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Self-similar Heat Transfer in a Turbulent Particle-laden Free Flow

Hamid Reza Zandi Pour and Michele Iovieno

Abstract The intermediate stage of the thermal interaction between two homothermal regions in a particle-laden turbulent flow is analyzed using data from direct numerical simulations (DNSs). The fluid and particle relevant thermal budget equations are provided and a self-similar analysis has been carried out. We show how particle thermal inertia introduces a lag in the dynamics which allows for self-similarity only in the asymptotic limit.

1 Introduction

The thermal interaction between turbulent flows and suspended particles is a multidisciplinary topic with implications for a wide range of natural and industrial processes. Ongoing research continues to refine our understanding of the phenomenology of these interactions. The main contributions arrive from the analysis of fluid-particle thermal interaction in homogeneous and isotropic turbulence. Bèc et al. [1] studied the effects of particle clustering on heat transfer in a turbulent flow, how particles tend to concentrate in strong front-like structures of the fluid temperature field in which particles sample high levels of temperature fluctuations resulting in fluid-particle heat transfer enhancement at small scales. Carbone et al. [2] showed that non-local thermal dynamics of particles play a significant role and illustrated through direct numerical simulations how the formation of particle thermal caustics dominates small-scale thermal dynamics. Saito et al [6] derived an analytical

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prediction for modulation of fluid temperature fluctuations by particles by using a Langevin model for fluid temperature fluctuations along each particle path.

Since the study of classical transport processes is crucial both from scientific and from engineering standpoints, in order to study the thermal interaction in an inhomogeneous flow, in our previous works [7, 8, 9] we have used direct numerical simulations to investigate the role of monodisperse spherical inertial particles in heat transfer in a simple archetypal situation, the developing mixing layer between two homothermal regions in a homogeneous and isotropic velocity field. A large set of direct numerical simulations has provided insights into the dynamics of the suspended inertial heavy particles on heat transfer in the time-developing thermal mixing layer which develops at the interface between the two homothermal regions, allowing to quantify the effect of inertia and thermal inertia and their interaction with background fluid turbulent field for a wide range of flow parameters: Stokes and thermal Stokes numbers up to 10 at a Taylor microscale Reynolds number from 56 to 124. Single point statistics of fluid and particle temperature fields and their correlations with velocity field have been provided in both the one- and two-way coupling regimes. Numerical results have shown, up to the numerical accuracy, it is possible to identify a self-similar stage of evolution of the thermal interaction zone, in which moments collapse when rescaled with a layer thickness, determined by the mean temperature, which grows as $t^{1/2}$ for all performed simulations. Self-similar solutions play a crucial role in understanding physical problems, particularly in the field of fluid dynamics and other dynamical systems, because they allow not only for a simplifying framework for analyzing complex phenomena, but also to identify universal behaviors and study the scaling properties of a system, facilitating the development of general models and insights that can be applied across a wide range of scales.

Therefore, in this work, we show, starting from the transport equations for particles and fluid, that a quasi self-similar solution can be obtained in the intermediate stage of evolution of the thermal interaction layer, where the large-scale Stokes number of the particles determines the asymptotic convergence to a self-similar solution in this transport problem. The non self-similar terms decay in time. The theory is confirmed by comparison with the statistics obtained from direct numerical simulations.

2 Physical model

We consider, as in [7], two unbounded regions at uniform temperatures T_1 and T_2 in a homogeneous and isotropic velocity field with zero mean velocity. Let us indicate the $x < 0$ the region at temperature T_1 and with $T_2 > 0$ the region at temperature T_2. We assume that the flow is seeded by a set of initially uniformly distributed identical inertial particles. Direction x is the only inhomogeneity direction, so that when collisions are not considered, the probability density function f that a particle is, at time t, in the plane x with velocity $\boldsymbol{v}$ and temperature ϑ is given by the following equation

