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# Introducing Combined Effects of Filtering and ASE Noise in Optical Links Supposing Different Equalization Algorithms

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**Abstract**—This paper presents a comprehensive analytical framework for evaluating filtering penalties in ASE-noise-limited coherent optical links. The model accounts for the cumulative effects of cascaded optical filters, amplifier-induced ASE noise, and transceiver noise, alongside digital equalization at the receiver. By developing a generalized channel representation, we derive closed-form expressions for signal-to-noise ratio degradation under various equalization strategies, including Zero-Forcing Equalizer, Minimum Mean Square Error Equalizer, and Fractionally Spaced Equalizer. These models capture the impact of colored noise resulting from linear filtering and provide both time- and frequency-domain insights. The proposed framework is validated through experimental comparisons using accurately modeled optical filters, demonstrating close agreement between theory and practice and offering a robust foundation for system-level performance evaluation in metro-access networks.

**Index Terms**—filtering penalty, equalization, MMSE, mathematical modeling

## I. INTRODUCTION

As the demand for high-capacity, low-latency communication continues to rise, metro and access optical networks have become a cornerstone of modern infrastructure. In these systems, signal quality is predominantly influenced by linear impairments rather than nonlinear effects, owing to the relatively short transmission distances and lower optical power levels typically employed [1]. Among these linear impairments, filtering penalties—introduced by cascaded optical filters throughout the transmission chain—play a critical role in shaping overall system performance [2].

Moreover, many steps have been made in view of an optical network digital twin (DT) development, thanks to accurate fiber and amplifiers modeling, through transparent lightpath approximation based on Gaussian Model [3]. What is still missing is how to take care for filtering effects in such framework, pursuing disaggregation and accurate mathematical modeling, which is the main target of this work.

This paper presents an analytical framework for the derivation and evaluation of filtering penalties in coherent optical

systems. The analysis begins with a detailed channel model that incorporates cascaded filter effects, additive noise contributions from optical amplifiers and transceivers, and the impact of receiver-side digital signal processing (DSP). In doing so, we assume the absence of nonlinear interference [1], thereby isolating the impact of linear filtering on system performance. To encompass filtering penalty and transceiver impairment we propose a transceiver model based on [4], [5], following the approach successfully adopted in [6]. Several equalization strategies are examined in depth, including the Matched Filter Bound (MFB), Zero-Forcing Equalizer (ZFE), Minimum Mean Square Error Equalizer (MMSE), and Fractionally Spaced Equalizer (FSE). For each case, closed-form expressions are derived to quantify the resulting signal-to-noise ratio (SNR) and associated filtering penalties. These derivations account for the colored nature of noise post-filtering, and they make use of both frequency and time-domain representations to accurately capture signal distortion and noise enhancement effects.

Crucially, the analytical results are corroborated by experiments, where the theoretical predictions are validated through practical measurements using accurately modeled optical filters, as in [6]. This experimental validation demonstrates a close agreement between theory and practice, confirming the robustness and applicability of the proposed framework in real-world metro-access network scenarios. The experiments not only substantiate the model’s accuracy but also highlight its utility in guiding the design and optimization of equalization algorithms and system components to mitigate filtering penalties effectively.

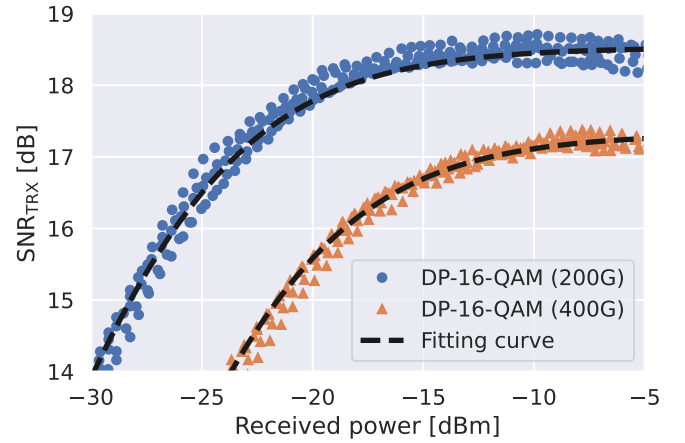
The proposed models offer a rigorous yet tractable means of assessing receiver performance in the presence of filtering impairments, and serve as a foundation for the quality of transmission evaluation considering filtering effect, transceiver impairment and equalization algorithms, which can be included in optical network DT transmission model.

## II. RELATED LITERATURE

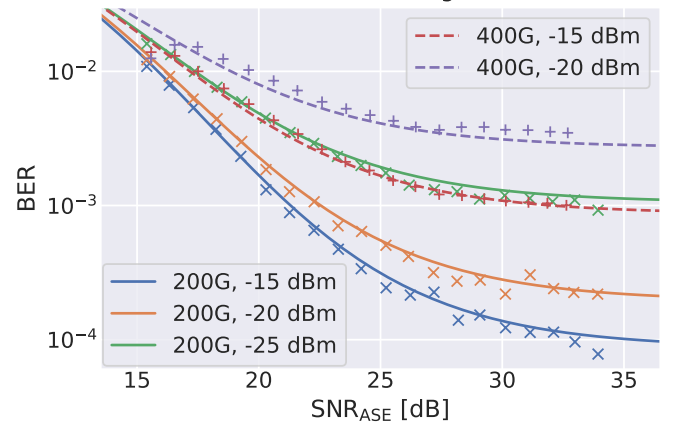
In Digital Communications applied to wireless technology the bandwidth-limited channels and equalization theory is widely known and extensively developed in many fundamental reference books [7], [8], [9], which are the basis of our proposed models. There is the need to apply such framework to optical communications, even if fundamental differences can be found compared to wireless. First, in optical networks it is possible to characterize the channel (here intended as the physical medium, not to be confused with WDM channels defined as the data modulated over a single WDM wavelength) transfer function and to model it with high accuracy, as it will be discussed in Section IV-A, which is a clear advantage compared to wireless. Second, in optical networks the alternation of amplification and filtering effect in optical devices such as ROADMs causes the injection of AWGN noise in distributed manner along a lightpath. Therefore, each noise contribution is affected by a different amount of filtering penalty, which is also different from the one affecting the transmitted signal. As a result, the overall noise affecting the link is colored, which has to be taken into account in the model derivation. Experimental characterization of filtering penalty in meshed optical networks with a large number of ROADMs and filtering penalty mitigation has been proposed in [10], by using optical wave shapers. Experiments has been performed considering metro-access network scenario [11] with a single ROADM and single noise source. The effect of optical filters frequency shift and bandwidth variation on Q-factor has been investigated through experiments and simulations in [12], underlining the importance of account for asymmetric filtering on the overall filtering effect.

