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# *Early Modern Age Fortifications*

Knowledge for Management, Conservation and Valorisation

*Marco Giorgio Bevilacqua, Roberta Spallone, Andrea Giordano, Michele Russo (Eds.)*



# PUBLICA

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MARCO GIORGIO BEVILACQUA, ROBERTA SPALLONE, ANDREA GIORDANO,  
MICHELE RUSSO (EDS.)

EARLY MODERN AGE FORTIFICATIONS.

KNOWLEDGE FOR MANAGEMENT, CONSERVATION AND VALORISATION

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The volume presents the outcomes achieved at the end of the first year in the project 'INFORTREAT. Reconstructing the Early Modern bastioned front. Information models for the fruition of constructive knowledge in FORTified architecture TREATises (16th-18th Century): a new integrated analysis tool for the interpretation, restoration and maintenance of Early Modern fortified heritage', financed by the European Union - Next-GenerationEU - National Recovery and Resilience Plan (NRRP) – MISSION 4 COMPONENT 2, INVESTMENT N. 1.1, CALL PRIN 2022 D.D. 104 02-02-2022 – CUP N. I53D23005420006.

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Immagine delle mura della Cittadella di Alessandria; in primo piano il baluardo di Santa Cristina.

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***Digital Informative Models of Early Modern  
Military Architecture Treatises.  
Methodological Approach and First Results***

*Piergiuseppe Rechichi, Virginia Miele, Enrico Pupi, Fabrizio Natta, Marco Vitali,  
Roberta Spallone, Marco Giorgio Bevilacqua*



## Introduction

The physical modern age fortifications realized over the centuries were influenced by preexisting fortified structures from the middle-age, topology, water availability, existing infrastructures, and resource limitations, but were still related to theoretical models defined in the circulating treatises. The ideal fortification model proposed in military architecture treatises can hardly be identified as an object but should be considered and studied as a complex architectural system characterized by a multitude of interrelated defensive components. The morphology of the fortification is the result of a convergence of defensive design criteria, and geometry is the key to ensure the effectiveness of the defense. The fortification model, precisely due to the attributes just described, is not expressed in the treatises in an accessible and linear fashion. In fact, the body of knowledge that contributes to the completeness and coherence of a fortification model, of both textual and iconographic nature, is disseminated within the treatises according to thematic subdivisions and in accordance with recurring content structures. This composition makes the process of acquiring knowledge of the military architectural model from the treatises extremely challenging, requiring analysis and interpretation that can only be carried out by those who have already acquired a sufficient level of expertise in the subject. To enable greater accessibility to this knowledge, it is necessary to process all the data present within the treatises to extrapolate geometrically complete and coherent fortification models. In a modern fortification system, defense is ensured if specific geometric conditions determined by the knowledge and expertise of the treatise writer are met. These conditions correlate variable quantities (e.g. proportion of the flanks), to parameters (e.g. number of base polygon sides), and allow the shaping of the fortification model's characterization to maintain optimal defense guarantee in different design conditions. The design logic of military architecture can surprisingly be combined with contemporary parametric design logic, highlighting the potential of using procedural modeling tools and visual programming languages to create digital models of fortification systems.

To ensure an effective correspondence between the geometric characterization of the fortification system and the complex network of textual information disseminated in the treatise, the applicability of a Building Information Modeling approach appears evident.

This research question is the central focus of WP5 in the INFORTREAT project. The first tests were conducted on treatises by Sardi, Guarini, Lorini, and Tensini, highlighting criticalities and potentialities for the next steps of the research project.

## *Digital Informative Models*

Recent advancements in digital technologies for architectural heritage documentation have improved large datasets processing capabilities (Chiabrando et al., 2017; Croce et al., 2019). However, critical aspects of architectural analysis emphasize the need for more informational depth and intelligibility in digital representations (Lo Buglio & De Luca, 2012). Research has focused on methodologies for interpreting models from architectural treatises (De Luca, 2014), formalizing architectural knowledge into structured model libraries (Carpo, 2016; De Luca et al., 2007; Cache, 2009), and developing information systems for semantic interpretations of theoretical frameworks (Apollonio et al., 2013).

Various methods have sought to digitally transcribe architectural proportional theories. Giovannini (2023) applied Visual Programming Language (VPL) within the HBIM framework to analyze Palladio's *The Four Books of Architecture*. Digital 3D reconstructions based on treatise descriptions enhance the understanding of narrated and idealized architecture through graphic analysis, adding depth to historical sources (Palestini, 2017). Further studies explore models from treatises, formalizing architectural knowledge into structured libraries (De Luca, 2014; Carpo, 2016; Cache, 2009), and developing information systems for semantic interpretations (Apollonio et al., 2013). Ontologies refine BIM methodologies and enable tailored queries for built heritage representation through Semantic Web technologies.

Simeone et al. (2019) addressed formalizing cultural heritage information models to support nuanced interpretative frameworks. BIM and VPL integration enables innovative architectural analysis, emphasizing HBIM's potential in heritage conservation. Lo Turco et al. (2022) showed how BIM's structured data management and VPL's flexibility allows dynamic heritage applications. This highlights the importance of disseminating such knowledge via web platforms (De Luca et al., 2011; Stefanini et al., 2013). Web-based BIM visualisation has evolved, prioritizing user-friendly interfaces and advanced metadata queries. Furuta et al. (2023), integrates geometric visualisation with semantic data updates. Interoperability efforts include El-Yamany et al. (2012), who developed a web application for replicating IFC objects, followed by research improving processing speeds for large models (Liu et al., 2015; Li et al., 2022). Despite progress, challenges remain in enhancing IFC model semantic accessibility for non-specialists, as Furuta et al. (2022) noted. Future studies ought to explore the integration of these technologies to improve interpretation, accessibility, and critical interaction with historical architectural data.

## *Methodology*

The methodology adopted for the development of digital fortification models, derived from treatises, is structured into five main phases (Fig. 1):

1. Critical interpretation of the treatise and graphical analysis.
2. Redrawing of the geometric models of fortification.
3. Identification of relationships between parts and VPL modelling of the fortification system.
4. Integration into a BIM environment and model information.
5. Sharing of the model in a web-BIM environment.

