Cost-Effective and Constructible Gridshell Design Through the Fusion of Form-Finding and Evolutionary Algorithms

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Abstract. The construction industry, despite its considerable environmental footprint, represents a crucial segment of the global economy. Consequently, even minor advancements in this field can generate significant global benefits. Structural optimization emerges as a promising strategy for addressing these challenges, particularly in the design of gridshell structures. Gridshells, known for their ability to span large spaces without intermediary supports, offer remarkable structural efficiency. However, their construction complexity has limited their widespread use. Optimization techniques can address this by improving material efficiency and simplifying construction processes. This study introduces an optimization framework for free-form gridshell structures, combining evolutionary algorithms with the enhanced Multi-body Rope Approach (i-MRA) for form-finding. The primary objectives minimize structural components, reduce material consumption, decrease production waste, and ensure compliance with structural standards. This approach provides an innovative and efficient solution for the conceptual design of lightweight and structurally efficient gridshells.

Keywords: Structural optimization · Multi-objective optimization · Form finding · Lattice structure · Gridshell.

1 Introduction

The construction industry is a major contributor to global environmental challenges, exerting a substantial impact on the world ecological footprint. It accounts for approximately 30% of greenhouse gas (GHG) emissions [1], consumes half of the raw materials, and utilizes 40% of global energy resources [2]. Moreover, the sector is responsible for contaminating 40% of drinking water sources, generating half of all landfill waste, and causing 23% of air pollution. Economically, the construction industry is a cornerstone of the global economy, representing 10% of the global Gross Domestic Product (GDP) and valued at roughly 7.5 trillion USD. With projections indicating a doubling of worldwide building floor area by 2050 - driven by population growth and increasing purchasing power in developing and emerging economies - the industry environmental impact is poised to grow significantly [3]. Given these challenges and the economic importance of the sector, integrating structural optimization techniques into standard construction practices [4] could yield substantial benefits [5]. This approach has the potential to mitigate environmental impacts while enhancing economic efficiency, offering both immediate and long-term advantages for the industry and global sustainability efforts [3]. Structural optimization plays a crucial role in developing efficient and cost-effective designs across various engineering disciplines [6]. By applying advanced optimization and machine learning methods [7, 8], engineers can significantly enhance structural performance and durability [9–11] while simultaneously reducing material consumption and construction costs [12, 13]. This approach not only improves the economic viability of projects but also contributes to sustainability efforts by minimizing resource use [14].

Gridshell structures, renowned for their architectural versatility and aesthetic appeal, present a unique set of challenges and opportunities in structural optimization [15]. For these complex systems, the optimization process takes on heightened importance, as it must balance multiple competing factors, such as: aesthetic considerations, functional requirements, construction feasibility and resource efficiency. By leveraging advanced optimization techniques, engineers can design gridshell structures that not only push the boundaries of architectural expression but also exemplify principles of sustainable and efficient design.

This paper presents an integrated approach for optimizing gridshell structures, combining advanced form-finding techniques with meta-heuristic multi-objective optimization methods [16]. The primary aim is to generate a range of optimal solutions that guide designers during the critical early stages of project development. The methodology integrates the improved Multi-body Rope Approach (i-MRA) for gridshell form-finding [17, 18] with a multi-objective genetic algorithm [19]. The approach employs the i-MRA method to determine structurally efficient gridshell shapes based on specific loading conditions, while a genetic algorithm minimizes material use and construction complexity by reducing the variety of elements required on-site. A custom MATLAB code [20] performs the entire optimization process and analyses. To enhance accessibility for architects and structural designers, an i-MRA component has been developed for Grasshopper, a parametric design tool within Rhinoceros3D [21]. The study focuses on simplifying gridshell construction management by minimizing the variety of structural components and optimizing material use to reduce consumption and waste. Additionally, the Cutting Stock Problem (CSP) is incorporated to optimize cutting patterns, further decreasing material waste during component production [22]. These efforts collectively aim to streamline construction, reduce costs, and promote sustainable design practices in gridshell projects. To validate the proposed approach, two case studies are presented. The first involves a square gridshell subjected to self-weight and vertical nodal forces, with optimization results focusing on the number of different structural element types and total material used. The second case study addresses a more complex gridshell, comparing results under various loading conditions, examining element typologies, material usage, and waste reduction. These case studies demonstrate the versatility and effectiveness of the proposed optimization methodology across different gridshell configurations and loading scenarios.

