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A Game-theoretic Approach to Cooperative Robust Nonlinear Model Predictive Control for a Network of Unmanned Ground Vehicles

Giovanni Marinello, Lorenzo Zino, Carlo Novara, Michele Pagone

***Abstract*— We propose a Pontryagin-based cooperative Robust Nonlinear Model Predictive Control (RNMPC) approach for the formation control of a network of Unmanned Ground Vehicles (UGVs). In the proposed scenario the UGV dynamics incorporates exogenous disturbances to capture real-world uncertainties, making the control problem nontrivial. The RNMPC foresees the solution of a min-max optimal control problem, where the goal is to optimize the control input to be delivered to the single UGV in the worst-case scenario. To this end, the Pontryagin-based min-max problem can be viewed as a zero-sum differential game, where the two players are the control input and the uncertainty whose solution is the Nash equilibrium. Additionally, collision avoidance for UGV is enforced through the implementation of artificial potential fields. The effectiveness and robustness of the proposed control technique have been assessed by means of numerical simulations.**

I. Introduction

Nonlinear Model Predictive Control (NMPC) has gained an extraordinary attention within the industrial and academic communities as one of the most promising techniques for achieving optimal control of constrained nonlinear systems. The reason for such a success lies on its capability to deal with nonlinear dynamics and to provide optimal control commands for multi-variable systems in the presence of inputs, outputs, and state constraints.

However, practical implementation of NMPC is often hindered by the presence of uncertainties in system models and external disturbances. In particular, the system's prediction is usually performed by employing a simplified and/or approximated model of the system's plant. Inaccurate predictions due to modeling errors and/or neglected external disturbances may seriously hamper the controller performance. For the above reasons, several approaches for robust NMPC (RNMPC) have been developed and studied in literature. RNMPC provides a framework for explicitly incorporating uncertainty within the optimization framework, leading to improved reliability in real-world systems.

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Many recent studies have focused on developing various methodologies to address this critical problem. A widely adopted RNMPC approach, introduced in [1], is the tube-based NMPC (see also [2]–[4]). As highlighted in [1] and [5], tube-based control relies on determining both a nominal (or reference) trajectory and an ancillary controller that constrains deviations of the uncertain system's state from the nominal trajectory. Specifically, the ancillary controller ensures that the system's trajectories remain within a tube surrounding the reference trajectory. Alternative approaches include learning-based [6], [7], stochastic [8], [9], and H_∞-based [10]–[13] RNMPC methods. The H_∞-based RNMPC, in particular, involves an offline computation of a pre-compensating H_∞ control law—obtained by solving the differential Riccati equation—which is then combined with a second control term derived from the min-max optimal control problem [10]–[13].

Another notable approach is the min-max RNMPC scheme based on the Pontryagin Minimum Principle (PMP) [14], [15], where the optimization problem is reformulated as a Two-Points Boundary Value Problem (TPBVP) [15]. This methodology formulates the min-max problem as a zero-sum differential game, where the control input and the uncertainty/disturbance act as opposing players [16]–[18]. Consequently, the min-max problem is framed as a zero-sum differential game with a Nash equilibrium (NE) [19]–[21].

In the context of network dynamics, distributed control is a suitable approach to overcome the limitations of centralized approaches in managing network systems. In fact, in network systems, agents may not always be able to share information with others, calling for control schemes that rely on local information.

According to [22], there are two main approaches to distributed control of network systems: i) cooperative controllers and ii) non-cooperative controllers, which differ in the way the state and the agents' input are shared. Specifically, in cooperative distributed control schemes, each controller optimizes an overall plant objective function, leading the system to its global (Pareto) optimum [23]–[25]). On the contrary, in non-cooperative control schemes, each agent takes its own decisions considering the other agents' information only locally [26], [27]. In this scenario, each agent behavior depends on how the adjacency matrix connects each single agents with the others of the network, leading to the definition of a non-cooperative differential game that converges to a Nash equilibrium (NE) that may differ from the global optimum [28], yielding further complexity to the problem of controlling network systems in a distributed

fashion. In this paper, we focus on this second approach.