From the analysis of the treatise, ontological, dimensional, and functional information related to the geometric construction and characterization of the parts is extracted. Preliminary redrawing guarantees a verification of the coherence between different

prescriptions inside the treatise text and different graphical representations in the treatises' figures.

By utilizing parametric modelling tools, it is possible to reconstruct the treatises' design logics, facilitating the creation and dynamic management of the models. These models can subsequently be transferred to the BIM environment for enhanced information management and consultation.

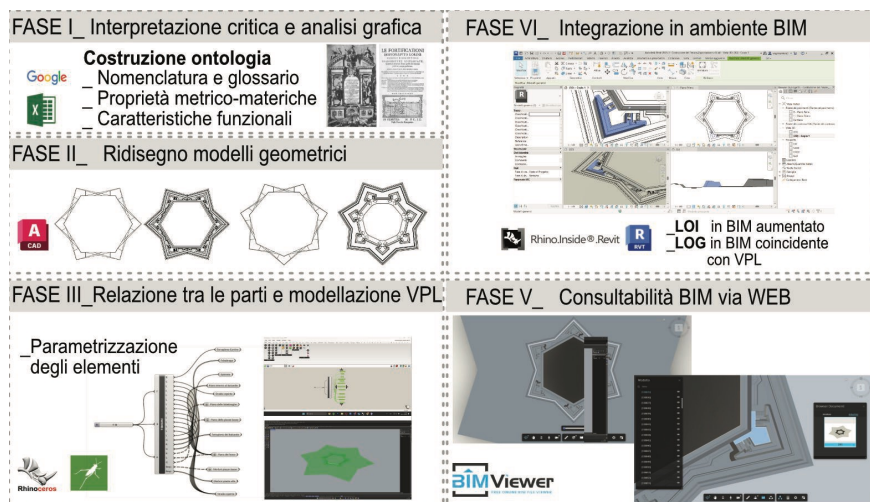


Fig.1 - Methodological diagram for the realization of digital informative models of modern-age fortified systems from the treatises (Souce: Rechichi, 2025; Elaborated by Miele, V.)

## Case Studies

### *Le Fortificazioni* by Bonaiuto Lorini

Lorini's treatise "*Le Fortificazioni*" represents a crucial bridge between early mechanical science and the science of weights, encompassing graphic scales and machines. This seminal work on military fortification highlights the intrinsic connection between theoretical principles and practical application. Lorini (1596) is recognized as the first Italian author to explicitly demonstrate the construction of axonometry (Scolari, 2005, p. 28). His treatise provides a thorough analysis of fortification advancements driven by innovations in artillery and weaponry.

Grounded in the mechanical principles of Archimedes—particularly the law of leverage—and the work of Guidobaldo del Monte (1545–1607), author of *Le Mecaniche* (1577–1581), Lorini integrates advanced mechanical theories into his discussion. The first edition of this five-book treatise on fortifications was published by Rampazzetto in Venice, following a limited print run of fifteen copies in 1596 (noted by Jordan; *Writing on Architecture*), which were dedicated to rulers and influential noble families. The treatise circulated widely, with a German translation printed in Frankfurt by Theodor de Bry in 1607. A second, expanded edition, with an additional sixth book dedicated to Grand Duke Cosimo II, was published in 1609 by Francesco Rampazzetto published a second, expanded edition in Venice, which included.

#### *Il Corno Dogale* by Pietro Sardi

Pietro Sardi, a Roman military engineer, authored two treatises on military architecture (*Corona Imperiale dell'Architettura militare*, 1618, and *Corno Dogale dell'Architettura militare*, 1639) and two treatises on artillery. The *Corno Dogale* represents an evolution of the fortification model previously proposed by Sardi in the *Corona Imperiale*. In addition to revising construction techniques, admitting the opportunity to build fortifications solely of earth under certain circumstances, Sardi reconfigures the external works of the fortification and the layout methods, acquiring and integrating knowledge and expertise related to the latest developments in Dutch-influenced military architecture.

In the seven books of the treatise, Sardi describes methods for regular and irregular fortification, defining two different fortification models: in the second book, he proposes his method of fortification, characterized by a defensive layout of Italian tradition; in the third book, he proposes the Dutch method of fortification, substantially coinciding with that already described by Marolois. The *Corno Dogale* is a treatise of great relevance in terms of geometric design of the defence system; the defensive systems are characterized by a low maintenance with a limited number of artillery pieces. Construction techniques and structural features of the fortification are only partially described.

*La Fortificatione* by Francesco Tensini

Francesco Tensini da Crema began his military career at a young age, amassing extensive hands-on experience and theoretical knowledge from treatises circulating in Italy and across Europe. His field experiences in Flanders and Friesland during the late 16th and early 17th centuries were particularly influential. His work “*La Fortificatione*,” comprising three books and ninety chapters in total, encapsulates the full spectrum of his military expertise, from fortification design to defense and offense, enriched by his own inventions and machines.

The first edition was printed in Venice by Evangelista Deuchino in 1624, followed by a second in 1630 in Venice by the Bariletti brothers, and a third in 1655 at Brogiollo.

Tensini’s genius is evident in his ability to synthesize the best of traditional Italian fortification models, integrating them with his own innovations, strongly influenced by his Dutch military experiences. This synthesis allowed him to refine state-of-the-art fortification models that are exceptionally effective and efficient in terms of construction and maintenance costs.

*Trattato di Fortificatione* by Guarino Guarini

Guarino Guarini’s *Trattato di Fortificatione* (1676) represents a significant contribution to early modern military architecture. Written during his tenure in Turin, the treatise reflects Guarini’s multidisciplinary expertise, blending mathematics, architecture, and military engineering. Book III of the work focuses on the “second delineations” of fortresses and their orthography, offering a systematic approach to fortress design through geometric principles and proportional relationships.

Guarini’s treatise follows the tradition of contemporary military theorists such as Marolois (1615) and Sardi (1639), presenting a manualistic approach to educate readers on fortification principles. A key characteristic of the work is its tabular system, which defines maximum, average, and minimum dimensions for fortification elements such as bastions, curtains, and ravelins. This method provides a comparative framework, allowing for variations in design while maintaining defensive effectiveness.

