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***Abstract*—This work describes a calibration method for mutual inductance standards using the INRIM-POLITO VersICaL fully-digital impedance bridge. Four mutual inductors with nominal values of $10\,\mathrm{mH}$, $1\,\mathrm{mH}$, $0.5\,\mathrm{mH}$, and $0.1\,\mathrm{mH}$ were measured with both the VersICaL bridge and a commercial LCR metre in the audio frequency range. The measurements carried out with the two methods are reasonably comparable within a preliminary evaluation of the uncertainty. The low frequency behaviour of mutual inductors is relevant for their potential application in the calibration of impedance analysers in the mHz range.**

***Index Terms*—calibration; inductance; impedance measurement; metrology; measurement uncertainty.**

I. Introduction

The accurate measurement of mutual inductance was considered a fundamental topic in electrical metrology from the introduction of ac measurement methods to about the 1960s. The calculable mutual inductor [1]–[3] allowed the realisation of the ohm, traceable to the mechanical measurement of its windings. The high purity of air-core mutual inductors was exploited to realise phase angle standards [4]. Bridge topologies specifically devoted to the measurement of mutual inductance [5, Ch. 6] were proposed and implemented.

Since about 1970, mutual inductance measurements have been performed with electronic self-balancing LCR metres, whose metrological traceability is achieved by calibration with resistors, capacitors and self-inductors. Proper measurement procedures are only briefly described in application notes [6, Sec. 5.3.3], without particular consideration of the stray effects affecting the measurement accuracy [7]. The manufacturing of mutual inductance standards was abandoned long ago and nowadays, to the authors' knowledge, these standards are no longer commercially available.

A need for accurate measurement of mutual inductance resurged in recent years. In primary metrology, the Joule balance [8], a variation of the Kibble balance, has been proposed as a way to realise the kilogram through a mutual inductance measurement. In applications, air-core mutual inductors are key components in wireless power transmission, every year more ubiquitous in wireless charging stations for battery-operated devices and electric vehicles [9], [10]. Very recently, mutual inductors have been proposed as calibration standards for electrical impedance spectroscopy (EIS) metres employed in the characterisation of electrical batteries and supercapacitors [11], [12]. In the low frequency range the mutual inductor, by reversing the polarity of its voltage output, simulates a high-valued capacitor. A proper traceability of the calibration method can be achieved from accurate measurements of the mutual inductance in the audio frequency range and calculated frequency corrections taking into account measurements of the inductor stray parameters [5, Sec. 3.3].

These new applications trigger the need for traceable and accurate calibrations of mutual inductance standards. Primary impedance metrology, traditionally based on manually-operated transformer ratio bridges, is progressively shifting to

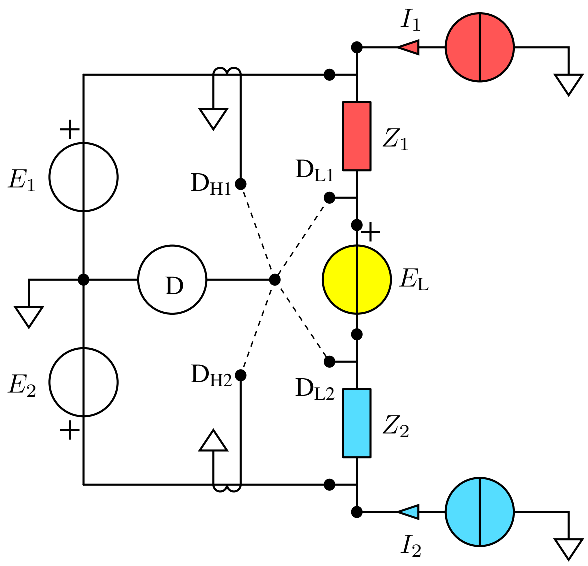

Fig. 1. Simplified schematic diagram of the VersICaL FDIB, see Sec. II for a description.

the use of digital impedance bridges [13, Sec. 5.5] [14, Sec. 5], to date mainly employed to perform ratio measurements between resistors, capacitors, and self-inductors.

In this work, we investigate the feasibility of accurate calibration of mutual inductance standards with a fully-digital impedance bridge (FDIB). The bridge was recently developed by INRIM and POLITO and originally designed for the realisation of the Italian national standards of capacitance and self-inductance [15].

