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The instability of low-temperature black holes in gauged $\mathcal{N} = 8$ supergravity

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Abstract: We consider the static planar black hole solutions in the STU model of the gauged $\mathcal{N} = 8$ supergravity in four dimensions. We give a straightforward derivation of the equation of state of the purely electric and purely magnetic solutions with four charges. Then we give a simple proof that the determinant of the Hessian of the energy is always negative below some critical finite temperature for the purely electric solutions. We compute the spinodal line for the usual planar Reissner-Nordström solution in four dimensions. Inspired by the magnetic superalgebra we show that the supersymmetric solutions are metastable if the energy is restricted to satisfy the topological twist condition ab initio and it is shifted to be zero on the BPS solutions.

Keywords: Black Holes in String Theory, Classical Theories of Gravity, M-Theory, Supergravity Models

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Contents

1 Introduction

Black holes in SO(8)-gauged $\mathcal{N}=8$ supergravity yield concrete examples of the physics of the low energy limit of M-theory. The four-dimensional theory was originally constructed in [1] and proven to describe the massless sector in the spontaneous compactification of eleven-dimensional supergravity on S^7 [2]. The consistent truncation of this theory to the four vectors gauging the Cartan subalgebra of SO(8) is the $\mathcal{N}=2$ U$(1)^4$-STU model, whose bosonic sector consists of the metric, four U(1)-vectors, three dilatons and three axions. The static-spherically symmetric black holes in this model were obtained with either four electric or four magnetic charges in [3]. These black holes were embedded in eleven dimensions and generalized to hyperbolic and planar horizons in [4]. The spinning solution, with two dilatons and two axions set to zero, was constructed in [5]. In [3], the BPS limit of the spherical black holes was found to yield naked singularities in the electric case and not to exist in the magnetic one. When the four U(1)-vectors are equal to one another, this theory reduces to the minimal gauged $\mathcal{N}=2$ supergravity, with the bosonic sector corresponding to the Einstein-Maxwell theory with a negative cosmological constant. In this limit, spherical black holes exhibit naked singularities [6], while finite-area black holes must have a locally hyperbolic horizon [7] and it was unknown whether running scalars would allow for spherical and planar black holes. Eventually, the first regular, finite-area spherically symmetric and

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planar supersymmetric black hole in $\mathcal{N}=8$ supergravity was constructed in [8]. The $\mathcal{N}=8$ black holes are known to contain different kinds of instabilities which have been studied restricting the number of charges [9–11].

In this paper, we provide a general and simple proof that the four-charge planar electric black holes of gauged $\mathcal{N}=8$ supergravity are unstable when the temperature is low enough. Indeed, there is a finite temperature at which the black holes are no longer equilibrium states in the thermodynamical sense. To this end, we construct the equation of state of these black holes and compute the determinant of the Hessian of the energy showing that, below a certain temperature, it is always negative. This means that there is a spinodal line for these black holes and we construct it explicitly in the pure Einstein-Maxell case.

This result is puzzling for the magnetic supersymmetric black hole. Indeed, from electromagnetic duality, one would expect that the same equation of state would apply in this case by replacing the electric charge squared with the magnetic charge squared. However, this would mean that the supersymmetric black hole is unstable. Following the results of [12, 13] we propose that the Hessian has to be computed on an energy that goes to zero in the BPS limit and that satisfies the topological twist condition ab initio, a condition necessary for the supersymmetric charge to exist asymptotically. Using this mass we find that the black hole is indeed meta-stable. Metastability of supersymmetric black holes should be expected as they are at the boundary of the allowed region of stability.

In the first section of the paper, we present the Lagrangian and our conventions. Then we provide the general non-extremal solutions of [3, 4] written in a slightly different way. Eventually, we derive the BPS limits of the electric and magnetic black holes. In the second section, we compute the equation of state of electric black holes and show that there is a critical temperature at which the Hessian is always negative. There we compute the spinodal line for the usual Reissner-Nordström black hole. Then we analyze the magnetic case, where there are supersymmetric black holes of finite area. We find that if the ensemble of black holes is required to have boundary conditions that allow for the existence of a finite supercharge asymptotically, and the mass is shifted, the Hessian is indeed positive definite. We also discuss the stability of the non-BPS extremal solutions which admit a first-order description in terms of a fake-superpotential [14–17]. There are appendices with details which are relevant to the calculations provided in the body of the paper.

2 STU model

We are interested in studying the dilatonic sub-sector of the STU model, where the complex scalars parameterizing the special Kähler manifold are purely imaginary $z_i = \mathrm{i}e^{-\phi_i}$, $i = 1, 2, 3$, which leads to a substantial simplification of the model. The effective action principle, that we consider for practical proposes, is given by

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{4}\sum_i \partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{4}\sum_\Lambda Y_\Lambda F^\Lambda_{\mu\nu}F^{\Lambda\mu\nu} + \frac{1}{L^2}\sum_i \cosh\phi_i\right). \tag{2.1}$$

The field equations coming from the action principle (2.1) are given by

$$\begin{aligned} \mathrm{d}(Y_\Lambda \star F^\Lambda) &= 0\,, \qquad \Lambda = 1,\dots,4 \\ \frac{1}{2}\Box\phi_i + \frac{1}{L^2}\sinh\phi_i - \frac{1}{4}\sum_{\Lambda=1}^{4}\frac{\partial Y_\Lambda}{\partial\phi_i}F^\Lambda_{\mu\nu}F^{\Lambda\mu\nu} &= 0\,, \qquad i = 1,2,3\,, \\ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= T^{(\phi)}_{\mu\nu} + T^{(F)}_{\mu\nu}\,, \end{aligned}$$

with

$$\begin{aligned} T^{(\phi)}_{\mu\nu} &= \frac{1}{2}\sum_{i=1}^{3}\left[\partial_\mu\phi_i\partial_\nu\phi^i - g_{\mu\nu}\left(\frac{1}{2}(\partial\phi_i)^2 - \frac{2}{L^2}\cosh\phi_i\right)\right]\,, \\ T^{(F)}_{\mu\nu} &= \sum_{\Lambda=1}^{4}Y_\Lambda\left(F^\Lambda_{\mu\rho}F^\Lambda{}_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F^\Lambda_{\rho\sigma}F^{\Lambda\rho\sigma}\right)\,. \end{aligned}$$

They partially reproduce the field equations of the dilatonic sector of the STU model. It is required to impose also the following equations

$$\begin{aligned} e^{2\phi_1}F^2\wedge F^3 - F^1\wedge F^4 &= 0\,, \\ e^{2\phi_2}F^1\wedge F^3 - F^2\wedge F^4 &= 0\,, \\ e^{2\phi_2}F^2\wedge F^2 - F^3\wedge F^4 &= 0\,, \end{aligned} \tag{2.2}$$

coming from the truncation of the axion fields to zero. These constraints are automatically fulfilled for purely electric and purely magnetic configurations. The quantities Y_Λ depends on the scalar fields and are minus the diagonal components of the generalized couplings $\mathcal{I}_{\Lambda\Sigma}$ in the purely dilatonic sector, see appendix A for further details. They are given explicitly by

$$Y_i = e^{\phi_1+\phi_2+\phi_3-2\phi_i}\,, \qquad Y_4 = e^{-\phi_1-\phi_2-\phi_3}\,, \qquad i = 1,2,3\,. \tag{2.3}$$

Reducing to the dilaton fields, some simplifications occur in the supersymmetry transformations: the Kähler connection vanishes $\mathcal{Q}_\mu = 0$, the first four components of the U(1)-section are real $L^\Lambda \in \mathbb{R}$, and the superpotential is real $\mathcal{W} \in \mathbb{R}$. Hence the Killing spinor equation coming from the supersymmetry variation of the gravitino reduces to

$$\begin{aligned} \delta\Psi^A_\mu\mathrm{d}x^\mu &\equiv \mathfrak{D}^A(\epsilon) \\ &\equiv \mathrm{d}\epsilon^A + \frac{1}{4}\omega_{ab}\gamma^{ab}\epsilon^A + \frac{1}{2}A^M\theta_M\varepsilon^{AB}\delta_{BC}\epsilon^C + \frac{1}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon^{AB}\epsilon_B + \frac{1}{2}\mathcal{W}\gamma\delta^{AB}\epsilon_B = 0\,, \end{aligned} \tag{2.4}$$

where we have defined the 1-form Clifford algebra valued $\gamma = \gamma_a e^a$. The supersymmetry variation for the gaugini are

$$\delta\lambda^{iA} = -\gamma^\mu\partial_\mu z^i\epsilon^A + \frac{1}{2}g^{i\bar{\jmath}}\overline{f}^\Lambda_{\bar{\jmath}}\mathcal{I}_{\Lambda\Sigma}F^\Sigma_{ab}\gamma^{ab}\varepsilon^{AB}\epsilon_B + g^{i\bar{\jmath}}\overline{\mathcal{U}}^M_{\bar{\jmath}}\theta_M\delta^{AB}\epsilon_B = 0\,, \tag{2.5}$$

where in this cases the quantities $z^i, \overline{f}^\Lambda_{\bar{\jmath}}$ and $\overline{U}_{\bar{\jmath}}{}^M\theta_M$ are purely imaginary in the dilatonic sector. The quantity $\overline{U}_{\bar{\jmath}}{}^M$ is the U(1) covariant derivative of the U(1) section $\overline{V}^M$ and $\overline{f}_{\bar{\jmath}}{}^\Lambda$ are the first four symplectic components of $\overline{U}_{\bar{\jmath}}{}^M$. The local supersymmetry parameters are chiral spinors satisfying $\gamma_5\epsilon_A = \epsilon_A$ and $\gamma_5\epsilon^A = -\epsilon^A$, and are related to each other through complex conjugation $(\epsilon^A)^* = \epsilon_A$. A^M are the symplectic gauge fields, and the embedding tensor is given by

$$\theta_M = \frac{1}{\sqrt{2}L}(1,1,1,1,0,0,0,0)\,. \tag{2.6}$$

The supercovariant derivative (2.4) is the object entering the Dirac bracket between supercharges that we outline in the next section. Since we are considering real γ-matrices, all the coefficients in the above equation are real. We can write an equation for the Majorana spinor $\psi^A = \epsilon^A + \epsilon_A$ by combining (2.4) with its complex conjugated. For simplicity, it is useful to define a complex spinor whose real and imaginary parts are the SU(2) components of the Majorana spinor $\zeta = \psi^1 + \mathrm{i}\psi^2$. The Killing spinor equation for the complex Killing spinor is

$$\left(\mathrm{d} + \frac{1}{4}\omega_{ab}\gamma^{ab} - \frac{\mathrm{i}}{2}A^M\theta_M - \frac{\mathrm{i}}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma + \frac{1}{2}\mathcal{W}\gamma\right)\zeta = 0\,. \tag{2.7}$$

The same can be done for the gaugino equations that implies the following equations for the complex spinor

$$\left(-\gamma^\mu\partial_\mu z^i - \frac{\mathrm{i}}{2}g^{i\bar{\jmath}}\overline{f}_{\bar{\jmath}}^T\mathcal{I}F_{ab}\gamma^{ab} + g^{i\bar{\jmath}}\overline{U}_{\bar{\jmath}}{}^M\theta_M\right)\zeta = 0\,, \tag{2.8}$$

See appendix A for the definitions and the explicit form of the γ-matrices that we are considering.