2 Form finding with improved Multi-body Rope Approach

The Multibody Rope Approach (MRA) [23, 24] represents a state-of-the-art technique for designing gridshell structures, particularly those involving free-form base geometries and standardized construction elements. This innovative method simulates a dynamic model of falling bodies in space and time, leveraging D'Alembert's principle to iteratively compute the equilibrium configuration of each node in the structure. In the MRA framework, structural components are conceptualized as ropes connecting masses at the nodes [25]. These rope elements exert forces to maintain a predetermined distance between connected masses, ultimately generating a geometry that is both structurally efficient and composed of a minimal variety of element typologies. The resulting configuration resembles an inverted hanging net, yielding a form that is visually striking and structurally optimized [26]. A crucial aspect of creating a construction-efficient gridshell design is minimizing the variety of beam elements and standardizing their lengths. The key metric in this process is determining how many structural elements in the final configuration match a target length (l_{rope}). As the assembly complexity of a gridshell increases with the number of distinct structural elements required, classifying these elements by length into distinct groups becomes an integral part of the design process. This categorization not only simplifies construction but also ensures a more manageable and cost-effective build. By focusing on standardization and element categorization, the improved Multibody Rope Approach (i-MRA) method addresses both the structural and practical aspects of gridshell design. This approach not only optimizes the form for structural efficiency but also considers the realities of construction, making it a powerful tool for creating innovative, buildable gridshell structures. The i-MRA enhances the original MRA method by incorporating introducing two significant advancements. The first is the Multiple Orders MRA (MO-MRA) feature that groups structural elements of identical lengths together, substantially reducing the variety of structural components required for construction. This grouping strategy simplifies inventory management and assembly processes on-site. Complementing this, the Repulsive Nodes MRA (RN-MRA) incorporates a repulsive force field into the dynamic model. This innovative addition allows for subtle geometric adjustments that further minimize the number of distinct components needed [27], enhancing construction efficiency without compromising structural integrity [28–30]. Tailored specifically to optimize gridshell geometries, the i-MRA method prioritizes construction simplification alongside structural efficiency. Initially developed as a custom MATLAB [20] code, the method has been made more accessible through its conversion into a Grasshopper component [21], as illustrated in Figure 1. This integration ensures that architects and structural designers can easily incorporate the i-MRA method into their workflow, bridging the gap between advanced optimization techniques and practical design tools.

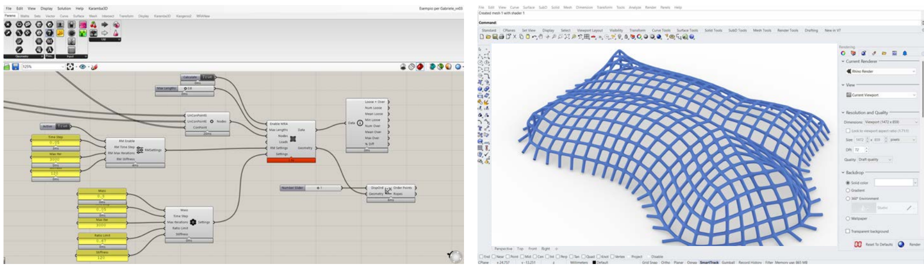


Fig. 1: i-MRA component developed for the Grasshopper3D environment.

3 The Optimization Methodology

This paper presents an innovative approach that integrates the improved Multi-body Rope Approach (i-MRA) with a multi-objective genetic algorithm (MOGA), utilizing MATLAB [20] "gamultiobj" function. The MOGA operates by generating a population of potential solutions and refining them through successive generations [31], providing a powerful tool for complex optimization problems. The optimization process focuses on two primary objectives: minimizing the number of distinct structural element types required for gridshell construction and reducing total material consumption, accounting for waste generated from cutting industrially produced 12-meter steel billets. This dual-objective approach ensures that the resulting gridshells are not only structurally optimized but also efficient in terms of material usage and construction management. Figure 2 illustrates the comprehensive optimization workflow. The process begins with a set of design variables that control the form-finding process. Starting from a base grid, the i-MRA method is applied to determine the optimal shape under specific loading conditions. Following the form-finding stage, the Cutting Stock Problem (CSP) [32, 33] is incorporated to calculate the number of 12-meter billets needed to produce all the structural elements. This crucial step addresses the industrial production process of steel beams, aiming to optimize cutting patterns and significantly reduce material waste during construction. By combining advanced form-finding techniques with multi-objective optimization and waste reduction strategies, this integrated approach offers a holistic solution to gridshell design. Following the optimization process, a finite element model (FEM) is developed based on the optimized structural shape. This model utilizes Circular Hollow Section (CHS) profiles made of S355 steel. To evaluate the structural performance, finite element analyses (FEA) are conducted using a custom MATLAB code [34]. These analyses compute critical structural parameters, including nodal displacements and stress distributions across individual beams. The primary purpose of

