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Structure and Shock Resistance in Financial Networks: A Numerical Study

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Abstract. We investigate the resilience of financial networks to exogenous shocks, which can trigger primary and secondary defaults that propagate through the network in a cascading manner. Building on previous studies that have developed tools for numerically assessing the resilience of financial networks to such shocks, we conduct a series of numerical experiments to analyze how the structure of financial networks affects their resilience. Specifically, we evaluate the worst-case loss scenarios of the financial system in relation to fluctuations in banks' cash inflows across different network structures. Our analysis explores the effects of network sparsity versus connectivity, as well as the impact of clustering and heterogeneity in liability patterns among financial institutions.

Keywords: Financial networks, Resilience, Robustness, Clearing Vector, Random Graph

1 Introduction

After the 2007-2008 financial crisis, scientific interest in the vulnerability of financial networks has grown significantly. Researchers employ mathematical tools from operation research, optimization, control theory, and network science to assess the resilience of financial systems against external shocks and default cascades [6,9]. Despite this expanding body of literature, a full understanding of how the resilience of the financial system is affected by the network ofliabilities among financial institutions remains elusive. For instance, the impact of the density of the network of liability relationships remains unclear. On the one hand, increased connections provide more pathways to absorb smaller shocks thus boosting resilience [3, 5, 18]. On the other hand, denser networks become more fragile to large shocks once a critical number of institutions have defaulted [1, 7, 20, 21, 27].

Numerous models have been developed to explore the repercussions of defaults in financial networks, many of which are inspired by the seminal work of Eisenberg and Noe [15] (later expanded by Elsinger et al. in [17]). If an external shock is weak, it is absorbed by the bank's net worth. However, if the shock is strong, the bank must reduce payments to others, propagating the shock

throughout the network. This requires reconciling the entire structure of mutual liabilities – a process called clearing, represented by a clearing vector that determines the total shortfall in the financial system. The concept of the clearing vector and the algorithms for its computation form the core of the Eisenberg-Noe theory. Clearing payments can be determined using various methods, including fictitious default algorithms, fixed-point iterations of a specialized monotone mapping, or by solving convex optimization problems [11,15,20,25,30]. All these methods assume that the full liability matrix and each bank's net cash inflow are known, which may not hold in the presence of the external shocks.

Exogenous shocks can originate from various sources [22, 24]. One potential source is a liquidity shortage in a bank [3], triggered by unexpected deposit withdrawals due to events such as an information campaign or a cyber attack. Another source of shocks comes from shared external assets (securities, commodities, real estate, etc.), where a price drop affects all shareholders [2, 12, 14, 21]. Such a price drop can be triggered, e.g., by stock market fluctuations or the fire sale of a substantial amount of assets by some holders [13, 14]. A similar effect arises when banks are linked by contracts like equity claims, which link their portfolios [16]. A drop in a bank's portfolio value reduces its own value, lowering thus the value of its equity holders. The interconnections between banks can amplify the impact of external shocks. Initial defaults by some banks can trigger secondary defaults, indirectly affecting other banks even if they were not directly exposed to the original shock. Such cascading effects can result in losses within the financial system that far exceed the magnitude of the initial shock, causing not only defaults but also insolvencies of some banks [1, 22].

Given the difficulty in predicting the exact value and structure of external shocks, worst-case consequence analysis is crucial. If the external sector imposes one or several simultaneous shocks at different nodes of the network, with a known total magnitude, the key questions are: What is the worst possible shortfall for the network, and what shock structure leads to this maximum shortfall? This problem is addressed in the recent work [12], which primarily focuses on shocks arising from asset commonality and analyzes their worst-case effects on the financial system. By leveraging linear programming duality, the paper [12] determines the maximum level of shocks that can occur without causing bank insolvencies and evaluates the total worst-case loss for shocks of a given magnitude. The resulting curve, which shows the dependence of the worst-case loss on the shock magnitude, characterizes the network's resilience to external shocks. In this paper, we apply the mathematical techniques from [12] to investigate the interplay between key features of financial network structures and the worst-case loss-magnitude curve. Specifically, we focus on four main network features.

First, we examine the role of network *density* by comparing different financial networks modeled as directed random regular graphs with increasing degree, where each bank has liabilities to a fixed number of other institutions. Our numerical results suggest that very sparse financial networks are more vulnerable to default cascades. However, the benefits of increased connectivity quickly reach a saturation point, consistent with observations from similar studies [1, 7, 20].