In this paper, we build on [21] and we take a step further by proposing a non-cooperative networked min-max RNMPC, whose robust optimal control problem is resembled as a differential game which admitting a NE, corresponding to the saddle point condition on the Hamiltonian. In this context, the mismatch between system's model and system's plant consists of exogenous disturbances or noise that directly affect the system, not depending on the system state or input. Such robust control approach is applied to a network of unmanned ground vehicle (UGV).

The novel contribution of the paper is twofold:

- Unlike the previous publication of the same authors [21], in this paper we directly take into account state constraints, which are coped by means of artificial potential fields (APF) (see, e.g, [29]).
- The proposed control approach is applied to a case study of a network system where, in principle, external disturbances may potentially propagate across the network.

The rest of the paper is organized as follows. In Section II, we present the problem statement, the UGV dynamics, and the topology definition. In Section III, the robust NMPC — based on PMP— is presented. In Section IV, the differential game solution is presented. Then, the proposed case study is illustrated and discussed in Section V. Finally, conclusions are drawn in Section VI.

A. Notation

We denote the set of real and strictly positive integer numbers as $\mathbb{R}$ and $\mathbb{N}_+$, respectively. Given $n, m \in \mathbb{N}_+$, a (column) real-valued vector is denoted as $x \in \mathbb{R}^n$, $x^\top$ denotes its transpose, and $\mathbf{A} \in \mathbb{R}^{n\times m}$ denotes a real-valued matrix. For any diagonal matrix $\mathbf{W} = \text{diag}(w_1, ..., w_n) \in \mathbb{R}^{n\times n}$, we define with a n-dimensional column vector $w^\dagger$ collecting the entries of the diagonal, such that $w^\dagger = [w_1, ..., w_n]^\top$. Given $z \in \mathbb{R}^n$ and $\mathbf{W} = \text{diag}(w_1, ..., w_n)$, $w_i \geq 0$, $\|z\|^2_{\mathbf{W}} \doteq z^\top \mathbf{W} z$ is the (square) weighted norm of z. Finally, $\nabla_z(\cdot)$ is the gradient operator with respect to the variable z. Given two vectors $z, v \in \mathbb{R}^n$, the Hadamard (element-wise) product is defined as $z \circ v = [z_1 v_1, ..., z_n v_n]^\top$. In a similar fashion, the element-wise division is $z \oslash v = [z_1/v_1, ..., z_n/v_n]^\top$.

II. Problem Statement

Consider a set of UGVs, labeled by integer numbers $V = \{1, \dots, n\}$. The pose of a generic j-th UGV at time $t \geq 0$ can be fully determined by its position with respect to the x- and y-axis of an inertial frame, denoted as $p_{x_j}(t)$ and $p_{y_j}(t)$, respectively, and by the angle $\theta_j(t)$ between the vector joining the center of gravity of the j-th UGV and the origin and the y-axis (see Fig. 1).

The dynamics of the UGV is governed by the following system of nonlinear differential equations, affected by external and bounded disturbance:

$$\begin{bmatrix} \dot{p}_{x_j} \\ \dot{v}_{x_j} \\ \dot{\theta}_j \end{bmatrix} = \begin{bmatrix} v_{x_j} \\ a_{x_j} \cos\theta_j \\ \omega_j \end{bmatrix} + \begin{bmatrix} 0 \\ w_{x_j} \\ w_{\theta_j} \end{bmatrix}. \tag{1}$$

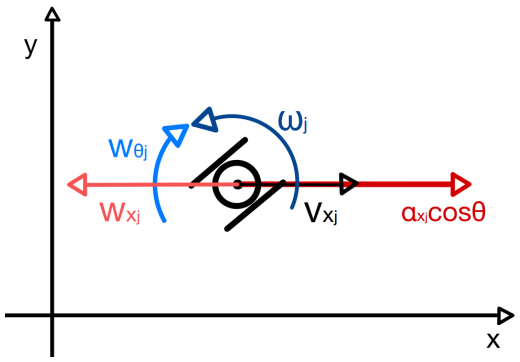


Fig. 1: Schematic of the law of motion of a single UGV.