The investigation focussed on the calibration of air-core mutual inductors from $10\,\mathrm{mH}$ to $0.1\,\mathrm{mH}$, in the audio frequency range, by using calibrated ac resistors as reference standards. A preliminary uncertainty analysis shows that the base calibration accuracy is in the $50\,\mu\mathrm{H/H}$ range at $1\,\mathrm{kHz}$, in line with that achieved on the calibration of self-inductors. These results confirm the behaviour of the mutual inductors in the audio frequency range observed using a commercial LCR metre. The positive outcome of the investigation extends the use of digital bridges to this calibration capability, not previously available.

II. INRIM-POLITO VersICaL FDIB

The FDIB used in this work was developed during the EMPIR project 17RPT04 VersICaL, *a versatile electrical impedance calibration laboratory based on digital impedance bridges* [16]. The bridge is described in detail in [15] together with an evaluation of the measurement uncertainty. In this section, we briefly summarise its operation with the help of the simplified schematic of Fig. 1.

The FDIB compares two impedances Z_1 and Z_2, defined as four-terminal-pair standards. The voltages E_1, E_2 and E_L and the currents I_1 and I_2 are generated by a polyphase sinusoidal synthesiser. The bridge is balanced when the voltages at the detection nodes D_L1, D_L2, D_H1 and D_H2 are zero, that is, when the voltages at the low potential terminal pairs of Z_1 and Z_2 are zero and the currents through the high potential terminal pairs, as measured by two current transformers, are zero too. This balance is achieved by iteratively adjusting E_2, E_L, I_1 and I_2. When the bridge is balanced, the relation $Z_1/Z_2 = -E_1/E_2$ holds, and the impedance ratio is directly determined from a voltage ratio.

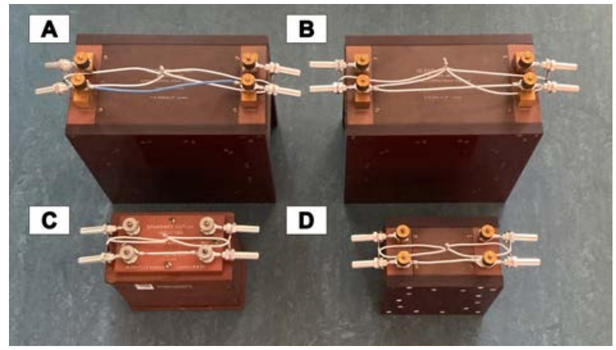

Fig. 2. (a) $10\,\mathrm{mH}$ Sullivan standard (M_a), (b) $1\,\mathrm{mH}$ Sullivan standard (M_b), (c) $0.5\,\mathrm{mH}$ Tinsley standard (M_c), and (d) $0.1\,\mathrm{mH}$ Sullivan standard (M_d). Adaptors of each standard were made with Post Office MUSA connectors for interfacing the mutual inductor with the VersICaL FDIB.

TABLE I
MEASURED PARAMETERS OF THE WINDINGS OF THE MUTUAL INDUCTANCE STANDARDS AT $120\,\mathrm{Hz}$.

M_i	L_1/mH	R_1/Ω	L_2/mH	R_2/Ω
M_a ($10\,\mathrm{mH}$)	$9.995(30)$	$11.4(34)$	$28.829(86)$	$21.5(65)$
M_b ($1\,\mathrm{mH}$)	$1.844(18)$	$1.76(53)$	$1.626(16)$	$1.67(50)$
M_c ($0.5\,\mathrm{mH}$)	$0.7237(72)$	$1.56(47)$	$0.7292(73)$	$1.65(49)$
M_d ($0.1\,\mathrm{mH}$)	$0.1821(91)$	$0.66(20)$	$0.1704(85)$	$0.67(20)$

III. Mutual Inductance Standards

The standards of mutual inductance employed in the experiment, shown in Fig. 2 and listed in decreasing order of nominal value, are:

- Sullivan R1960-R1968 series, with nominal values of $10\,\mathrm{mH}$ (M_a), $1\,\mathrm{mH}$ (M_b), and $0.1\,\mathrm{mH}$ (M_d), designed to have low temperature dependence ($5\,\mu\mathrm{H/(H\,K)}$), low frequency coefficient ($\Delta M/M_0$) and low phase defects [17].
- Tinsley 4190AM (M_c), with nominal value of $0.5\,\mathrm{mH}$.