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$$\frac{\partial f(t,x,\boldsymbol{v},\vartheta)}{\partial t}+\frac{\partial}{\partial x}[f(t,x,\boldsymbol{v},\vartheta)\langle \boldsymbol{V}_p|x,\boldsymbol{v},\vartheta\rangle]+\frac{\partial}{\partial \boldsymbol{v}}[f(t,x,\boldsymbol{v},\vartheta)\langle \boldsymbol{A}_p|x,\boldsymbol{v},\vartheta\rangle] + \frac{\partial}{\partial \vartheta}[f(t,x,\boldsymbol{v},\vartheta)\langle \dot{\Theta}_p|x,\boldsymbol{v},\vartheta\rangle]=0 \tag{1}$$

where the acceleration $\boldsymbol{A}_p$ and thermal acceleration $\dot{\Theta}_p$ are given, for particles much denser than the fluid, by $\boldsymbol{A}_p=(\boldsymbol{u}(t,\boldsymbol{X}_p)-\boldsymbol{v})/\tau$ and $\dot{\Theta}_p=(T(t,\boldsymbol{X}_p)-\Theta)/\tau_\vartheta$, respectively [5]; $\boldsymbol{u}$ and T are the fluid velocity and temperature at the particle position $\boldsymbol{X}_p$, while $\tau=(2/9)(\rho_p/\rho_f)R^2/\nu$ and $\tau_\vartheta=(1/3)(\rho_p c_{pp})/(\rho_f c_{pf})R^2/\kappa$ are the relaxation and thermal relaxation times of the particles, which depend on particle radius R, fluid momentum and thermal diffusivities ν and κ as well as on the particle to fluid density ratio ρ_p/ρ_f and specific heat ratio c_{pp}/c_{pf}. By multiplying times ϑ and integrating over $(\boldsymbol{v},\vartheta)$, it is possible to obtain an equation for the particle mean temperature,

$$\frac{\partial\langle\vartheta\rangle}{\partial t}+\frac{\partial}{\partial x}\langle v\vartheta\rangle=\frac{1}{\tau_\vartheta}\left(\langle T\rangle-\langle\vartheta\rangle\right) \tag{2}$$

At the same time, the temperature of the carrier flow is given by an ensemble average of the passive scalar advection-diffusion equation

$$\frac{\partial\langle T\rangle}{\partial t}+\frac{\partial}{\partial x}\langle uT\rangle=\kappa\frac{\partial^2}{\partial x^2}\langle T\rangle \tag{3}$$

No particle thermal feedback is considered in this work.

3 Self-similarity

In this section we analyze the possibility that the system (3-2) has self-similar solutions. Let us introduce a length-scale $\delta(t)$, proportional to the thickness of the interaction zone, and we assume that the solution can be expressed in terms of $\eta=x/\delta(t)$, so that we define

$$\langle T\rangle(t,x)=\frac{T_1+T_2}{2}-\frac{T_2-T_1}{2}\varphi_f(\eta) \tag{4}$$

$$\langle uT\rangle=\frac{T_1-T_2}{2}\kappa\delta^{-1}\psi_f(\eta) \tag{5}$$

so that ψ_f vanishes for $\eta\to\pm\infty$ while $\varphi_f(-\infty)=1$, $\varphi_f(\infty)=-1$. By introducing these relations into (3) one obtains

$$-\frac{\delta'(t)}{\delta(t)}\eta\varphi_f'(\eta)+\frac{\kappa}{\delta^2(t)}\psi_f'(\eta)=\frac{\kappa}{\delta^2(t)}\varphi_f''(\eta) \tag{6}$$

which allows for a self-similar solutions if all time dependent coefficients are proportional, i.e. if $\delta'/\delta\propto\kappa/\delta^2$, i.e. when $\delta(t)=2\sqrt{\kappa t}$ by setting the proportionality

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constant equal to 2. With this assumption, equation (6) allows to determine the convective heat flux as a function of the mean fluid temperature,

$$\psi'_f(\eta) = 2\eta\varphi'_f(\eta) + \varphi''_f(\eta) \tag{7}$$

which, by considering that $\psi_f(\pm\infty) = 0$, $\varphi_f(-\infty) = 1$, $\varphi_f(\infty) = -1$, can be integrated to obtain ψ_f. Note that equation (7) is invariant under the transformation $\eta \to -\eta$, $\varphi_f \to -\varphi_f$, $\psi_f \to \psi_f$, so that φ_f is an odd function of η and ψ_f is an even function. The unknown value of $\psi_f(0)$ determines the convective heat flux between the two regions and, thus, the convective Nusselt number of the flow, which, by using the definition in [7], eq. (21), can be expressed in self-similar variables as $\mathrm{Nu}_c = \psi_f(0)/|\varphi'_f(0)|$. Let us now consider the particle temperature equation (2). The right hand side of the equation prevents to have a solution which depends only on η, because, while all the other terms would scale as $1/t$, it would scale as t^0. i.e. since the convective heat flux reduces in time as $1/t$, this implies that the difference between the fluid and particle temperature reduces as $1/t$, and therefore the particle mean temperature cannot be function of η alone. Therefore, we relax the self-similarity hypothesis, and look for a solution in the form