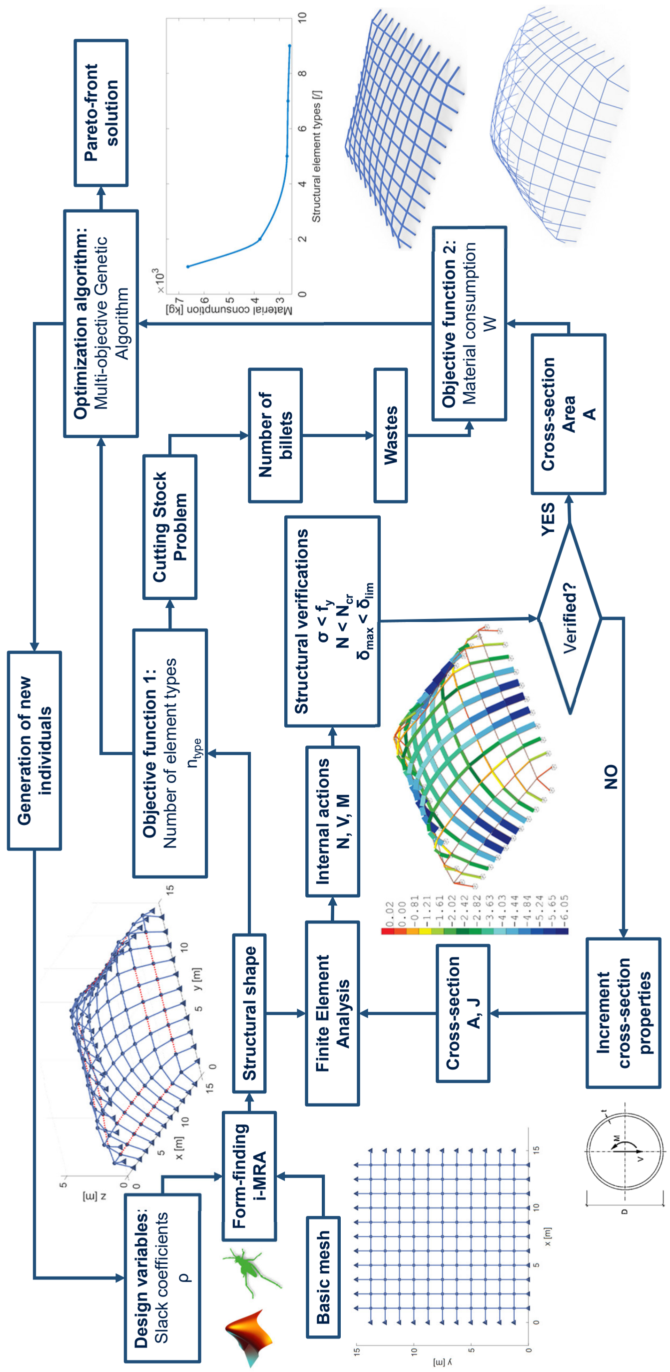


Fig. 2: Workflow of the optimization process.

these analyses is to assess whether the generated structure meets the necessary performance criteria. These criteria, as outlined in equation 1, encompass three key aspects: limit displacement, yield strength, and Eulerian buckling. The limit displacement criterion ensures that the structure deformation under load remains within acceptable bounds. The yield strength check verifies that the stresses in the structural elements do not exceed the material yielding stress. Finally, the Eulerian buckling assessment ensures that the slender elements of the gridshell are not at risk of buckling under compressive loads [35].

$$\delta_{max} < \delta_{lim} \qquad |\sigma_{max}| < f_y \qquad N_{max}^{Compression} < N_{Buckling} \tag{1}$$