The standards are cylindrical air-core inductors wound on an insulating frame, defined as four-terminal (4T) impedances [13, Sec. 2.1.3]. A simple adapter [18], visible in Fig. 2, is added to each standard to convert the 4T definition to a four-terminal-pair definition [13, Sec. 2.2.5] with British Post Office MUSA connectors, making the standards suitable for measurements with the VersICaL FDIB.

The parameters of the primary and secondary windings of the standards were measured using an LCR metre (Agilent E4980A), following the literature recommendations [6]. These measured values are shown in Table I. The results are consistent with the specification provided by the manufacturer in the catalogue [17].

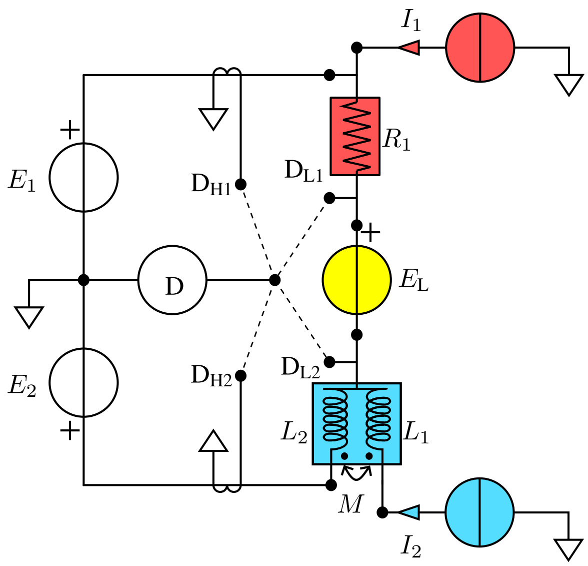

Fig. 3. Schematic diagram of the VersICaL FDIB for measuring a mutual inductor M using a resistor R_1 as the reference standard.

TABLE II
MEASUREMENT CONFIGURATION OF THE VERSICAL FDIB FOR EACH STANDARD.

M_i	f	Z(nom)	R_1	I_1
M_a (10 mH)	120 Hz	7.54 Ω	10 Ω	10 mA
	1 kHz	62.8 Ω	100 Ω	3 mA
	10 kHz	628 Ω	1 kΩ	1 mA
M_b (1 mH)	120 Hz	754 mΩ	1 Ω	30 mA
	1 kHz	6.28 Ω	10 Ω	10 mA
	10 kHz	62.8 Ω	100 Ω	3 mA
M_c (0.5 mH)	120 Hz	377 mΩ	1 Ω	30 mA
	1 kHz	3.14 Ω	10 Ω	10 mA
	10 kHz	31.4 Ω	100 Ω	10 mA
M_d (0.1 mH)	1 kHz	628 mΩ	1 Ω	30 mA
	10 kHz	6.28 Ω	10 Ω	10 mA

IV. MEASUREMENTS

The mutual inductors were measured with the VersICaL FDIB, connected as shown in Fig. 3. The primary winding L_1 is connected to the current port and the secondary winding L_2 to the voltage port. The experimental setup is presented in Fig. 4. During the measurements special care must be taken, such as isolating the mutual inductors from metallic materials to reduce eddy currents, and twist current and voltage leads to avoid stray mutual inductance coupling. The following calibrated reference resistors with known frequency coefficients were used: 1 Ω (Tinsley 5695), 10 Ω, 100 Ω and 1 kΩ (Tinsley 5685 series). The mutual inductance standards were measured at 120 Hz, 1 kHz and 10 kHz. The reference resistor was selected so that its nominal value and the impedance of the mutual inductor were as close as possible, to minimise the error due to the source nonlinearity. M_d was not measured at 120 Hz because its impedance is significantly less than the lowest reference available. The current I_1 was set equal to the calibration current of the reference resistors R_1. These measurement configurations and the nominal impedance of the standard $Z(\mathrm{nom}) = 2\pi f M(\mathrm{nom})$ are reported in Table II.