Second, we investigate the effect of *clustering*, by simulating Watts–Strogatz small-world networks with varying re-wiring probabilities [26]. Our findings show no noticeable impact of the clustering coefficient on network resilience.

The last two studies explore different aspects of *heterogeneity*, focusing on the degree distribution and the amount of liabilities to each bank. Our results reveal a nonlinear behavior: small to moderate heterogeneity in the degree distribution tends to enhance the resilience of the financial network. However, for highly heterogeneous networks, the resilience decreases, consistent with findings from other model-based studies [8, 10, 23].

2 Preliminaries: Financial networks and clearing vectors

We denote strictly positive integers, nonnegative integers, and strictly positive real numbers as $\mathbb{N}_{>0}$, $\mathbb{N}_{\geq 0}$, and $\mathbb{R}_{>0}$, respectively. We use bold lowercase font to denote a vector $\boldsymbol{x}$, with x_i its ith entry; and the uppercase for matrices $A = (a_{ij})$. The all-0 and all-1 vectors are denoted as $\mathbf{0}$ and $\mathbf{1}$, respectively, and the identity matrix as I, where dimensions are omitted when unnecessary. Given two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ with the same dimension, we denote by $\boldsymbol{x} \leq \boldsymbol{y}$ the entry-wise inequality (i.e., $x_i \leq y_i$, for all entries). Given a vector $\boldsymbol{x}$, we define its entry-wise positive part as $\boldsymbol{x}^+$, with $x_i^+ = \max\{x_i, 0\}$.

2.1 Financial network

We consider an extension of the Eisenberg–Noe model [15], as introduced in [17]. Specifically, we consider $n \in \mathbb{N}_{>0}$ banks, represented by the set $\mathcal{V} := \{1, \ldots, n\}$. Each bank $i \in \mathcal{V}$ may have obligations to other banks, represented by a nonnegative *liability* matrix $\bar{P} \in \mathbb{R}_{\geq 0}^{n \times n}$. Namely, bank i has an obligation to pay $\bar{p}_{ij}$ currency units to bank $j \neq i$; the diagonal entries are all zeros $\bar{p}_{ii} = 0$. These interconnections between banks are naturally represented by a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \bar{P})$, referred to as a *financial network*, where $(i, j) \in \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ exists if and only if bank i has an obligation to bank j, i.e., $\bar{p}_{ij} > 0$ (Fig. 1).

The row sums of $\bar{P}$, represented by the vector $\bar{\boldsymbol{p}} = [\bar{p}_1, \ldots, \bar{p}_n]^\top := \bar{P}\mathbf{1}$, correspond to the total liabilities (or the weighted out-degrees) of the nodes. Similarly, the column sums of $\bar{P}$, form the vector $\mathbf{1}^\top \bar{P}$ where each entry i represents the total debt owed by other banks to node i (or its weighted in-degree). We also use the matrix of *relative liabilities* $A \in [0, 1]^{n \times n}$ whose (i, j)-entry is defined as

$$a_{ij} = \begin{cases} \frac{\bar{p}_{ij}}{\bar{p}_i} & \text{if } \bar{p}_i > 0, \\ 1, & \text{if } \bar{p}_i = 0 \text{ and } i = j, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

denoting the proportion of debt to j in the total debt of bank i.

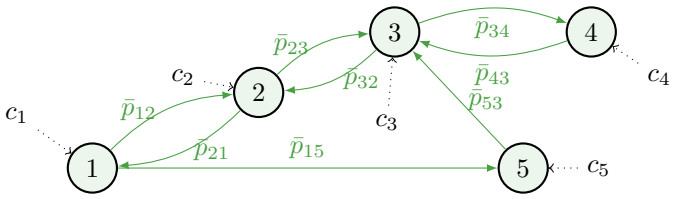


Fig. 1: Illustration of a financial network with 5 banks.

2.2 The net cash inflow and net worth

In addition to mutual liabilities, each bank $i \in \mathcal{V}$ is characterized by its net cash inflow $c_i \in \mathbb{R}$, defined as the difference between total incoming liquidity from the non-financial sector (e.g., operations, investments) and the total debt owed to the non-financial sector, which is considered senior to interbank claims[1] [17]. The net cash inflows constitue a vector $\boldsymbol{c} = [c_1, \dots, c_n]^\top$.

