Nonlinear Numerical Analyses of Reinforced Concrete Structures

CHAPTER 11

Chapter 11: Non-linear numerical analyses of reinforced concrete structures: safety formats, aleatory and epistemic uncertainties

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Chapter Abstract
The non-linear numerical analyses (NLNAs) have become the preferred instrument devoted to simulate the actual physical response of civil reinforced concrete structures and infrastructures. As for the numerical outcomes from this kind of onerous calculations, they have to be analysed and processed to satisfy target safety levels required by design codes. In this regard, different methods (i.e., safety formats - SFs), have been described and reported by literature and codes. All the different SFs are able to define a general approach to compute the design value of parameters describing the overall structural behaviour (i.e., structural resistance) including the relevant sources of uncertainties: the aleatory and the epistemic ones. This chapter reports and describes the current methodologies proposed by design codes and recent scientific advances to perform reliability evaluations of both existing and new RC structures by means of NLNAs.

11.1 Introduction

Reliability analysis of reinforced concrete (RC) members and structures requires methodologies able to fulfil safety requirements expected by the society. These requirements, as defined by international codes (*fib* Model Code 2010, 2013; ISO 2394, 2015; EN 1990. 2013), are represented by limits on the likelihood that structural failure may occur in a given reference period. These limits are dependent on the typology, the use and the lifetime along which a structure should carry out its serviceability. In this context, approaches and methodologies, aimed to design and assess RC systems in compliance with target reliability levels, are provided. With the progress of the last twenty years, non-linear numerical analyses (NLNAs) have become the preferred instrument devoted to simulate the actual physical response of civil reinforced concrete structures and infrastructures under the relevant loading configurations. A huge number of modelling hypotheses to perform NLNAs of RC structures and members is available for engineers and practitioners, who more and more should become confident with such complex calculation methods (*fib* Bulletin 45, 2008). As discussed within this chapter, a modelling hypothesis collects all the assumptions in terms of equilibrium, kinematic compatibility and constitutive laws related to a specific non-linear numerical model. In the near future, design codes of practice will allow to perform design and assessment using NLN simulations and, for instance, appropriate methodologies are necessary to include safety issues.

This chapter introduces and describes how the main sources of uncertainty affecting structural engineering problems (i.e., aleatory and epistemic), with particular care to assessment and design of RC structures, can be accounted for when NLNAs are used. First of all, the global resistance format (GRF) for evaluation of structural reliability is described and, then, the corresponding safety formats are introduced. The comparison among the different safety formats is proposed in order to highlight their limits of applicability. To overcome these limits, specific proposals are introduced with particular reference to the aleatory uncertainty influence. Next, a methodology to characterise the epistemic uncertainties associated to the establishment of non-linear numerical models of RC structures is described and appropriate partial safety factors are proposed to fulfil specific target levels of reliability.

11.2 Aleatory and epistemic uncertainties within the global resistance format (GRF)

The global resistance format (GRF) (*fib* Model Code 2010, 2013; Allaix, Carbone & Mancini, 2013) deals with the uncertainties associated to structural behavior in line to the limit states design method (*fib* Model Code 2010, 2013; EN 1990, 2013) at the level of global structural behavior (i.e., structural resistance). The main sources of uncertainties are accounted for to determine the design structural resistance R_d and are included within the calculation through appropriate partial safety factors. In detail, the aleatory uncertainties relate to the intrinsic randomness of both materials (e.g., concrete compressive strength and reinforcement yielding strength) and geometric (e.g., members size, concrete cover and reinforcement location) properties, while, the epistemic ones are mainly referred to missing knowledge, simplification and hypotheses adopted establishing the resistance numerical model itself (see also Castaldo, Gino & Mancini, 2019; Der Kiureghian & Ditlevsen, 2009). The GRF has been proposed by design codes and literature (*fib* Model Code 2010, 2013; Allaix, Carbone & Mancini, 2013) with the scope to perform structural assessment and design of RC structures by means of NLNAs. In general, the evaluation of safety of existing and new structures is performed through the local verification of cross sections design capacity (R_d) compared to the design value of the effect of the actions (E_d) deriving from linear elastic analysis: $E_d \leq R_d$ (EN 1990, 2013). This procedure for the structural reliability assessment is denoted as "local", as it requires just verification of different cross sections of the structural elements composing the structure disregarding from the overall structural response.