In (1), v_{x_j} is the UGV velocity along the x-axis, a_{x_j} and ω_j are, respectively, the acceleration along the x-axes and the angular velocity, and w_{x_j}, w_{θ_j} are external disturbances that may affect the UGV dynamics (e.g., friction, aerodynamic drag, noisy state estimation, etc.).

In the field of multi-agent systems, a critical problem is to guarantee that a set of UGVs performs operations in a coordinated and efficient manner [30]. In this context, the formation control problem is extremely important, which requires that the UGVs can achieve and maintain – while moving – a prescribed formation structure.

We employ the so-called leader-follower approach: a single UGV is picked as the leader, "deciding" the global trajectory and movement directives, the other UGVs (followers) adapt their positions relative to the leader, based on planned formation patterns and inter-vehicle distances. This methodology simplifies the overall control strategy by reducing the complexity of the control laws required for each agent while maintaining high coordination efficiency.

The leader-follower topology can be represented as an undirected graph $G = (V, \mathcal{E})$, where $V = \{1, \dots, n\}$ is the set of nodes (UGVs), and $\mathcal{E} \subseteq V \times V$ is the set of edges (connections between UGVs), whereby $(i, j) \in \mathcal{E}$ if and only if agents i and j are connected. The graph consists of one leader UGV, which we denote with index 1, and $n-1$ follower UGVs, denoted with indices $2, \dots, n$.

In this leader-follower configuration, the leader agent is connected to all follower agents and each follower is only connected to the leader, i.e., all elements of the sets of edges are of the form $(1, i)$, with $i \in \{2, \dots, n\}$. The adjacency matrix $\mathbf{A} \in \{0, 1\}^{n\times n}$, which is defined as

$$a_{ij} = \begin{cases} 1 & \text{if } (i, j) \in V, \\ 0 & \text{otherwise,} \end{cases}$$

takes the form:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Hence, the aim of the controller is to guide the set of UGVs in a desired shape, while accounting for external disturbances. We assume that the selected formation structure is a regular polygon with the leader in the center. If the leader faces obstacles or deviations, the followers dynamically adapt their positions to maintain the decided formation structure.

Given the position of the leader (henceforth, the leader will be flagged as agent '1') at time t, $(p_{x_1}(t), p_{y_1}(t))$, its angular displacement $\theta_1(t)$, and a radius $r > 0$ used to guarantee a distance r of the followers from the leader, which acts as a reference formation center, the objective is for the generic j-th follower to reach position

$$\begin{aligned} \bar{p}_{x_j}(t) &= p_{x_1}(t) + r\cos(\theta_1(t) + \theta_j(t)) \\ \bar{p}_{y_j}(t) &= p_{y_1}(t) + r\sin(\theta_1(t) + \theta_j(t)), \end{aligned} \tag{2}$$

Hence, the goal of our controller is to optimally design the inputs in terms of $a_{x_j}(t)$ and $w_j(t)$ to achieve the prescribed formation, taking into account the uncertainty and the disturbances that may affect the UGVs' dynamics. Moreover, for the regular polygonal formation, the angle position $\theta_j(t)$ is defined as $\theta_j(t) = 2\pi j/(n-1)$.

III. Robust Nonlinear MPC

Consider the following affine-in-the-input nonlinear system describing the dynamics of the $j-$th agent, affected by an exogenous and bounded disturbance $w(t) \in \mathcal{W} \subset \mathbb{R}^{n_x}$:

$$\dot{x}_j(t) = f(x_j(t)) + g(x_j(t))u_j(t) + w_j(t). \tag{3}$$

Here, $x(t) \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the state vector at time $t \in \mathbb{R}$ and $u(t) \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ is the input vector (where $\mathcal{U} \subseteq \mathbb{R}^{n_u}$ is a compact set such that $0 \in \mathcal{U}$). Note that, f and g are generic nonlinear functions.

Assumption 1: Assume that $f \in \mathcal{C}^1(\mathcal{X} \to \mathcal{X})$ and $g \in \mathcal{C}^1(\mathcal{X} \to \mathcal{X} \times \mathcal{U})$.

Assumption 2: System (3) is assumed to be controllable and the measurements of the state vector are available with period $T_S > 0$.