2.1 Dirac bracket between supercharges

In gauge theories, the conserved charges associated with large gauge transformations are obtained by the integration of a suitable conserved current on a co-dimension 2 spacelike surface, due to the fact that the Hodge dual of the Noether current is a closed form, up to imposing the field equations [18]. In [12] was developed a method to evaluate the superalgebra in backgrounds that have an asymptotic Killing spinor for $\mathcal{N} = 2$ gauged supergravity solutions. This prescription was applied to compute the BPS bound for configurations that asymptote to AdS or mAdS, resolving the tension between the BPS bound propose in [19] and the explicit BPS configurations found by Romans [6]. The same analysis was carried out in [13] for $D = 4$, $\mathcal{N} = 2$ gauged supergravity coupled to matter fields, where it was shown that the 3-form dual to the Noether supercurrent can be written as the exterior derivative of a 2-form up to imposing the field equations. This leads the supercharge to be expressed as an integral of a 2-form over a spacelike surface in the asymptotic region. The Dirac bracket between the supercharges is computed by acting with the supersymmetry transformation on the supercharge leading to

$$\{\mathcal{Q},\mathcal{Q}\} = -2\int_{\partial\Sigma}\left(\bar{\epsilon}^A\gamma\wedge\mathfrak{D}_A(\epsilon) - \bar{\epsilon}_A\gamma\wedge\mathfrak{D}^A(\epsilon)\right)\,. \tag{2.9}$$

where $\mathfrak{D}^A(\epsilon)$ is the supercovariant derivative defined in (2.4), and $\mathfrak{D}_A(\epsilon)$ is its complex conjugated. We defined the Clifford algebra valued 1-form $\gamma = \gamma_a e^a$. We consider $\mathcal{Q}$ being the product between the Grassmann-odd supercharge and the spinorial parameters ϵ^A, i.e. there

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are no free indices and it is a Grassmann-even quantity. If the superalgebra (2.9) is computed in a supersymmetric background, the spinor ϵ_A is chosen to be the Killing spinor of the background, leading to zero in the right-hand side. However, one can consider a configuration that breaks supersymmetry in the bulk but it has an asymptotic Killing spinor defined in the asymptotic region by imposing certain boundary conditions on the gravitino and the gaugini

$$\delta\Psi^A_\mu = o(1/r^n)\,, \qquad \delta\lambda^{Ai} = o(1/r^{n_i})\,, \tag{2.10}$$

where the constants $n, n_i > 0$ must be chosen in such a way that the conserved charges in (2.9) are finite. Configurations having the same boundary condition as a supersymmetric background can be understood as excitations of it. We will compute the superalgebra in backgrounds belonging to the dilatonic sector of the STU model. In that case (2.9) can be written in terms of the Majorana Killing spinor ψ^A as (see appendix C for a detailed derivation.)

$$\{\mathcal{Q},\mathcal{Q}\}=2\int\overline{\psi}^A\gamma_5\gamma\wedge\left(\delta_{AB}\mathrm{d}+\frac{1}{4}\omega_{ab}\gamma^{ab}\delta_{AB}+\frac{1}{2}A^M\theta_M\varepsilon_{AB}+\frac{1}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon_{AB}+\frac{1}{2}\gamma\mathcal{W}\delta_{AB}\right)\psi^B. \tag{2.11}$$

2.2 Electric dilatonic black holes and their singular supersymmetric limits

The four charges electric black hole in the dilatonic sector in the STU model, with a spherical horizon, was found in [3], and its generalizations to planar and hyperbolic horizons were constructed in [4]. Here, we present the solution for arbitrary horizon geometry controlled by the parameter $k = -1, 0, 1$ leading to hyperbolic, planar and spherical horizons. The BPS limit of the configurations can be analysed in a simple way by introducing a parameter q through a change of coordinates. The metric reads

$$\mathrm{d}s^2 = -\frac{f(r)}{\sqrt{H(r)}}\mathrm{d}t^2 + \frac{\sqrt{H(r)}}{f(r)}\mathrm{d}r^2 + r^2\sqrt{H(r)}\left(\frac{\mathrm{d}x^2}{1-kx^2} + (1-kx^2)\mathrm{d}y^2\right)\,, \tag{2.12}$$

$$f(r) = k + \frac{r^2}{L^2}H(r) - \frac{m}{r} - \frac{q}{r^2}\,, \qquad H(r) = H_1H_2H_3H_4\,, \qquad H_\Lambda = 1 + \frac{q_\Lambda}{r}\,. \tag{2.13}$$

The dilatons and the gauge fields are

$$\phi_1 = \frac{1}{2}\log\left(\frac{H_2H_3}{H_1H_4}\right)\,, \qquad \phi_2 = \frac{1}{2}\log\left(\frac{H_1H_3}{H_2H_4}\right)\,, \qquad \phi_3 = \frac{1}{2}\log\left(\frac{H_1H_2}{H_3H_4}\right)\,, \tag{2.14}$$

$$A^\Lambda = \left(\frac{Q_\Lambda}{\sqrt{2}rH_\Lambda} - \mu_\Lambda\right)\mathrm{d}t\,, \qquad Q_\Lambda^2 = q_\Lambda^2 k + q_\Lambda m - q\,. \tag{2.15}$$

Where μ_Λ is related to the electric chemical potential which is fixed in terms of the rest of the parameters in such a way that the gauge fields are regular in the Euclidean configuration. A necessary condition, and sufficient in this case, to preserve some amount of supersymmetry is that the matrices of the gaugino equations (2.8) are not invertible. Imposing that the matrices have zero determinant implies the following relation between the parameters

$$-kq = \frac{m^2}{4}\,. \tag{2.16}$$

The BPS configurations exist only in the spherical and planar case, even though they are naked singularities, they have a Killing spinor defined on the geometry. For the hyperbolic case the condition (2.16) leads to purely imaginary gauge fields, which only can make sense by Wick rotating the time coordinate leading to a Euclidean configuration without a Lorentzian limit.

The Majorana Killing spinor for the planar BPS configuration can be found and is given by

$$\psi^A_{\rm pl}(r) = \frac{f_{\rm pl}^{1/4}(r)}{2H^{1/8}(r)}(\alpha_{\rm pl}(r) - \beta_{\rm pl}(r)\gamma_1)(\delta_{AB} + \varepsilon_{AB}\gamma_0)\psi_0^B\,, \tag{2.17}$$

where ψ_0^B is Majorana constant spinor and the relevant functions are

$$\alpha_{\rm pl}(r) = \left(1 - \frac{\sqrt{-q}}{r f_{\rm pl}^{1/2}}\right)^{1/2}, \qquad \beta_{\rm pl}(r) = \left(1 + \frac{\sqrt{-q}}{r f_{\rm pl}^{1/2}}\right)^{1/2}, \qquad f_{\rm pl}(r) = \frac{r^2}{L^2}H - \frac{q}{r^3}\,. \tag{2.18}$$

The projector $\delta_{AB} + \varepsilon_{AB}\gamma_0$ has matrix rank equal 2, which implies that planar BPS configuration preserves four real supercharges. Note also that the Killing spinor diverges at the curvature singularities of the manifold which are located at $H(r) = 0$.

The Majorana Killing spinor for the spherical BPS configuration is explicitly given by

$$\psi^A_{\rm sp}(t,r,x,y) = \frac{f_{\rm sp}^{1/4}(r)}{2H^{1/8}(r)} e^{\frac{it}{2L}} e^{\frac{i}{2}\gamma_{012}\arccos x} e^{-\frac{1}{2}y\gamma_{23}}(\alpha_{\rm sp}(r) + \beta_{\rm sp}(r)\gamma_1)(\delta_{AB} + \varepsilon_{AB}\gamma_0)\psi_0^B\,, \tag{2.19}$$

where the radial functions are

$$\alpha_{\rm sp}(r) = \left(1 + \frac{1-\frac{m}{2r}}{f_{\rm sp}^{1/2}}\right)^{1/2}, \qquad \beta_{\rm sp}(r) = \left(1 - \frac{1-\frac{m}{2r}}{f_{\rm sp}^{1/2}}\right)^{1/2}, \qquad f_{\rm sp}(r) = \frac{r^2}{L^2}H + \left(1 - \frac{m}{2r}\right)^2. \tag{2.20}$$

This background preserves four real supercharges, hence it is 1/2 BPS, and the Killing spinor diverges at the singularity. The spinor depends on all the coordinates, which is also the case in the purely AdS background.

2.3 Magnetic dilatonic black holes

The first example of, asymptotically globally AdS spacetime, supersymmetric static black holes was given by Cacciatori and Klemm [8] by considering extremal magnetically charged black holes. They also considered hyperbolic and planar horizon topology. The non-extremal version of the spherical black holes with an arbitrary number of vector multiplets and FI terms was constructed in [20]. In [17] a family of non-extremal solutions was constructed which contain, in certain limits, the solutions of [8] and of [3, 4]. Here we present the dilatonic, magnetic black hole configurations in a slightly different parametrization which allows us to straightforwardly connect, through a suitable BPS limit, the magnetic version of the four-dimensional black holes constructed in [3, 4] to the BPS configurations of [8], when the theory allows an embedding in the maximal gauged supergravity.

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The metric of the configuration is

$$\mathrm{d}s^2 = -\frac{f(r)}{\sqrt{H(r)}}\mathrm{d}t^2 + \frac{\sqrt{H(r)}}{f(r)}\mathrm{d}r^2 + r^2\sqrt{H(r)}\left(\frac{\mathrm{d}x^2}{1-kx^2} + (1-kx^2)\mathrm{d}y^2\right), \tag{2.21}$$

$$f(r) = \frac{r^2}{L^2}H(r) + k - \frac{m}{r} - \frac{q}{r^2}, \qquad H(r) = H_1H_2H_3H_4\,, \qquad H_\Lambda = 1 + \frac{q_\Lambda}{r}, \tag{2.22}$$

and the matter fields are given by

$$\phi_1 = -\frac{1}{2}\log\left(\frac{H_2H_3}{H_1H_4}\right), \qquad \phi_2 = -\frac{1}{2}\log\left(\frac{H_1H_3}{H_2H_4}\right), \qquad \phi_3 = -\frac{1}{2}\log\left(\frac{H_1H_2}{H_3H_4}\right),$$
$$A^\Lambda = \frac{1}{\sqrt{2}}P_\Lambda x\mathrm{d}y\,, \qquad P_\Lambda^2 = q_\Lambda^2 k + q_\Lambda m - q\,, \tag{2.23}$$

Note that the magnetic charges P_Λ are related to the rest of the parameters in order to satisfy the field equations. This configuration corresponds to the magnetic black holes constructed in [3] with different topologies for the horizon. The parameter q allow to analyze the BPS equations in a simple way, and it was introduced through a diffeomorphism by shifting the radial coordinate.

The vanishing of the determinant of matrices entering in the gaugini variations (2.8) implies a relation between the parameters that can be established in a simple way in terms of the variable p_Λ related to the q_Λ as

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}. \tag{2.24}$$

The supersymmetry conditions are given by

$$m = -2kp_1 + \frac{8}{L^2}p_2p_3p_4\,, \tag{2.25}$$

$$q = -\frac{1}{4}k^2L^2 + k(-p_1^2 + p_2^2 + p_3^2 + p_4^2) - \frac{4}{L^2}(p_3^2p_4^2 + p_2^2p_3^2 + p_2^2p_4^2 - 2p_1p_2p_3p_4)\,. \tag{2.26}$$

These are sufficient conditions to have a non-trivial Killing spinor satisfying (2.4). In this limit, the magnetic charges and the metric function can be expressed as follows:

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix} = \frac{1}{2L}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} kL^2 \\ 4p_3p_4 \\ 4p_2p_3 \\ 4p_2p_4 \end{pmatrix}, \tag{2.27}$$

$$f(r) = \frac{1}{L^2r^2}\left(\frac{k}{2}L^2 + (r+p_1)^2 - p_2^2 - p_3^2 - p_4^2\right)^2, \tag{2.28}$$

where p_Λ are real provided:[1]

$$(P_1 + P_2 - kL)(P_1 + P_3 - kL)(P_2 + P_3 - kL) < 0\,.$$

[1]If this condition is not met, we need to change the last three signs in the last column of the matrix on the right-hand side of eq. (2.27).