$$\langle\vartheta\rangle = \frac{T_1 + T_2}{2} - \frac{T_2 - T_1}{2}\varphi_p(t,\eta),$$
$$\langle v\vartheta\rangle = \frac{T_1 - T_2}{2}\kappa\delta^{-1}\psi_p(t,\eta),$$

where $\delta(t)$ is the same length-scale already defined. The introduction of these expressions of the solution into (2) together with the definition of δ leads to

$$\frac{\partial\varphi_p}{\partial t}(t,\eta) + \frac{1}{4t}\left(-2\eta\varphi'_p(t,\eta) + \psi'_p(t,\eta)\right) = \frac{1}{\tau_\vartheta}\left(\varphi_f(\eta) - \varphi_p(t,\eta)\right), \tag{8}$$

where the sign $'$ indicates the derivative with respect to η. Equation (8) allows to determine the particle heat flux as a function of the particle mean temperature, which in turn depends on the carrier flow temperature. In the tracer limit, when particle inertia vanishes, $\tau_\vartheta \to 0^+$, the mean particle temperature becomes equal to the mean fluid temperature in the same position x and time t, so that $\varphi_p(t,\eta) = \varphi_f(\eta)$ and the particle temperature-velocity correlation are given by (8) as $\psi'_p = 2\eta\varphi'_f(\eta)$. In this limit the ratio between particle and fluid temperature-velocity correlation becomes

$$\frac{\langle v\vartheta\rangle}{\langle uT\rangle} = \frac{\psi_p}{\psi_f} = \left[1 + \frac{1}{2}\varphi'(\eta)/\int_{-\infty}^{\eta} s\varphi'_f(s)\mathrm{d}s\right]^{-1}.$$

When the Reynolds number increases, the difference between particle and fluid the velocity-temperature tends to vanish because the difference is only in the diffusive term. A finite inertia, i.e. finite τ_ϑ, introduces a lag between the between the temperature of the carrier flow and the particle temperature, proportional to τ_ϑ. Since the timescale of the process, $\delta/\delta' \propto t$, increases with time, the ratio τ_ϑ and the process timescale becomes smaller and smaller, so that the particle thermal inertia has a

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smaller and smaller effect on particle thermal dynamics. Therefore, let us expand φ_p and ψ_p in a power series of τ_ϑ/t, i.e. let us write

$$\varphi_p(t,\eta) = \sum_{k=0}^{\infty} (t/\tau_\vartheta)^{-k} \varphi_k(\eta) = \varphi_0(\eta) + (t/\tau_\vartheta)^{-1}\varphi_1(\eta) + (t/\tau_\vartheta)^{-2}\varphi_2(\eta) + \dots$$

$$\psi_p(t,\eta) = \sum_{k=0}^{\infty} (t/\tau_\vartheta)^{-k} \psi_k(\eta) = \psi_0(\eta) + (t/\tau_\vartheta)^{-1}\psi_1(\eta) + (t/\tau_\vartheta)^{-2}\psi_2(\eta) + \dots$$

By introducing these expansions into (8) and by equating to zero the coefficients of all powers of t one obtains the following equations,

$$\varphi_0 - \varphi_f = 0 \tag{9}$$

$$\frac{1}{2}\left(\psi_0' - 2\eta\varphi_0'\right) = -\varphi_1 \tag{10}$$

$$-(k-1)\varphi_k + \frac{1}{2}\left(\psi_k' - 2\eta\varphi_k'\right) = -\tau_\vartheta^{k-1}\varphi_{k+1} \quad \forall k \geq 1 \tag{11}$$

This set of equations allows to determine the non self-similar part of the heat flux from the non self-similar part of the mean temperature distribution. Since the system is not closed, it is not possible to solve the problem unless a parametrization of velocity-temperature correlations is introduced. Anyway, any parametrization should be compatible with equations (7) and (9-11). This is what was done, for example, in [3], where a gradient parametrization was introduced and thus a self-similar solution for particle concentration could be analytically obtained.