The fortification model outlined in Guarini's treatise emphasises the integration of geometric rigour with practical considerations of military defence.

His approach considers factors such as soil type, ballistic trajectories, and structural stability, ensuring adaptability to different terrains and warfare strategies. These principles highlight Guarini's role in codifying fortification theory within a broader architectural and scientific discourse.

## Results

The components of fortifications are widely described within military architecture treatises in a non-systematized manner. For instance, the description of an element might be located in a different chapter compared to its dimensioning, or its material and constructive characterization. For this reason, it is fundamental to develop a systematized knowledge structure of the text's contents, which allows for defining the ontology of the fortification presented by the treatise and for systematizing the information related to individual entities. To this end, a specific Excel spreadsheet containing all the necessary information has been constructed (Fig. 2). The quality and consistency of the iconographic apparatus presented within individual treatises can vary drastically. However, the figures are fundamental for an accurate interpretation of the text, for the integration of geometric characteristics not explicitly stated in textual form, and for verifying the overall coherence between textual prescriptions and the dimensions represented in the figures. For this reason, the critical redrawing of the ichnographies and orthographies of the fortifications allows for resolving the geometric characterization of the fortifications themselves with coherence and completeness, before proceeding with the actual creation of digital models (Figs. 6-8). Particular attention must be paid to the critical interpretation of the geometric construction methods of the defensive profiles, a fundamental element in defining the overall form of the fortification system (Figs. 3-5).

Titolo		Corno Dogale dell'Architettura Militare							
Autore		Pietro Sardi							
Anno		1638							
Voce	Tipologia	Libro	Capitolo	Pag.	Nota	Unità	Misura		
Angolo del centro	Nomenclatura	Terzo	Quarto Principale	118	Vedere figura Prima Olandese, segnato Csu	x	x		
Angolo del fianco	Nomenclatura	Secondo	Quarto Principale	48	Vedere figura prima, segnato E	x	x		
Angolo della spalla	Nomenclatura	Secondo	Quarto Principale	48	Vedere figura prima, segnato D	x	x		
Angolo di congiunzione del fianco del baluardo	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato DFG	x	x		
Angolo di congiunzione della spalla del baluardo	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato DGF	x	x		
Angolo di direzione	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato D	x	x		
Angolo d'asse del baluardo	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato DCG	x	x		
Angolo esterno del baluardo	Nomenclatura	Secondo	Quarto Principale	48	Vedere figura prima, segnato C	x	x		
Angolo esterno del baluardo	Descrizione	Secondo	Quarto Principale	71	Sottocapo Discorso sopra l'angolo esterno del baluardo/Opinione cca	x	x		
Angolo interno del baluardo	Nomenclatura	Secondo	Quarto Principale	48	Vedere figura prima, segnato A	x	x		
Angolo interno del poligono	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato Nca	x	x		
Baluardo a sua piazza	Nomenclatura	Secondo	Quarto Principale	48	Vedere figura prima, segnato B	x	x		
Banca del cavaliere	Nomenclatura	Secondo	Quarto Principale	51	Vedere figura seconda, segnato XIX	piade geometrico	lunghezza 200		
Bocca (Cannoniera)	Nomenclatura	Secondo	Quarto Principale	73	Vedere figura nona, segnato G	piade geometrico	largia 4		
Cammerera (parapetto)	Nomenclatura	Secondo	Quarto Principale	73	Vedere figura nona, segnato AA. In pianta segnata HH. Cannoniera seg.	x	x		
Cammerera (parapetto)	Descrizione	Secondo	Quarto Principale	73	Spazio cammerera va armato delle sue bande di garofoli fatti di viti	x	x		
Cammerera (Ranchi)	Descrizione	Secondo	Quarto Principale	65	Sottocapo De Parapetti/Le cammerera dei Ranchi saranno 3 e non pu	x	x		
Cavaliere	Nomenclatura	Secondo	Quarto Principale	51	Vedere figura seconda, segnato D in figura terza H	piade geometrico	altezza sul piano del terrazo		
Cavaliere	Descrizione	Secondo	Quarto Principale	57	Rigo 22/in mezzo alla cortine per vedere e battere il nemico da lontano	x	x		
Cavaliere	Descrizione	Secondo	Quarto Principale	56	Rigo 24/Importanza del cavaliere	x	x		
Circolo moto	Nomenclatura	Terzo	Primo	120	Vedere figura decima Olandese, segnato punteggiato BCDEF	x	x		
Controscarpa	Nomenclatura	Secondo	Quarto Principale	54	Vedere figura terza, primo profilo, segnato E	piade geometrico	altezza 15		
Controscarpa	Nomenclatura	Terzo	Primo	120	Vedere figura Dodicesima Olandese segnato M	x	x		
Corbelli pieni di terra (terzoia)	Nomenclatura	Secondo	Quarto Principale	54	Vedere figura terza, segnato V	x	x		
Cortine	Nomenclatura	Secondo	Quarto Principale	54	Vedere figura terza, secondo profilo, segnato P	piade geometrico	Ad un quarto dell'altezza di		
Cortina	Nomenclatura	Terzo	Primo	118	Vedere figura Prima Olandese, segnato F4	Scala del Reno	lunghezza libera 360		
Cortina libera	Nomenclatura	Secondo	Quarto Principale	50	Vedere figura prima, segnato GH	piade geometrico	lunghezza 500		

Fig. 2 - Table containing information extracted from the textual analysis of the Corno Dogale dell'Architettura Militare by Pietro Sardi. (Source: Rechichi, 2024; Elaborated by Rechichi, P.)

Different authors propose geometric constructions based on methods of varying complexity and sophistication. In many cases, within the same treatise, multiple methods for tracing the defense are proposed, referencing the traditions of different authors or geographical areas.

Once all components of the fortification are ontologically defined and characterized by definitions of an informative, geometric, and dimensional nature, it is possible to proceed with the digital modeling of the fortifications.