The metric function factorizes in a perfect square, then if there is any horizon, it will be an extremal horizon. For the BPS configuration it follows that

$$\sum_{\Lambda} P_{\Lambda} = 2kL \quad \Longleftrightarrow \quad \frac{1}{\sqrt{2}}\Gamma^{M}\theta_{M} = k\,, \tag{2.29}$$

which was recast in a symplectic-invariant way by using Γ^M that is the symplectic vector of the topological charges $\Gamma^M = (P^\Lambda, Q_\Lambda)$, and Q_Λ are the electric charges that in the present case are zero. The condition (2.29) is known as the topological twist condition, and it was shown in [14] that (2.29) is a necessarily condition to have a BPS static configuration with dyonic topological charges in $D = 4$, $\mathcal{N} = 2$ supergravity with vector multiples and FI terms.

The configurations satisfying the BPS conditions (2.25) and (2.26) have a well-defined Majorana Killing spinor given by

$$\psi^A(r) = \frac{f(r)^{1/4}}{H(r)^{1/8}}\frac{1}{4}(1+\gamma_1)(\delta_{AB} - \varepsilon_{AB}\gamma_{23})\psi_0^B\,, \tag{2.30}$$

where ψ_0^B is a constant Majorana spinor. The projector $(1+\gamma_1)(\delta_{AB} - \varepsilon_{AB}\gamma_{23})$ rule out 6 of the 8 components of the doublet ψ_0^B. Hence, the magnetic BPS black holes for any geometry of the horizon have 2 real supercharges, which corresponds to 1/4 of the total supersymmetry.

3 Thermodynamic analysis for planar configurations

In this section, we will analyze the thermodynamic stability of the planar black holes in both the electric and magnetic cases. In this case, the equation of state can be written analytically in terms of the horizon "area-density", given by

$$A = r_+^2\sqrt{H(r_+)}\,. \tag{3.1}$$

From now on, with a slight abuse of notation, we will refer to it as the horizon area. In our units the length dimension of A is 2 and the length dimension of the electric charges Q_Λ is 1.

3.1 Stability of electric black holes

Considering the electric configurations with arbitrary parameters given in (2.13), we can solve the parameter m, which up to a numerical factor corresponds to the boundary term given by the on-shell value of the Hamiltonian density [21, 22], in terms of the horizon area and the electric charges Q_Λ,

$$E \equiv m = \frac{1}{A^{1/2}}\left[\prod_{\Lambda=1}^{4}\left(\frac{A^2}{L^2} + Q_\Lambda^2\right)\right]^{1/4}\,. \tag{3.2}$$

The energy density given in (3.2) reproduces the Hawking temperature, up to a numerical factor, by computing its derivative with respect to the horizon area

$$T_H = \frac{m}{4\pi A}\left[\frac{A^2}{L^2}\sum_{\Lambda}\left(\frac{A^2}{L^2} + Q_\Lambda^2\right)^{-1} - 1\right] = \frac{1}{2\pi}\frac{\partial E}{\partial A}\,. \tag{3.3}$$

For the electric black holes, the regularity of the gauge fields at the horizon fixes the chemical potentials μ_Λ in (2.13). This can be also reproduced by the computation of the partial derivative of the energy (3.2) with respect to the electric charges

$$\mu_\Lambda = \frac{Q_\Lambda}{\sqrt{2} r_+ H_\Lambda(r_+)} = \frac{m Q_\Lambda}{\sqrt{2}(\frac{A^2}{L^2} + Q_\Lambda^2)} = \frac{1}{\sqrt{2}} \frac{\partial E}{\partial Q_\Lambda} . \tag{3.4}$$

Then, we can construct the first law for the electric black holes

$$\frac{1}{8\pi} \delta E = T_H \delta S + \frac{1}{8\pi\sqrt{2}} \sum_\Lambda \mu_\Lambda \delta Q_\Lambda , \tag{3.5}$$

where the factors can be reabsorbed in the definition of the energy density E and the electric charges Q_Λ.

Having an expression for the energy density as a function of the physical charges and the entropy of the electric black holes, we can analyse the stability of the system by studying the Hessian matrix

$$\mathcal{H}_{ab} = \frac{\partial E}{\partial l^a \partial l^b} , \qquad l^a = (A/L, Q_\Lambda) . \tag{3.6}$$

which is a 5×5 symmetric matrix, and hence there is a limitation to find the eigenvalues in a closed form for a generic value of the parameters. However, the determinant of the Hessian can be written in a simple form in terms of the temperature as follows

$$\det \mathcal{H} = \frac{\pi L^2}{4A^3} T_H - \frac{1}{16 E^3 L^4} \sum_\Lambda Q_\Lambda^2 . \tag{3.7}$$

Clearly, for all the extremal black holes, i.e. $T_H = 0$, the Hessian has at least one negative eigenvalue, indicating an instability. Consequently, all the extremal electrically charged black holes in this ensemble are thermodynamically unstable. Furthermore, even above extremality, there is a finite gap for which these black holes are all unstable.

Now we will specialize in the computation for electrically charged Reissner-Nordström black hole with a planar horizon, which is obtained by setting $q_\Lambda = 0$ and defining $Q^2 = -q$. The metric function and gauge fields reduce to its standard form

$$f(r) = \frac{r^2}{L^2} - \frac{m}{r} + \frac{Q^2}{r^2} , \qquad A^\Lambda = \left(\frac{Q}{\sqrt{2} r} - \mu_\Lambda \right) \mathrm{d}t , \tag{3.8}$$

and the temperature of the black hole becomes

$$T_H = \frac{3A^2 - L^2 Q^2}{4\pi A^{3/2} L^2} . \tag{3.9}$$

In this case, it is possible to compute the eigenvalues of the Hessian matrix (3.6) which are given by

$$\lambda = \frac{A^2 - L^2 Q^2}{2\sqrt{A}(A^2 + L^2 Q^2)} , \tag{3.10}$$

$$\lambda_\pm = \frac{1}{8A^{1/2}} \left(5 + \frac{3L^2 Q^2}{A^2} \pm \sqrt{1 + 22 L^2 \frac{Q^2}{A^2} + 9 L^4 \frac{Q^4}{A^4}} \right) , \tag{3.11}$$

where λ has multiplicity three. The eigenvalues $\lambda_\pm$ are always positive for any value of $A > 0$ and $Q \in \mathbb{R}$, while the triple eigenvalue λ is positive for large black holes compared with the AdS radius and the electric charge, namely $A > L|Q|$. Whence the spinodal line, which separates the stable from the unstable region, is located at $A = L|Q|$. Consistently with our previous discussion, the extremal black holes are obtained at $A = L|Q|/\sqrt{3} < L|Q|$, namely outside the stability region.

3.2 Stability of magnetic planar black holes

The planar magnetic black holes presented in (2.21) have a well-defined BPS limit that generically represents extremal BPS black holes with a globally defined Killing spinor (2.30). One can notice that for the magnetic black holes (2.21), it is also possible to solve the integration constant m in terms of the horizon area A, defined in (3.1), and the magnetic charges P_Λ as

$$m = \frac{1}{A^{1/2}} \left[\prod_\Lambda \left(\frac{A^2}{L^2} + P_\Lambda^2 \right) \right]^{1/4} . \tag{3.12}$$

Then, if m is identified with the energy density of the configurations, we can run the same argument that we outline for the electric black holes and conclude that the extremal BPS black holes are unstable. This is in tension with the fact that the magnetic planar BPS black holes are vacuum states of the theory, and therefore are believed to be stable. In what follows, we will show that the configurations (2.21) with $k = 0$ asymptote to the BPS configurations, in the sense that they admit an asymptotic Killing spinor which leads to finite conserved charges, if and only if the topological twist condition (2.29) is satisfied. Then, we compute the Dirac bracket between the supercharges for the asymptotic Killing spinor, showing that the quantity that we would like to identify with the energy density should vanish in the BPS limit. We will take this fact into account to propose an energy density that leads to a semi-positive defined Hessian matrix on backgrounds that satisfy the topological twist condition imposed at the beginning of the analysis.

To simplify the analysis we consider the supersymmetry transformation for the complex spinors, and denote the complex gravitino and complex gaugini as the chiral one but erasing the SU(2) index. The leading order of the gaugini equations expanded at $r \to \infty$ goes as $1/r$ and is a matrix equation that can be solved by imposing the following projector

$$\zeta_\infty = \frac{1}{2}(1 + \gamma_1)\chi_\infty \,, \tag{3.13}$$

then the subleading term of the complex gaugini equations read

$$\delta\lambda^i = \frac{1}{2r^2} \left[\frac{(P_i^2 - P_4^2)^2 - (P_j^2 - P_k^2)^2}{2Lm^2} + \sum_\Lambda \Omega_{i\Lambda} P_\Lambda \mathrm{i}\gamma_{23} \right] \zeta_\infty + o(r^{-3}) \,, \qquad i \neq j \neq k \neq 4 \,, \tag{3.14}$$

$$\Omega_{i\Lambda} = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix} . \tag{3.15}$$

It is interesting to notice that the matrix in bracket in the subleading term is invertible unless the P_Λ and m satisfy the BPS conditions; the same will happen for the subleading terms in the gravitino equations. Using the projector (3.13) the gravitino equations for the complex spinor in the asymptotic region are given by

$$\delta\Psi_t = \partial_t\zeta_\infty + \frac{\mathrm{i}}{8Lr}\sum_\Lambda P_\Lambda\gamma_{023}\zeta_\infty + o(r^{-2})\,, \tag{3.16}$$

$$\delta\Psi_r = \partial_r\zeta_\infty - \frac{1}{2r}\zeta_\infty + o(r^{-2})\,, \tag{3.17}$$

$$\delta\Psi_x = \partial_x\zeta_\infty + \frac{\mathrm{i}}{8r}\sum_\Lambda P_\Lambda\gamma_3\zeta_\infty + o(r^{-2})\,, \tag{3.18}$$

$$\delta\Psi_y = \partial_y\zeta_\infty - \frac{\mathrm{i}}{4}x\sum_\Lambda P_\Lambda\zeta_\infty + \frac{\mathrm{i}}{8r}\sum_\Lambda P_\Lambda\gamma_2\zeta_\infty + o(r^{-2})\,. \tag{3.19}$$

Note that the second term in the equation (3.19) is leading in the expansion on r, thus, a necessarily condition on the background to have asymptotic Killing spinors is that the topological twist condition (2.29) must be fulfilled. Solving the leading order of the above equations equal zero, and going back to the Majorana spinor, we find that the asymptotic Killing spinor on a background that fulfills the topological twist condition is given by

$$\psi^A_\infty(r) = \frac{1}{2}r^{1/2}(1+\gamma_1)\psi^A_0 + o(r^{-1/2}) \equiv r^{1/2}\mathbb{P}_{AB}\psi^B_0\,, \tag{3.20}$$

where ψ^A_0 is a constant doublet of Majorana spinors. Observe that the radial dependency agrees with both the expansion at infinity and the projection $1+\gamma_1$ of the Killing spinor of the BPS background given in (2.30), and there are no further projections of the asymptotic Killing spinor. Therefore, the asymptotic spinor (3.20) has 4 independent real components which represent an enhancement of the 2 independent real components with respect to the global Killing spinor defined in the vacuum (2.30). Indeed we can assert that the asymptotic Killing spinor can be split into two independent spinors obtained by projecting (3.20) with the projector

$$\mathbb{P}^\pm_{AB} \equiv \frac{1}{4}(1+\gamma_1)(\delta_{AB} \pm \varepsilon_{AB}\gamma_{23})\,. \tag{3.21}$$

Computing the right-hand-side of the algebra (2.9) on the background (2.21) satisfying the topological twist condition for the asymptotic Killing spinor (3.20) we get the following result

$$\{\mathcal{Q},\mathcal{Q}\} = -\frac{1}{2}\int_{\partial\Sigma}\sum_\Lambda H^{1/4}\left[\left(-\frac{\mathrm{i}r^2f^{1/2}}{H_\Lambda} + \frac{\mathrm{i}r^3}{L}H_\Lambda\right)\overline{\psi}^A_0\gamma_0\mathbb{P}_{AB}\psi^B_0 - \frac{rP_\Lambda}{H_\Lambda}\overline{\psi}^A_0\gamma_5\varepsilon_{AB}\mathbb{P}_{BC}\psi^C_0\right]\mathrm{d}x\wedge\mathrm{d}y. \tag{3.22}$$