Concerning the constraint sets, we consider that $\mathcal{W} \subset \mathbb{R}^{n_w}$, with $n_w = n_x$. In particular, we assume that w is confined within a polytope

$$\mathcal{W} \doteq \{w \in \mathbb{R}^{n_w} : \mathbf{H}w \leq \bar{w}\},$$

where $\mathbf{H}$ is a coefficient matrix of suitable dimension and $\bar{w}$ is a vector with nonnegative entries. Moreover, we define the state constraint set as

$$\mathcal{X} \doteq \{x \in \mathbb{R}^{n_x} : C(x) \leq 0\},$$

where $\mathcal{X}$ is a sublevel set of some $\mathcal{C}^1$-smooth function $C : \mathbb{R}^{n_x} \to \mathbb{R}$ (generally, non-convex), and the set of admissible inputs as the polytope

$$\mathcal{U} \doteq \{u \in \mathbb{R}^{n_u} : \mathbf{G}u \leq \bar{u}\},$$

where $\mathbf{G}$ is a coefficient matrix of suitable dimension and $\bar{u}$ is a vector with nonnegative entries.

We thus define the finite-horizon RNMPC quadratic cost function as

$$\begin{aligned} J = \int_{t_k}^{t_k+T_p} \big(\|u(\tau)\|^2_{\mathbf{R}} + \|\tilde{x}(\tau)\|^2_{\mathbf{Q}} - \gamma\|w(\tau)\|^2_2\big)\mathrm{d}\tau + \\ + \|\tilde{x}(t_k + T_p)\|^2_{\mathbf{P}}, \end{aligned} \tag{4}$$

where $\tilde{x}(t) = x_r - \hat{x}(t)$ is the predicted tracking error, $\hat{x}$ the predicted state, x_r is a (constant) reference signal of interest, $\mathbf{P}, \mathbf{Q} \in \mathbb{R}^{n_x}$, $\mathbf{R} \in \mathbb{R}^{n_u}$ are positive diagonal matrices weighting the contributions of the entries on the cost function arguments, and $\gamma \in \mathbb{R}_{\geq 0}$ is meant as a parameter for attenuating the effect of the uncertainty.

In cooperative distributed MPC (see, e.g., [25]), each agent j calculates its corresponding input u_j by solving a decentralized optimization problem. Based on this, the solution of each agent at the next iteration is calculated from the solution of the robust NMPC optimization problem. Hence, the min-max problem is formulated by defining a cost function to be minimized with respect the control signal u over the worst-case adversary input w. The optimal pair (u^*, w^*) is the solution of the following problem:

$$(u^*, w^*) = \arg\min_u \max_w J(x, u, w) \tag{5}$$

subject to the following constraints:

$$\begin{aligned} &\dot{\hat{x}}(t) = f(\hat{x}(t)) + g(\hat{x}(t))u(t) + w(t), \\ &\hat{x}(t_k) = x(t_k), \\ &u(\tau) \in \mathcal{U},\ w(\tau) \in \mathcal{W},\ \hat{x}(\tau) \in \mathcal{X},\ \forall \tau \in [t_k, t_k + T_p]. \end{aligned} \tag{6}$$

The optimal control problem defined in (5)-(6), with the cost function given by (4), can be interpreted as a zero-sum differential game, whose players are the control input u and the uncertainty w, respectively, with payoff functions equal to J_w and $-J_w$, respectively. In this particular setting, the pair (u^*, w^*) is the corresponding saddle point of the game, i.e., the NE, and the optimal value of J_w corresponds to the value of the payoff for the control input at such NE (see, e.g. [21]).

State constraints are coped by employing the APF methodology. The APF purposes is to enforce the collision avoidance among the UGVs by including an artificial repulsion force within the system dynamics equations (1). Given agents j and i, where $j \neq i$, the artificial potential is hereby defined as:

$$F_{\text{rep}_j} = \begin{cases} k\exp\left(\dfrac{p_{x_i} - p_{x_j}}{\|p_{x_i} - p_{x_j}\|_2}\right), & \text{if } \|p_{x_i} - p_{x_j}\|_2 \leq \tilde{r}, \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

where k represents the intensity of the repulsion force and $\tilde{r}$ is a predefined radius in which the repulsion force is activated. Therefore, the inclusion of the term F_{rep_j} within the system dynamics (3) incorporates soft state constraints (III) within problem (5); this allows to remove the hard state constraints at the price of obtaining a slight approximation of the optimum (u^*, w^*).