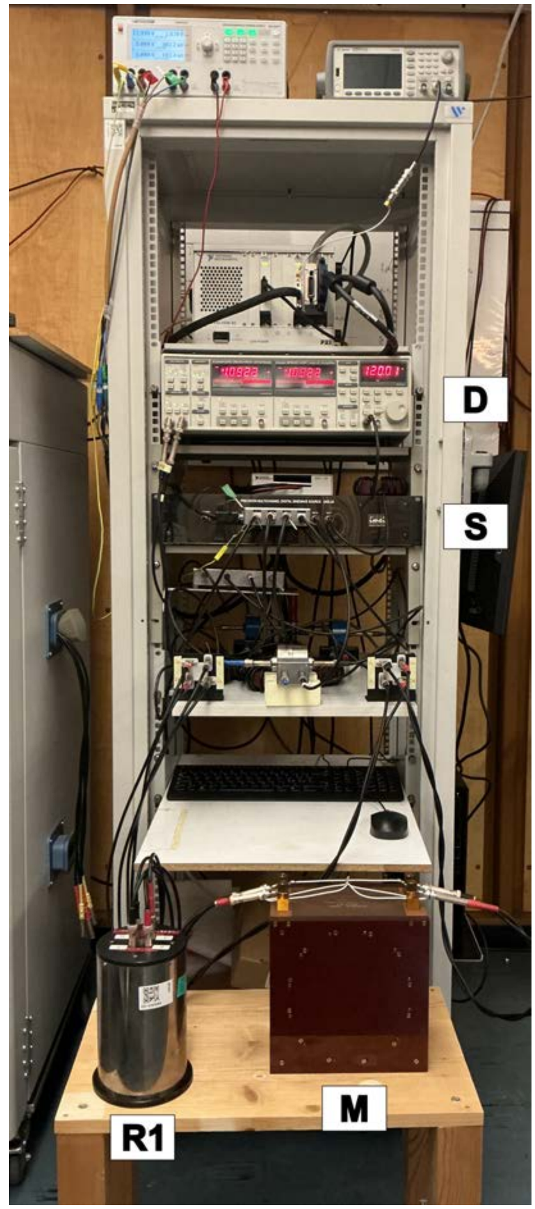

Fig. 4. Photograph of the experimental setup of the VersICaL FDIB. The mutual inductance standard M is kept isolated from the rest of the bridge and any metallic surface to avoid magnetic interactions; R1 is the reference resistance standard; S is the polyphase sinusoidal synthesiser that generates the voltage and current sources; and D is the lock-in amplifier used as null detector.

The mutual inductance standards were also measured with the LCR metre mentioned in Sec. III. This measurement in a frequency range from 20 Hz to 20 kHz gives a general trend of the behaviour of the standards.

V. RESULTS

Fig. 5 reports the results of the measurements made with the VersICaL FDIB (blue circles) and the LCR metre (black solid