From modeling point of view, many approaches have been proposed. In [13] it is shown that system performance degradation due to in-line optical filtering by wavelength routing nodes in WDM networks can be correctly modeled using digital communications theory of band-limited channels with linear equalization. Experimental validation using a commercial transponder and fiber nonlinear transmission is presented. The equalizer operates in continuous time and is modeled through Fourier transform analysis. Also in [14] the proposed equalizer operates in continuous time and it is validated experimentally on a setup composed by a single ROADM and a single ASE noise source, including optical filter bandwidth variation and central frequency shift in the analysis. Finally, [15] proposes MMSE equalization based on DTFT, underlining the importance of relative placement of ASE and filters and on the implementation-dependent capabilities of the DSP receiver. Also in this last case bandwidth variation is considered; validation has been performed through numerical time-domain simulations.

The models proposed in this work are a step forward compared to the previously listed ones. First, equalizers (ZFE, MMSE, FSE) are assumed to operate after sampling, so in discrete domain, as it happens in real commercial transceivers. The reference optical system is general, admitting any kind of



(a) Transceiver SNR evolution with the received power  
Back-to-back configuration



(b) Validation of the TRX model in presence of ASE noise

Fig. 1: Complete transceiver characterization

optical filter shape, and colored noise. Strong focus is on the interplay between filtering experienced on optical link side and transceiver impairment, whose model is plugged in the main derivation. All the key assumption related to the particular use case of optical amplified links, and the full derivation, are provided. Some highly technical passages are omitted, but referenced during the derivation.

## III. MODELS DERIVATION

### A. Transceiver model and SNR definition

This section introduces a measurement-based methodology for transceiver (TRX) performance modeling, aimed at taking into account the transceiver impairment when evaluating filtering penalty. The focus is on accurately modeling the signal-to-noise ratio (SNR) and its impact on bit error rate (BER), particularly in scenarios dominated by linear impairments, such as metro networks.

The bit error rate is expressed as:

$$\text{BER} = k_1 \cdot \text{erfc} \left( \sqrt{k_2 \cdot \text{SNR}} \right) \quad (1)$$

where  $k_1$  and  $k_2$  depend on the modulation format. The total system SNR is modeled as the inverse sum of individual contributions:

$$\text{SNR}^{-1} = \text{SNR}_{\text{ASE}}^{-1} + \text{SNR}_{\text{TRX}}^{-1} \quad (2)$$

Where we consider:

$$\text{SNR}_{\text{ASE}} = \frac{P_{\text{RX}}}{P_{\text{ASE}}} \Big|_{Bw=R_s} \quad (3)$$

The only impairment in optical domain (other than filtering, which will be taken into account later on) is ASE noise, because we neglect the fiber nonlinearity. Moreover, we take the symbol rate as a reference bandwidth to compute the noise power. The transceiver's contribution is further decomposed into transmitter and receiver components:

$$\text{SNR}_{\text{TRX}}^{-1} = \text{SNR}_{\text{TX}}^{-1} + \text{SNR}_{\text{RX}}^{-1} \quad (4)$$

The transmitter impact is considered constant due to fixed output power (typically 0 dBm), and it is assumed to have a negligible impact with respect to receiver's. The receiver contribution varies with input power  $P_{\text{RX}}$ , primarily due to photodetector noise sources including thermal, shot, and dark current noise.

To model transceiver performance, the effective transceiver  $\text{SNR}_{\text{TRX}}$  is extracted via curve fitting, based on the equation proposed in [4].

$$\text{SNR}_{\text{TRX}} = N \cdot \frac{P_{\text{RX}}}{P_{\text{RX}} + D} \quad (5)$$

Where  $N$  and  $D$  are two fitting parameters to obtain the dashed line in Figure 1a.

A sensitivity setup can be exploited by excluding external noise sources in a back-to-back transceiver connection whose signal can be attenuated using a Variable Optical Attenuator (VOA). By changing the attenuation it is possible to test different  $P_{\text{RX}}$  conditions, which enables the generation of  $\text{SNR}(P_{\text{RX}})$  curves by mapping BER measurements to theoretical SNR values by inverting Equation 1 with the proper modulation format coefficients  $k_{1,2}$ . The resulting curves demonstrate monotonic SNR growth towards saturation at high  $P_{\text{RX}}$ .

Analysis across C-band frequencies shows minimal impact on  $\text{SNR}_{\text{TRX}}$ , validating a frequency-agnostic modeling approach. Consistently, in Figure 1a the measurements are taken at different central frequencies in the C-band. On the contrary, modulation formats affect BER significantly, but the derived SNR vs.  $P_{\text{RX}}$  curves remain consistent by selecting format-specific  $k_1$  and  $k_2$ .

Finally, the described transceiver characterization has been validated in presence of ASE noise: considering the additive contributions, the BER can be computed using Equation 1 in which the SNR is obtained as in Equation 2, and by testing different  $\text{SNR}_{\text{ASE}}$  values it is possible to obtain the curves in Figure 1b.

## B. D-transform and power spectra evaluation for wide-sense stationary processes

Since this work deals with equalizers operating in discrete domain, it is necessary to introduce an effective tool which allows to operate with discrete sequences and discrete filters, avoiding the complexity that a time-domain representation. Such tool is the D-transform [9], which is defined for sequences  $x_k$  defined  $\forall k \in (-\infty, \infty)$  with  $k \in \mathbb{Z}$  as ( $x_k$  can be complex):

$$X(D) = \sum_{k=-\infty}^{\infty} x_k \cdot D^k \quad \forall D \in \mathcal{D}_x, \quad D \in \mathbb{C} \quad (6)$$

Where  $\mathcal{D}_x$  is the region of convergence of complex D values, for which the sum  $X(D)$  converges, and  $X(D)$  is analytic in  $\mathcal{D}_x$ . The inverse transform is a clockwise line integral around any closed circle in the region of convergence:

$$x_k = \frac{1}{2\pi j} \oint_{D \in \mathcal{D}_x} X(D) \cdot D^{k-1} dD \quad (7)$$