For instance, if the structural reliability is evaluated using refined NLNAs, the possibility of RC structural systems to develop internal redistribution of stresses within the selected loading configuration should not be disregarded. In detail, the adoption of NLN requires to consider a "global" evaluation of structural safety, performing the comparison between the actions simultaneously present on the structure to the associated overall capacity, denoted as structural resistance. The main features of the mentioned above approaches are showed in Figure 11.1. In accordance with the GRF, the safety condition can be formulated as follows:

$$R_d \geq F_d \tag{11.1}$$

where the design value of actions F_d can be evaluated in accordance with the EN 1990 (2013) according to the proper combination, while, the design structural resistance R_d can be assessed through NLNAs adopting the safety formats (*fib* Model Code 2010, 2013; Allaix, Carbone & Mancini, 2013) based on the GRF as follows:

$$R_d = \frac{R_{NLNA}\left(X_{rep};a_{rep}\right)}{\gamma_R \gamma_{Rd}} \qquad (11.2)$$

In Eq.(11.2), $R_{NLNA}(X_{rep};a_{rep})$ denotes the structural resistance computed by means of NLNAs using the representative values X_{rep} and a_{rep} for material and geometrical properties, respectively, in accordance with the adopted safety format, as commented in the next. The level of structural reliability is involved by means of two different partial safety factors:

- the *global resistance partial safety factor* γ_R, which takes into account, at the level of global structural behaviour, the aleatory uncertainties influence associated to the material properties and even geometry. This partial safety factor may be evaluated in line to specific target level of reliability according to the methodology described by the selected safety format (*fib* Model Code 2010, 2013 and Allaix, Carbone & Mancini, 2013);
- the *resistance model uncertainty partial safety factor* γ_{Rd}, which takes into account the level of epistemic uncertainty associated to simplifications, assumptions and choices performed to define of the NLN model. This partial safety factor is horizontal and independent on the safety format (*fib* Model Code 2010, 2013; Allaix, Carbone & Mancini, 2013) adopted to carry out the global structural verification.

In the next sections, the principal features of the most common safety formats based on the GRF are described.

11.2.1 Safety formats for NLNAs of RC structures

As previously discussed, the safety verification by means of NLNAs can be performed according to the GRF. In scientific literature and codes of practice, different safety formats based on the GRF have been proposed. In the present chapter, the following safety formats are considered:

1. Partial Factor Method (PFM) (*fib* Model Code 2010, 2013);
2. Global Resistance Methods (GRMs):
   - Global Resistance Factor (GRF) (*fib* Model Code 2010, 2013);
   - Method of estimation the coefficient of variation of the structural resistance (ECoV) (*fib* Model Code 2010, 2013);
   - Global Safety Format related to mean values of mechanical properties (GSF) (Allaix, Carbone & Mancini, 2013);

In addition, also the general probabilistic approach is considered:

3. Probabilistic Method (PM) (*fib* Model Code 2010, 2013).

Next, a brief outline of the mentioned above methods is reported.

11.2.1.1 *Partial Factor Method (PFM)*

The partial factor method (PFM) has been reported firstly in the *fib* Model Code 2010 (2013). The design value associated to overall structural resistance R_d derives from a single NLNA according to Eq.(11.3):

$$R_d = \frac{R_{NLNA}(X_d\,;a_d)}{\gamma_{Rd}} \qquad (11.3)$$

where $R_{NLNA}(X_d\ ;\ a_d)$ represents the structural resistance estimated with a NLNA defined adopting the design values of both geometric (a_d) material (X_d) characteristics; γ_{Rd} is the safety factor associated to uncertainty in the resistance model definition. Within PFM, the influence of the aleatory uncertainties is accounted for applying partial safety factors γ_M to divide, and then, reduce, the characteristic values of materials properties (X_k) (e.g., characteristic value of cylinder concrete strength f_{ck} and reinforcement characteristic tensile strength f_{yk}), and the adoption of design values of geometric properties (a_d). The partial safety factors for material properties γ_M may be defined accounting for the appropriate target level of reliability for new structures, see EN 1990 (2013), *fib* Model Code 2010 (2013) and existing structures, see *fib* Bulletin 80 (2016). The design value of geometrical properties (a_d) can be defined starting from their nominal values a_{nom} modified with appropriate deviation (Δa) to account for construction imperfections (EN 1990, 2013).