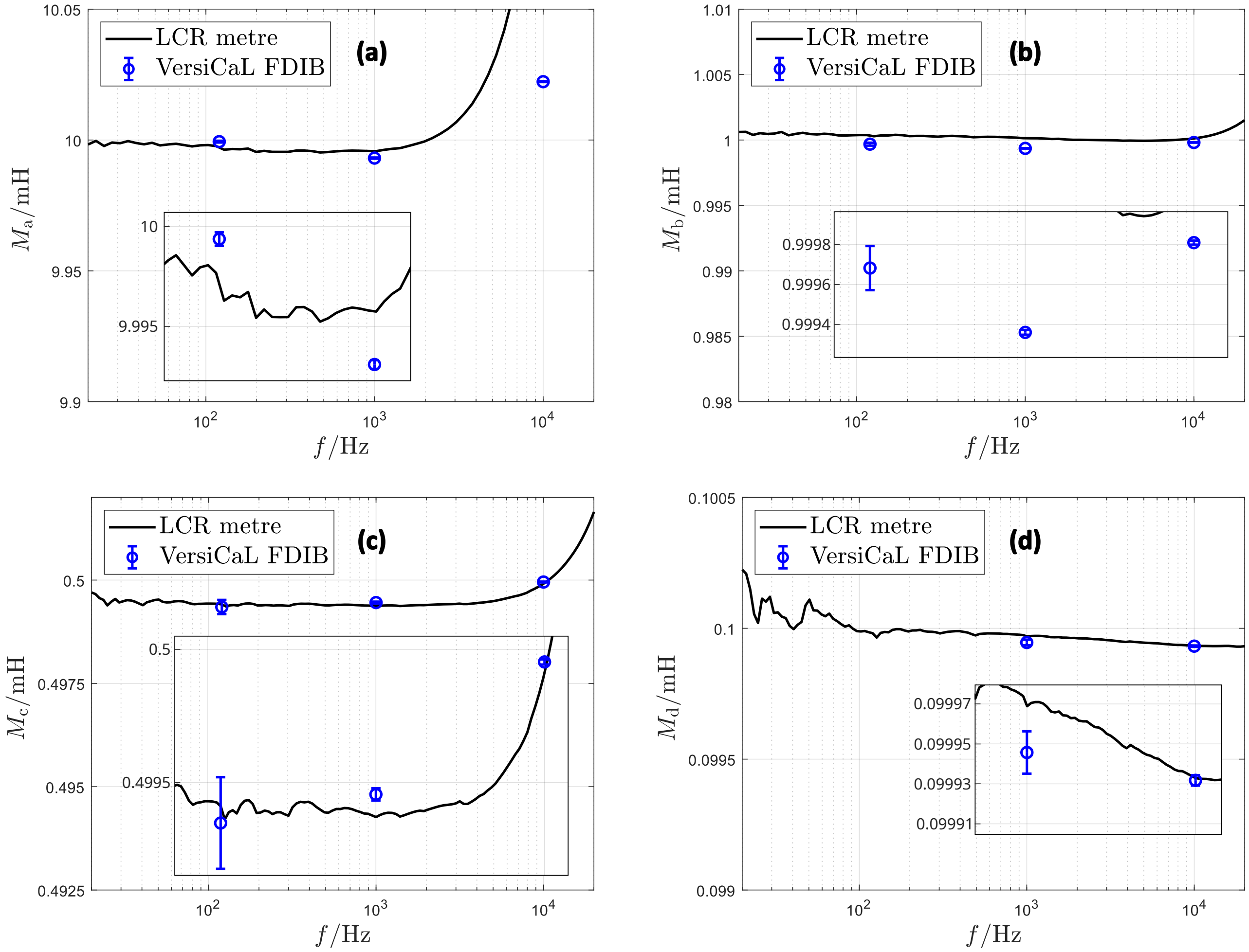

Fig. 5. Measurements with the VersICaL FDIB (blue circles) and E4980A LCR metre (black solid line) of (a) M_a (10 mH), (b) M_b (1 mH), (c) M_c (0.5 mH), and (d) M_d (0.1 mH) mutual inductance standards. The insets in each plot provide a magnified view of the VersICaL measurements. The error bars correspond to the combined ($k = 1$) uncertainties of the VersICaL FDIB measurements. For the given impedance and frequency ranges, the specified accuracy of the LCR metre measurements is within 0.1% and 0.3%. The uncertainty bars of the frequency sweep were omitted for clarity in the graph.

line). This figure is divided into four plots (a), (b), (c) and (d) that correspond to (a) M_a (10 mH), (b) M_b (1 mH), (c) M_c (0.5 mH), and (d) M_d (0.1 mH) mutual inductance standards, respectively.

The LCR metre measurements (black solid line) are shown in Fig. 5 over a frequency range from 20 Hz to 20 kHz. The behaviour of M_d, which is the mutual inductance standard with the lowest nominal value (0.1 mH), shows a flat response in the whole frequency range, with increasing noise at low frequency. In contrast, M_a, M_b, and M_c remain constant in the low frequency range and increase at higher frequencies. Measurements above 20 kHz, not shown in Fig. 5, reveal a resonance effect which becomes proportionally more significant for higher value standards. For the measured impedance and frequency ranges, the specified accuracy of the LCR metre is within 0.1% and 0.3%.