For applications requiring Z-transform, it is sufficient to impose  $Z = D^{-1}$ . The sequence  $x_{-k}^*$  has D-transform  $X^*(D^{-*}) = \sum_{k=-\infty}^{\infty} x_k^* D^{-k}$ . An important property of D-transform is that, when the region of convergence includes the unit circle (which is true for all the sequences considered in this work), and the discrete sequence is obtained by sampling a continuous time signal at the symbol rate  $T$ , the discrete-time sequence's Fourier transform exists as:

$$X(e^{-j2\pi fT}) = X(D)|_{D=e^{-j2\pi fT}} \quad (8)$$

The previous property is important when dealing with sequences representing wide-sense stationary processes, for which the evaluation of autocorrelation and power spectrum is often needed. Following [9], we recall that if  $x_k$  is any stationary complex sequence, its autocorrelation function is  $r_{xx,j} = \mathbb{E}[x_k x_{k-j}^*]$  with D-Transform:

$$R_{xx}(D) = \mathbb{E}[X(D) \cdot X^*(D^{-*})] \quad (9)$$

The power spectrum of a stationary sequence is the Fourier transform of its autocorrelation function:

$$R_{xx}(e^{j2\pi fT}) = R_{xx}(D)|_{D=e^{-j2\pi fT}} \quad -\frac{1}{2T} < f < \frac{1}{2T} \quad (10)$$

By stationarity  $r_{xx,j} = r_{xx,-j}^*$  and  $R_{xx}(D) = R_{xx}^*(D^{-*})$ , so the power spectrum is real and nonnegative  $\forall f$ . The random process energy can be computed as:

$$\begin{aligned} \mathcal{E}_x &= \mathbb{E}[|x_k|^2] = T \cdot r_{xx,0} \\ &= T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} R_{xx}(e^{j2\pi fT}) df \end{aligned} \quad (11)$$

If the sequence is deterministic then the power spectrum is the squared magnitude of its Fourier transform.

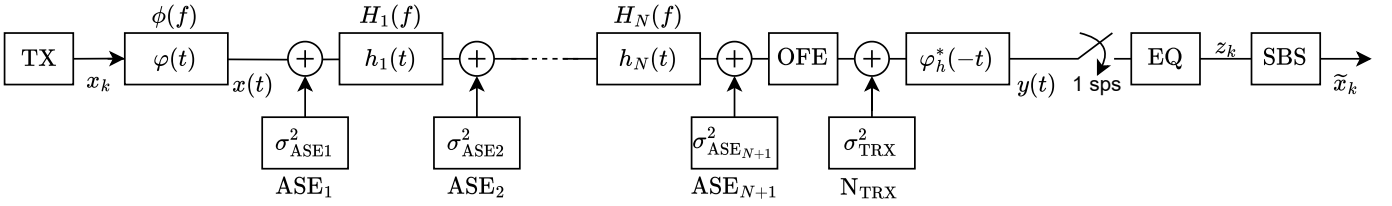


Fig. 2: Reference optical link abstraction for model derivation.

### C. Channel model

The starting point for the analytical determination of the filtering penalty in optical systems is the block diagram shown in Figure 2. A general optical link in case of metro network scenario has been theoretically replicated. We consider dual-polarization coherent transmission, and we focus on evaluating the SNR for the symbol sequence transmitted through one of the two polarizations. This approach is justified, since filtering effect acts in the same way on both the polarizations. Therefore, when we compute the total SNR as the average for the SNR on the two polarizations, we obtain the same result. In addition, we assume the modulation adopts In-phase and Quadrature components, as is done in modern optical transceivers.

The transmitted symbols are  $\{x_k\}_{k=0}^{K-1}$ , for  $K$  successive transmissions, for each polarization and they are generated at symbol rate equal to  $R_s$  (from now on, we will refer also to the symbol period as:  $T = \frac{1}{R_s}$ ), therefore we can write the sequence energy as:

$$\mathcal{E}_x = \mathbb{E} [|x_k|^2] \quad (12)$$

Consequently, the total transmitted signal power, counting for both the polarizations is:

$$P_{\text{TX}} = 2 \cdot \frac{\mathcal{E}_x}{T} = 2 \cdot \mathcal{E}_x \cdot R_s \quad (13)$$

For derivation purpose it is useful to write also the per-dimension energy, namely the energy of the In-phase or Quadrature component of one of the two polarizations:

$$\bar{\mathcal{E}}_x = \frac{1}{2} \mathcal{E}_x = \frac{P_{\text{TX}}}{4 \cdot R_s} \quad (14)$$

The  $x_k$  are modulated by the shaping filter  $\varphi(t)$  (assumed to satisfy the Nyquist criterion for inter symbol interference (ISI) free channels and symbol-by-symbol (SBS) detection, and with Fourier transform  $\Phi(f) = \mathcal{F}[\varphi(t)]$ ), so we can write the modulator output as:

$$x(t) = \sum_{k=0}^{K-1} x_k \cdot \varphi(t - kT) \quad (15)$$

Then the modulated symbols pass through a cascade of  $N$  optical filters described by their transfer functions  $H_i(f)$  ( $i = 1, \dots, N$ ) experiencing the injection of noise in a distributed manner along the line, before the matched filter  $\varphi_h^*(-t)$ . The signal is then sampled, equalized and SBS detection is performed on the  $z_k$  ( $k = 0, \dots, K - 1$ ) outputs

of the equalizer, providing the final estimates  $\hat{x}_k$  of the input symbols.

This model incorporates some simplifying assumptions. As anticipated in the Introduction, we assume the absence of Non Linear Interference (NLI) because metro and access networks are characterized by short links and low optical power [1]. Attenuation has been taken into account, but it could be neglected, since it operates equally on signal and noise, without affecting the SNR or the filtering penalty.

In addition, it is assumed that transmitter impairments are generally much smaller than receiver impairments, according to [6]. Therefore, all the filtering penalty affecting the transceiver SNR is considered on the receiver side, and is taken into account through the  $\text{SNR}_{\text{TRX}}$  value, provided by the transceiver characterization. As in [6], the optical front end (OFE), converts the optical signal to electrical signal without additional filtering impairments. This implies that both the ASE introduced at the receiver side (source number  $N + 1$  in Figure 2, where  $N$  is the number of filters) and the noise introduced by the transceiver are not filtered, so the equalizer applies the same effect on each of them.