11.2.1.2 *Global resistance methods (GRMs)*

The global resistance methods (GRMs) are the safety formats based on the evaluation of the design structural resistance R_d by means of partial safety factors applied directly to the global structural resistance estimated throughout NLNAs. In particular, three main safety formats can be recognized:

I. *Global resistance factor method (GRF)*

As for the global resistance factor (GRF) method (*fib* Model Code 2010, 2013; EN 1992-2, 2005), the design structural resistance R_d is defined as follows:

$$R_d = \frac{R_{NLNA}(f_{cmd}, f_{ym}; a_{\mathrm{nom}})}{\gamma_{GL}} \qquad (11.4)$$

In Eq.(11.4), the global safety factor γ_{GL} is set to 1.27 with reference to target reliability level identified by reliability index β_t=3.8 referred to 50 years of working life. With the scope to link the GRF method with the other GRMs and safety formats, the fixed value of γ_{GL} can be represented by the product between global resistance partial safety factor γ_R and the partial safety factor for model uncertainty γ_{Rd}. To evaluate the value representing the structural resistance $R_{NLNA}(f_{cmd}\ , f_{ym}\ ;\ a_{nom})$, the mean value of the reinforcement tensile yield stress f_{ym} and the reduced value of concrete cylinder strength f_{cmd} in compression have to be employed as reported below:

$$f_{ym} = 1.1 f_{yk}\ ;\ f_{cmd} = 0.85 f_{ck} \qquad (11.5)$$

where f_{yk} and f_{ck} represent, respectively, the 5% characteristic values of the reinforcement tensile yield stress and cylinder concrete compressive strength.

II. *Method of estimation the coefficient of variation of the structural resistance (ECoV)*

The method of estimation of the coefficient of variation (ECoV), as suggested by *fib* Model Code 2010 (2013), allows the estimation of the design value of structural resistance R_d as follows:

$$R_d = \frac{R_{NLNA}(X_m; a_{nom})}{\gamma_R \cdot \gamma_{Rd}} \quad (11.6)$$

where $R_{NLNA}(X_m;a_{nom})$ is the structural resistance determined through a NLN simulation established adopting the mean material properties (X_m) and nominal geometric ones (a_{nom}) to define the numerical structural model; γ_R is the partial safety factor taking into account uncertainties associated to the properties of materials; γ_{Rd} denotes the partial safety factor associated to uncertainty in definition of the numerical model. The ECoV method is founded on the assumptions that the aleatory variability of the structural resistance can be represented by a lognormal probabilistic distribution and that the related mean value is almost equal to the value of $R_{NLNA}(X_m;a_{nom})$. According to these hypotheses, the global resistance partial safety factor γ_R may be evaluated as follows:

$$\gamma_R = \exp(\alpha_R \cdot \beta_t \cdot V_R) \quad (11.7)$$

where V_R denotes the CoV (i.e., coefficient of variation) of the probabilistic distribution associated to structural resistance; α_R represents the FORM (first-order-reliability-method) sensitivity coefficient, adopted as 0.8 according to *fib* Model Code 2010 (2013) for dominant resistance variables; β_t is the target value of the reliability index which allows to consider the required level of reliability for the investigated structure. In accordance to the hypothesis of lognormally distributed variable for structural resistance, the estimate of coefficient of variation V_R can be easily determined according to the expression:

$$V_R = \frac{1}{1.65}\ln\left(\frac{R_{NLNA}\left(X_m;a_{nom}\right)}{R_{NLNA}\left(X_k;a_{nom}\right)}\right) \tag{11.8}$$

where $R_{NLNA}(X_k;a_{nom})$ denotes the value of structural resistance estimated using NLNA established adopting characteristic material properties (X_k) and nominal (a_{nom}) for geometric ones.

III. *Global safety format (GSF) related to mean values of material properties*

The method of Allaix, Carbone & Mancini (2013), denoted as global safety format (GSF), to allows to compute the design value of overall structural resistance R_d basing NLN simulation on mean material properties (X_m) and nominal geometric ones (a_{nom}) grounding on the same assumptions of the ECoV method. In fact, the estimation of the value R_d can be performed according to Eq.(11.6), as well as, the global resistance partial safety factor γ_R can be determined according to Eq.(11.7). The GSF is different from the method of estimation of coefficient of variation (ECoV). In fact, the level of refinement in the estimation of $V_R=\sigma_R/\mu_R$ in the hypothesis of lognormal distribution (with μ_R and σ_R the mean value and the standard deviation of the probabilistic distribution representing structural resistance, respectively). Specifically, within the GSF the statistical parameters are evaluated through reduced Monte Carlo simulation technique employing the Latin Hypercube Sampling technique (from Mckey et al.,1979) with a proper number of samples (in general, 30 samples are sufficient if $V_R \leq 0.20$). The probabilistic model for the random variables representing material properties and even geometrical ones can be assumed with reference to JCSS Probabilistic Model Code (2001) or *fib* Model Code 2010 (2013).