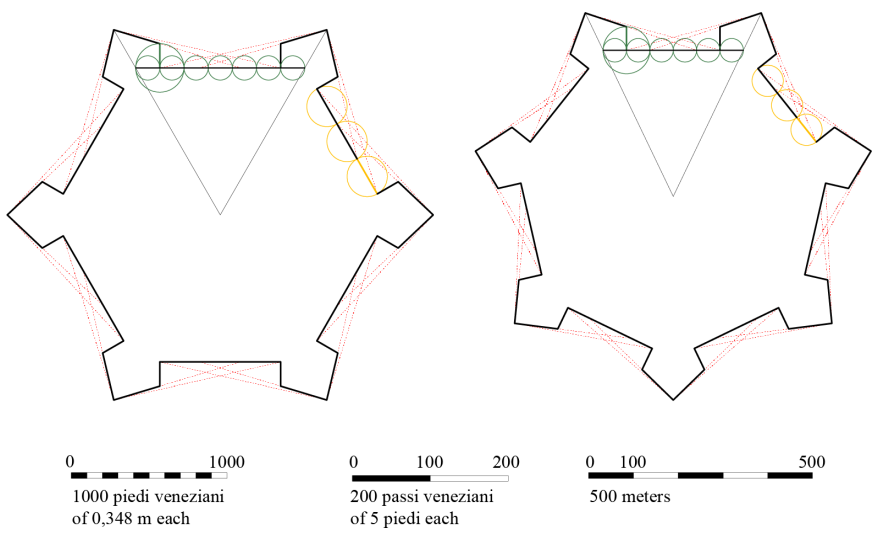


Fig. 3 - Geometric construction of the defended polygon from the treatises by Tensini (hexagon) and Lorini (heptagon). (Source: Rechichi, 2025; Elaborated by Rechichi, P.)

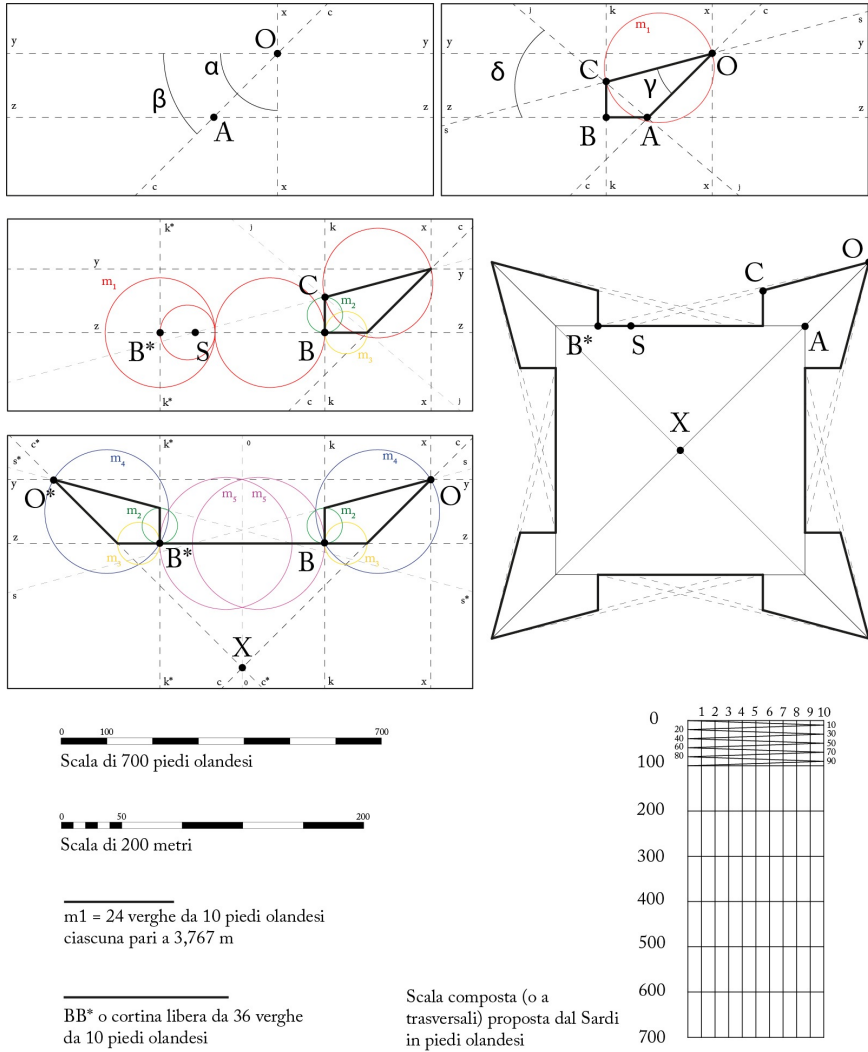


Fig. 4 - Geometric construction of the defensed polygon from the third book of the *Corno Dogale* by Pietro Sardi (Source: Rechichi, 2024; Elaborated by Rechichi, P.)

The analysis of geometric tracing methods for the defensed polygons allows for defining a script in Grasshopper that precisely replicates the tracing operations of the defense following the prescriptions of the treatises. The shape of the defensed polygon dynamically adapts to the variation in the number of sides of the reference regular polygon.

The knowledge framework developed through the analysis of the text and the critical redesign operations makes it possible to geometrically relate every part of the fortification to its original defended profile.