All the noise sources are assumed to be characterized by a flat power spectral density (PSD) with a value of  $\sigma_{\text{ASE}_i}^2$  (where  $i = 1, \dots, N + 1$ ) for each ASE source and  $\sigma_{\text{TRX}}^2$  for the transceiver, at least within the signal bandwidth. Therefore, each noise contribution is an additive white Gaussian noise when injected into the link. Nevertheless, the filtering effect makes the noise to be colored, after crossing one or more filtering devices. The PSD in case of ASE noise due to optical amplification can be computed through the well-known formula [16] in Equation 16:

$$\sigma_{\text{ASE}_i}^2 = \frac{1}{4} \sum_{j=1}^{N_i} h f_0 (G_{j,i} - 1) N F_{j,i} \quad (16)$$

Where  $N_i$  is the number of amplified span located in-between each optical filters pair,  $h$  is the Planck constant,  $f_0$  is the central frequency of the transmitted signal,  $G_{j,i}$  and  $N F_{j,i}$  are the amplifier gain and noise figure of each amplifier. The factor  $\frac{1}{4}$  is needed because we consider the noise PSD for each modulation component (In-phase and Quadrature) and for each polarization.

The final assumption concerns the ISI elimination strategy at receiver side: after matched filtering, the DSP is assumed to perform equalization operating at a sampling rate equal to 1 Sample per Symbol (SpS). The matched filtering assumption

is not realistic in optical systems. In general, the presence of a matched filter before sampling is not necessary, since modern commercial transceiver implement both matched filtering and equalization in discrete domain. We will start our discussion with this assumption to derive models for Zero Forcing Equalizer (ZFE) and Minimum Mean Square Error Equalizer (MMSE) and finally we will get rid of matched filtering when considering Fractionally Spaced Equalizer (FSE). In this way, FSE model derivation will follow straightforward after ZFE and MMSE, justifying our approach. All the equalizer models assume an infinite number of taps.

The initial step of the analysis involves evaluating the noise at receiver side. As anticipated, the overall effect of filtering is that the data signal becomes distorted, and the noise is no longer white. Specifically, at the receiver side, the perceived noise is the sum of multiple ‘‘colored’’ components due to the additive nature of noise. Each of these components is colored based on the number and shape of filters it has passed through. Using the well-known formula for computing the output PSD  $S_{out}(f)$  of a random process crossing a filter with transfer function  $U(f)$ , having an input PSD  $S_{in}(f)$ :

$$S_{out}(f) = |U(f)|^2 \cdot S_{in}(f) \quad (17)$$

It is possible to derive the receiver-side equivalent noise normalized PSD before sampling:

$$S_{ASE+TRX}(f) = \frac{\sum_{i=1}^{N+1} \sigma_i^2 \cdot \prod_{n=i}^N |H_n(f)|^2}{\sigma_{ASE+TRX}^2} \quad (18)$$

where:

$$\begin{cases} \sigma_i^2 = \sigma_{ASEi}^2 & i = 1, \dots, N \\ \sigma_{N+1}^2 = \sigma_{ASE_{N+1}}^2 + \sigma_{TRX}^2 \end{cases} \quad (19)$$

The total noise PSD in Equation 18 has been normalized by the factor in Equation 20:

$$\sigma_{ASE+TRX}^2 = \sum_{i=1}^{N+1} \sigma_i^2 \quad (20)$$

This operation is performed in order to derive the optical link white noise equivalent channel model, whose block diagram is provided in Figure 3a. Since the penalty on the link is due to linear filtering, the colored PSD expressed by Equation 18 has an invertible square root, therefore it is possible to whiten the noise without loss of information [7], as depicted in Figure 3b. Thanks to the normalization factor, the equivalent white noise source at receiver side PSD is  $\sigma_{ASE+TRX}^2$ .

We introduce some terms that will be useful in the discussion, following the notation of [8], in time domain. We denote the white noise equivalent pulse response of the optical link as the inverse Fourier transform of the white-noise equivalent channel frequency response:

$$h(t) = \mathcal{F}^{-1}[H(f)] \quad (21)$$

$$H(f) = \frac{\Phi(f) \cdot \prod_{i=1}^N |H_i(f)|}{\sqrt{S_{ASE+TRX}(f)}} \quad (22)$$

Where the numerator in Equation 22 has been obtained from the cascade of shaping filter and optical filters, while the denominator is due to the whitening filter effect according to Figure 3b.

Since the pulse energy is not necessary normalized to 1, it is useful to introduce the normalized pulse response as:

$$\varphi_h(t) \triangleq \frac{h(t)}{\|h\|} \quad (23)$$

Where

$$\|h\| = \sqrt{\int_{-\infty}^{\infty} h(t) \cdot h^*(t) dt} = \sqrt{\langle h(t), h(t) \rangle} \quad (24)$$

So that the band limited channel output before noise injection is:

$$x_h(t) = \sum_{k=0}^{K-1} x_k \cdot \|h\| \cdot \varphi_h(t - kT) \quad (25)$$

Finally, we define the deterministic autocorrelation function  $q(t)$  as:

$$q(t) \triangleq \varphi_h(t) * \varphi_h^*(-t) = \frac{h(t) * h^*(-t)}{\|h\|^2} \quad (26)$$

It is useful to note that  $q(t)$  is Hermitian ( $q(t) = q^*(-t)$ ) and that  $q(0) = 1$ .

Since in the considered optical system we assume to know the expression of each filter and each noise contribution, it is possible to rewrite  $Q(f) = \mathcal{F}[q(t)]$  in explicit manner.

$$Q(f) = \frac{|H(f)|^2}{\|h\|^2} = \frac{|\Phi(f) \cdot \prod_{i=1}^N H_i(f)|^2}{\|h\|^2 \cdot S_{ASE+TRX}(f)} \quad (27)$$

In view of the following derivations, it is useful to compute the discrete output after sampling  $y(kT)$ , which is fed into the equalizer. Following the steps in [8] we find that, after the sampler (assuming  $y_k = y(kT)$ ,  $q_k = q(kT)$ ,  $n_k = n(kT)$ , with  $n(t) = n_h(t) * \varphi_h^*(-t)$ ):

$$y_k = \|h\| \cdot x_k + n_k + \|h\| \sum_{m \neq k} x_m \cdot q_{k-m} \quad (28)$$

Which is indeed composed by the scaled input symbol, noise filtered by  $\varphi_h^*(-t)$  and sampled and ISI. Applying the D-transform to Equation 28 we finally obtain:

$$Y(D) = X(D) \cdot \|h\| \cdot Q(D) + N(D) \quad (29)$$

Equation 29 is the starting point of all the following equalizer models development, and it fully characterizes the considered optical link.