11.2.1.3 ***Probabilistic method (PM)***

Finally, the probabilistic method (PM) reported in the *fib* Model Code 2010 (2013) is founded on several NLNAs delineated adopting Latin Hypercube or Monte Carlo's sampling techniques to select input data from the relevant random variables characterizing the aleatory uncertainties (i.e., material and geometric). The results from numerical analyses represented by the overall structural resistance should be statistically processed to characterize the most likely probabilistic model assessing the statistics of the distribution of structural resistance. The statistical parameters can be represented by the mean value μ_R and coefficient of variation V_R. Successively, it can be suitable to directly derive the quantile associated to the design value R_d which should correspond to a specific target value of the reliability index β_t as follows:

$$R_d = \frac{F_R^{-1}\left[\Phi\left(-\alpha_R \beta_t\right)\right]}{\gamma_{Rd}} \qquad (11.9)$$

where F_R is the cumulative probabilistic distribution representing structural resistance; Φ denotes the cumulative standard normal distribution; α_R represent the FORM (first order reliability method) sensitivity coefficient, adopted the value of 0.8 in accordance to *fib* Model Code 2010 (2013) for dominant variables associated to structural resistance; γ_{Rd} denotes the partial safety factor associated to the uncertainty when the numerical model is defined. The PM differs from the method of GSF for the hypothesis of lognormal distribution on the latter one. Moreover, the GSF grounds on mean material properties and nominal geometric ones adopting, as a simplification, the first order a Taylor expansion of the function describing the structural response (Allaix, Carbone & Mancini, 2013), while, the PM is able to ensure the estimation of the quantile of the most likely probabilistic distribution as reported by Eq.(11.9).

11.3 Influence of aleatory uncertainties in the use of the safety formats

The present sections discusses and highlights relevant aspects associated to the use of the safety formats in relation to the aleatory uncertainties influence (i.e., material and geometric) on the computation of the design value associated to structural resistance R_d .

11.3.1 Comparison between the safety formats based on the GRF

In the next, the comparison of the outcomes in terms of design value of structural resistance R_d deriving from application of the different safety formats based on the GRF is discussed highlighting limits and possible enhancements. In particular, the investigation of Castaldo, Gino & Mancini (2019) reports the outcomes from NLN models established to simulate the ultimate behaviour of different structural members and assess the design structural resistance in accordance to previously described methods. In addition, particular attention is devoted to the accuracy of each safety format to predict the relevant failure modes associated to the computation of the design structural resistance R_d.

11.3.1.1 *Structural members considered to compare the safety formats*

Castaldo, Gino & Mancini (2019) reports the application of the several safety formats for estimation of design value of structural resistance R_d of four simply supported beams with transverse openings in the web tested in laboratory by Aykac et al. (2013) and one isostatic "T" beam designed to fulfil the requirements of EN 1992-1-1 (2014). Aykac et al. (2013) reports the results from experimental tests performed on four beams having rectangular cross section with dimensions 15x40 cm and span between supports of of 390 cm. The first three beams present n°12 20x20 cm transverse web holes having squared shape. These beams have been realized with increasing values of the reinforcement ratio and are denoted with SL, SM, SH. The last beam of Aykac et al. (2013) presents n°12 circular transverse holes having a radius of 10 cm and casted with crossed reinforcements between openings in the web. The concrete compressive strength ranges between 20 and 22 MPa. The details of geometrical properties and reinforcements arrangements are acknowledged in Aykac et al. (2013) and Castaldo, Gino & Mancini (2019). All the specimens present a symmetrical "six points bending" statically determined test scheme. The last beam considered from Castaldo, Gino & Mancini (2019) has been designed according to EN 1992-1-1 (2014). The beam is defined with a "T" cross section (i.e., T-Beam) with height of 50 cm, top flange width of 50 cm and width of the web equal to 15 cm. The thickness of the top flange is 10 cm. The concrete compressive strength, representing the mean value, is 28 MPa. More details may be found in Castaldo, Gino & Mancini (2019). The NLN models have been realized by Castaldo, Gino & Mancini (2019) using the software ATENA 2D (2014). The main modelling hypotheses adopted by Castaldo, Gino & Mancini (2019) to define the numerical models are listed in the Table 11.1. The NLN models have been calibrated with the purpose to reproduce the response of the mentioned above RC members adopting experimental/mean values for materials properties. The comparison of the experimental investigation of Aykac et al. (2013) with the numerical simulations of Castaldo, Gino & Mancini (2019) is reported in Figure 11.2 as well as the identification of the main features of the failure modes of the RC members in relation to the systems denoted as SL and T-Beam.