4 Numerical simulations

The results of the previous section can be compared with information extracted by direct numerical simulations [7, 8], carried out within the point-particle approach in a parallelepiped domain with periodic boundary conditions by decomposing temperature in a linear part and a residual part.

In those simulations, fluid temperature and velocity moments have been found to collapse when rescaled with the thickness of the thermal interaction layer δ_*, conventionally defined on the basis of the inverse temperature gradient as $\delta_* = (T_1 - T_2)/\max|\partial T/\partial t|$. This thickness grows as $\delta_*/\ell \sim (t/\tau)^{1/2}$, where $\tau = \ell/u'$ is the large-scale eddy turnover time, ℓ is the integral scale of the turbulence and u' is the root mean square value of the velocity fluctuations. When the position x is rescaled with δ_*, i.e. in terms of the variable $\eta_* = x/\delta_*(t)$, the temperature profile is independent from the Reynolds number. This shows that the thermal mixing is driven mainly by the large scales of the flow [7, 4], which can correlate velocity and temperature fluctuations over longer distances. Since that $\delta_*/\delta \sim \mathrm{Pe}^{1/2}/2$, where $\mathrm{Pe} = u'\ell/\kappa$ is the Peclet number of the flow based on the integral scale. When data

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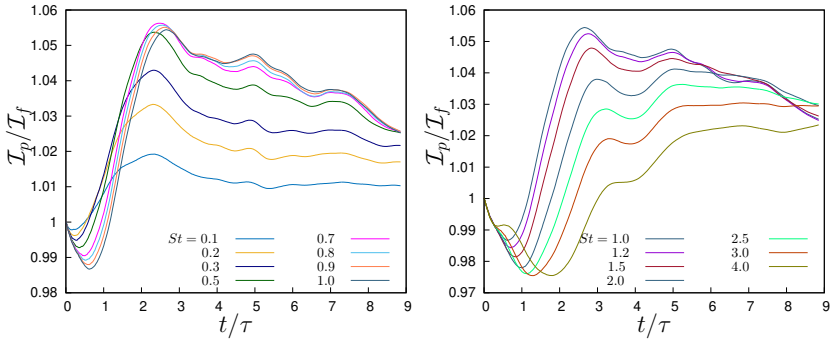


Fig. 1 Mean temperature moment ratio $\mathcal{I}_p/\mathcal{I}_f$ for (a) particles with $St \leq 1$; (b) particles with St ≥ 1.

at the same Reynolds number are considered, this scaling factor is constant, so that we will continue to use the same variables introduced in section 3. To use δ_* as a scale for the self-similar equations is equivalent to the transformation $\eta \to \eta \mathrm{Pe}^{1/2}/2$ and $\psi_f \to \psi_f \mathrm{Pe}^{1/2}/2$, $\psi_p \to \psi_p \mathrm{Pe}^{1/2}/2$. $\varphi_f \to \varphi_f$, $\varphi_p \to \varphi_p$. The main results of the simulations were the Nusselt number and the ratio between particle and fluid temperature-velocity correlation, which gives a measure of particle contribution to the overall heat flux across the thermal layer. With present variables, the convective Nusselt number is given by $\mathrm{Nu}_c = \psi_f(0)/|\varphi_f'(0)|$, so that it is contained in the values of the rescaled flux ψ_f. while the ratio between particle and fluid temperature-velocity correlation is $\psi_p(0)/\psi_f(0) = F(\mathrm{Re}, \mathrm{Pr}, \mathrm{St})$, where $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number of the fluid.

A direct comparison of the temperature moments is made difficult by the intrinsic uncertainty in data coming from numerical simulation, where the noise due to numerical constraints and the limited sample size prevents the computation of derivatives. Therefore, a better comparison can be obtained by deriving integral quantities which describe the thermal mixing layer. We can observe that the following integrals, which represent the spatial moment of the mean temperature,

$$\mathcal{I}_f = \int_0^{+\infty} x(\langle T \rangle - T_2)\mathrm{d}x = \frac{T_1 - T_2}{2}\delta^2 \int_0^{+\infty} \eta(1 + \varphi_f(\eta))\mathrm{d}\eta$$
$$\mathcal{I}_p = \int_0^{+\infty} x(\langle \vartheta \rangle - T_2)\mathrm{d}x = \frac{T_1 - T_2}{2}\delta^2 \int_0^{+\infty} \eta(1 + \varphi_p(t, \eta))\mathrm{d}\eta$$

scale as δ^2, i.e. grow linearly in time when the solution behaves in a self-similar way. Therefore, by replacing the expansion of φ_p and using (9)-(11), we have