Note that the last term would be combined with the first term if the asymptotic spinor would satisfy the extra projection condition that we are lacking to reproduce from the asymptotic analysis. We proceed as follows, let us consider the two independent Killing spinors obtained by projecting the asymptotic Killing spinor (3.20) with (3.21) and then computing Dirac bracket between supercharges twice, one for each independent spinor. To emphasize this fact we include a subscript in the supercharge $\mathcal{Q}_\pm$. The resulting algebra is the following

$$\{\mathcal{Q}_\pm, \mathcal{Q}_\pm\} = \frac{\mathrm{i}}{2}\int_{\partial\Sigma}\sum_\Lambda H^{1/4}\left(-\frac{r^2 f^{1/2}}{H_\Lambda} + \frac{r^3}{L}H_\Lambda \pm \frac{rP_\Lambda}{H_\Lambda}\right)\overline{\psi}_0^A\gamma^0\mathbb{P}^\pm_{AB}\psi_0^B \mathrm{d}x\wedge\mathrm{d}y\,, \tag{3.23}$$

$$= \mathrm{i}\mathbf{M}_\pm\overline{\psi}_0^A\gamma^0\mathbb{P}^\pm_{AB}\psi_0^B\int \mathrm{d}x\wedge\mathrm{d}y\,, \tag{3.24}$$

where

$$\mathbf{M}_\pm = \frac{L}{2m^3}(m^2 \pm m^2_{\mathrm{BPS}})(2m^2 \pm m^2_{\mathrm{BPS}})\,, \tag{3.25}$$

$$m^2_{\mathrm{BPS}} \equiv \frac{1}{2\sqrt{2}L}\prod_{i=1}^{3}\left(\sum_\Lambda \Omega_{i\Lambda}P_\Lambda^2\right)^{1/2}\,, \tag{3.26}$$

and $\Omega_{i\Lambda}$ is defined in (3.15). We will show that indeed, the charges are such that $m^2_{\mathrm{BPS}} > 0$ in the region of the phase space where the black holes exist. As expected the BPS bound computed with the spinors that asymptote to the Killing spinor of the background (2.30), i.e. with the lower sign, leads to a non-trivial constraint on the parameter m

$$\mathbf{M}_- > 0 \implies m > m_{\mathrm{BPS}}\,. \tag{3.27}$$

While the BPS bound coming from the asymptotic Killing spinor with the upper sign is trivially fulfilled. Now, we go back to the issue of the definition of the energy density in this configuration.

Note that the derivative of (3.12) with respect to the horizon area correctly reproduces the Hawking temperature of the black hole. To avoid spoiling the above relation, any reasonable attempt to define a new thermodynamic energy density can only differ from (3.12) by the addition of a function of the magnetic charges.

For configurations satisfying the topological twist condition, we propose the following definition of energy density

$$E = m(A, P_\Lambda) - m_{\mathrm{BPS}}(P_\Lambda)\,, \tag{3.28}$$

where $m(A, P_L)$ is given in (3.12) and m_{BPS} is given by (3.26). Under these considerations, it is straightforward to prove that the 4×4 Hessian matrix is semi-positive defined on extremal configurations.

Now we move to the discussion on the existence of black hole configuration in the extremal limit that satisfies the topological twist condition. First of all, observe that the location of the horizon in terms of the horizon area and the location of singularities in terms of the integration constant m and q are

$$r_0 = \frac{A^2}{L^2 m} - \frac{q}{m}\,, \qquad r_\Lambda^{(\mathrm{sing})} = -q_\Lambda = -\frac{P_\Lambda^2}{m} - \frac{q}{m}\,, \tag{3.29}$$

respectively. The black hole configuration exist if $r_0 > r_\Lambda^{(\mathrm{sing})}$ which implies that

$$\frac{1}{m}\left(\frac{A^2}{L^2} + P_\Lambda^2\right) > 0\,. \tag{3.30}$$

The existence of a zero of the metric function $f(r_0)=0$ implies the existence of a horizon with horizon area $A>0$. The equation (3.30) implies that if such a condition is fulfilled, a horizon automatically covers the singularities.

Extremal configurations have Hawking temperature equal to zero. The Hawking temperature for the magnetic black hole configurations is obtained by computing the derivative of the energy (3.28) with respect to the entropy over 8π, leading to

$$T_H = \frac{m(A,P_\Lambda)}{4\pi A}\left[\frac{A^2}{L^2}\sum_\Lambda\left(\frac{A^2}{L^2}+P_\Lambda^2\right)^{-1}-1\right]. \tag{3.31}$$

Replacing the topological twist condition on (3.31), we find that the right-hand-side gets factorized and consequently

$$T_H=0 \qquad \Longrightarrow \qquad \mathrm{Pol}_+(A,P_\Lambda)\mathrm{Pol}_-(A,P_\Lambda)=0\,, \tag{3.32}$$

where

$$\mathrm{Pol}_\pm(A,P_\Lambda)=(2\pm1)\frac{A^4}{L^4}+\frac{A^2}{2L^2}\sum_\Lambda P_\Lambda^2\mp\prod_\Lambda P_\Lambda \qquad \text{satifying} \qquad \sum_\Lambda P_\Lambda=0\,. \tag{3.33}$$

The greater root for A coming from $\mathrm{Pol}_-(A,P_\Lambda)=0$, which we call A_-, correctly reproduces the horizon area of BPS black holes. While the greater root for A from $\mathrm{Pol}_+(A,P_\Lambda)=0$, which we call A_+, corresponds to the horizon area of non-BPS extremal black holes. In the surface with $T_H=0$ for configurations satisfying the topological twist conditions, there are three magnetic charges as remaining free parameters. The horizon areas A_+ and A_- take values on this space. It is interesting to note that in the region where $A_->0$ we have $A_+<0$ and vice versa, therefore, the location where $A_-=0$ coincides with $A_+=0$. This is depicted in figure 1.

One can interpret this result as follows. For extremal black holes, there are certain boundary conditions that allow the existence of supersymmetric magnetic black holes, and its complementary region will lead to extremal non-BPS black holes. These two essentially different boundary conditions lead to the horizon area A_- and A_+, respectively, that can be written in terms of quartic invariant quantities constructed out of the embedding tensor θ_M, the topological charges symplectic vector Γ^M and K_{MNPQ} rank-4 completely symmetric tensor of $\mathrm{Sp}(8,\mathbb{R})$. The explicit form of the horizon area for the extremal BPS black holes is[2]

$$A_-^2=\frac{3}{2}\frac{I_2}{I_0}+\sqrt{\left(\frac{3}{2}\frac{I_2}{I_0}\right)^2-\frac{1}{4}\frac{I_4}{I_0}}\,, \tag{3.34}$$

since $I_2<0$ then the area is real an if $I_4<0$. The horizon area for extremal non-BPS black holes are

$$A_+^2=\frac{1}{2}\frac{I_2}{I_0}+\sqrt{\left(\frac{1}{2}\frac{I_2}{I_0}\right)^2+\frac{1}{12}\frac{I_4}{I_0}}\,. \tag{3.35}$$

[2]For the expression of A_- in terms of $\mathrm{SL}(2,\mathbb{R})^3$-invariants see [23]. Here we also give the analogous expression for the horizon-area A_+, corresponding to new extremal non-BPS solutions.

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again $I_2 < 0$ then the above area is real if $I_4 > 0$. We have defined

$$I_0 = -2\mathcal{I}_1(\theta,\theta,\theta,\theta) = \frac{1}{L^4}\,, \tag{3.36}$$

$$I_4 = -2\mathcal{I}_1(\Gamma,\Gamma,\Gamma,\Gamma) = 4\prod_{\Lambda} P_\Lambda\,, \tag{3.37}$$

$$I_2 = -\frac{2}{3}\sum_{i=1}^{3}\mathcal{I}_i(\Gamma,\theta,\Gamma,\theta) = -\frac{1}{6L^2}\sum_{\Lambda} P_\Lambda^2\,, \tag{3.38}$$

where the last relation was obtained provided that $\sum_\Lambda P_\Lambda = 0$, see appendix D for the definition of the tensor $\mathcal{I}_i$.

3.3 First-order description and stable extremal non-BPS solutions

The BPS solutions discussed above admit a first-order description in terms of gradient-flow equations defined by a suitable black-hole superpotential. The general form of the latter was found, in the spherical horizon case, in [14]. A general discussion of the first-order description of extremal solutions in the STU model was performed in [17]. Using the following standard notation for the spacetime metric:

$$ds^2 = -e^{-2U}\,dt^2 + e^{-2U}\,dr^2 + e^{2(\psi-U)}\,d^2\Omega\,, \tag{3.39}$$

where $d^2\Omega$ is the metric on the horizon and $U = \mathrm{U}(r)$, $\psi = \psi(r)$, the superpotential for the BPS case can be written in the form:

$$\mathscr{W}(U,\psi,z^i,\bar{z}^{\bar{\imath}}) = e^U\,(\mathcal{Z}+i\,e^{2(\psi-U)}\mathcal{W})\,. \tag{3.40}$$

where $\mathcal{Z} \equiv V^T\mathbb{C}\,\Gamma$ is the $\mathcal{N}=2$ central charge and $\mathcal{W} \equiv V^T\,\theta$ is the gauge superpotential. In our solutions:

$$\mathrm{U}(r) = \frac{1}{2}\log\left(\frac{f(r)}{H(r)^{1/2}}\right)\,, \qquad \psi(r) = \log\left(r f(r)^{1/2}\right) \tag{3.41}$$

The scalar fields satisfy the gradient flow equations:

$$\frac{\mathrm{d}z^i}{\mathrm{d}r} = -2\,e^{-2\psi}\,g^{i\bar{\jmath}}\partial_{\bar{\jmath}}|\mathscr{W}|\,.$$

There is a non-BPS branch of extremal solutions whose first-order description was studied in [15–17]. The fake-superpotential for the dilatonic solutions has the form

$$\mathscr{W}_{\text{non-BPS}}(U,\psi,z^i,\bar{z}^{\bar{\imath}}) = e^U\left(\frac{1}{2}\left(\mathcal{Z}+\sum_{\hat{\imath}=1}^{3}\mathcal{D}_{\hat{\imath}}\mathcal{Z}\right)+i\,e^{2(\psi-U)}\mathcal{W}\right)\,, \tag{3.42}$$

where:

$$\mathcal{D}_{\hat{\imath}}\mathcal{Z} \equiv e_{\hat{\imath}}^{j}\left(\partial_i + \frac{1}{2}\,\partial_i\mathcal{K}\right)\mathcal{Z}\,, \tag{3.43}$$

are the three matter charges, $e_{\hat{\imath}}^{j}$ being the inverse vielbein matrix, see appendix A for the relevant definitions related to the special geometry of the model.

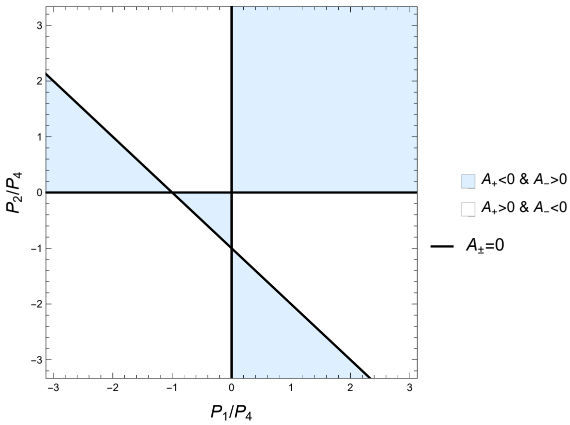

Figure 1. Considering $P_3 = -P_1 - P_2 - P_4$, we consider the plane $(P_1/P_4, P_2/P_4)$ where the coloured regions correspond to the existence of susy black holes with $A_- > 0$. We defined $A_\pm$ as the grater root for A of $\mathrm{Pol}_\pm(A, P_\Lambda) = 0$.