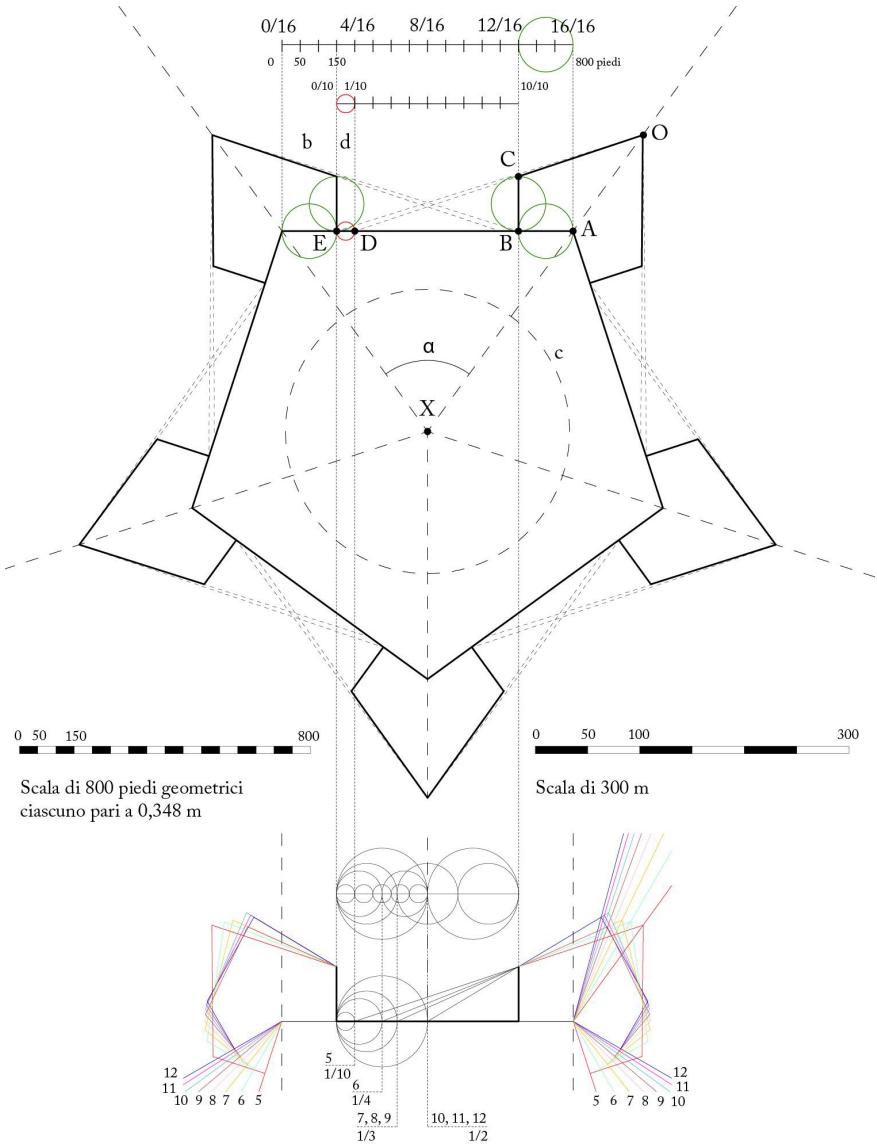


Fig. 5 - Geometric construction of the defended polygon from the second book of the *Corno Dogale* by Pietro Sardi (Source: Rechichi, 2024; Elaborated by Rechichi, P.)

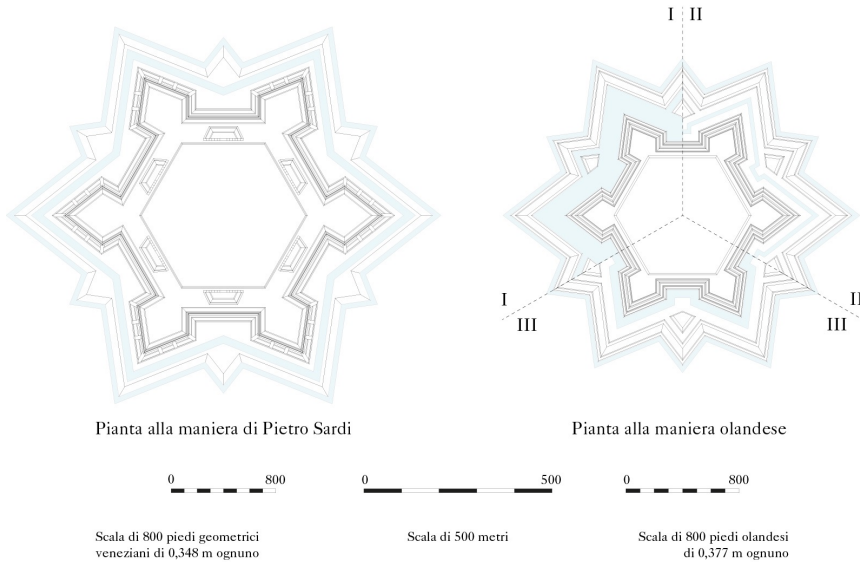


Fig. 6 - Critical redrawing of icnographic projections of fortifications from the second and third book of the *Corno Dogale* by Pietro Sardi. (Rechichi, 2024; Elaborated by Rechichi, P.)

The computational model of the fortification system is developed by replicating these relationships in VPL within Grasshopper (Fig. 9). Concerning the treatises of Sardi, Tensini and Lorini, the three-dimensional forms of the components of the fortification system are obtained from a composition system of surfaces, generated by two or more closed polyline curves.

These closed polyline curves are obtained as translations, offsets, and intersections starting from only the initial defended profile. With this technique, it is possible to create a script capable of containing all the geometric characterizations of the fortification system, which dynamically adapts to the variation of the parameter representing the number of sides of the regular polygon.

The result of this process consists of a system of closed three-dimensional forms, each corresponding to one of the entities of the fortification system.

It's relevant to note that for the *Trattato di Fortificatione* by Guarini, a different approach to parametric modelling was adopted compared to the general workflow outlined for the other treatises.