Once the channel model has been established, it is possible to derive the filtering penalty model according to the chosen equalization strategy as in [8].

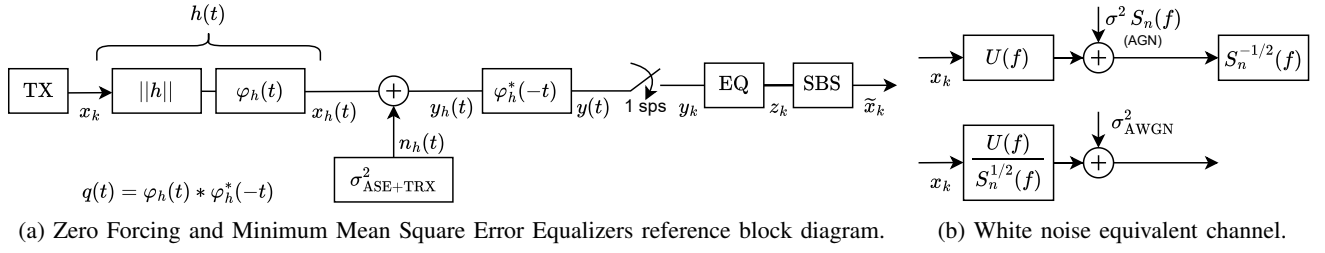


Fig. 3

#### D. The unfiltered SNR bound

In the optical system under study optical filters band limit the channel, causing ISI which affects the performance in terms of SNR degradation. The maximum SNR is obtained when filtering effect is negligible, therefore when signal is not distorted. Such situation can verify when filters' band is large compared to the signal bandwidth. When this happens, the SNR computation is straightforward, because ASE noise is flat in the signal bandwidth. We conclude that in this case the SNR computed taking into account ASE noise and transceiver noise, neglecting NLI, must be equal to:

$$\text{SNR}_{max} = (\text{SNR}_{ASE}^{-1} + \text{SNR}_{TRX}^{-1})^{-1} = \text{SNR} \quad (30)$$

Where  $\text{SNR}_{TRX}$  is given in Section III-A. Since the signal and noise power can be computed, we can explicitly express the SNR bound according to the notation developed up to now, which will be useful when deriving equalizers models and comparing their performance:

$$\text{SNR} = \frac{P_{RX}}{P_{ASE} + P_{TRX}} = \frac{4 \cdot \bar{\mathcal{E}}_x \cdot R_s \cdot ||h||^2}{4 \cdot \sigma_{ASE+TRX}^2 \cdot R_s} = \frac{\bar{\mathcal{E}}_x \cdot ||h||^2}{\sigma_{ASE+TRX}^2} \quad (31)$$

Where we point out that bandwidth terms cancels out since both signal and noise power are computed with respect to the same band equal to the symbol rate.

#### E. Zero Forcing Equalizer

The reference of the current derivation is Figure 3a. ZFE is the simplest and most intuitive equalizer among the ones considered in this work. Nevertheless, it is also the worst in terms of performance, due to its noise enhancement effect. Nevertheless, its model allows the expression of filtering penalty in disaggregated manner, as it will be clear in the following and as it has been shown in [6]. Such feature is pivotal when including the filtering penalty in DT transmission model, therefore we included ZFE in the discussion.

ZFE acts as a linear time invariant filter, with discrete pulse response  $w_k$ , which eliminates the ISI component in Equation 28, ignoring the noise term:

$$y_k = ||h|| \cdot x_k * q_k \quad (32)$$

Then, if we apply the D-transform:

$$Y(D) = ||h|| \cdot X(D) \cdot Q(D) \quad (33)$$

The equalizer output is then:

$$Z(D) = W(D) \cdot Y(D) = W(D) \cdot ||h|| \cdot X(D) \cdot Q(D) \quad (34)$$

Which is ISI-free if  $X(D) = Z(D)$ , so:

$$W(D) = \frac{1}{Q(D) \cdot ||h||} \quad (35)$$

In the derivation of noise autocorrelation and noise variance after the equalizer is carried on. The final value of noise variance is:

$$\sigma_{ZFE}^2 = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{\sigma_{ASE+TRX}^2}{||h||^2 \cdot Q\left(e^{-j\frac{2\pi f}{R_s}}\right)} df \quad (36)$$

Where, compared to [8], we substituted the symbol period with the symbol rate  $T = \frac{1}{R_s}$ , we used the frequency instead of angular velocity  $w = 2\pi f$  and we substituted the noise PSD with the one previously computed in Equation 20, namely:  $\frac{N_0}{2} = \sigma_{ASE+TRX}^2$ . The term  $Q\left(e^{-j\frac{2\pi f}{R_s}}\right)$  is the folded spectrum of  $Q(f)$  expressed in Equation 27, it is periodic with period  $R_s$  and it can be expressed as:

$$Q\left(e^{-j\frac{2\pi f}{R_s}}\right) = R_s \cdot \sum_{n=-\infty}^{\infty} Q(f + nR_s) \quad (37)$$

The ZFE output SNR is then:

$$\text{SNR}_{ZFE} = \frac{\bar{\mathcal{E}}_x}{\sigma_{ZFE}^2} = \frac{\text{SNR}}{k_{ZFE}} \quad (38)$$

Where  $k_{ZFE}$  is the penalty due to filtering effect, since SNR is experienced in absence of filters as pointed out in Section III-D.

$$k_{ZFE} = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{1}{Q\left(e^{-j\frac{2\pi f}{R_s}}\right)} df \quad (39)$$

The disaggregation of ZFE can be performed by considering each noise source characterized by  $\sigma_i^2$  ( $i = 1, 2, 3, 4$ ) computed in Equation 19 individually, and computing the corresponding constant  $k_i$  through Equation 39, with the other sources injecting no noise. It is important to note that, in this case, the disaggregated colored PSD to be considered in the channel block is solely the contribution corresponding to  $\sigma_i^2$  in the

general PSD equation 18. The final formula for computing  $k_i$  is then:

$$k_i = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{|\Phi(e^{j\frac{2\pi f}{R_s}})|^2 \prod_{n=i}^N |H_n(e^{j\frac{2\pi f}{R_s}})|^2}{\prod_{m=1}^N |H_m(e^{j\frac{2\pi f}{R_s}})|^2} df \quad (40)$$

#### F. Minimum Mean Square Error Equalizer

The MMSE equalizer balances the ISI reduction and noise enhancement. In this case the reference system is the same as ZFE, reported in Figure 3a.