The expressions for the relevant quantities for these non-BPS solutions are obtained from the corresponding ones derived above for the BPS black holes, upon changing $P_4 \to -P_4$. The topological twist condition (2.29), for instance, for the flat-horizon case, becomes:

$$P_1 + P_2 + P_3 - P_4 = 0\,. \tag{3.44}$$

Just as it happened for the BPS case, the above condition implies a factorization of the expression of the temperature T_H. The horizon area now corresponds to a root A'_- given by that of A_- by changing $P_4 \to -P_4$:

$$A'^2_- = \frac{3}{2}\frac{I_2}{I_0} + \sqrt{\left(\frac{3}{2}\frac{I_2}{I_0}\right)^2 + \frac{1}{4}\frac{I_4}{I_0}}\,, \tag{3.45}$$

since $I_2 < 0$ then the area is real an if $I_4 > 0$. The expression of the Hessian, as a function of the charges, is the same as that of the BPS case. By the same token, we then conclude that also these non-BPS extremal solutions, described by first-order gradient flow equations, are stable. Stability seems then to be implied by the existence of a first-order description of the solution.

By the same token, we also find, for suitable values of the magnetic charges, extremal non-BPS solutions with area A'_+ whose expression is obtained from that of A_+ by changing $P_4 \to -P_4$:

$$A'^2_+ = \frac{1}{2}\frac{I_2}{I_0} + \sqrt{\left(\frac{1}{2}\frac{I_2}{I_0}\right)^2 - \frac{1}{12}\frac{I_4}{I_0}}\,. \tag{3.46}$$

Reality of A'_+ then requires $I_4 < 0$. We conclude that the condition $I_4 < 0$ does not uniquely define the BPS configuration. This is to be expected since, in the two cases, the linear conditions on the charges, (2.29) and (3.44), are different.

The stability of the magnetic configuration can be compared with perturbative stability analysis existent in the literature [24, 25] where the authors studied the perturbative stability of magnetic configurations in the context of the purely dilatonic sector of the STU model. In particular, they studied the stability of magnetically charged $\mathrm{AdS}_2 \times \mathbb{R}^2$ geometry, which corresponds to the near horizon region of the extremal magnetically charged black holes considered here.

Considering the extremal limit of the magnetically charged black hole solutions in (2.21)–(2.23), we can find analytically an expression for the value of the scalars at the horizon

$$\phi_i(r_+) = -\log\left[\frac{m^2 A}{(P_i^2 + A^2/L^2)(P_4^2 + A^2/L^2)}\right], \qquad i = 1, 2, 3. \tag{3.47}$$

In [25] they studied a perturbation of the STU model of $\mathrm{AdS}_2 \times \mathbb{R}^2$ background in the T^3-truncation of the STU model, and gave an analytic expression for the eigenvalues of the mass matrix. These backgrounds can be obtained from our black holes by taking the extremal limit and then the near horizon geometry for $\pm P_1 = \pm P_2 = \pm P_3 \equiv \mathcal{P}$, for any choice of the relative signs. The smallest eigenvalue of the mass matrix of the perturbation is given by [25][3]

$$L^2_{\mathrm{AdS}_2} m^2_{\mathrm{min}} = -\frac{1}{48}\frac{(5 + 3\overline{X}^4)^2}{2 + 3\overline{X}^4 + \overline{X}^8}, \tag{3.48}$$

$$\overline{X} \equiv \frac{m A^{1/2}}{(\mathcal{P}^2 + A^2/L^2)^{1/2}(P_4^2 + A^2/L^2)^{1/2}}, \tag{3.49}$$

where L_{AdS_2} is the AdS radius of the AdS_2 near the horizon and $\overline{X} = Y_1^{-1/2} = Y_2^{-1/2} = Y_3^{-1/2} = Y_4^{1/6}$ evaluated at the horizon. The BF bound on the AdS_2 is violated when $\overline{X} < (-1 + \frac{2}{\sqrt{3}})^{1/4}$. We found that for the BPS configuration $\overline{X}$ saturates the bound $\overline{X}|_{\mathrm{BPS}} = (-1 + \frac{2}{\sqrt{3}})^{1/4}$ in agreement with [25], while for the non-BPS configuration $\overline{X}|_{\mathrm{non-BPS}} = 1 > (-1 + \frac{2}{\sqrt{3}})^{1/4}$, which is above the BF bound indicating that the instability is not triggered by the particular perturbation considered in [25]. For the non-BPS, thermodynamically stable configurations, we found that they also saturate the BF bound, as they are obtained from the BPS configurations by mapping $P_4 \to -P_4$, and the value of the scalars at the horizons depends on the P_Λ^2.

4 Conclusion

In this work, we presented a simple argument to prove the thermodynamic instability of extremal planar 4-charges electric black holes in the STU model of the maximal theory in $D = 4$. This result constitutes a generalization of the instabilities of electric black holes studied in [11]. The argument can be made sharper to conclude that there is a finite critical lower temperature for which electrically charged black holes are unstable, indicating that the mechanics that trigger the instability do not rely on particular features of extremal black holes. We note that indeed it is well known that extremal black holes are unstable under charged perturbations [26]. However, the phenomenon presented here is different because it does not require extremality and occurs at fixed charges. It seems, in other words, to somehow capture the non-linearity of gauged $\mathcal{N} = 8$ supergravity.

[3] The exact mapping between conventions is the following: $\phi_i^{\mathrm{here}} = -\phi_i^{\mathrm{there}}$, $A_\Lambda^{\mathrm{here}} = 2A_\Lambda^{\mathrm{there}}$, and $L = 1/\sqrt{2}$.

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It would be interesting to investigate which kind of instability is at work. There are at least two candidates: a Gregory-Laflamme type of instability, due to the presence of flat directions; a superradiance effect that could appear in rotating black holes, as in our case, from the point of view of 11D supergravity, the system corresponds to rotating M2-branes along the $\mathrm{U}(1)^4$ isometries of the S^7.

We also considered 4-charge magnetic black hole configurations with different horizon topologies constructed in [3, 4], and presented them in a slightly different form that allows us to prove that they do have a regular black hole BPS limit. This result shows that, when the embedding in the maximal theory exists, the non-extremal configurations which generalize [8], correspond to the black holes discussed in [3, 4] and, in the spherical case, they coincide with the non-extremal black holes presented in [20].

Within the context of extremal magnetically charged black holes with flat horizon geometry, we identified three families of extremal black holes classified by their boundary conditions. One of them corresponds to the family of extremal BPS black holes, whose fields can be written as a solution of a first-order system of equations controlled by a superpotential [14], and the quartic invariant in the charges turns out to be negative. The thermodynamic stability analysis of the Hessian matrix imposing the topological twist condition ab initio indicates these configurations are metastable. The remaining two families of black holes are extremal non-BPS and they differ in the sign of the quartic invariant; this suggests that the sign of the quartic invariant is not a sufficient condition to identify a black hole configuration as supersymmetric or not. One of these families is obtained from the extremal BPS family by flipping the sign of the magnetic charge of the graviphoton, and thus satisfies a topological twist condition with one sign flipped. This class is also described by a first-order system controlled by a fake-superpotential [16]. We assert thermodynamic quasi-stability for the black holes belonging to this family, and the perturbative stability analysis carried out in [25] shows that the mass of the perturbed scalars saturate the BF bound in the AdS_2. The last family corresponds to extremal non-BPS configurations that exist for certain choices of boundary conditions in a complementary region in the space of the magnetic charges where the two latter families exist. These black holes are not described by a first-order system and its Hessian has generically negative eigenvalues indicating thermodynamic instability. Applying the analysis in [25] to these backgrounds leads to scalars whose mass squared is above the BF bound of the AdS_2, indicating that the instability is not triggered by the perturbation considered in [25]. A more exhaustive perturbative analysis is required to explore this further. In general, all these instabilities open the question of what is the phase diagram relevant for supergravity and its dual field theory, something that we expect to explore in the future along the lines of [27].

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A The STU model from maximal supergravity

Let us recall in this appendix the basic facts about the STU model and its embeddings in the SO(8)-gauged maximal supergravity [3]. This STU model describes $\mathcal{N}=2$ supergravity coupled to 3 vector multiplet, with a suitable Fayet-Iliopoulos (FI) term. The four vector fields gauge a Cartan subalgebra of the $\mathfrak{so}(8)$ gauge algebra. The corresponding gauge group $\mathrm{SO}(2)^4 = \mathrm{SO}(2)_0 \times \mathrm{SO}(2)_1 \times \mathrm{SO}(2)_2 \times \mathrm{SO}(2)_3 \subset \mathrm{SO}(8)$ can be chosen so that each factor act on one of the four couples (A_I), $I = 0, \ldots, 3$:

$$A_0 = (1,2); \qquad A_1 = (3,4); \qquad A_2 = (5,6); \qquad A_3 = (7,8).$$

in which the R-symmetry index of the eight gravitini $\psi_{\mathtt{i}\mu}$, $\mathtt{i} = 1, \ldots, 8$, of the maximal theory, can be split:

$$\Psi_{\mathtt{i}\mu} = \{\Psi_{A_I\,\mu}\}_{I=0,\ldots,3}\,,$$

so that the couple of gravitini $\psi_{A_I\,\mu}$ transform as a doublet under $\mathrm{SO}(2)_I$. The bosonic sector of the STU model is defined by the fields of the $\mathcal{N}=8$ theory which are singlets under $\mathrm{SO}(2)^4$. The 28 gauge fields $A^{\mathtt{IJ}}_\mu$, $\mathtt{I},\mathtt{J} = 1, \ldots 8$, of the parent model, transforming in the $\mathbf{28}$ of $\mathrm{SL}(8,\mathbb{R})$, together with their magnetic duals $A_{\mathtt{IJ}\,\mu}$, yield, upon truncation, the following singlet vector fields

$$(A^M_\mu) = \mathbf{\Omega}\cdot(A^{12}_\mu,\, A^{34}_\mu,\, A^{56}_\mu,\, A^{78}_\mu,\, A_{12\,\mu},\, A_{34\,\mu},\, A_{56\,\mu},\, A_{78\,\mu})\,. \tag{A.1}$$

The above fields transform as a symplectic vector in the $(\mathbf{2},\mathbf{2},\mathbf{2})$ of the classical global symmetry group $\mathrm{SU}(1,1)^3$ of the $\mathcal{N}=2$ theory and $\mathbf{\Omega}$ is a constant 8×8 symplectic matrix encoding the freedom in the choice of the basis of the symplectic representation (*symplectic frame*). We shall discuss the relevant symplectic frames below.

The 70 scalar fields $\phi^{\mathtt{ijkl}}$ of the maximal theory, transforming in the $\mathbf{70}$ of SU(8),[4] upon reduction to $\mathrm{SO}(2)^4$-singlets, reduce to three complex scalars

$$(\phi_{(12)(34)}, \phi_{(12)(56)}, \phi_{(12)(78)}) = z^i = \chi_i + i e^{-\phi_i}\,,\; i = 1,2,3$$

spanning the scalar manifold of the STU model:

$$\mathscr{M}_{\text{scal.}} = \left(\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\right)^3\,.$$

This is a *special Kähler* manifold[5] whose isotropy group $\mathrm{U}(1)^3$ being the subgroup of SU(8) commuting with the $\mathrm{SO}(2)^4$ residual gauge group. The isometry group $\mathrm{SU}(1,1)^3$ defines the on-shell global symmetry group of the classical theory, acting on the eight A^M_μ in the $(\mathbf{2},\mathbf{2},\mathbf{2})$ symplectic representation, as mentioned above.