11.3.1.2 *Results from probabilistic analysis and application of the safety formats*

The NLN models so far introduced have been adopted to realize probabilistic analysis of the structural resistance. The probabilistic model has been defined as suggested by JCSS Probabilistic Model Code (2001) in relation to the description of the main random variables (i.e., concrete cylinder compressive strength, reinforcements yielding strength and Young modulus). Probabilistic analysis has been carried out using the Latin Hypercube Sampling (LHS) technique (Mckey et al., 1979) to achieve 30 sampled NLN models for each one of the investigated RC members. More information about the probabilistic analysis may be found in Castaldo, Gino & Mancini (2019). The results of the 30 NLN simulations are briefly commented in the following and reported in Figures 11.3 and 11.4 for RC members SL and T-Beam, respectively. The failure mechanism of beam SL is denoted by a bending failure presenting crushing of concrete in compression and yielding of the tensed reinforcements.

The T-Beam shows two different possible mechanisms of failure denoted by a bending failure with crushing of concrete and yielding of the tensed reinforcement and a shear failure caused by crushing of concrete close to support devices. According to Castaldo, Gino & Mancini (2019), the lognormal probabilistic distributions can be used to represent the structural resistance of the considered RC members. The characterization of the design structural resistance for the SL and T-Beam RC members has been evaluated in accordance to the methods and safety formats described in this chapter. The design value of structural resistance has been estimated adopting the resistance model uncertainty safety factor γ_{Rd} set to 1.00, whereas, the target value of reliability index β_t is set to 3.8 for structures with normal consequences of failure for a working life of 50 years (*fib* Model Code 2010, 2013; ISO 2394, 2015; EN 1990, 2013). The sensitivity coefficient α_R is set to 0.8 according to the assumption of dominant variable associated to structural resistance (*fib* Model Code 2010, 2013). Table 11.2 lists the assumptions adopted to apply of the safety formats. Figure 11.5 shows, for members SL and T-Beam, the cumulative distribution functions (CDFs) estimated according to probabilistic analysis. These CDFs derived from the PM are adopted as reference curves and the design values of structural resistance obtained by the other safety formats based on the GRF are marked and compared. The values of the achieved reliability index β are also reported on the ordinates axis and compared with the target value set equal to

3.8. The values of β have been evaluated as $-\Phi(p)/\alpha_R$, where Φ denotes the cumulative standard normal distribution and p is the not-exceedance probability. The results show that, in case the estimation of the design value of structural resistance is performed using a safety format which differs from the PM, the required level of reliability is not properly addressed. From the results of Castaldo, Gino & Mancini (2019), arises the necessity to define a methodology able to include the uncertainty related to sensitivity of the numerical model to modify the prediction of the failure mode depending on the aleatory uncertainties also in safety formats that do not require probabilistic analysis of structural resistance.

11.3.2 Sensitivity of the numerical model to aleatory uncertainties within the GRF

With the scope to use for practical cases the safety formats defined according to the GRF, Castaldo, Gino & Mancini (2019) propose a methodology able to identify the mentioned above sensitivity of the numerical model. In particular, two preliminary NLNAs are suggested with the scope to understand if the PFM and GRMs are able to provide results consistent with the more accurate PM. The two mentioned above preliminary numerical simulations can be defined as follows:

1) the first NLN analysis may be performed adopting the mean concrete properties and the design reinforcement properties;
2) the second NLN analysis may be performed adopting the design concrete properties and the mean reinforcement properties.

The design values associated to material characteristics should be evaluated according to EN 1990 (2013), EN1992-1-1 (2014), *fib* Model Code 2010 (2013) and *fib* Bulletin 80 (2016) with reference to preferred target safety level. In case the results in terms of failure mode (i.e., characterized by the region of the structure where the local failure occurs and by the material that governs the failure mechanism) of the two preliminary NLN simulations are different, the numerical model is sensitive to modify the prediction of the failure mode depending on the aleatory uncertainties. In this circumstance, this additional source of uncertainty, which is not included in the partial safety factors γ_R and γ_{Rd}, should be accounted for when methods different from the PM (i.e., PFM and GRMs) are adopted to compute the design structural resistance. In particular, a further failure mode-based safety factor denoted as γ_{FM} is suggested and applied according to following expression:

$$R_d = \frac{R_{rep}}{\gamma_R \cdot \gamma_{FM} \cdot \gamma_{Rd}} \qquad (11.10)$$

This safety factor γ_{FM} has been defined in order to meet the outcomes of R_d computed with the PFM and GRMs in line to the values of R_d estimated with the PM. The results of Castaldo, Gino & Mancini (2019) lead to values of γ_{FM} ranging between 1.00-1.18. The value of 1.15 is proposed for γ_{FM} adopting the hypotheses of β_t=3.8 with α_R=0.8, normal consequences of failure, working life of 50 years.