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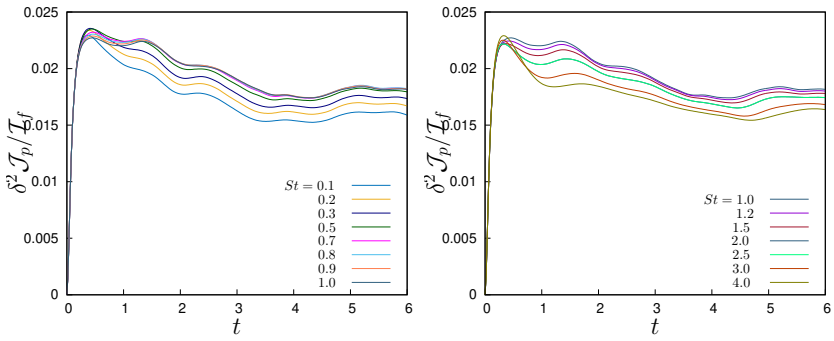


Fig. 2 Integral flux to mean temperature moment ratio, $\delta^2 \mathcal{I}_p/\mathcal{I}_f$, for (a) particles with $St \leq 1$; (b) particles with St ≥ 1.

$$
\begin{aligned}
\mathcal{I}_p &= \mathcal{I}_f + \frac{T_1 - T_2}{2}\delta^2 \left(\frac{\tau_\vartheta}{t} \int_0^{+\infty} \eta \varphi_1(\eta) \mathrm{d}\eta + o\left(\frac{\tau_\vartheta}{t}\right)\right) \\
&= \mathcal{I}_f - \frac{T_1 - T_2}{2}\delta^2 \frac{\tau_\vartheta}{t} \left(\left(\frac{1}{2}\int_0^{+\infty} \psi_0 \mathrm{d}\eta - \int_0^{+\infty} \eta(1+\varphi_0)\mathrm{d}\eta \right) + o\left(\frac{\tau_\vartheta}{t}\right)\right) \\
&, = \mathcal{I}_f - \frac{T_1 - T_2}{2}\delta^2 \frac{\tau_\vartheta}{t} \left(\left(\frac{1}{2}\int_0^{+\infty} \psi_p \mathrm{d}\eta - \int_0^{+\infty} \eta(1+\varphi_0)\mathrm{d}\eta \right) + o\left(\frac{\tau_\vartheta}{t}\right)\right) \\
&= \mathcal{I}_f - \frac{\tau_\vartheta}{t}\left(\frac{\delta^2}{2}\mathcal{J}_p - \mathcal{I}_f \right) + o\left(\frac{\tau_\vartheta}{t}\right)
\end{aligned}
$$

where $\mathcal{J}_f$ and $\mathcal{J}_p$ are the integral fluxes

$$
\begin{aligned}
\mathcal{J}_f &= \frac{1}{\kappa}\int_0^{+\infty} \langle uT \rangle \mathrm{d}x = \frac{T_1 - T_2}{2}\int_0^{+\infty} \psi_f(\eta)\mathrm{d}\eta \\
\mathcal{J}_p &= \frac{1}{\kappa}\int_0^{+\infty} \langle v\vartheta \rangle \mathrm{d}x = \frac{T_1 - T_2}{2}\int_0^{+\infty} \psi_p(t,\eta)\mathrm{d}\eta
\end{aligned}
$$

We can therefore write

$$
\frac{\mathcal{I}_p}{\mathcal{I}_f} = 1 + \frac{\tau_\vartheta}{t}\left(1 - \frac{\delta^2}{2}\frac{\mathcal{J}_p}{\mathcal{I}_f} \right) + o\left(\frac{\tau_\vartheta}{t}\right). \tag{12}
$$

Equation (12) can be easily verified from the numerical simulations, because integral $\mathcal{I}_f$, $\mathcal{I}_p$, $\mathcal{J}_f$, and $\mathcal{J}_p$ can be obtained directly by the flow statistics without the introduction of any hypothesis about the solution. Note that also the $\mathcal{I}_p$, $\mathcal{J}_p$ admit an expansion in series of the inverse time when the expansion of φ_p and ψ_p are introduced in the integrals. The first terms of such series, corresponding to the integrals of φ_0 and ψ_0, can be obtained from the limits of $\mathcal{I}_p$ and $\mathcal{J}_p$ for large t.