In this context, δ_{max} represents the maximum nodal displacement experienced by the structure, while δ_{lim} is the allowed limit, set at $\frac{1}{250}$ of the structure largest dimension, assuming no external constraints are applied. The term $|\sigma_{max}|$ refers to the absolute maximum stress within the structure, which must remain below f_y, the yield stress of the S355 steel used. Finally, $N_{max}^{Compression}$ denotes the maximum compressive axial force in the structure, and $N_{Buckling}$ corresponds to the critical axial force for Eulerian buckling. The optimization process incorporates an iterative approach to determine the optimal cross-section dimensions of the CHS profiles. These dimensions are incremented systematically until a configuration is achieved that satisfies all verification criteria. This process ensures the structural integrity of the gridshell while also determining the minimum required cross-sectional area of the structural elements. To calculate the total material necessary for constructing the structure, the optimized cross-sectional area is multiplied by the number of billets derived from the Cutting Stock Problem (CSP) solution.

The fitness function evaluated by the multi-objective genetic algorithm (MOGA) is designed to consider two critical aspects of the design: the number of different structural element types (n_type) of varying lengths, and the total amount of material used (W). By considering both these factors, the optimization process aims to minimize construction complexity while also reducing material consumption. This iterative optimization continues until a diverse set of solutions is generated, forming a Pareto front. Each point on this front represents an optimal structural geometry, offering a balance between the number of element types and material usage. The Pareto front serves as a decision-making tool for designers, allowing them to select the most appropriate structural configuration based on a comprehensive set of considerations including architectural vision, structural performance, economic constraints, and project management factors. By providing this range of optimized solutions, the method empowers designers to make informed decisions that best align with the specific requirements and constraints of each project.

4 Results and discussion

This section presents two exemplary cases that illustrate the application of the proposed optimization process to gridshell form-finding. The first case study fo-

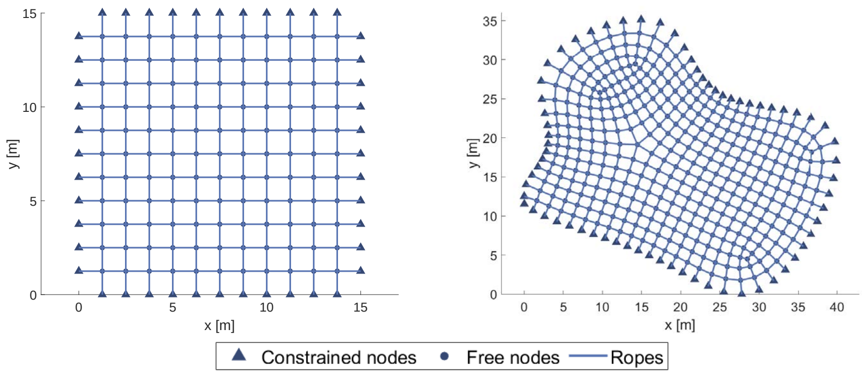
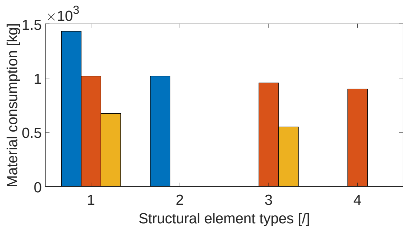


Fig. 3: Base meshes for the two case studies.

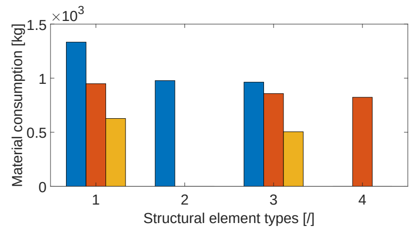
cuses on a gridshell structure with a square plan, featuring a side length of 15 meters. The initial configuration of this structure is depicted in Figure 3(a). The optimization outcomes for this square gridshell are comprehensively displayed in Figure 4. These results showcase the structural responses under various loading conditions, specifically examining the effects of self-weight combined with different magnitudes of nodal overloads. The analysis considers three distinct nodal overloads equal to 1kN, 5kN, and 10kN.

