clear way. It is therefore evident how the load paths evolve as the damage of the rod #7 increase, redestribuiting within the structure

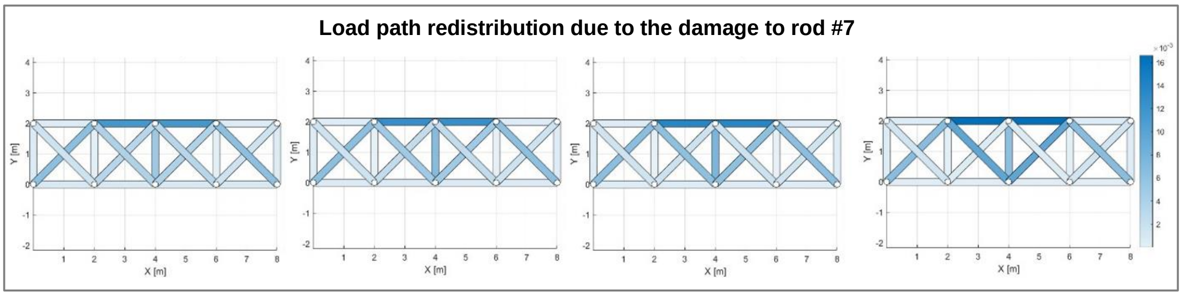


__Figure 7.__ Damage to rod #7: Variation in deformation work (∂W) in the case of load node 5. Colour variations indicate how load paths change in response to incremental damage. Each colour represents a different level in the variation of deformation work in members.

5. Conclusions

In this study the examination of the load path variations in statically indeterminate structures under active damage conditions is investigated. A comprehensive mathematical model has been developed to analyze changes in load paths as random structural elements undergo damage; an energy-based metric for the analysis introduced by De Biagi and Chiaia in 2013 [4] has been implemented ; the simulation of damage progression in the structure has been modeled by discretizing the process of the cross-sectional area reduction into steps. The mathematical model can determine whether the system is operating in a series or parallel regime, thus, it identifies when the element under damage is part of the preferential load path (series regime) or when it is part of the load path but not preferential (parallel regime). Knowing whether a specific element acts as a preferred load path has a direct consequence on the optimal sensor placement and monitoring.

The applicability and efficiency of the proposed methodology is investigated through the case study of the 21-bars planar truss, in which the changes in load paths in the case of different load configurations and considering the damage of random elements within the structure were studied. The results of the numerical application to the planar truss bar are consistent with those obtained from the analytical application of the model to simple rod systems. For each element whose damage was analyzed, it is possible to determine whether, for a given load configuration, that element is a preferred part of the load path or not. The results produced also show, as the damage condition progresses, which groups of elements load and which groups of elements unload and in which measure.

The methodology presented here significantly contributes to the enhancement of structural integrity and safety, offering potential strategies for improving structural health monitoring systems, particularly in terms of optimal sensor placement.

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