IV. PMP-BASED DIFFERENTIAL GAME SOLUTION

According to the NE definition, the pair (u^*, w^*) is the solution of the min-max problem if and only if the following conditions are simultaneously satisfied:

$$u^* = \arg\min_u J_w(u, w) \tag{8a}$$
$$w^* = \arg\min_w -J_w(u, w) = \arg\max_w J_w(u, w) \tag{8b}$$

over the choice of all inputs $u(t) \in \mathcal{U}$ and adversary inputs $w(t) \in \mathcal{W}$, for all $t \in [t_k, t_k + T_p]$.

According to [21], the pair of Hamiltonians $H^{(u)}, H^{(w)} \in \mathcal{C}^1(\mathcal{X} \times \mathcal{X} \times \mathcal{U} \times \mathcal{W} \to \mathbb{R})$ are defined as

$$\begin{aligned} H^{(u)} &= \|u\|_R^2 + \|\tilde{x}\|_Q^2 - \gamma\|w\|_2^2 + \\ &\quad + \lambda^{(u)\top}\big[f(x(t)) + g(x(t))u(t) + w(t) + F_{\text{rep}_j}\big], \end{aligned} \tag{9a}$$
$$\begin{aligned} H^{(w)} &= -\|u\|_R^2 - \|\tilde{x}\|_Q^2 + \gamma\|w\|_2^2 + \\ &\quad + \lambda^{(w)\top}\big[f(x(t)) + g(x(t))u(t) + w(t) + F_{\text{rep}_j}\big], \end{aligned} \tag{9b}$$

where $\lambda^{(u)}, \lambda^{(w)} \in \mathbb{R}^{n_x}$ are the covectors of the minimization and maximization problem, respectively. Hence, the necessary conditions for optimality are given by

$$\begin{aligned} &H^{(u)}(x^*, u^*, w^*, \lambda^{*(u)}, \lambda^{*(w)}) = \\ &\qquad \min_u H^{(u)}(x^*, u, w^*, \lambda^{*(u)}, \lambda^{*(w)}), \end{aligned} \tag{10a}$$
$$\begin{aligned} &H^{(w)}(x^*, u^*, w^*, \lambda^{*(u)}, \lambda^{*(w)}) = \\ &\qquad \min_w H^{(w)}(x^*, u^*, w, \lambda^{*(u)}, \lambda^{*(w)}), \end{aligned} \tag{10b}$$
$$\dot{\lambda}^{(u)} = -\nabla_x H^{(u)}, \tag{10c}$$
$$\dot{\lambda}^{(w)} = -\nabla_x H^{(w)}, \tag{10d}$$
$$\lambda^{(u)}(t_k + T_p) = 2\tilde{x}^\top(t_k + T_p)\mathbf{P}, \tag{10e}$$
$$\lambda^{(w)}(t_k + T_p) = -2\tilde{x}^\top(t_k + T_p)\mathbf{P}, \tag{10f}$$

where the minimization operators with respect to u and w in (10a) and (10b) should be intended over all inputs u such that $u(t) \in \mathcal{U}$, for all $t \in [t_k, t_k + T_p]$, and adversary inputs w such that $w(t) \in \mathcal{W}$, for all $t \in [t_k, t_k + T_p]$.

In the following, we expand the results from [21], where a tight correlation between $\lambda^{(u)}$ and $\lambda^{(w)}$ has been found in some special condition. In this context, the min-max problem resembles in defining a common Hamiltonian and finding the solution (i.e., the saddle point) by minimization of the Hamiltonian with respect to u.

Theorem 1: Consider the min-max necessary conditions in (10a)–(10f) and let Assumptions 1-2 hold. Then, $\forall t \in [t_k, t_k + T_p]$:

$$\lambda^{(w)}(t) = -\lambda^{(u)}(t). \tag{11}$$

Proof: For the sake of readability, we omit the dependence on time whenever it is clear from the context.