The derivation of filtering penalty for optical system in this case is based on the computation of the mean square error between the transmitted sequence and the equalizer output, for linear equalization:

$$e_k = x_k - w_k * y_k = x_k - z_k \quad (41)$$

Consequently, the minimum mean square error is:

$$\sigma_{\text{MMSE}}^2 = \min_{w_k} \mathbb{E}[|x_k - z_k|^2] \quad (42)$$

The error can be expressed in D-transform domain as:

$$E(D) = X(D) - W(D) \cdot Y(D) \quad (43)$$

We proceed as in [8] by exploiting the orthogonality principle, since at each discrete time sample  $k$ , the error sample  $e_k$  must be uncorrelated with any equalizer input signal  $y_m$ , therefore:

$$\mathbb{E}[E(D) \cdot Y^*(D^{-*})] = 0 \quad (44)$$

Following the same steps of [8], starting from Equation 43 and 44 the equalizer transfer function in D-domain is:

$$W(D) = \frac{1}{\|h\| \left( Q(D) + \frac{1}{\text{SNR}} \right)} \quad (45)$$

As widely known in digital communications community, MMSE equalizer's difference compared to ZFE is only due to the positive constant  $\frac{1}{\text{SNR}}$  at the denominator of Equation 45.

The second step after obtaining the transfer function in D-domain is the MMSE evaluation, through the error autocorrelation time-0 coefficient computation, which leads to [8]:

$$\sigma_{\text{MMSE}}^2 = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{\sigma_{\text{ASE+TRX}}^2}{\|h\|^2 \left( Q(e^{-j\frac{2\pi f}{R_s}}) + \frac{1}{\text{SNR}} \right)} df \quad (46)$$

We again substituted  $\omega = 2\pi f$  and  $T = \frac{1}{R_s}$  compared to [8].

As did in Section III-E it is possible to define a penalty due to filtering  $k_{\text{MMSE}}$  with respect to the SNR at unfiltered bound:

$$\text{SNR}_{\text{MMSE}} = \frac{\bar{\mathcal{E}}_x}{\sigma_{\text{MMSE}}^2} - 1 = \frac{\text{SNR}}{k_{\text{MMSE}}} - 1 \quad (47)$$

Where the term  $-1$  is needed to obtain the unbiased SNR, since MMSE equalizer is by definition biased [8]. The explicit expression of  $k_{\text{MMSE}}$  is:

$$k_{\text{MMSE}} = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{1}{\left( Q(e^{-j\frac{2\pi f}{R_s}}) + \frac{1}{\text{SNR}} \right)} df \quad (48)$$

#### G. Fractionally Spaced Equalizer

In this final derivation the reference channel model block scheme is 4a. The FSE exploits the MMSE equalization strategy, with some additional features that allow increasing the performance by a slight increment in terms of complexity.

Finally, we get rid of the matched filter before sampling. The DSP performs both equalization and matched filtering in discrete domain, according to what happens in commercial transceivers deployed in optical systems. In addition, the FSE increases the sampling rate by some rational number  $\ell$ , ( $\ell > 1$ ), which we assume to be an integer. Such sampling rate is sufficient to fulfill the Shannon-Nyquist sampling theorem, so all the operations after sampling are performed at a rate equal to  $\ell \cdot R_s$ . In this configuration, the noise samples variance after sampling will be  $\ell \cdot \sigma_{\text{ASE+TRX}}^2$ . Also in this final case, we follow the notation of [8].

Since  $\ell$  is an integer number, before DSP stage the sampler output decomposes into  $\ell$  interleaved sequences sampled at rate  $R_s = \frac{1}{T}$  with D-transforms  $Y_0(D), \dots, Y_i(D), \dots, Y_{\ell-1}(D)$ , where  $Y_i(D)$  corresponds to the sample sequence  $y(kT - i\frac{T}{\ell})$  and can be expressed as:

$$Y_i(D) = H_i(D) \cdot X(D) + N_i(D) \quad (49)$$

Where  $H_i(D)$  is the D-transform of the symbol-rate-spaced  $i^{\text{th}}$  phase of  $h(t)$  sequence  $h(kT - (i-1)\frac{T}{\ell})$ :

$$H_i(D) = \sum_{k=-\infty}^{\infty} h\left(kT - (i-1)\frac{T}{\ell}\right) \cdot D^k \quad (50)$$

Similarly,  $N_i(D)$  is the D-transform of a symbol-rate-sampled white noise sequence with autocorrelation function  $R_{nn}(D) = \ell \cdot \sigma_{\text{ASE+TRX}}^2$ . Each noise sequence is independent of the other. Following the notation of [8] we define the transform column vector as:

$$\mathbf{Y}(D) = \begin{bmatrix} Y_0(D) \\ \vdots \\ Y_{\ell-1}(D) \end{bmatrix} = \begin{bmatrix} H_0(D) \\ \vdots \\ H_{\ell-1}(D) \end{bmatrix} \cdot X(D) + \begin{bmatrix} N_0(D) \\ \vdots \\ N_{\ell-1}(D) \end{bmatrix} \\ = \mathbf{H}(D) \cdot X(D) + \mathbf{N}(D) \quad (51)$$

Since the FSE output is at sampling rate  $R_s$ , its interleaved coefficients rewrite in a row vector  $\mathbf{W}(D) = [W_0, \dots, W_{\ell-1}]$  so that the FSE output is:

$$Z(D) = \mathbf{W}(D) \cdot \mathbf{Y}(D) \quad (52)$$

Applying the orthogonality principle, after some basic linear algebra computations [8] the MMSE in case of FSE is:

$$\sigma_{\text{MMSE}, \text{FSE}}^2 = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{\ell \cdot \sigma_{\text{ASE+TRX}}^2}{\left\| \mathbf{H}\left(e^{j\frac{2\pi f}{R_s}}\right) \right\|^2 + \frac{\ell}{\text{SNR}}} df \quad (53)$$