Under regular operations, each bank $i \in \mathcal{V}$ acquires a net *nominal* cash inflow of $c_i = \bar{c}_i$, sufficient to cover its liabilities. The difference between a bank's asset side (cash inflow and payments from other banks) and its liability side $\bar{\beta}_i := \bar{c}_i + \sum_{k\in\mathcal{V}} \bar{p}_{ki} - \bar{p}_i \geq 0$ is termed the nominal net worth, or equity.

Assumption 1 The quantities A, $\bar{\boldsymbol{p}}$, and $\bar{\boldsymbol{c}}$ are such that $\bar{\boldsymbol{\beta}} := \bar{\boldsymbol{c}} + A^\top \bar{\boldsymbol{p}} - \bar{\boldsymbol{p}} \geq \boldsymbol{0}$.

A central goal of the Eisenberg-Noe theory was to examine the impact of shocks that reduce the net inflow of some banks below the nominal level, i.e., $c_i < \bar{c}_i$. If such a shock is sufficiently large, a bank may be unable to fully pay its debts to other banks, as its asset side falls short of its total liabilities

$$c_i + \sum\nolimits_{k\in\mathcal{V}} \bar{p}_{ki} < \bar{p}_i. \tag{2}$$

In this default scenario, bank i must reduce its payments to other banks from the nominal values $\bar{p}_{ij}$ to smaller amounts $p_{ij} \in [0, \bar{p}_{ij}]$, aiming to maintain non-negative equity. This reduction imposes a secondary shock on creditor banks, reducing the value of their assets below nominal values and potentially triggering defaults and further payment reductions. This process causes shocks to propagate through the network, leading to chains of defaults, potentially returning to the initially defaulting bank and further worsening its financial position.

This raises the problem of determining the matrix of "fair" payments $P = (p_{ij})$ when some banks default due to exogenous shocks. In this paper, we adopt the approach from [15, 17], which is based on the concept of the *clearing vector*.

[1] As noted in [15], external liabilities of the same seniority as interbank claims can be redirected to a fictitious node with only incoming arcs. Consequently, such external debts are excluded from the net inflow calculation. The original Eisenberg-Noe model [15], which considers only these debts, thus assumes $c_i \geq 0$.

2.3 Clearing Vectors

According to [15,17], the actual payment p_{ij} that each bank makes to maximally settle its debts is determined by the following three rules: i) The actual net worth of each bank remains nonnegative, i.e., $\beta_j := c_j + \sum_{k\in\mathcal{V}} p_{kj} - \sum_{k\in\mathcal{V}} p_{jk} \geq 0$, except for the case where the bank is *insolvent* and has negative asset side $c_j + \sum_{k\in\mathcal{V}} p_{kj} < 0$. Insolvent banks, obviously, cannot make payments to other banks. ii) Debts take priority over equity growth, meaning $\beta_i \leq 0$ unless the debt of node i is fully settled, i.e., $p_{ij} = \bar{p}_{ij}$ for all j; iii) All inter-bank liabilities have the same priority, so the total payment is split proportionally to the relative liability. Rules i)–iii) lead to the following definition.

Definition 1. *Given a financial network $(\mathcal{V}, \mathcal{E}, \bar{P})$ and the cash inflow vector $\boldsymbol{c}$, a vector that satisfies $\boldsymbol{p} = (\min(\bar{\boldsymbol{p}}, (\boldsymbol{c} + A^\top \boldsymbol{p}))^+$ is a clearing vector. The matrix of clearing payments P corresponding to $\boldsymbol{p}$ is defined as $p_{ij} = a_{ij} p_i$ for all $i \neq j$.*

The clearing vector always exists [17, 22], being unique in the generic situations [1, 22]. However, even if it is non-unique, one can always find the (elementwise) *maximal* clearing vector [12]. In the most interesting situation where the shock does not lead to insolvencies, this maximal clearing vector is the minimizer in the following linear program[2] (see [11, 12] for more details).

Proposition 1 (Proposition 2 from [12]). *Consider the linear program:*

$$\begin{aligned} \eta^* = \eta^*(c) = \min_{\boldsymbol{p}\in\mathbb{R}^n} \ & \mathbf{1}^\top(\bar{\boldsymbol{p}} - \boldsymbol{p}) \\ s.t.: \ & \mathbf{0} \leq \boldsymbol{p} \leq \bar{\boldsymbol{p}} \\ & \boldsymbol{c} + A^\top \boldsymbol{p} \geq \boldsymbol{p}. \end{aligned} \tag{3}$$

If the constraints are feasible, then the optimal solution $\boldsymbol{p^*}$ is unique, being the maximal clearing vector. For the corresponding matrix of clearing payments, no bank is insolvent. Contrary, if Eq. (3) is infeasible, then there will exist banks which are insolvent to the external sector for every choice of the clearing vector.