The fermionic sector, on the other hand, depends on the factor $\mathrm{SO}(2)_I$, within $\mathrm{SO}(2)^4$, which is chosen to lie within the R-symmetry group of the $\mathcal{N}=2$ model. This sector of the

[4]Recall that the $\phi^{\mathtt{ijkl}}$ satisfy the reality condition $\phi^{\mathtt{ijkl}} = \frac{1}{24}\,\epsilon^{\mathtt{ijkl}\mathtt{i}'\mathtt{j}'\mathtt{k}'\mathtt{l}'}\,(\phi^{\mathtt{i}'\mathtt{j}'\mathtt{k}'\mathtt{l}'})^*$.

[5]See [28], and references therein, for a review of special geometry and of the gauging of $\mathcal{N}=2$ models in the embedding tensor formalism.

corresponding STU model, which will be referred to as STU_I, is then defined as the singlet sector, within the maximal theory, with respect to the remaining group $\mathrm{SO}(2)^3 = \mathrm{SO}(2)_J \times \mathrm{SO}(2)_K \times \mathrm{SO}(2)_L$, $I \neq J \neq K \neq L \neq I$ and $\psi_{A_I\,\mu}$ are the spin-3/2 fields of the truncation. The consistency of this truncation is guaranteed by the fact that the STU_I model, so defined, is the singlet sector with respect to the subgroup $\mathrm{SO}(2)^3 = \mathrm{SO}(2)_J \times \mathrm{SO}(2)_K \times \mathrm{SO}(2)_L$ of SO(8). Choosing, for instance, the STU_0 model, whose gravitini are $\psi_{A_0\,\mu} = (\psi_{1\,\mu},\, \psi_{2\,\mu})$, the spin-1/2 fields which are singlets with respect to $\mathrm{SO}(2)_1 \times \mathrm{SO}(2)_2 \times \mathrm{SO}(2)_3$, are:

$$(\lambda^{A_0 34}, \lambda^{A_0 56}, \lambda^{A_0 78}) \equiv \lambda^{iA_0}\,, \quad i = 1,2,3,\ A_0 = 1,2.$$

and enter the three vector multiplets together with the complex scalar fields and three of the vector fields.

Upon reducing the gauge group of the maximal theory to $\mathrm{SO}(2)^4$ and the electric/magnetic duality index to the eight $\mathrm{SO}(2)^4$-singlets, the gauge connection reduces to:[6]

$$A^M_\mu\, X_M = A^M_\mu\, \Theta_M{}^I\, t_I\,, \tag{A.2}$$

where t_I are the four generators of $\mathrm{SO}(2)^4$. In the STU_I model, only t_I has a non-trivial action on the fermionic fields and thus the only minimal couplings involve the spin-1/2 and 3/2 fields and the single vector combination $A^M_\mu\, \Theta_M{}^I$. The 8-component symplectic vector $\Theta_M{}^I$ defines the FI term of the model and the corresponding scalar potential. For the sake of concreteness, we shall work in the STU_0 model, which we shall simply refer to as the STU model, denoted by θ_M the corresponding FI term $\Theta_M{}^0$ and by $A, B = 1,2$ the R-symmetry indices A_0, B_0.

geometry of the spacial Kähler manifold $\mathscr{M}_{\text{scal.}}$ is described in terms of a holomorphic symplectic section $\Omega^M(z_i)$ which, in the symplectic frame that we adopt and modulo multiplication by a non-vanishing holomorphic function, reads:

$$\Omega^M = (z_2 z_3, z_1 z_3, z_1 z_2, -1, z_1, z_2, z_3, -z_1 z_2 z_3)\,, \tag{A.3}$$

The Kähler potential, for instance, reads:

$$\mathcal{K}(z,\bar{z}) = -\log(i\,\overline{\Omega}^T \mathbb{C}\,\Omega) = 8\,\mathrm{Im}(z_1)\mathrm{Im}(z_2)\mathrm{Im}(z_3) = 8\,e^{-\phi_1-\phi_2-\phi_3}\,. \tag{A.4}$$

where $\mathbb{C}$ is the $\mathrm{Sp}(8,\mathbb{R})$-invariant matrix:

$$\mathbb{C} = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{1} & \mathbf{0} \end{pmatrix} \tag{A.5}$$

The metric reads:

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K} = -\frac{1}{(z_i - \bar{z}_{\bar{i}})^2}\delta_{i\bar{j}} = \sum_{\hat{i}=1}^{3} e_i^{\hat{i}} (e_j^{\hat{i}})^*\,, \tag{A.6}$$

where $e_i^{\hat{i}}$ is the complex vielbein matrix. We also define the covariantly holomorphic symplectic section $V^M(z,\bar{z}) \equiv e^{\mathcal{K}/2}\,\Omega(z)^M$, in terms of which the $\mathcal{N}=2$ central charge $\mathcal{Z}$ on a given solution and the gauge superpotential $\mathcal{W}$ induced by the FI term, read:

$$\mathcal{Z} \equiv \Gamma^T \cdot \mathbb{C} \cdot V\ ,\ \ \mathcal{W} \equiv V^T \cdot \theta\,, \tag{A.7}$$

$\Gamma = (\Gamma^M) = (P^\Lambda,\, -Q_\Lambda)$ being the symplectic vector of quantized charges.

[6] We absorb the gauge grouping constant g in the embedding tensor.

The relationship between the different STU_I models within the maximal one can be inferred from inspection of the gravitino shift-matrix $\mathrm{A}_{1\mathtt{ij}}$ as a function of the singlet scalars, whose only non-vanishing entries are:

$$\begin{aligned}
\mathrm{A}_{1\,A_0,A_0}\,(L^{-1}) &= \frac{1}{\sqrt{2}}\,\overline{\mathcal{W}} = \frac{1}{\sqrt{2}}\,\overline{V}^T\cdot\theta\,,\\
\mathrm{A}_{1\,A_i,A_i}\,(L^{-1}) &= \frac{1}{\sqrt{2}}\,\overline{\mathcal{W}}_{(i)} = \frac{1}{\sqrt{2}}\,\mathcal{D}_{\hat{i}}V^M\,\Theta_M{}^i\,,\quad i=1,2,3\,,
\end{aligned} \tag{A.8}$$

where $\mathcal{D}_{\hat{i}}V^M$ are the Kähler-covariant derivatives of V^M:

$$\mathcal{D}_{\hat{i}}V^M \equiv e_{\hat{i}}^i\,\mathcal{D}_iV^M = e_{\hat{i}}^i\left(\partial_i+\frac{1}{2}\,\partial_i\mathcal{K}\right)V^M\ ,\ e_{\hat{i}}^i = -(z_i-\bar{z}_{\bar{\imath}})\delta_{\hat{i}}^i\,.$$

If $\mathcal{W}_{(0)}\equiv\mathcal{W}$ is the gauge superpotential in the STU=STU_0 model, $\mathcal{W}_{(i)}$ is the corresponding function in the STU_i model. In the chosen symplectic frame, we have:

$$\begin{aligned}
\theta_M \equiv \Theta_M{}^0 &= \frac{1}{\sqrt{2}L}\,(1,1,1,1,0,0,0,0)\,,\\
\Theta_M{}^1 = \mathcal{O}_{1\,M}{}^N\,\theta_N\,,\qquad &\Theta_M{}^2 = \mathcal{O}_{2\,M}{}^N\,\theta_N\,,\qquad \Theta_M{}^3 = \mathcal{O}_{3\,M}{}^N\,\theta_N\,,
\end{aligned} \tag{A.9}$$

where the symplectic matrices $\mathcal{O}_i$, $i=1,2,3$ read:[7]

$$\begin{aligned}
\mathcal{O}_1 &= \mathrm{diag}(1,-1,-1,1,1,-1,-1,1)\,,\\
\mathcal{O}_2 &= \mathrm{diag}(-1,1,-1,1,-1,1,-1,1)\,,\\
\mathcal{O}_3 &= \mathrm{diag}(-1,-1,1,1,-1,-1,1,1)\,.
\end{aligned}$$

Moreover, one can verify that:

$$\begin{aligned}
\mathcal{O}_i\cdot V(z,\bar{z}) &= \overline{\mathcal{D}}_{\hat{i}}\overline{V}\Big|_{(z_i\to z_i\,,\ z_{j\neq i}\to -\bar{z}_{\bar{\jmath}})}\,,\\
\mathcal{O}_i\cdot\mathcal{D}_{\hat{\jmath}}V &= |\epsilon_{ijk}|\ \mathcal{D}_{\hat{k}}V\big|_{(z_i\to z_i\,,\ z_{j\neq i}\to -\bar{z}_{\bar{\jmath}})}\,.
\end{aligned} \tag{A.10}$$

Using the first of the above properties, one can verify that:

$$\mathcal{W}_{(i)} = \overline{\mathcal{W}}\Big|_{(z_i\to z_i\,,\ z_{j\neq i}\to -\bar{z}_{\bar{\jmath}})}\,. \tag{A.11}$$

We then conclude that solutions to the STU $=\mathrm{STU}_0$ model are mapped to solutions of the STU_i one through the following transformation:

$$\mathrm{STU}_0\to\mathrm{STU}_i\ \Leftrightarrow\ \begin{cases}\Gamma\to\mathcal{O}_i\cdot\Gamma\,, & \theta\to\Theta^i=\mathcal{O}_i\cdot\theta\\ z_i\to z_i\,, & z_{j\neq i}\to -\bar{z}_{\bar{\jmath}}\end{cases} \tag{A.12}$$

Under this transformation, a BPS solution of the STU model is mapped into a BPS solution of the STU_i one, which is non-BPS in the original truncation but BPS in the maximal theory. The action of $\mathcal{O}_i$ for going from STU_0 to STU_i, at the level of R-symmetry indices, as an SU(8) compensating transformation, has the effect to exchanging the couples $(A_0)=(1,2)$ with (A_i) and (A_j) with (A_k), $i\neq j\neq k\neq i$. For instance, $\mathcal{O}_1$ implies $(1,2)\leftrightarrow(3,4)$ and $(5,6)\leftrightarrow(7,8)$.

[7]The matrices $\mathcal{O}_i$ $(i=1,2,3)$,together with the identity matrix $\mathbb{1}_{8\times8}$ define a Klein group $\mathbb{Z}_2\times\mathbb{Z}_2$. They satisfy the relation

$$\mathcal{O}_i\cdot\mathcal{O}_j = \delta_{ij}\,\mathbb{1} + |\epsilon_{ijk}|\mathcal{O}_k\,.$$

Action and supersymmetry transformations. The $\mathcal{N}=2$ gauged STU model has the following bosonic action principle (we set $8\pi G_N=1$):

$$\mathcal{S}=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2}-g_{i\bar{\jmath}}\partial_\mu z^i\partial^\mu\bar{z}^{\bar{\jmath}}-V(z,\bar{z})+\frac{1}{4}\mathcal{I}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu}+\frac{1}{8e}\mathcal{R}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}\right). \tag{A.13}$$

where $i,j=1,2,3$, $\Lambda,\Sigma=1,\dots,4$ and we are considering the symplectic frame described by a holomorphic section of the form (A.3). This frame is associated with a prepotential of the form

$$F(X)=2i\sqrt{X^0X^1X^2X^3}\,. \tag{A.14}$$

Indeed $\Omega^M(z)$, modulo multiplication by a non-vanishing holomorphic function, can be written in the form $\Omega^M=(X^\Lambda,\partial F/\partial X^\Lambda)$ provided we identify:

$$z_1=-i\sqrt{\frac{X^1X^2}{X^0X^3}}\,,\quad z_2=-i\sqrt{\frac{X^0X^2}{X^1X^3}}\,,\quad z_3=-i\sqrt{\frac{X^0X^1}{X^2X^3}}\,. \tag{A.15}$$

The Kähler potential and the metric are given in eqs (A.4) and (A.6), respectively.