We use the results for direct numerical simulations of the thermal layer advected by a homogeneous and isotropic velocity field with a Taylor macroscale Reynolds number equal to 56, when the ratio between the thermal relaxation time τ_ϑ to the

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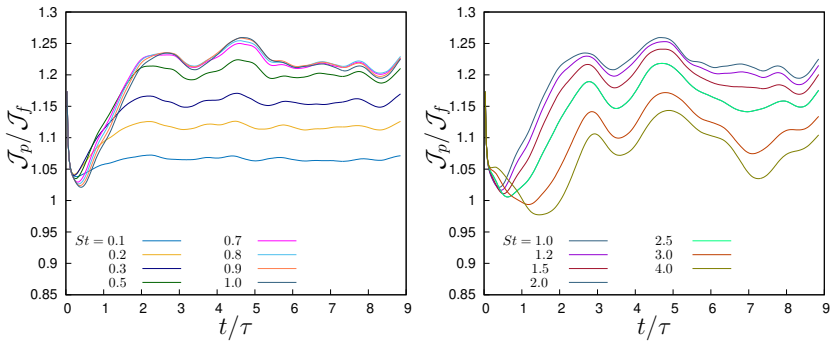


Fig. 3 Integral flux ratio $\mathcal{I}_p/\mathcal{I}_f$ for (a) particles with $St \leq 1$; (b) particles with St ≥ 1.

Stokes relaxation time is equal to 4.43 and the Prandtl number is equal to 0.71 (see [7] for more details about the simulated flow). Figure (1) shows the ratio $\mathcal{I}_p/\mathcal{I}_f$. After an initial transient of about two eddy turnover times τ, the ratio evolves showing a gradual decay toward a constant value, according to equation (12). Since the fluid temperature evolution is driven by the large-scales of the motion, t scales over τ, so that t/τ_ϑ scales as the inverse of the large-scale Stokes number $\mathrm{St}_\ell = \tau_\vartheta/\tau$, and the corrections due to the particle thermal inertia are small, so that the non self-similar terms tend to be affected by the noise unavoidably present in any experimental statistics, and we cannot check higher order terms in (12). The ratio increases when the particle inertia increases, reaches a maximum when the Stokes number of the particles, $\mathrm{St} = \tau_\vartheta/\tau_\eta$, where τ_η is the Kolmogorov timescale,is equal to one, then reduces for larger inertia. This is because, notwithstanding the correction is proportional to τ_ϑ, the coefficient of τ_ϑ/t depends on the particle flux, which reduces when the Stokes number becomes larger than one [7, 8]. We can check this behaviour by looking at the second term in expansion (12), the leading term of the non self-similar correction, which is shown in figure 2. After the initial transient, the ratio $\delta^2\mathcal{J}_p/\mathcal{I}_f$ slowly decays in time according to the inverse power series expansion in time, showing the same non monotonic dependence on particle inertia, with a maximum when the particle Stokes number is equal to one. However, the time variations appear to be smaller and after about five eddy turnover times the ratio is already constant. Indeed, if we look at the integral flux ratio $\mathcal{J}_p/J_f$ (see figure 3), we can observe that it remains almost constant in time. This suggests that the first term in the series expansion of $\psi_f(t,\eta)$ is zero. This implies also that the higher terms in the expansion of $\varphi_p(t,\eta)$ do not depend on the particle heat flux. Simulations at higher Stokes numbers present more marked fluctuations due to the reduced number of particles in the simulations, which were carried out at a constant volume fraction.

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5 Conclusions

We have carried out an analysis of the heat transport at the interface between two homothermal regions advected by a homogeneous and isotropic velocity field in presence of suspended particles and fluid-particle thermal interaction. We have shown that particle inertia prevents a full self-similar evolution of the system, but that a quasi self-similar stage of evolution is achieved. Comparison with direct numerical simulations confirm the theoretical analysis.

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