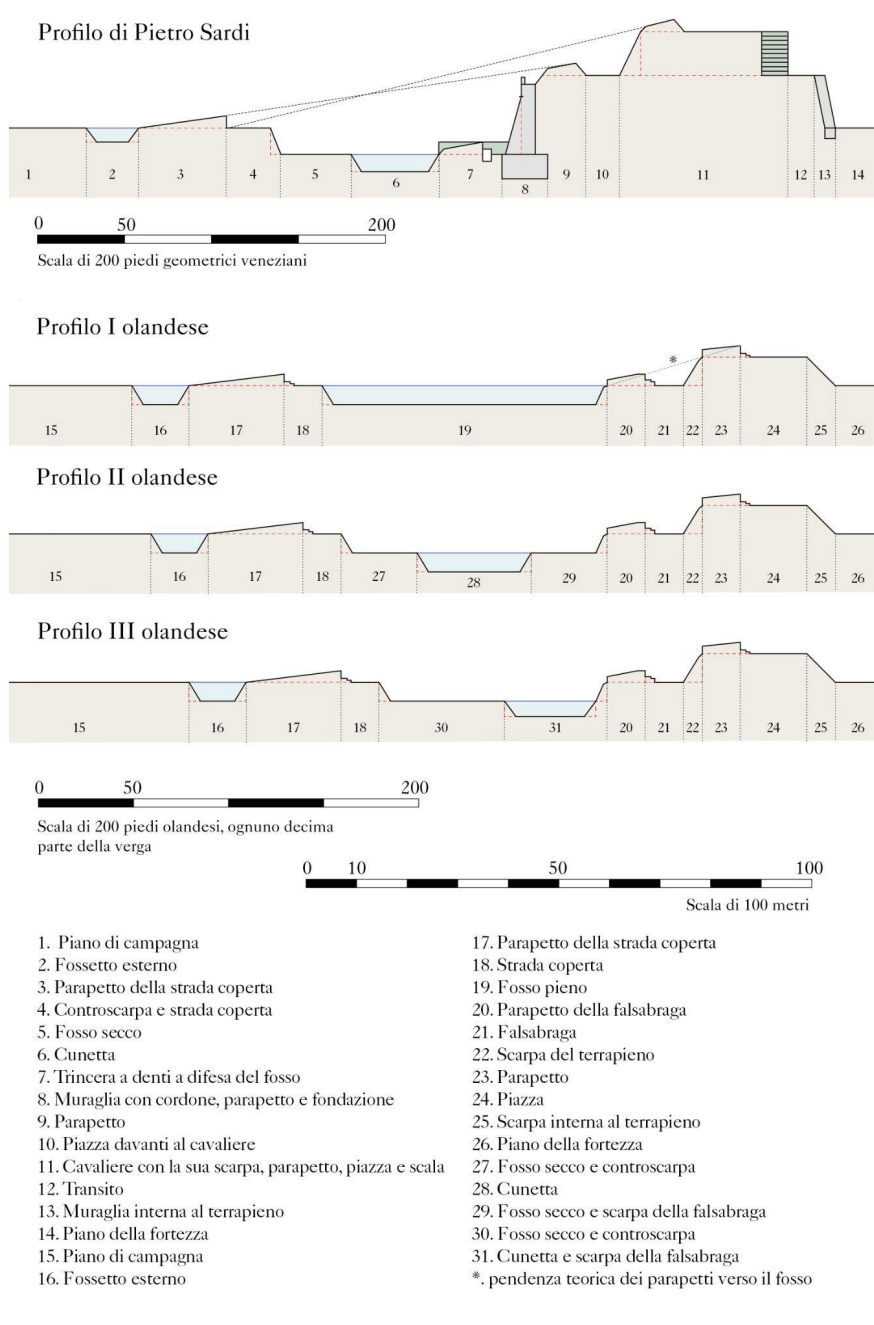


Fig. 7 - Critical redrawing of orthographic projections of fortifications from the second and third book of the Corno Dogale by Pietro Sardi. (Source: Rechichi, 2024; Elaborated by Rechichi, P.)

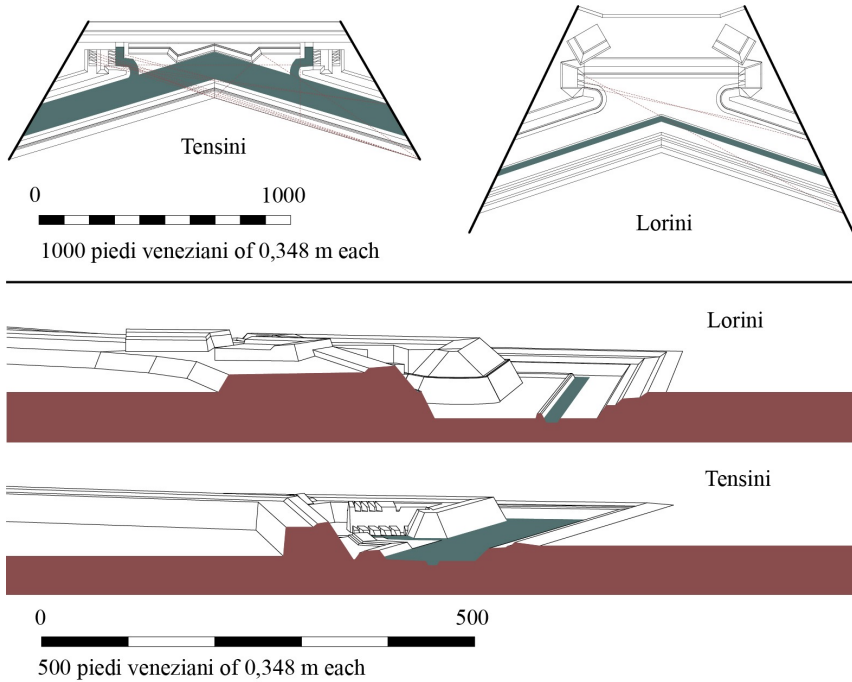
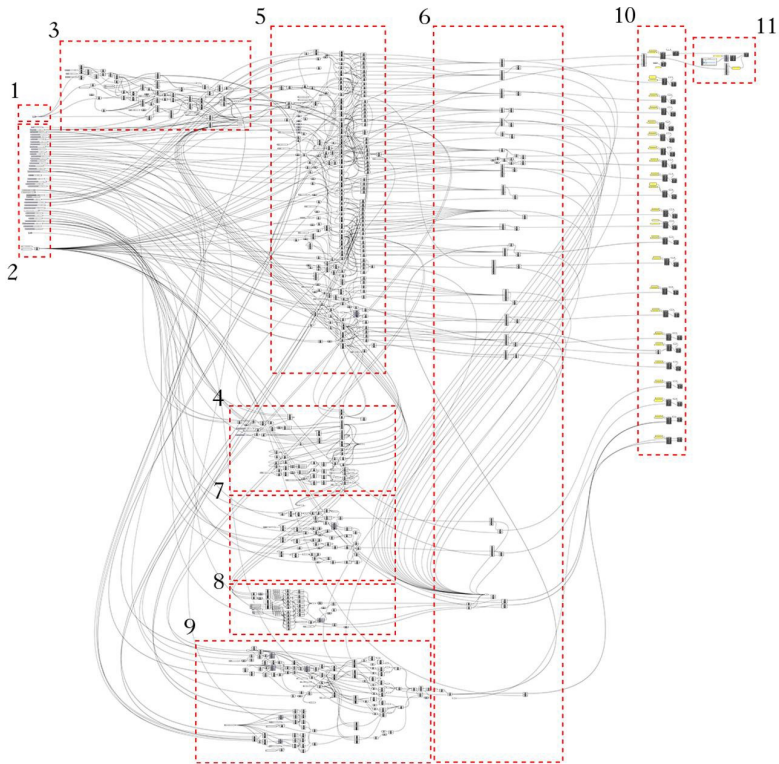