The measurements performed with the VersICaL FDIB are reported in Table III. The same points are shown in Fig. 5 (blue circles). The values show reasonable agreement with the general trend indicated with the LCR metre. The inlets of each section of Fig. 5 provide a magnified view of the measurements and the differences within the two methods.

TABLE III
RESULTS OF THE VERSICAL FDIB MEASUREMENTS FOR EACH STANDARD AT EACH FREQUENCY. THE REPORTED UNCERTAINTIES ARE WITH A COVERAGE FACTOR $k = 1$.

f/Hz	M_a/mH	M_b/mH	M_c/µH	M_d/µH
120	9.999 37(34)	0.999 68(11)	499.35(86)	
1000	9.993 09(22)	0.999 360(12)	499.455(23)	99.946(11)
10000	10.022 28(13)	0.999 809(10)	499.9526(97)	99.9317(26)

Regarding the resonance effect in the high frequency range and the discrepancy observed, for example, on M_a in 10 kHz, it is necessary to take into account the loading of the secondary winding by the high voltage port and the capacitance of the coaxial cable [7]. This effect depends on the measured system employed and is different for the VersICaL FDIB and the LCR metre. In the case of the VersICaL FDIB, this error can be modelled because it includes a null detector D, indicated in Fig. 3, and the loading error is due only to the capacitance of the coaxial cable connected to the high voltage port. The estimation of the loading error, shown in Fig. 6, was calculated for the VersICaL FDIB using the measured values L_2 and R_2 of each standard, reported in Table I, and the measured capacitance of the coaxial cable. This error is negligible for M_d $(0.1\,\mathrm{mH})$ but dominant for the higher value standards in the high frequency range. In the case of the LCR metre, this is not easily calculable because it also depends on the input impedance of the amplifier.

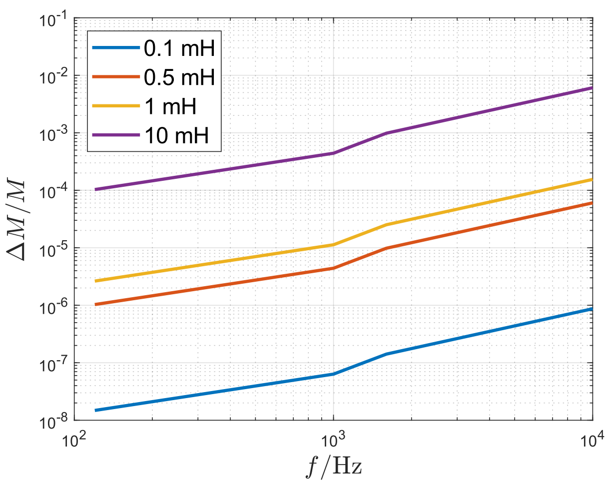

Fig. 6. Estimation of the loading error for each mutual inductance standard in the frequency range from $120\,\mathrm{Hz}$ to $10\,\mathrm{kHz}$. This error was estimated using the parameters of the mutual inductors reported in Table I and the measured capacitance of the cable.

The uncertainties of the VersICaL FDIB measurements reported in Table III and Fig. 5 were evaluated with a dedicated Matlab script [19]. This script computes the uncertainty with a Monte Carlo method, according to [20], on the basis of the bridge parameters characterised for self-inductance measurements. Since these parameters change for mutual inductance measurements, the reported uncertainties may be possibly underestimated and should be considered as provisional.

These measurements were performed in a shielded room, temperature-controlled at $23.0(5)\,^\circ\mathrm{C}$. The contribution of temperature changes to the uncertainty was neglected due to the low temperature coefficient $(5\,\mu\mathrm{H}/(\mathrm{H\,K}))$ of the standards.

VI. Conclusions

In this work, the results for the calibration of mutual inductance standards with a fully digital impedance bridge are presented. The measurements with the FDIB and the LCR metre are in reasonable agreement and confirm the behaviour of the standards within the frequency range of interest. This piece of information is relevant because EIS metres, which are proposed to be calibrated using mutual inductors, perform measurements in the low frequency range (typically down to the mHz range), where direct measurements are not feasible.

An initial uncertainty estimation for these measurements was provided and a more detailed analysis together with a possible strategy for the mitigation of the loading error effect will be discussed at the conference.

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