11.4 Epistemic uncertainties according to the safety formats for NLNAs

The numerical models used for NLNAs with the prediction of structural behavior are merely estimations of the actual structural response. In general, the structural numerical model definition grounds on physical principles associated to constitutive relationships, equilibrium of forces and kinematic compatibility that, all together, denote a specific modelling hypothesis. Specifically, focusing on NLNAs, the mentioned above basic principles are met by means of iterative solution methods (e.g., Newton-Raphson algorithm, Arch-length algorithm) that lead, inevitably, to an approximation of the exact solution for the structural response. Then, the multiplicity of choices that may be performed defining a NLN model induces to an increasing level of uncertainty having an epistemic nature. It implies that, in accordance with the methodologies proposed by the methods and safety formats for NLNAs described in this chapter, the reliability-based evaluation of the modelling uncertainty safety factor associated to structural resistance γ_{Rd} is also an important topic for future codes implementation.

11.4.1 Methodology to estimate epistemic uncertainty related to NLNAs

The computation of the epistemic uncertainty related to the use of NLN simulations, as widely discussed by Holický, Retief & Sikora (2016), Castaldo et al. (2018), Castaldo et al. (2020) and Engen et al. (2017) should be performed in agreement with several aspects. First of all, an appropriate dataset of experimental outcomes have to be collected, including the information required to realize the numerical models effective to reproduce the experimental tests. A wide variety of the failure modes and the typology of structural members should be identified to cover as much as possible the current construction practice. Finally, a probabilistic characterization of resistance model uncertainty associated to NLNAs should be performed with the scope to assess the proper probabilistic distribution and the associated statistical parameters. According to Holický, Retief & Sikora (2016), the resistance model uncertainty random, denoted as θ, can be described comparing the i^{th} outcome of structural resistance from tests $R_i(X,Y)$ to the i^{th} structural resistance deriving from NLNA $R_{NLNA,i}(X)$ as follows:

$$R_i\left(X,Y\right) \approx \vartheta_i R_{\mathrm{NLNA},i}\left(X\right) \tag{11.11}$$

where X collects the variables explicitly included in the resistance model definition, Y collects variables that relates to the resistance mechanism but are disregarded by the numerical model. In order to estimate the partial safety factor γ_{Rd}, the following steps can be followed, as suggested by Castaldo et al. (2018) and Castaldo et al. (2020):

1) *Selection of the benchmark experimental tests*: the collection of the benchmark set of

experimental results should be performed collecting members having different nature, geometry and different failure modes;

2) *Differentiation between modelling hypotheses*: the plausible modelling hypotheses available to engineers and practitioners able to simulate specific RC structures by means of NLNAs should be involved in the calibration procedure;

3) *Probabilistic calibration (Bayesian approach)*: accounting for the differentiation between modelling hypotheses, the probabilistic distribution devoted to represent the random variable ϑ should be characterized and the mean value μ_ϑ and the variance σ^2_ϑ evaluated. The processing of the model uncertainties can be done in accordance to the Bayes theorem grounding on assumption of equiprobable modelling hypotheses (Castaldo et al., 2018; Castaldo et al., 2020);

4) *Characterization of the partial safety factor γ_{Rd}*: grounding on log-normality hypothesis for the random variable representing the model uncertainty ϑ, the corresponding partial safety factor γ_{Rd} is determined in accordance with the following expression:

$$\gamma_{Rd} = \frac{1}{\mu_\vartheta \exp\left(-\alpha_R \beta_t V_\vartheta\right)} \qquad (11.12)$$

where μ_ϑ denotes the mean value of the resistance model uncertainty ϑ; V_ϑ denotes the coefficient of variation of the variable ϑ computed as $\sigma_\vartheta/\mu_\vartheta$; α_R is set to 0.32 as reported by *fib* Model Code 2010 (2013) for non-leading random variables; β_t denotes the target value of the reliability index.