Fig. 8 - Critical redrawing of icnographic projections of fortificatory systems from the treatises by Bonaiuto Lorini and Francesco Tensini. Perspective sections of the same fortificatory systems (Source: Rechichi, 2025; Elaborated by Rechichi, P.)

The modelling of Guarini's treatise was undertaken focusing on the dimensional prescriptions provided in tabular form within the treatise (maximum, medium, and minimum).

The methodological approach for Guarini involved structuring an algorithm organized into thematic clusters, each dedicated to a specific element of the fortification. This cluster-based organization facilitated the separate management of different modelling phases, from initial delineations to the generation of second delineation elements (Fig. 10).

Dimensional control was implemented using multiline data panels, allowing for simultaneous modification of correlated parameters through a single selector, thus ensuring model coherence across dimensional variations. Geometry generation was achieved through monorail sweeps, with particular attention paid to defining section curves.



1. Numero di lati
2. Parametri dimensionali (espressi in piedi geometrici veneziani) e fattore di scala (modellato metrico)
3. Prima delineazione planimetrica
4. Tracciamento in pianta dei cavalieri
5. Polilinee spezzate chiuse per ogni variazione di pendenza
6. Modelli 3D delle singole entità della fortificazione (closed polysurfaces)
7. Corpo e parapetto dei cavalieri
8. Scale dei cavalieri
9. Denti della trincera a difesa del fosso
10. Importazione Rhino.Inside.Revit
11. Aggiunta di un parametro di informazione con il suo valore

*Fig. 9 - Grasshopper graph for the BIM integrated procedural model of the fortified system of the second book of the Corno Dogale by Pietro Sardi (Source: Rechichi, 2024; Elaborated by Rechichi, P.)*

These curves were constructed as closed polylines based on control points driven by parametric vectors, whose direction and magnitude varied according to selected dimensional values (maximum, medium, or minimum). A key challenge addressed was ensuring geometric continuity between adjacent sections to avoid both penetrations

and discontinuities, achieved through a control point concatenation system where adjacent section curves shared tangent points, ensuring a coherent transition of generated solids.

To digitally inform these entities, the Rhino.Inside.Revit plugin was utilized. This plugin allows for opening a script created in Grasshopper directly from the Revit software.

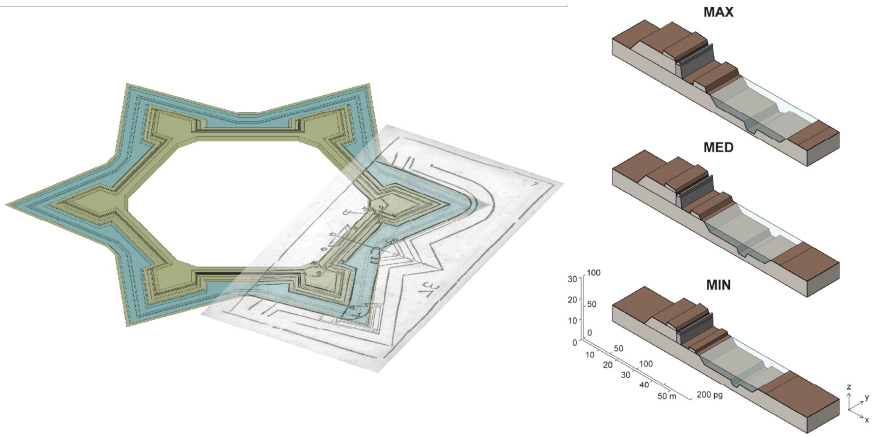


Fig. 10 - Parametric model of Guarini's fortress based on maximum dimensions (L). Axonometric section of Guarini's fortress; variation in size of the consistencies through maximum, average and minimum values taken from Guarini's tables (R). (Source Processing: E. Pupi, F. Natta)

Specific VPL components enable the visualization of the fortification system's geometries as direct shapes in Revit and manage the insertion of custom instance parameters, to which values can be associated that can dynamically vary with the number of sides of the regular polygon.

The model thus created can be used dynamically in Revit or saved as a standalone model constrained to the defined parameter values. Once saved, the standalone model can be freely modified and informed in Revit (Fig. 11).

To facilitate the visualization, querying, and management of fortified models directly within the browser – eliminating the need for dedicated software – this project prioritizes web-based access to BIM models through an intuitive, user-centered interface.

While current solutions allow both visualization and data interrogation online, challenges remain in optimizing performance and enhancing semantic structuring.