From (10e) and (10f), one has that $\lambda^{(w)}(t_k + T_p) = -\lambda^{(u)}(t_k + T_p)$. By taking into account (3), (10c) and (10d), the variation of the covectors along the prediction horizon is equal to

$$\dot{\lambda}^{(u)}(t) = -2\mathbf{Q}\tilde{x}(t) \tag{12a}$$
$$- \left(\frac{\partial f}{\partial x}(x(t)) + \frac{\partial g}{\partial x}(x(t))u(t) + \frac{\partial F_{\text{rep}_j}}{\partial x}\right)\lambda^{(u)}(t), \tag{12b}$$
$$\dot{\lambda}^{(w)}(t) = 2\mathbf{Q}\tilde{x}(t) \tag{12c}$$
$$- \left(\frac{\partial f}{\partial x}(x(t)) + \frac{\partial g}{\partial x}(x(t))u(t) + \frac{\partial F_{\text{rep}_j}}{\partial x}\right)\lambda^{(w)}(t). \tag{12d}$$

By introducing the auxiliary variables

$$\varrho \doteq \lambda^{(u)} \oslash \lambda^{(w)}, \tag{13}$$
$$\xi(t) \doteq -\Big(\frac{\partial f}{\partial x}(x(t)) + \frac{\partial g}{\partial x}(x(t))u(t) + \frac{\partial F_{\text{rep}_j}}{\partial x}\Big), \tag{14}$$

the time derivative of ϱ is then computed as

$$\dot{\varrho} = \dot{\lambda}^{(u)} \oslash \lambda^{(w)} - (\lambda^{(u)} \circ \dot{\lambda}^{(w)}) \oslash (\lambda^{(w)} \circ \lambda^{(w)}). \tag{15}$$

Upon substituting (14) and (12a)-(12c) into (15) one has that

$$\begin{aligned} \dot{\varrho} = \big[&\lambda^{(w)} \circ (\xi \circ \lambda^{(u)} - 2q^\dagger \circ x) + \\ &- \lambda^{(u)} \circ (\xi \circ \lambda^{(w)} + 2q^\dagger \circ x)\big] \oslash (\lambda^{(w)} \circ \lambda^{(w)}), \end{aligned} \tag{16}$$

Whereby, after some simple mathematical manipulations

$$\dot{\varrho} = -2q^\dagger \circ (\lambda^{(w)} + \lambda^{(u)}) \oslash (\lambda^{(w)} \circ \lambda^{(w)}). \tag{17}$$

By collecting ϱ on the right-hand side of (17)

$$\dot{\varrho} = -2q^\dagger \circ (1 + \varrho) \oslash \lambda^{(w)}. \tag{18}$$

Equation (18) provides a potential infinite number of NE equilibria for any admissible variation of $\lambda^{(w)}$. Necessarily, if a couple $(\lambda^{(u)}(\tau), \lambda^{(v)}(\tau))$ solves the TPBVP (10a)-(10f), then it satisfies Eq. (11), $\forall\, \tau \in [t_k, t_k + T_p]$. The most immediate way to fulfill such a necessary condition (consistently with the boundary conditions in Eqs. (10e) and (10f)) consists of letting $\dot{\varrho} = 0$. Hence, from the solution properties of the Lipschitz-continuous differential equations, we have that

$$\varrho(t) = \varrho(t_k + T_p) - \int_{t_k + T_p}^{\tau} \dot{\varrho}(\tau)\mathrm{d}\tau. \tag{19}$$

By accounting (10e) and (10f), the boundary conditions on ρ at $t = t_k + T_p$ are $\varrho(t_k + T_p) = -1$. Hence, recalling that $\dot{\varrho} = 0$ the following expression hold:

$$\varrho(t) = \lambda^{(u)}(t) \oslash \lambda^{(w)}(t) = -1, \quad \forall t \in [t_k, t_k + T_p], \tag{20}$$

which yields the statement of the Theorem. ■

Remark 1: From Theorem 1, we observe that:

- A common Hamiltonian H and covector set λ of the min-max problem can be defined, such that H can be picked as $H^{(u)}$ and $\lambda(t) = \lambda^{(u)}(t) = -\lambda^{(w)}(t)$, or vice-versa[1]. Hence,

$$u^*(t) = \arg\min_u H(x(t), \lambda(t), u(t), w(t)), \tag{21a}$$
$$w^*(t) = \arg\min_w H(x(t), -\lambda(t), u(t), w(t)), \tag{21b}$$

[1]One can pick $H^{(w)}$ as joint Hamiltonian. In this case, the signs of the covectors are inverted.

for u and w such that $u(t) \in \mathcal{U}$ and $w(t) \in \mathcal{W}$, for all $t \in [t_k, t_k + T_p]$.

- The NE of the min-max problem coincides with the Hamiltonian saddle point: there exists a pair (u^*, w^*) such that $H(u^*, w) \leq H(u^*, w^*) \leq H(u, w^*)$.

V. Numerical Example

We consider a network of six UGVs (one leader and five followers)[2]. Each agent of the network is affected by unknown (but bounded) disturbances. Referring to Eq. (1), the system's state vector is $x \doteq [p_x, v_x, \theta]^\top$ and the command input vector $u \doteq [a_x, \omega]^\top$. The system dynamics is sampled at $T_s = 0.01\,s$, with $T_p = 10T_s$. For each agent, the RNMPC weighting parameters are set as $\mathbf{Q} = \text{diag}([10000, 500, 200])$, $\mathbf{R} = \text{diag}([15, 15])$, $\mathbf{P} = \text{diag}([30, 50, 50])$, and $\gamma = 4$. The disturbance vector w is sampled at each k-th instant from a uniformly distributed random variable over the interval $[-0.2, 0.2]$.

Concerning the system's constraints, for each agent, we set $\|u\|_\infty \leq 20$ and the APF parameters for enforcing collision avoidance are set to $\tilde{r} = 1$, $k = 17$. These latter parameters are chosen empirically to ensure that the repulsive forces are neither too weak nor overly aggressive, maintaining a balance between collision avoidance and smooth motion.

The UGVs maneuver is subdivided into two different phases. In the first one, the UGVs are asked to achieve a prescribed formation, according to the task demanded to the leader. Once that the formation is achieved, the UGVs are asked to coordinately move towards a different prescribed reference, while keeping the formation. The initial conditions for the UGVs are chosen as:

$$x_1 = [5; 0; 0], \quad x_2 = [6; 0; 0.2], \quad x_3 = [7; 0; 0.3],$$

$$x_4 = [3; 0; 0.4], \quad x_5 = [2; 4; 0.5], \quad x_6 = [1; 0; 0].$$

Finally, the leader's constant reference is chosen as $x_r = [10; 0; \frac{\pi}{6}]$ for the first formation, and $x_r = [15; 0; \frac{\pi}{4}]$ for the second formation. We recall that, in order to achieve a regular polygonal formation, the angle position $\theta_j(t)$ —with respect to the leader UGV— is defined as $\theta_j(t) = 2\pi j/(n-1)$.

A comparison between the performance of nominal NMPC and RNMPC are given in Tables I-II. It is worth to notice that, in presence of persistent disturbances affecting the nonlinear network systems, RNMPC is able to deliver a reduced tracking error with respect to the nominal NMPC. The trajectories of the UGVs are depicted in Fig. 2. A more detailed plot (Fig. 3) shows the pivotal phases of the UGVs maneuver, reporting: i) the initial conditions initial condition; ii) the instant at which the desired formation is achieved; and iii) translation while keeping the prescribed formation. Finally, in order to evaluate the coherence of the fleet formation and reformation for the Euclidean norm of the tracking error along the maneuvers illustrated in Fig. 4.

[2]Note that, the numerical values of the example are meant and given in International System of Units.