Note that $\mathbf{1}^\top(\bar{\boldsymbol{p}} - \boldsymbol{p})$ is the total difference between the liability vector and the actual payments. Hence, the optimal value of the objective function of Eq. (3), $\eta^* = \mathbf{1}^\top(\bar{\boldsymbol{p}} - \boldsymbol{p^*})$ is the *total loss* of the financial network due to defaults. Clearly, if $\boldsymbol{c} = \bar{\boldsymbol{c}}$, then there are no defaults due to Assumption 1, and $\eta^* = 0$.

3 Problem setup

Proposition 1 offers a method for finding the maximal clearing vector given the net cash inflow vector $\boldsymbol{c}$. As discussed earlier, a bank's actual cash inflow may fall below its nominal value due to external shocks, which come from various sources and are difficult to predict. This motivates a *robust* problem setup. Let the net cash inflow vector $\boldsymbol{c}$ fluctuate as $\boldsymbol{c} = \bar{\boldsymbol{c}} + \boldsymbol{\delta}$, where $\bar{\boldsymbol{c}}$ is the nominal value and $\boldsymbol{\delta} \in$

[2] In the case where Eq. (3) is infeasible, the maximal clearing vector can be found by an iterative algorithm [12]. This situation is beyond the scope of this work.

$\mathbb{R}^n$ represents the uncertain fluctuation from a certain class $\Delta = \{\boldsymbol{\delta}\}$. We aim to predict the worst-case consequences of such fluctuations across all possible shocks within the set Δ. Specifically, we seek to understand (a) whether the financial network absorbs these shocks without triggering defaults, and (b) if some shocks do lead to defaults, determine the worst-case systemic loss $\mathbf{1}^\top(\bar{\boldsymbol{p}} - \boldsymbol{p}(\boldsymbol{c}))$ over all $\boldsymbol{\delta} \in \Delta$. These questions are addressed in the next subsection for a specific class of shocks Δ, namely, an ℓ_1-norm ball.

3.1 Resilience margin and worst-case loss

Henceforth, we assume that the external sector generates shocks that are unknown and may affect one or several banks, but the total "magnitude" (i.e., the loss of liquidity entering the financial system) is bounded by a known amount ε. Mathematically, we assume that $\|\boldsymbol{\delta}\|_1 \leq \varepsilon$, where $\|\boldsymbol{\delta}\|_1 = \sum_{i\in\mathcal{M}} |\delta_i|$.

This problem setup is a special case of the one in [12], assuming that the "asset shares" matrix S is the identity matrix. As shown in [12], there are two critical values of ε: i) ε^*, termed the *resilience margin*, which is the largest shock magnitude that the network can absorb without causing any defaults[3], and ii) ε_{ub}, which is the maximum norm of a fluctuation vector such that Eq. (3) is feasible[4]. The latter can be computed using [12, Proposition 5] (with $S = I_n$).

Shocks of magnitude $\varepsilon \in (\varepsilon^*, \varepsilon_{ub})$ may cause defaults, but the clearing procedure prevents insolvencies. The optimal clearing vector is found by solving Eq. (3), and we can assess the severity of the defaults by computing the worst-case total loss of the financial network η_{wc} that can be caused by shocks with $\|\boldsymbol{\delta}\|_1 \leq \varepsilon$ (recall that $\boldsymbol{c} = \bar{\boldsymbol{c}} + \boldsymbol{\delta}$). This worst-case loss, computed by solving a max-min problem, where we maximize the optimal solution η^* of Eq. (3) over all possible fluctuations $\boldsymbol{\delta}$ with $\|\boldsymbol{\delta}\|_1 \leq \varepsilon$. In [12], it is shown that the problem further simplifies to solving n independent linear programs, as follows.