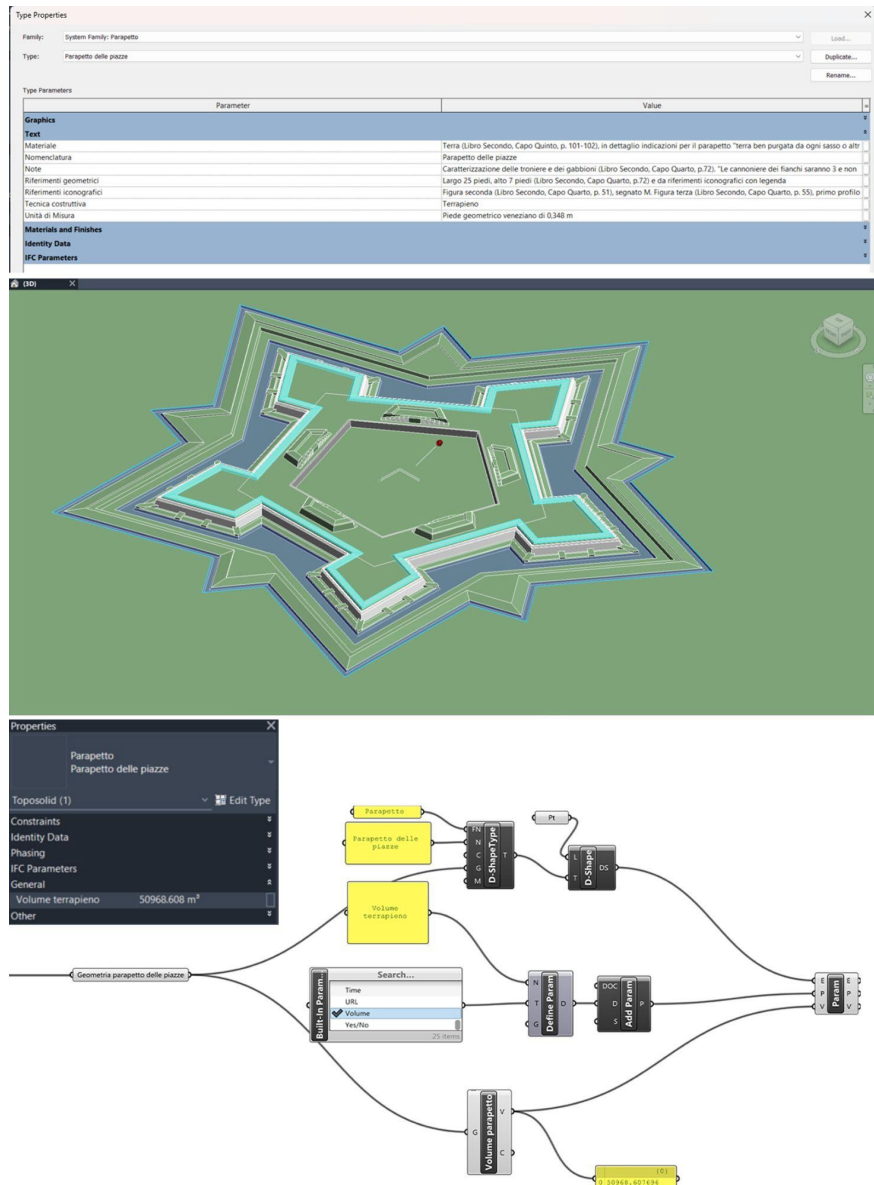


Fig. 11 - Revit visualization and information of the model of the fortified system of the second book of the Corno Dogale by Pietro Sardi (Rechichi, 2024; Elaborated by Rechichi, P)

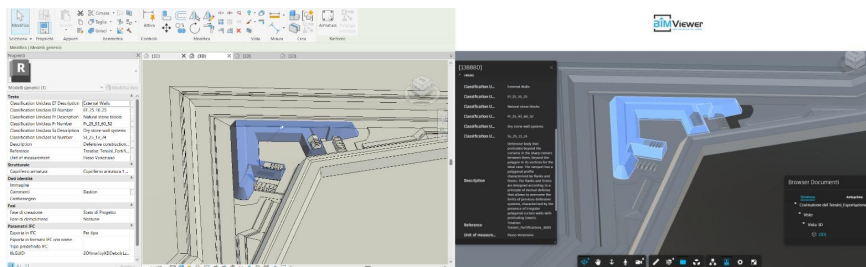


Fig. 12 - BIM Viewer online access to the digital model from the treatise by Tensini (Source: Rechichi, 2025; Elaborated by Miele, V.)

WebGL-powered platforms play a crucial role in this context, enabling browsers to render 3D content without external plugins. The most widely used tools include Autodesk Forge Viewer, Unity with WebGL, and CesiumJS. Ensuring model conversion to the IFC format preserves essential geometric and metadata attributes, making information accessible to non-expert users. Integrating these visualization tools into a streamlined web environment allows users to explore fortification elements with real-time access to metadata and descriptive content displayed in an adjacent panel. Following a comparative evaluation of available solutions, BIM Viewer was chosen for its flexibility in handling multiple file formats, eliminating the need for IFC conversion and enabling seamless sharing across different platforms (Fig. 12).

## Discussion and Conclusion

The creation of digital information models for the fortification systems proposed in the four described treatises has made it possible to verify the capacity of the proposed methodology to adapt or not to the variety of the landscape of early modern military architecture treatises.

The four models present different geometric and functional characterizations, which elevate or reduce the richness and complexity of the elements and their forms.

In this sense, Sardi's *Corno Dogale* is characterized by great descriptive clarity and great simplicity of the fortification model; the experiments conducted on Lorini and Tensini, instead, have demonstrated how increasing the number and geometric complexity of the parts of the fortification can make the development of a script capable of returning a dynamic digital model of the fortification system extremely demanding and complex.

Guarini's *Trattato di Fortificatione* offered a further modelling opportunity due to its tabular format specifying maximum, medium, and minimum dimensions for each element. This distinctive feature shaped a slightly different parametric approach.

Unlike models relying on lofted surfaces, Guarini's model uses sweeps to create forms based on these dimensional sets dynamically, showcasing design variations inherent in the text. This highlights the methodology's adaptability to different dimensional data types and geometric construction techniques, enabling the representation of both singular designs and ranges of design possibilities encoded in historical treatises.

This comparative analysis across the four treatises reveals a spectrum of modelling challenges and opportunities.

The varying levels of geometric and functional complexity encountered in these texts underscore the need for a flexible and adaptable methodological framework. Moving forward, this suggests the potential for tailoring modelling approaches to the specific characteristics of each treatise.

This could involve strategies like developing models with different levels of detail depending on the treatise's complexity, or combining dynamic parametric control for key elements with more traditional modeling for others.

This observation is particularly significant considering the creation of digital information models of treatises that are more focused on the constructive aspects of fortification, which will be the subject of future studies within the INFORTREAT project.

The four treatises studied, in fact, are characterized by a partial and non-exhaustive treatment of these subjects. Furthermore, in the selection of future case studies, the need to broaden the temporal span of publication of the treatises will be taken into account.

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