The study compares outcomes with and without the consideration of material waste. Figures 4(a) and (c) illustrate the results where material waste was accounted for in the optimization process. In contrast, Figures 4(b) and (d) display the outcomes when waste was not factored into the calculations. This comparison allows for a clear assessment of how waste consideration affects the overall design and material efficiency of the gridshell structure. Then, the optimizations were performed using either one or two design variables. Figures 4(a) and (b) show the results obtained using a single design variable, while Figures 4(c) and (d) depict the outcomes when two design variables were employed. These design variables correspond to the slack coefficients assigned to the ropes during the form-finding process utilizing the improved Multibody Rope Approach (i-MRA) method.

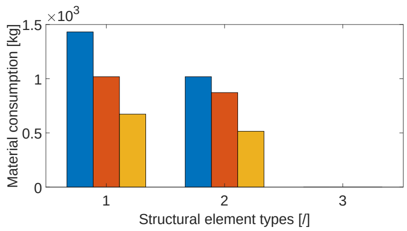
The analysis reveals a significant inverse relationship between the number of distinct structural element types and the total material required for construction. This trade-off is a crucial insight for gridshell design optimization. As the variety of structural elements decreases, typically leading to simpler construction and assembly processes, the overall material consumption tends to increase. Conversely, designs that minimize material usage often require a greater diversity of structural element types, potentially complicating the construction process. This relationship is visually represented in the Pareto front illustrated in Figure 5.



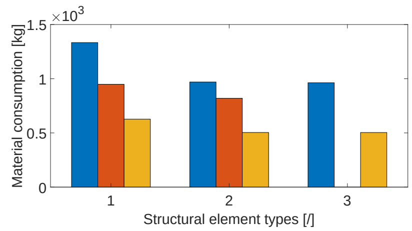
(a) 1 Design variable with CSP



(b) 1 Design variable without CSP

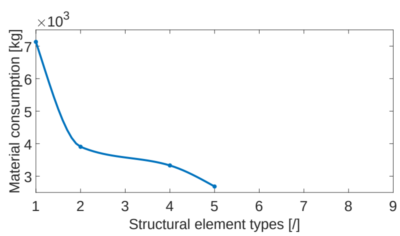


(c) 2 Design variables with CSP

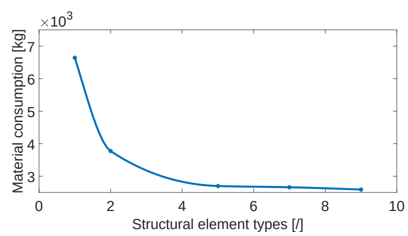


(d) 2 Design variables without CSP

Fig. 4: Results of the optimization of the square gridshell case study.

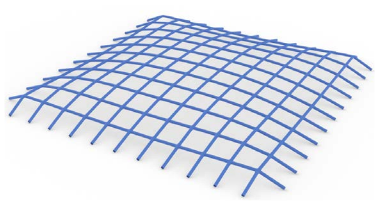


(a) CSP

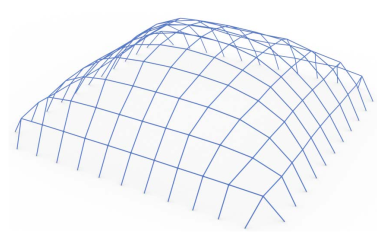


(b) No CSP

Fig. 5: Results of the optimization reported as Pareto front solutions related to the square gridshell case study with a nodal overload equal to 50kN.