Proposition 2. *The worst-case loss over all fluctuation vectors $\|\boldsymbol{\delta}\|_1 \leq \varepsilon$ equals*

$$\begin{aligned} \eta_{wc} = \max_{i=1,\dots,n} \max_{\boldsymbol{\beta},\boldsymbol{\lambda}\geq 0} \; & (\mathbf{1}-\boldsymbol{\beta})^\top \bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}^\top \boldsymbol{\lambda} + \varepsilon\lambda_i \\ \textit{s.t.:} \; & \boldsymbol{\beta} - \mathbf{1} + (I - A)\boldsymbol{\lambda} \geq 0. \end{aligned} \tag{4}$$

To illustrate our approach, we present the following example.

Example. Consider the financial network with $n = 5$ banks illustrated in Fig. 1, with nominal parameters $\bar{P}$ and $\bar{\boldsymbol{c}}$ reported in Fig. 2. For this network, after finding $\varepsilon^* = 5$ and $\varepsilon_{ub} = 11$, we use Proposition 2 to compute the worst-case loss for different values of $\varepsilon \in (\varepsilon^*, \varepsilon_{ub})$, as reported in the cyan curve in Fig. 2. From this plot, we observe a piecewise linear profile: initially, the total loss is 0, until the resilience margin ε^* is reached. Then, the curve proceeds with changes in slope at critical points where the set of banks that have a default changes (as found empirically by observing the solution to the optimization problem). Then,

[3] For such shocks, the minimum in Eq. (3) is zero.

[4] A closer analysis of the proofs in [12] reveals that ε_{ub} is the supremum of ε, for which the maximal value in Eq. (4) is finite.

$$\bar{P} = \begin{bmatrix} 0 & 20 & 0 & 0 & 50 \\ 30 & 0 & 40 & 0 & 0 \\ 0 & 60 & 0 & 20 & 0 \\ 0 & 0 & 40 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 \end{bmatrix} \quad \bar{\boldsymbol{c}} = \begin{bmatrix} 45 \\ 5 \\ -30 \\ 25 \\ 10 \end{bmatrix}$$

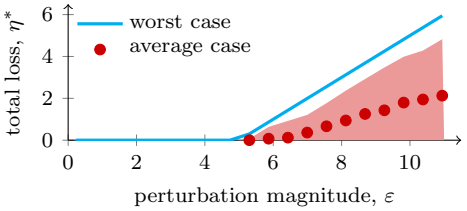


Fig. 2: The worst-case total loss (cyan curve) and the average (red dots) total loss computed over 1,000 realizations of the fluctuation vector $\boldsymbol{\delta}$ for different values of its magnitude ε. The red area represents the range of all simulations.

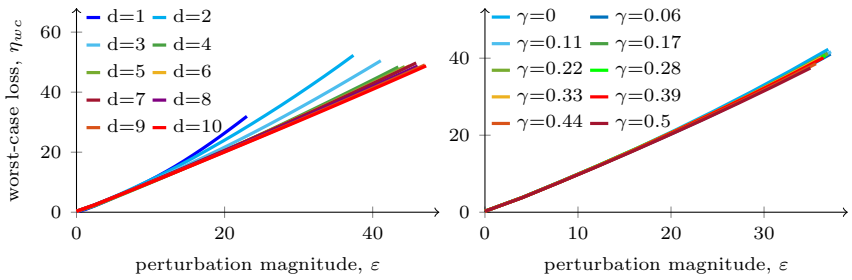
for each value of ε, we generate 1,000 independent realizations of the perturbation vector $\boldsymbol{\delta}$ with magnitude $\|\boldsymbol{\delta}\|_1 = \varepsilon$, and, for each realization, we compute the total loss by using Proposition 1. The average total loss and the total range of outcomes are reported in Fig. 2. We observe that, while the worst-case loss can be conservative with respect to the average total loss, some realizations in our simulations have outcomes that are quite close to the worst-case scenario.

3.2 Research problem

The worst-case loss curve, calculated using Proposition 2, characterizes the financial network's resistance to exogenous liquidity shocks. The aim of this work is to explore *how the structure of the financial network impacts its resilience*.

Specifically, we perform a set of studies, each focusing on a network features, viz. connectivity, presence of clusters, degree distribution, and heterogeneity of liabilities. In each study, we consider a network of $n = 100$ banks, generated with the desired characteristics detailed in the following. Once the matrix $\bar{P}$ is generated, for each bank $i \in \mathcal{V}$, we define the nominal value of its assets as $\bar{c}_i = \bar{p}_i - \sum_{k \in \mathcal{V}} \bar{p}_{ki} + \xi_i$, where ξ_i is a realization of a uniformly distributed random variable with values in $[0, 100]$, each realization independent of the others. Such a synthetic network obeys Assumption 1 and exhibits no defaults in the absence of fluctuations, since bank i has nominal net worth $\bar{\beta}_i = \xi_i > 0$, all with the same expected value but with different realizations to account for natural stochasticity.