The supersymmetry variations of the fermions in a bosonic background are given by[8]

$$\delta\Psi^A_\mu=D_\mu\epsilon^A+\frac{1}{4}L^\Lambda\mathcal{I}_{\Lambda\Sigma}F^{\Sigma+}_{\rho\sigma}\gamma^{\rho\sigma}\gamma_\mu\varepsilon^{AB}\epsilon_B+\frac{1}{2}\underbrace{i(\sigma^2)^A{}_C\varepsilon^{BC}}_{\delta^{AB}}\mathcal{W}\gamma_\mu\epsilon_B\,, \tag{A.16}$$

$$\delta\lambda^{iA}=-\partial_\mu z^i\gamma^\mu\epsilon^A+\frac{1}{2}g^{i\bar{\jmath}}\overline{f}^\Lambda_{\bar{\jmath}}\mathcal{I}_{\Lambda\Sigma}F^{\Sigma-}_{\mu\nu}\gamma^{\mu\nu}\varepsilon^{AB}\epsilon_B+W^{iAB}\epsilon_B\,. \tag{A.17}$$

where

$$D_\mu\epsilon^A=\partial_\mu\epsilon^A+\frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon^A+\frac{1}{2}A^M_\mu\theta_M\underbrace{i(\sigma^2)^A{}_B\epsilon^B}_{\varepsilon^{AB}\delta_{BC}\epsilon^C}+\frac{i}{2}\mathcal{Q}_\mu\epsilon^A\,, \tag{A.18}$$

$$\mathcal{Q}_\mu=\frac{i}{2}(\partial_{\bar{\imath}}\mathcal{K}\partial_\mu\bar{z}^{\bar{\imath}}-\partial_i\mathcal{K}\partial_\mu z^i)=\frac{1}{2}\sum_{i=1}^{3}e^{\phi_i}\partial_\mu\chi_i\,, \tag{A.19}$$

$$V^M=e^{\mathcal{K}/2}\Omega^M=(L^\Lambda,M_\Lambda)\,,\quad \mathcal{K}=-\log[-i\Omega(z)^T\mathbb{C}\overline{\Omega}(\bar{z})]\,, \tag{A.20}$$

$$\mathcal{W}=V^M\theta_M\,, \tag{A.21}$$

$$V(z,\bar{z})=g^{i\bar{\jmath}}\mathscr{D}_i\mathcal{W}\overline{\mathscr{D}}_{\bar{\jmath}}\mathcal{W}-3\mathcal{W}\overline{\mathcal{W}}\,, \tag{A.22}$$

$$W^{iAB}=\underbrace{i(\sigma^2)_C{}^B\varepsilon^{CA}}_{\delta^{AB}}g^{i\bar{\jmath}}\overline{\mathscr{D}}_{\bar{\jmath}}(\overline{V}^M\theta_M)\,. \tag{A.23}$$

and the embedding tensor θ_M was given in (A.9).

[8]We consider $(\sigma^2)^A{}_C=-i\delta^A_1\delta^2_C+i\delta^A_2\delta^1_C$, for which the following identities hold $i(\sigma^2)^A{}_C\varepsilon^{BC}=\delta^{AB}$, $i(\sigma^2)^A{}_B\epsilon^B=\varepsilon^{AC}\delta_{CB}\epsilon^B$, $i(\sigma^2)_C{}^B\varepsilon^{CA}=\delta^{AB}$. Requiring that $(\sigma^2)^1{}_2=(\sigma^2)_1{}^2$, meaning that they are the same matrices, implies that $(\sigma^2)_B{}^A=-i\delta^1_B\delta^A_2+i\delta^2_B\delta^A_1$ leading to the identity $i(\sigma^2)_C{}^B\varepsilon^{CA}=\delta^{AB}$.

Relation to other symplectic frames. Let us now give the explicit relation between the symplectic frame used in the present work and other frames commonly used in the literature.

Frame 1. The first is the symplectic frame, which we shall refer to as *cubic frame*, in which the prepotential function $\tilde{F}(\tilde{X})$ has the following cubic form:

$$\tilde{F}(\tilde{X}) = \frac{d_{ijk}}{3!}\,\frac{\tilde{X}^i\tilde{X}^j\tilde{X}^k}{\tilde{X}^0} = -\frac{\tilde{X}^1\tilde{X}^2\tilde{X}^3}{\tilde{X}^0}\,. \tag{A.24}$$

The corresponding holomorphic section $\tilde{\Omega}(z) = (\tilde{X}^\Lambda,\, \partial\tilde{F}/\partial\tilde{X}^\Lambda)$ reads, modulo multiplication by a non-vanishing holomorphic function:

$$\tilde{\Omega}(z) = (1, z_1, z_2, z_3, z_1z_2z_3, -z_2z_3, -z_1z_3, -z_1z_2)\,, \tag{A.25}$$

where

$$z_i = \frac{\tilde{X}^i}{\tilde{X}^0} = \chi_i + i\,e^{-\phi_i}\ ,\quad i = 1,2,3\,.$$

The two frames are related by the following symplectic matrix $\mathbf{E} = (E^M{}_N)$:

$$\Omega(z) = \mathbf{E}\cdot\tilde{\Omega}(z)\ ,\quad \mathbf{E} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}. \tag{A.26}$$

The charges $\Gamma^M = (P^\Lambda,\, Q_\Lambda)$ in our frame are then related to those $\tilde{\Gamma}^M = (\mathfrak{p}^\Lambda,\, \mathfrak{q}_\Lambda)$ as follows:

$$\Gamma^M = (-\mathfrak{q}_1, -\mathfrak{q}_2, -\mathfrak{q}_3, -\mathfrak{p}^0, \mathfrak{p}^1, \mathfrak{p}^2, \mathfrak{p}^3, -\mathfrak{q}_0)\,. \tag{A.27}$$

The quartic invariant, see appendix D, in the cubic frame, reads:

$$I_4(\tilde{\Gamma}) = -(\mathfrak{p}^\Lambda\mathfrak{q}_\Lambda)^2 - 4\,\mathfrak{q}_0\,\mathfrak{p}^1\,\mathfrak{p}^2\,\mathfrak{p}^3 + 4\,\mathfrak{p}^0\,\mathfrak{q}_1\,\mathfrak{q}_2\,\mathfrak{q}_3 + 4\left(\sum_{i<j}\mathfrak{p}^i\mathfrak{q}_i\mathfrak{p}^j\mathfrak{q}_j\right). \tag{A.28}$$

In light of eq. (A.27), we can write the quartic invariant in our frame, in the magnetic case $Q_\Lambda = 0$, in terms of the charges in the cubic one as follows

$$I_4(\Gamma) = 4P^1P^2P^3P^4 = 4\mathfrak{p}^0\,\mathfrak{q}_1\,\mathfrak{q}_2\,\mathfrak{q}_3\,.$$

Frame 2. The second symplectic frame is the one which naturally arises from direct truncation of the SO(8) gauged maximal theory. It is a special coordinate frame with a prepotential function $\hat{F}(\hat{X})$ of the form:

$$\hat{F}(\hat{X}) = -2\sqrt{\hat{X}^0\hat{X}^1\hat{X}^2\hat{X}^3}\,. \tag{A.29}$$

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The holomorphic section $\hat{\Omega} = (\hat{X}^\Lambda, \partial\hat{F}/\partial\hat{X}^\Lambda)$, modulo multiplication by a non-vanishing holomorphic function, can be written in the form:

$$\hat{\Omega}(z) = (z_1 z_2 z_3, z_1, z_2, z_3, -1, -z_2 z_3, -z_1 z_3, -z_1 z_2)\,, \tag{A.30}$$

where:

$$z_i = \sqrt{\frac{\hat{X}^0 \hat{X}^i}{\hat{X}^j \hat{X}^k}}\ ,\ \ i \neq j \neq k \neq i\,. \tag{A.31}$$

The symplectic transformation relating this frame with the cubic one is straightforward:

$$\hat{X}^0 = \frac{\partial\tilde{F}}{\partial\tilde{X}^0}\,,\ \hat{X}^i = \tilde{X}^i\,,\ \frac{\partial\hat{F}}{\partial\hat{X}^0} = -\tilde{X}^0\,,\ \frac{\partial\hat{F}}{\partial\hat{X}^i} = \frac{\partial\tilde{F}}{\partial\tilde{X}^i}\,.$$

B Spinor conventions

We use the Majorana basis for the Clifford algebra

$$\gamma^0 = -i\begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}\,,\quad \gamma^1 = -\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}\,,\quad \gamma^2 = i\begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix}\,,\quad \gamma^3 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}\,. \tag{B.1}$$

The charge conjugation matrix and γ^5 matrix are given by

$$C = \gamma_0\,, \qquad\qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{B.2}$$

We use $\mathcal{N} = 2$ chiral supersymmetry parameters ϵ^A, ϵ_A with $A = 1, 2$, satisfying

$$\gamma^5\epsilon^A = -\epsilon^A\,, \qquad\qquad \gamma^5\epsilon_A = \epsilon_A\,, \tag{B.3}$$

that are defined as the chiral components of doublet of Majorana spinors $\psi^A = \epsilon^A + \epsilon_A$. The relation between the chiral spinors is $\epsilon_A = (\epsilon^A)^*$. It is useful to define the complex spinors

$$\zeta = \psi^1 + \mathrm{i}\psi^2 \tag{B.4}$$

that allows one to write down a set of differential equations. The general rule to go from an equation with real coefficients for the Majorana spinor ψ^A to an equation for the complex spinor ζ is by replacing $\delta_{AB} \to 1$ and $\varepsilon_{AB} \to -\mathrm{i}$, and vice-versa.

C Dirac bracket between supercharges

The author of [13] showed that the Dirac bracket between the supercharges is expressed as

$$\{\mathcal{Q}, \mathcal{Q}\} = \int_{\partial\mathscr{M}} \mathrm{d}\Sigma_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\left(\bar{\epsilon}^A\gamma_\rho\mathfrak{D}_{A\sigma}(\epsilon) - \bar{\epsilon}_A\gamma_\rho\mathfrak{D}^A{}_\sigma(\epsilon)\right)\,, \tag{C.1}$$

where $\mathfrak{D}^A{}_\sigma(\epsilon)$ is generically defined by the variation of the gravitino Ψ^A_σ. Since we are interested in the purely dilatonic sector, $\mathfrak{D}^A{}_\sigma(\epsilon)$ is given by (2.4) and $\mathfrak{D}_{A\sigma}(\epsilon)$ is its conjugated, explicitly

$$\mathfrak{D}_A(\epsilon) = \mathrm{d}\epsilon_A + \frac{1}{4}\omega_{ab}\gamma^{ab}\epsilon_A + \frac{1}{2}A^M\theta_M\varepsilon_{AB}\delta^{BC}\epsilon_C + \frac{1}{4}\overline{L}^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon_{AB}\epsilon^B + \frac{1}{2}\overline{W}\gamma\delta_{AB}\epsilon^B\,, \tag{C.2}$$

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we used the fact that the Kähler connection vanishes. The 2-form volume is defined as

$$\mathrm{d}\Sigma_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathrm{d}x^{\rho}\wedge \mathrm{d}x^{\sigma}\,. \tag{C.3}$$

First, we simplify the contraction appearing in the integral, namely

$$\mathrm{d}\Sigma_{\mu\nu}\epsilon^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\lambda\delta}\epsilon^{\mu\nu\rho\sigma}\mathrm{d}x^{\lambda}\wedge \mathrm{d}x^{\delta}\,. \tag{C.4}$$