11.4.2.1 ***Partial safety factor γ_{Rd} for uncertainties in definition of numerical model***

The methodology so far described to characterize the partial safety factor γ_{Rd} associated to uncertainty in the definition of the numerical model has been established by Castaldo et al. (2018) and Castaldo et al. (2020) in order to quantify the epistemic uncertainties corresponding to NLNAs in case of incremental monotonic and cyclic loading processes. The main outcomes related to the mentioned investigations are present in Table 11.3.

In the literature, also other studies have been devoted to the characterization of the resistance model uncertainty random variable ϑ with reference to NLNAs of RC structures. In particular, the results of Cervenka, Cervenka & Kadlek (2018) and Engen et al. (2017) leaded to the characterization of the resistance model uncertainty random variable ϑ distinguishing between different failure mechanisms. Table 11.3 also reports the main results of these other investigations. The partial safety factors γ_{Rd} are listed in Table 11.3 are calculated from the statistical parameters according to Eq.(11.12) and are related to the value of 3.8 in terms of target reliability index β_t for RC structure having 50 years of working life with the assumption of non-dominant resistance variable (i.e., $\alpha_R=0.32$).

11.5 Conclusions

The present chapter reports the description and the discussion of the methodologies proposed by literature to perform safety verifications using NLN simulations. These methodologies, denoted as safety formats, are able to include, more or less explicitly, the influence of both aleatory (i.e., material and geometric) and epistemic (i.e., missing knowledge, simplifications and approximations related to definition of the numerical model) uncertainties. A comparison between different safety formats has been reported highlighting the necessity of a further partial safety factor, denoted as failure-mode based partial safety factor γ_{FM}, which is able to consider the uncertainty associated to the sensitivity of the numerical structural model in the prediction of the failure mechanism depending on aleatory uncertainty. The sensitivity of the numerical model is suggested to be checked through two preliminary simulations. If the numerical model turns out to be sensitive, a failure-mode based partial safety factor γ_{FM} set to 1.15 is proposed to be included for evaluation of design value of global structural resistance.

Moreover, the general methodology to estimate epistemic uncertainties and partial safety factor γ_{Rd} related to uncertainty in the numerical model definition is also described. Finally, appropriate values of γ_{Rd} have been proposed grounding on several literature results.

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CHAPTER 11

Figures

Figure 11.1: Local and global approaches for structural analysis (modified from Castaldo, Gino & Mancini, 2019, with permission).

Figure 11.2: Results from numerical simulations and experimental load-displacement curves with identification of the failure modes (modified from Castaldo, Gino & Mancini (2019), with permission).

Figure 11.3: Beam SL: representation of the failure mechanisms identified from NLN simulations (a); results in terms of structural resistance for the NLN simulations deriving from the 30 samples from LHS (b); probabilistic distribution suitable to represent the structural resistance R (c). Figure modified from Castaldo, Gino & Mancini (2019), with permission.

Figure 11.4: Beam designed according to EN1992-1-1 (2014) (T-Beam): representation of the failure mechanisms identified from NLN simulations (a); results in terms of structural resistance for the NLN simulations deriving from the 30 samples from LHS (b); probabilistic distribution suitable to represent the structural resistance R (c). Figure modified from Castaldo, Gino & Mancini (2019), with permission.

Figure 11.5: Representation of probability plot of the design value of structural resistance associated to the several safety formats: Beam SL (a); T-Beam (b). (γ_{Rd} =1.00; β_t= 3.8; α_R = 0.8). Figure modified from Castaldo, Gino & Mancini (2019), with permission.

CHAPTER 11

Tables

Table 11.1: Modelling hypotheses adopted by Castaldo, Gino & Mancini (2019) to define the NLN models.

	Software: ATENA 2D
Equilibrium	- Solution method based on standard Newton-Raphson - Convergence criteria referred to strain energy - Definition of the loading process in line with the experimental tests set
Kinematic compatibility	- Iso-parametric finite elements for plane stress analysis with 4 nodes (the integration grid of 2x2 Gauss points using linear interpolation is used) - Discrete reinforcements - Mesh refinement evaluated by validation process to get numerical accuracy
Constitutive laws	*Model for concrete properties:* - Fixed crack model, smeared cracking, constant shear retention factor assumed as 0.2 - Uni-axial model extended to biaxial stress state - Compressive response: non-linear presenting post peak linear softening - Tensile response: elastic presenting post peak linear tension softening (LTS) *Model for reinforcement properties:* - bi-linear relationship for tensile and compressive response

Table 11.2: Assumptions for application of the safety formats in Castaldo, Gino & Mancini (2019).