Where:

$$\left\| \mathbf{H}\left(e^{j\frac{2\pi f}{R_s}}\right) \right\|^2 = \sum_{i=1}^{\ell} \left| H_i\left(e^{j\frac{2\pi f}{R_s}}\right) \right|^2 \quad (54)$$

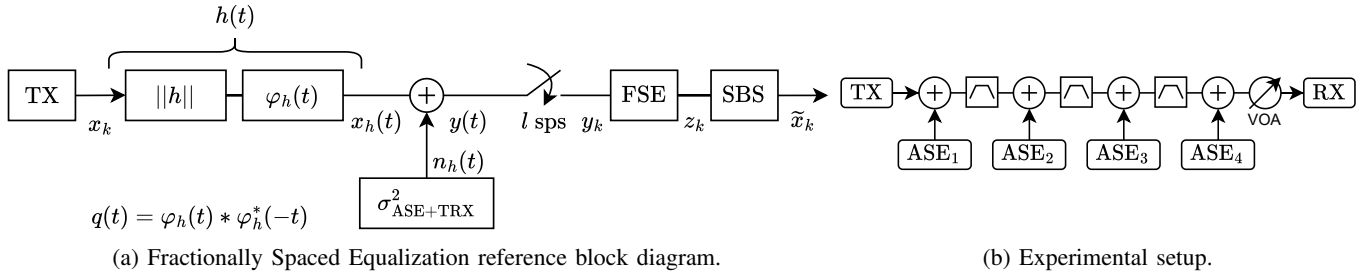


Fig. 4

Where  $H_i \left( e^{-j \frac{2\pi f}{R_s}} \right)$  are determined by evaluating  $H_i(D)$  expressed in Equation 50 in the unit circle. In practical applications, these functions can be obtained by computing the Discrete Fourier Transform (DFT) of the symbol-rate-spaced phases  $h(kT - (i-1)\frac{T}{\ell})$  of  $h(t)$ .

Then the unbiased SNR follows as in Section III-F:

$$\text{SNR}_{\text{MMSE,FSE}} = \frac{\bar{\mathcal{E}}_x}{\sigma_{\text{MMSE,FSE}}^2} - 1 = \frac{\text{SNR}}{k_{\text{MMSE,FSE}}} - 1 \quad (55)$$

$$k_{\text{MMSE,FSE}} = \frac{1}{R_s} \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} \frac{\ell}{\left\| \mathbf{H} \left( e^{-j \frac{2\pi f}{R_s}} \right) \right\|^2 + \frac{\ell}{\text{SNR}}} df \quad (56)$$

#### IV. EXPERIMENTAL VALIDATION

##### A. Optical filters model

The proposed models are general and can embed any optical filter shape. For experimental validation purpose, the spectral response of the optical filters was modeled following the approach in [17], using the same parameters of [6]. Such optical filter model is well-suited to represent accurately the shape of filtering effect when commercial ROADMs are displaced in the optical link, and in general is preferable compared to supergaussian filter model. The filter transfer function amplitude is given by:

$$S(f) = \frac{1}{2} \sigma \sqrt{2\pi} \left[ \text{erf} \left( \frac{\frac{B_{\text{ch}}}{2} - f}{\sigma \sqrt{2}} \right) - \text{erf} \left( \frac{-\frac{B_{\text{ch}}}{2} - f}{\sigma \sqrt{2}} \right) \right] \quad (57)$$

Where  $B_{\text{ch}}$  is the channel bandwidth and  $\sigma = \frac{BW_{\text{OTF}}}{2\sqrt{2\ln 2}}$ . The OSA-measured traces of the filter spectra were fitted by adjusting the  $BW_{\text{OTF}}$  parameter. A conservative fitting approach was adopted, resulting in modeled spectra that are slightly narrower than the experimental data near the edges.

##### B. Comparison between models and experiment

The experimental validation of the models developed in Section III is provided in Figure 5. The measurements are the same used in [6] to validate the ZFE model. In this work we perform the validation of MMSE and FSE, since the ZFE model of Section III-E is equivalent to the one of [6].

The experimental setup used in [6] to investigate filtering effects is recalled in Figure 4b and it was composed by a cascade of three ROADM filters aligned to the channel

under test. Filter bandwidths were reconfigurable from 37.5 to 75 GHz in 12.5 GHz steps, creating a symmetric dual-sided filtering condition. A booster amplifier compensated for insertion losses, maintaining a constant launch power of 0 dBm. Experiments were conducted using DP-16QAM at 200 and 400 Gbps with 31.6 and 63.1 Gbaud signals. ASE noise was independently controlled and calibrated across four sources to ensure  $\text{SNR}_{\text{ASE}}$  is equal between the four noise sources when measured at the receiver. BER was measured at different received optical power (ROP) levels and  $\text{SNR}_{\text{ASE}}$  values, while an optical spectrum analyzer (OSA) was used to capture the signal spectrum and measure OSNR.

Figure 5 is organized as follows: from left to right the optical filters bandwidth changes and also the symbol rate of the transmitted signal. From top to bottom the received optical power changes from -15 dBm to -25 dBm. The plots are presented in terms of the well-known Q factor [18], which is derived from the measured BER values using the following equations:

$$Q = \sqrt{2} \cdot \text{erfcinv}(2 \cdot \text{BER}) \quad (58)$$

$$Q_{\text{dB}}^2 = 10 \cdot \log_{10}(Q^2) \quad (59)$$

For the analytical model, the electrical SNR is calculated for MMSE and FSE equalizers using respectively Equations 47 and 55. The bit error rate (BER) is then determined using the standard theoretical expression [19]:

$$\text{BER} = \frac{3}{8} \cdot \text{erfc} \left( \sqrt{\frac{1}{10} \cdot \text{SNR}} \right) \quad (60)$$

This analysis considers DP-16QAM modulation, where SNR is evaluated for the various filter configurations under investigation. The resulting BER values are then converted to  $Q_{\text{dB}}^2$  using Equations 58 and 59.

Additionally, theoretical reference curves (shown as dashed lines in Figure 5 represent the ideal case without any filter-induced penalty. These curves are derived exploiting the transceiver characterization, as done in [6] and using Equation 60 to evaluate the corresponding BER.