Considering the definitions of the symbolic with curved indices

$$\epsilon_{\mu\nu\rho\sigma} = e^{-1}e^{a}{}_{\mu}e^{b}{}_{\nu}e^{c}{}_{\rho}e^{d}{}_{\sigma}\epsilon_{abcd}\,, \qquad \epsilon^{\mu\nu\rho\sigma} = ee_{a}{}^{\mu}e_{b}{}^{\nu}e_{c}{}^{\nu}e_{d}{}^{\rho}\epsilon^{abcd}\,, \tag{C.5}$$

where the symbol with flat indices is $\epsilon_{0123} = 1 = -\epsilon^{0123}$. Hence,

$$\epsilon_{\mu\nu\lambda\delta}\epsilon^{\mu\nu\rho\sigma} = e^{c}{}_{\lambda}e^{d}{}_{\delta}e_{g}{}^{\rho}e_{h}{}^{\sigma}\epsilon_{abcd}\epsilon^{abgh}\,. \tag{C.6}$$

using the identity $\epsilon_{abcd}\epsilon^{abgh} = -4\delta^{g}_{[c}\delta^{h}_{d]}$, it follows that $\epsilon_{\mu\nu\lambda\delta}\epsilon^{\mu\nu\rho\sigma} = -2(\delta^{\rho}_{\lambda}\delta^{\sigma}_{\delta} - \delta^{\sigma}_{\lambda}\delta^{\rho}_{\delta})$, which leads to the following simplified form of (C.4)

$$\mathrm{d}\Sigma_{\mu\nu}\epsilon^{\mu\nu\rho\sigma} = -(\delta^{\rho}_{\lambda}\delta^{\sigma}_{\delta} - \delta^{\sigma}_{\lambda}\delta^{\rho}_{\delta})\mathrm{d}x^{\lambda}\wedge \mathrm{d}x^{\delta} = -2\mathrm{d}x^{\rho}\wedge \mathrm{d}x^{\sigma}\,. \tag{C.7}$$

Replacing this result into the bracket between supercharges (C.1) we obtain (2.11). The expression (2.11) can be obtained by defining a Majorana spinor

$$\psi^{A} = \epsilon_{A} + \epsilon^{A} \tag{C.8}$$

which implies the following relations

$$\epsilon_{A} = \mathtt{P}_{+}\psi^{A}\,, \qquad \epsilon^{A} = \mathtt{P}_{-}\psi^{A}\,, \qquad \mathtt{P}_{\pm} = \frac{1}{2}(1\pm\gamma_5)\,. \tag{C.9}$$

We recall that the conjugated of the chiral spinors is defined as $\bar{\epsilon}_{A} = \mathrm{i}(\epsilon^{A})^{\dagger}\gamma^{0}$ and $\bar{\epsilon}^{A} = \mathrm{i}(\epsilon_{A})^{\dagger}\gamma^{0}$, in order to preserve the chirality. In our basis $\gamma_5^{\dagger} = \gamma_5$, so one can check that

$$\bar{\epsilon}_{A} = \overline{\psi}^{A}\mathtt{P}_{+}\,, \qquad \bar{\epsilon}^{A} = \overline{\psi}^{A}\mathtt{P}_{-}\,. \tag{C.10}$$

Now, we use the fact that the electric components of the U(1) section and the superpotential are real functions, i.e. $L^{\Lambda}, \mathcal{W}\in\mathbb{R}$. Then, the supercovariant derivatives $\mathfrak{D}^{A}(\epsilon), \mathfrak{D}_{A}(\epsilon)$ can be written in terms of the Majorana spinor as follows

$$\begin{aligned}\mathfrak{D}^{A}(\epsilon) = {}& \mathtt{P}_{-}\mathrm{d}\psi^{A} + \frac{1}{4}\omega_{ab}\gamma^{ab}\mathtt{P}_{-}\psi^{A} + \frac{1}{2}A^{M}\theta_{M}\varepsilon^{AB}\delta_{BC}\mathtt{P}_{-}\psi^{C} + \\ & + \frac{1}{4}L^{T}\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon^{AB}\mathtt{P}_{+}\psi^{B} + \frac{1}{2}\mathcal{W}\gamma\delta^{AB}\mathtt{P}_{+}\psi^{B}\,,\end{aligned} \tag{C.11}$$

$$\begin{aligned}\mathfrak{D}_{A}(\epsilon) = {}& \delta_{AB}\mathtt{P}_{+}\mathrm{d}\psi^{B} + \frac{1}{4}\omega_{ab}\gamma^{ab}\delta_{AB}\mathtt{P}_{+}\psi^{B} + \frac{1}{2}A^{M}\theta_{M}\varepsilon_{AB}\mathtt{P}_{+}\psi^{B} + \\ & + \frac{1}{4}L^{T}\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon_{AB}\mathtt{P}_{-}\psi^{B} + \frac{1}{2}\mathcal{W}\gamma\delta_{AB}\mathtt{P}_{-}\psi^{B}\,,\end{aligned} \tag{C.12}$$

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The 2-form appearing in the integral of the Dirac bracket are

$$\begin{aligned}\bar{\epsilon}^A\gamma\wedge\mathfrak{D}_A(\epsilon) &= \overline{\psi}^A\gamma\wedge\mathtt{P}_-\left(\mathrm{d}\psi^A+\frac{1}{4}\omega_{ab}\gamma^{ab}\psi^A+\frac{1}{2}A^M\theta_M\varepsilon^{AB}\delta_{BC}\psi^C+\right.\\ &\left.\quad+\frac{1}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon^{AB}\psi^B+\frac{1}{2}\mathcal{W}\gamma\delta^{AB}\psi^B\right)\,,\\ \bar{\epsilon}^A\gamma\wedge\mathfrak{D}_A(\epsilon) &= \overline{\psi}^A\gamma\mathtt{P}_+\wedge\left(\delta_{AB}\mathrm{d}\psi^B+\frac{1}{4}\omega_{ab}\gamma^{ab}\delta_{AB}\psi^B+\frac{1}{2}A^M\theta_M\varepsilon_{AB}\psi^B+\right.\\ &\left.\quad+\frac{1}{4}L^T\mathcal{I}F_{ab}\gamma^{ab}\gamma\varepsilon_{AB}\psi^B+\frac{1}{2}\mathcal{W}\gamma\delta_{AB}\psi^B\right)\,,\end{aligned}$$

Clearly, its subtraction cancels the identity factor in $\mathtt{P}_\pm$ leading to (2.11).

D Quartic invariant of $\mathbf{SL(2,\mathbb{R})^3}$

To construct the quartic invariants of $\mathrm{SL}(2,\mathbb{R})^3\subset\mathrm{Sp}(8,\mathbb{R})$ we consider its generators given by

$$\mathtt{X}_i=\frac{\partial\mathcal{M}}{\partial\chi_i}\bigg|_{\chi,\phi=0}\,,\qquad \mathtt{Y}_i=\frac{\partial\mathcal{M}}{\partial\phi_i}\bigg|_{\chi,\phi=0}\,,\qquad \mathtt{Z}_i=[\mathtt{Y}_i,\mathtt{X}_i]\,,\qquad (\text{no sum over } i) \tag{D.1}$$

with $i=1,2,3$. They span the three commuting $\mathfrak{sl}(2,\mathbb{R})$ algebras. It is convenient to consider the basis with nilpotent generators $\mathtt{e}_i^2=\mathtt{f}_i^2=0$ and the generators $\mathtt{h}_i$ of the Cartan subalgebra, given by

$$\begin{aligned}&\mathtt{h}_i=\frac{1}{2}\mathtt{Y}_i\,, &&\mathtt{e}_i=\frac{1}{4\sqrt{2}}(\mathtt{Z}_i+2\mathtt{X}_i)\,, &&\mathtt{f}_i=\mathtt{e}_i^T\,,\\ &[\mathtt{h}_i,\mathtt{e}_i]=\mathtt{e}_i\,, &&[\mathtt{e}_i,\mathtt{f}_i]=\mathtt{h}_i\,, &&[\mathtt{h}_i,\mathtt{f}_i]=-\mathtt{f}_i\,.\end{aligned} \tag{D.2}$$

We collect all the generators as $t_{(i)\alpha}=\{\mathtt{e}_i,\mathtt{h}_i,\mathtt{f}_i\}$ with $\alpha=1,2,3$. By definition the positions of the symplectic indices are $t_{(i)\alpha}=(t_{(i)\alpha})_M{}^N$ and we lower them by the symplectic matrix $\mathbb{C}=\mathbb{C}_{MN}$ defining $t_{(i)\alpha}\mathbb{C}=(t_{(i)\alpha})_{MN}$. We construct the Cartan-Killing form $\eta_{(i)\alpha\beta}=\mathrm{Tr}(t_{(i)\alpha}t_{(i)\beta})$ and its inverse, denoted by $\eta^{\alpha\beta}_{(i)}$. These allow us to define $(t^\alpha_{(i)})_{MN}=\eta^{\alpha\beta}_{(i)}(t_{(i)\beta})_{MN}$ and construct the following tensors of $\otimes^4\underline{8}$

$$\mathcal{C}_{MNPQ}=\mathbb{C}_{MN}\mathbb{C}_{PQ}\,, \tag{D.3}$$
$$(\mathcal{I}_i)_{MNPQ}=(t^\alpha_{(i)})_{MN}(t_{(i)\alpha})_{PQ}\,, \tag{D.4}$$
$$(\mathcal{L}_{ij})_{MNPQ}=(t^\alpha_{(i)})_{M\bullet}(t^\beta_{(j)})^\bullet{}_N(t_{(i)\alpha})_{P\bullet}(t_{(j)\beta})^\bullet{}_Q\,, \tag{D.5}$$
$$(\mathcal{Z}_{ijk})_{MNPQ}=(t^\alpha_{(i)})_{M\bullet}(t^\beta_{(j)})^\bullet{}_\bullet(t^\gamma_{(k)})^\bullet{}_N(t_{(i)\alpha})_{P\bullet}(t_{(j)\beta})^\bullet{}_\bullet(t_{(k)\gamma})^\bullet{}_Q\,. \tag{D.6}$$

which are invariant under $\mathrm{SL}(2,\mathbb{R})^3$. Among all the above tensors only eight of them are independent, and one can pick these to be $\{\mathcal{C},\mathcal{I}_1,\mathcal{I}_2,\mathcal{I}_3,\mathcal{L}_{12},\mathcal{L}_{13},\mathcal{L}_{23},\mathcal{Z}_{123}\}$. Nevertheless, when these tensors act on two arbitrary symplectic vectors the eight invariants reduce to seven independent functions.

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In our case we have two symplectic vectors Γ^M and $\theta^M = \mathbb{C}^{MN}\theta_N$, hence we can construct the following independent invariants

$$\mathcal{I}_1(\Gamma,\Gamma,\Gamma,\Gamma) = -2\prod_\Lambda P_\Lambda\,, \tag{D.7}$$

$$\mathcal{I}_1(\theta,\theta,\theta,\theta) = -\frac{1}{2L^2}\,, \tag{D.8}$$

$$\mathcal{I}_i(\Gamma,\theta,\Gamma,\theta) = \frac{1}{16L^2}\left(\sum_\Lambda \Omega_{i\Lambda}P_\Lambda\right)^2\,. \tag{D.9}$$

The rest are zero or functionally dependent on the above ones. Note that the BPS mass can be expressed in terms of the invariants as

$$m^4_{\text{BPS}} = 64L^3\prod_i \mathcal{I}_i(\Gamma,\theta,\Gamma,\theta). \tag{D.10}$$

Other invariants that are dependent on the above are the following.

$$\mathcal{C}(\Gamma,\theta,\Gamma,\theta) = \frac{1}{2L^2}\left(\sum_\Lambda P_\Lambda\right)^2\,, \tag{D.11}$$

$$\mathcal{I}_1(\Gamma,\Gamma,\theta,\theta) = -\frac{1}{2L^2}(P_2P_3+P_1P_4)\,, \tag{D.12}$$

$$\mathcal{I}_2(\Gamma,\Gamma,\theta,\theta) = -\frac{1}{2L^2}(P_1P_3+P_2P_4)\,, \tag{D.13}$$

$$\mathcal{I}_3(\Gamma,\Gamma,\theta,\theta) = -\frac{1}{2L^2}(P_1P_2+P_3P_4)\,, \tag{D.14}$$

$$\mathcal{Z}_{123}(\Gamma,\Gamma,\theta,\theta) = -\frac{1}{128L^2}\prod_{\Lambda\geq\Sigma} P_\Lambda P_\Sigma\,, \tag{D.15}$$

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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