In our studies, using Proposition 2, we compute the worst-case loss η_{wc} for evenly-spaced values of the magnitude of the fluctuation in the range $\varepsilon \in (\varepsilon^*, \varepsilon_{ub})$. For the sake of robustness, we introduce stochasticity in the network formation process and we average our results over 100 independent realizations of each network in a Monte Carlo fashion. All simulations are performed using MATLAB, and the code is available at https://github.com/lzino90/financial_cn.



(a) Connectivity (b) Clustering

Fig. 3: Impact of the structure on the resilience of the financial network, considering (a) random regular graphs with different degree d and (b) small-world networks with different clustering coefficient γ.

4 Results

4.1 Density

In the first study, we investigate the effect of the density of the financial network. The literature shows no consensus on the impact of this feature, as it can produce two contrasting effects, where resilience could either increase [3,5,18] or decrease [1,7,20,21,27]. We explore this question by computing the worst-case total loss curve for different financial networks, all modelled as (weighted) directed regular random graphs with varying degree $d \in \{1,\dots,10\}$, where larger values of d correspond to denser networks. In other words, each bank has liabilities to exactly d other banks, selected uniformly at random as a d-uple from the other nodes (with independent selection for each bank). The total liability is evenly divided among the (out)-neighbors, with each entry equal to $100/d$.

The results of our simulations, reported in Fig. 3a, confirm the intuition that the network connectivity influences the resilience of the financial network. In fact, very sparse networks with $d \leq 3$ have visibly smaller values of ε_{ub} and the corresponding curves are above the others, which means that they typically result in larger worst-case losses with respect to denser structures. However, the benefit of increasing network density quickly saturates, and only minimal improvements are observed when $d > 4$.

4.2 Clustering

In the second study, we investigate the role of clustering, a common feature of many financial networks: banks that share common connections are often also connected by mutual liability [28,29]. In our experiments, we use Watts–Strogatz small-world networks to generate financial networks with different levels of clustering. Specifically, we consider different Watts–Strogatz networks with the

same average degree ($d = 4$), but with different re-wiring probability, resulting in different levels of clustering, captured by the (global) clustering coefficient γ (in plain words, the larger γ, the higher the level of clustering) [26]. For each network generated, we set nonzero liabilities to $\bar{p}_{ij} = 25$.

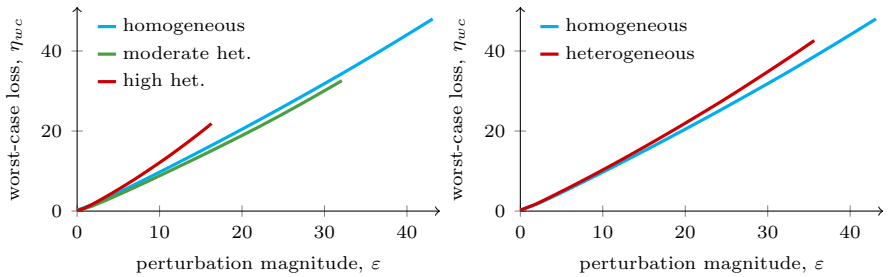
Our findings are reported in Fig. 3b. From the results of our numerical simulations, we conclude that clustering seems to have a marginal impact on the resilience of the financial network, although highly clustered networks appear to be slightly less resilient to large shocks. We conjecture that these observations are due to the presence of two contrasting effects. On the one hand, clusters amplify the impact of defaults, as once one or multiple defaults occur within a cluster, all the other banks in that cluster have large exposure to them. On the other hand, clusters attenuate default avalanches by containing the impact primarily within the cluster, reducing exposure for banks outside of it.

4.3 Degree heterogeneity

In the third study, we focus on another important feature of real-world financial networks: heterogeneity among banks. Specifically, we compare the worst-case loss for three different network structures, with different degree distributions: i) homogeneous, where each node has the same number of neighbors $d = 4$; ii) moderately heterogeneous, where the number of neighbors d_i for each node is sampled from a zero-truncated Poisson distribution [31] with a mean of 4; and iii) highly heterogeneous, with each node having number of neighbors d_i sampled from a Zipf distribution with a mean of 4. All networks are generated randomly: for each node i, the set of (out-)neighbors is selected uniformly at random as a d_i-uple of nodes, with independent selection for each node. Similar to previous studies, we set the nonzero liabilities to $\bar{p}_{ij} = 25$.