(a) $n_{type} = 1$ $m = 7.13ton$

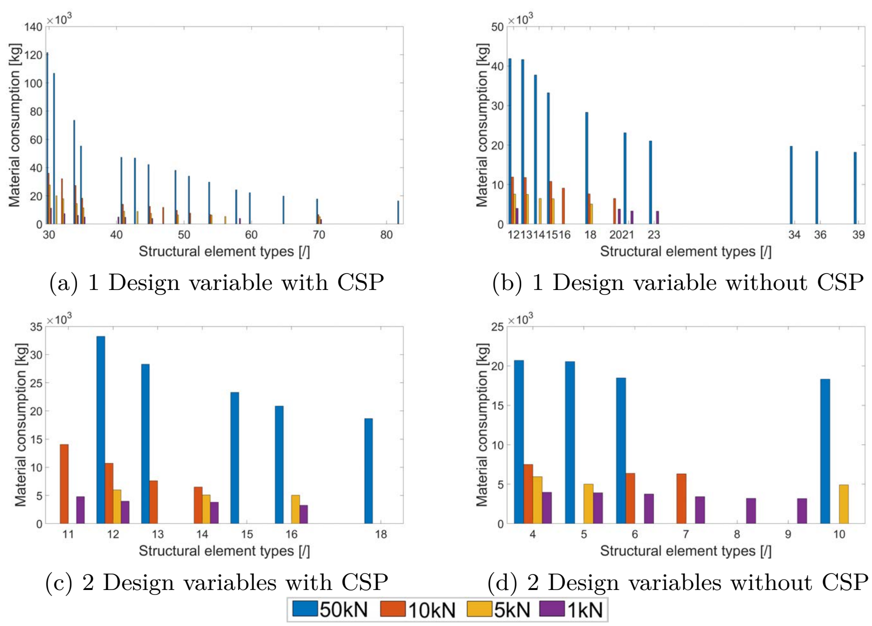


(b) $n_{type} = 2$ $m = 2.55ton$

Fig. 6: Optimized structural designs for the square gridshell case study, considering the effects of self-weight along with a 50kN nodal overload.

Furthermore, the analysis reveals a key trade-off in gridshell design between structural profile and element characteristics. Lower-profile structures require thicker elements to withstand higher compression forces but use fewer element types, simplifying construction. Conversely, taller configurations allow for slimmer, more lightweight elements, optimizing material efficiency at the cost of increased element variety. Figure 6 illustrates two optimal configurations, each representing a different balance between material efficiency and construction simplicity.

The study further demonstrates the versatility of the proposed optimization process through a second case study. This example applies the methodology to a more complex gridshell structure, using the initial layout depicted in Figure 3(b) as its starting point. The optimization outcomes for this case study are illustrated in Figure 7 as bar graphs. These results reflect various nodal overload conditions and account for production deviations computed using the Cutting Stock Problem (CSP).

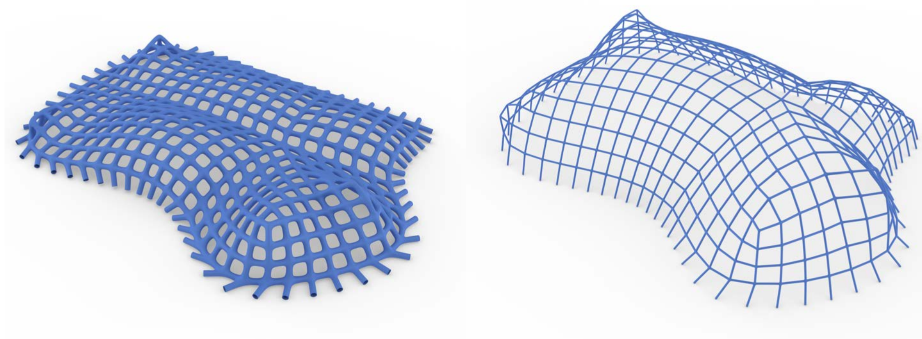


(a) 1 Design variable with CSP (b) 1 Design variable without CSP

(c) 2 Design variables with CSP (d) 2 Design variables without CSP

Fig. 7: Results of the optimization of the free-form base gridshell case study.

Two examples of optimal configurations for the second case study are presented in Figure 8. As with the previous case, the lower configuration features thicker structural elements and higher material usage, while the elevated configuration is characterized by slimmer structural elements and reduced material consumption.



(a) $n_{type} = 30$ $m = 121.5ton$ (b) $n_{type} = 60$ $m = 22.2ton$

Fig. 8: Optimized structural shapes for the free-form base gridshell considering self-weight and a nodal overload of 50kN.

5 Conclusions

This study presents an innovative optimization procedure for gridshell structures that integrates advanced form-finding techniques with metaheuristic optimization techniques. At the core of this approach is the improved Multibody Rope Approach (i-MRA), which serves as the primary form-finding method. The optimization process focuses on two key objectives: minimizing the variety of structural element types and reducing overall material usage. A significant feature of this methodology is the incorporation of the Cutting Stock Problem (CSP) to determine optimal cutting patterns for structural components. This addition addresses a critical aspect of the fabrication process, substantially reducing material waste and improving overall resource efficiency. The multi-objective genetic algorithm (MOGA) employed in this study generates a Pareto front, offering a visual representation of the trade-offs between various design criteria. By providing a spectrum of optimal structural configurations, the approach empowers designers to make informed decisions based on project-specific requirements and constraints. The practical application of this optimization process is demonstrated through case studies on gridshell structures.

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