Overall, the analytical models align well with the experimental data, maintaining a slightly conservative margin, particularly under stronger filtering conditions. The models' validity is first confirmed by the overlap between the analytical curves and experimental points when filter bandwidths are

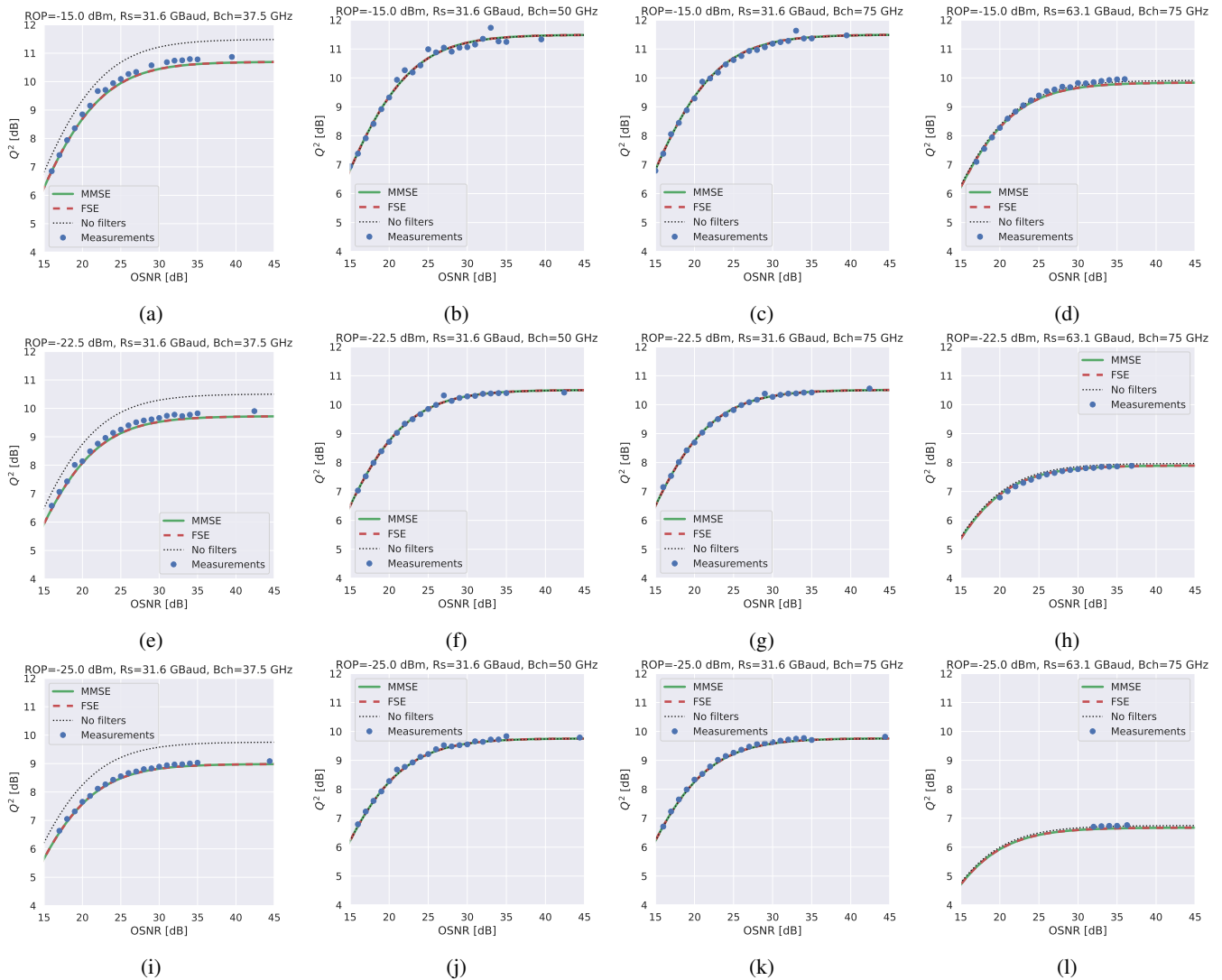


Fig. 5: Experimental validation of MMSE and FSE equalizer models.

significantly wider than the symbol rate: consistent with the theoretical no-filtering reference. The conservative margin visible in Figures 5a, 5e and 5i arises from the conservative filter modeling approach described in Section IV-A. Additionally, discrepancies between the experimental results and the model can be attributed to the use of ideal, infinite-length equalizers in the model, which differ from the practical implementation in the actual transceiver. Furthermore, Figures 5d, 5h and 5l display a lower Q factor, which is attributed to the higher bitrate configuration. This setup introduces a more significant transceiver penalty, as already stated in [6].

Another interesting consideration comes out by comparing the MMSE and FSE equalizers performance, which are expressed by identical superposed curves in Figure 5. As stated in the derivation in Section III, FSE is the more realistic version of MMSE equalizer, since it does not rely on matched filtering in continuous domain. Since also FSE is based on mean square error as optimization metric, it is

expected to have similar performance. The advantage of FSE in commercial transceivers is that it allows an improvement in sensitivity to sampling-phase errors, which is important to limit noise enhancement that would occur if using MMSE and ZFE equalizers. Since in the derivation we did not consider sampling-phase deviation, the overlapping of MMSE and FSE performance curves is correct and expected.

## V. CONCLUSIONS

This work presents a comprehensive analytical framework for modeling the combined impact of linear filtering and ASE-induced noise on the performance of coherent optical transmission systems. By leveraging a modular channel abstraction and discrete-time equalization theory, we derived closed-form expressions for the signal-to-noise ratio degradation under various equalizer configurations, including ZFE, MMSE, and FSE. The analytical insights are not only general and extensible but also align closely with experimental data, validating

the robustness of the proposed models across realistic optical filtering scenarios.

A key contribution of this study is the disaggregation of filtering penalties across cascaded filter elements and noise sources, enabling the identification of dominant impairment factors in system design. Moreover, the framework accommodates diverse filter shapes and noise coloring effects, making it suitable for modeling real-world optical links—particularly in metro and access networks where linear impairments prevail.

Experimental validation using commercial-grade ROADMs and transceivers confirms the model’s accuracy and practical relevance. The demonstrated consistency between theoretical predictions and empirical measurements underscores the value of the proposed framework for both performance evaluation and design optimization in future digital twin implementations and quality-of-transmission estimators.

Ultimately, this work provides a foundation for integrating accurate filtering penalty models into system-level planning tools and digital twin platforms, paving the way toward more efficient and intelligent optical network operation.

Further developments will extend the current derivation to finite-taps equalizers, filling the gap that still remains between modeling effort and real commercial transceivers operation.

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