Figure 4a presents our results, which reveal complex and nontrivial insights. On the one hand, heterogeneity appears to be detrimental in terms of the maximal shocks that the system can absorb before a bank is not able to pay its external debts (larger ε_{ub}). On the other hand, moderate levels of heterogeneity (green curve) might increase the resilience in terms of worst-case loss, consistent with the results in other model-based studies [10]. However, such a beneficial effect diminishes with increasing the heterogeneity (see the red curve), in accordance with the results in [19].

A possible explanation for this nonlinear behavior is the following. Nodes with low degree may be less robust to fluctuations, but they typically trigger smaller cascades, limiting the total loss of the network. However, when heterogeneity is large, the presence of hubs can lead extremely large cascades in the worst-case scenarios considered in this study, yielding larger total loss and more defaults. In summary, these results suggest that diversity in the network structure has a nontrivial impact (corroborating findings from previous studies [8,23]), and pave the way for future studies towards designing financial networks with the optimal level of diversity to achieve maximal resilience.



(a) Degree heterogeneity (b) Liability heterogeneity

Fig. 4: Impact of heterogeneity on the resilience of the financial network, considering (a) random regular graphs (homogeneous, cyan curve), Erdos-Renyi random networks (low heterogeneity, green curve) and Albert–Barabasi scale-free networks (high heterogeneity, red curve); and (b) regular random graphs with homogeneous (cyan curve) and heterogeneous (red curve) liabilities.

4.4 Liability heterogeneity

In the fourth study, we focus on another aspect of heterogeneity in the financial network. In all previous studies, we assumed that all liabilities between banks are of the same amount. In the real world, the size of each liability is usually different. For this reason, we again consider (directed) random regular graphs with a degree of $d = 4$ (to avoid confounding due to different sources of heterogeneity). We compare two scenarios: i) homogeneous liabilities, where, for each bank, the four nonzero liabilities are set to $p_{ij} = 25$; and ii) heterogeneous liabilities, where each bank's four nonzero liabilities are sampled independently from a uniform distribution over $[0, 50]$. The results reported in Fig. 4b suggest that heterogeneity in the amount of mutual liabilities slightly weakens the network. This can be explained by the fact that in heterogeneous networks, a bank's default is more likely to cause larger fluctuations in the credit of some neighbors, potentially generating more secondary defaults than in a balanced network.

5 Conclusion

In this paper, we investigated the impact of the structure of a financial network on its resilience to default cascades. Using an extended formulation of the well-studied Eisenberg-Noe model [15] and analytical tools developed in [12], we performed a simulation campaign to assess resilience of the financial network in terms of worst-case total loss in the presence of defaults due to shocks in the value banks' net value. By systematically considering different network structures, we have determined that the network density and its heterogeneity have a critical role in shaping the network resilience: very sparse networks are more keen to amplify shocks, yielding cascades of secondary defaults, whereas a moderate level

of heterogeneity in degree distribution seems to increase the network resilience. These observations are consistent with similar results in the literature [8, 10, 23].

The results of our numerical studies pave the way for several lines of research. First, our results are obtained under the assumption that all shocks are independent. However, in many real-world scenarios, banks have shares in common assets, which means that fluctuations in their values are often correlated. Further studies should be performed considering the nontrivial structure of the network of bank-to-asset ownership (similar to [12]), to consider correlated shocks. Some preliminary results in this direction have been obtained in [32] using a more general formulation of the model and the techniques from [12], but further studies are required to test robustness of the findings presented in this paper in the more general scenario of correlated shocks.

Second, the theoretical framework from [12], which we used to compute the worst-case total loss, should be extended to incorporate further features of financial systems, such as illiquid assets and costs associated with defaults [14, 30]. Illiquid assets lead to higher-order interactions between groups of banks, where the fire sale of one bank's share in an illiquid asset depresses the asset's overall price, imposing a nonlinear shock on all other shareholders.

Third, while the network features explored in this paper are motivated by empirical observations in financial systems [28,29], further effort should be placed into analyzing case studies of real-world financial systems and validating our findings against other stress test methodologies presented in the literature [4, 9].

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