Safety format	$R_{NLNA}(X_{rep};a_{rep})$	γ_R [-]	γ_{Rd} [-]
PFM	$R_{NLNA}(f_{cd},f_{yd};a_{nom})$ *1	1.00	1.00
ECOV	$R_{NLNA}(f_{cm},f_{ym};a_{nom})$	$exp(\alpha_R\beta V_R)$ *2	
GRF	$R_{NLNA}(f_{cmd}$ *3$,f_{ym};a_{nom})$	1.27	
GSF	$R_{NLNA}(f_{cm},f_{ym};a_{nom})$	$exp(\alpha_R\beta V_R)$ *4	
PM	$R_{NLNA,m}$ *4	$exp(\alpha_R\beta V_R)$ *4	

*1 The partial safety factors used to calculate f_{cd} and f_{yd} are respectively set equal to 1.5 and 1.15 according to EN1992-1-1 (2014), while, the design vale of geometrical properties is a_d is set equal to the nominal one a_{nom} which corresponds to the size of the experimental set up.

*2 Coefficient of variation of global resistance V_R is estimated with a simplified approach according to the hypothesis of lognormal distribution performing two NLNAs using mean ($R_{NLNA}(f_{cm},f_{ym};a_{nom})$) and characteristic values ($R_{NLNA}(f_{ck},f_{yk};a_{nom})$) of material properties. The geometrical properties are set qual to the nominal one a_{nom} which corresponds to the size of the experimental set up.

*3 The representative value for concrete compressive strength is evaluated as: $f_{cmd}=0.85\,f_{ck}$.

*4 The mean value $R_{NLNA,m}$ and coefficient of variation V_R of structural resistance can be estimated performing a reduced Monte Carlo simulation such as the Latin Hypercube Sampling with at least 30 samples. Lognormal distribution has been assumed to represent the structural resistance.

Table 11.3: Characterization of model uncertainty random variable ϑ from different literature references; the resistance model uncertainty partial safety factor γ_{Rd} is calculated adopting target reliability index β_t set equal to 3.8 for RC structures having 50 years of reference period with the assumption of non-dominant resistance variable (i.e., $\alpha_R=0.32$)

<table>
<tr><th rowspan="2">Ref.</th><th rowspan="2">Type of NLNA</th><th rowspan="2">Failure mode</th><th colspan="3">Resistance model uncertainty random variable ϑ</th><th rowspan="2">Resistance model uncertainty partial safety factor γ_{Rd}</th></tr>
<tr><th>Probabilistic distribution</th><th>Mean value μ_ϑ</th><th>Coefficient of variation V_ϑ</th></tr>
<tr><td>Castaldo et al. (2018)</td><td>Plane stress non-linear finite element analysis, incremental monotonic loading</td><td>Several failure modes inclusive of concrete crushing and reinforcement yielding</td><td>lognormal</td><td>1.01</td><td>0.12</td><td>1.15</td></tr>
<tr><td>Castaldo et al. (2020)</td><td>Plane stress non-linear finite element analysis, cyclic loading</td><td>Several failure modes inclusive of concrete crushing and reinforcement yielding</td><td>lognormal</td><td>0.88</td><td>0.13</td><td>1.35</td></tr>
<tr><td rowspan="4">Cervenka, Cervenka & Kadlek (2018)</td><td rowspan="4">3D non-linear finite element analysis, incremental monotonic loading</td><td>Punching</td><td rowspan="4">lognormal</td><td>0.97</td><td>0.08</td><td>1.16</td></tr>
<tr><td>Shear</td><td>0.98</td><td>0.07</td><td>1.13</td></tr>
<tr><td>Bending</td><td>1.07</td><td>0.05</td><td>1.01</td></tr>
<tr><td>All</td><td>0.98</td><td>0.08</td><td>1.16</td></tr>
<tr><td rowspan="3">Engen et al. (2017)</td><td rowspan="3">3D non-linear finite element analysis, incremental monotonic loading</td><td>Ductile (significant reinforcements yielding)</td><td rowspan="3">lognormal</td><td>1.04</td><td>0.05</td><td>1.02</td></tr>
<tr><td>Brittle (concrete crushing without reinforcements yielding)</td><td>1.14</td><td>0.12</td><td>1.02</td></tr>
<tr><td>All</td><td>1.10</td><td>0.11</td><td>1.04</td></tr>
</table>