	Nominal			**Robust**		
	p_x	v_x	θ	p_x	v_x	θ
L	0.0260	0.0000	0.0539	0.0200	0.0000	0.0455
F	0.0385	0.0000	0.0539	0.0270	0.0000	0.0455
	0.0361	0.0000	0.0539	0.0257	0.0000	0.0455
	0.0232	0.0000	0.0539	0.0183	0.0000	0.0455
	0.0226	0.0000	0.0539	0.0180	0.0000	0.0455
	0.0276	0.0000	0.0539	0.0208	0.0000	0.0455

TABLE I: First formation: comparison between Nominal and Robust Scenarios in terms of final tracking error. L stands for Leader and F stands for Followers.

	Nominal			**Robust**		
	p_x	v_x	θ	p_x	v_x	θ
L	0.0378	0.0000	0.0539	0.0268	0.0000	0.0455
F	0.0570	0.0000	0.0538	0.0372	0.0000	0.0455
	0.0519	0.0000	0.0539	0.0344	0.0000	0.0455
	0.0355	0.0000	0.0539	0.0027	0.0000	0.0455
	0.0335	0.0000	0.0539	0.0241	0.0000	0.0455
	0.0421	0.0000	0.0539	0.0289	0.0000	0.0455

TABLE II: Second formation: Comparison between Nominal and Robust Scenarios in terms of final tracking error. L stands for Leader and F stands for Followers.

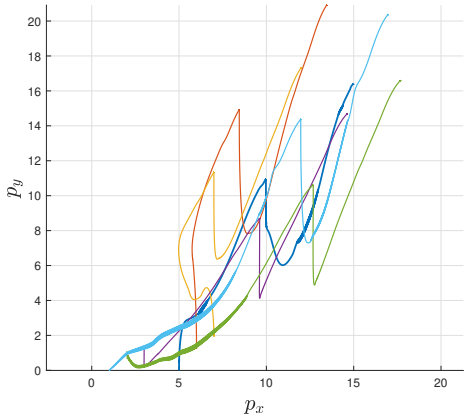


Fig. 2: Complete trajectory of the controlled UGVs.

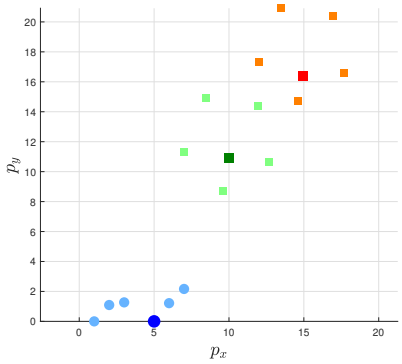


Fig. 3: Position of the UGVs at different time instants of the simulation: i) initial condition (leader in blue, followers in cyan); ii) formation maneuver (leader in dark green, followers in light green); and iii) translation while keeping the prescribed formation (leader in red, followers in orange).

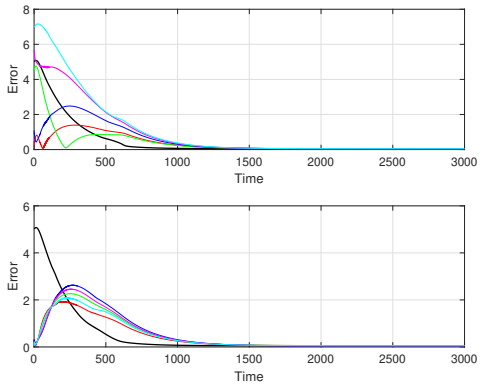


Fig. 4: Reference tracking error along the two phases of the maneuver.

VI. Conclusion

In this work, we proposed a networked RNMPC approach to leader-follower UGVs formation control, integrating the APF method to enforce collision avoidance. The proposed NMPC incorporates external disturbances modeled as uniformly distributed random noise. The robust approach consists of solving of a min-max optimal control problem by means of the Pontryagin principle. As consequence, the min-max problem consists of finding the Nash Equilibrium of a zero-sum differential game, where the two players are the control input and the uncertainty. As test-case example, we proposed numerical simulation where the UGVs are asked to i) achieve a prescribed formation and, then, ii) to move while preserving the formation. The numerical simulations confirm that the RNMPC is able to handle the external disturbance in an appropriate fashion, delivering more accurate results, in terms of tracking error, with respect to the non-robust NMPC. As future work, the proposed approach could be extended by encapsulating parametric uncertainties in order to address more physically realistic scenarios, such as complex interaction topologies and real-physical model.

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