# Mathematical modelling and optimal control of epidemiological systems across scales

**Abstract.** This manuscript provides an overview of different modelling approaches in epidemics, spanning from a microscale to a macroscale perspective. Problems of various natures are addressed, involving a variety of analytical and numerical methods, aiming to explore and contribute to the management of epidemic dynamics.

In the first part a compartmental model comprising a system of ODEs is proposed as a starting point to investigate an optimal control problem aimed at answer to which is the best non-pharmaceutical policy that balances economic and epidemic repercussions. In particular, I have shown how imposing a state constraint on infected population, that translates into the idea of an Intensive Care Units capacity, has repercussion on the social cost and on the epidemic size. Utilizing ODEs theory, existence results on the optimal control problems with state constraint and Pontryagin Minimum Principle, the investigation is carried out, see [4].

In the second part, a new SI epidemic model model is established as an individual-based model, formulated as a system comprising discrete time Markov processes, tracking the stochastic evolution of two continuous labels over time, the resistance and viral load levels on each agent, as well as the compartment switching. The corresponding *mesoscopic* model, consisting of an IDEs system that describes the evolution of distribution of resistance in susceptible individuals and the distribution of viral load in infectious individuals, is formally derived through mechanical statistics arguments. Under appropriate assumptions, also a *macroscopic* counterpart of the system, comprising a system of ODEs, has been derived. The analytical results on the asymptotic behavior, carried out by the study of the integral curves of the system, depicts an accurate prediction of the scenario that the model formulation will lead to, see [3].

In the third part, a generalization of the model formulated in the second one has been proposed to assess the effect of a pharmaceutical control policy. Specifically, after following the same procedure in deriving the model across scales as in the previous setting, analytical results on the ODEs system have been established. Based on the dynamics depicted from a macro-scale perspective, an optimal control problem aimed at containing the epidemic size under a vaccination supply, has been formulated. Making use of analytical results on existence of the optimal control and on Pontryagin Minimum Principle, the optimal control has been characterized at least for a sufficiently small time horizon, [2].

In the fourth part, a compartmental model comprising a system of nonlocal IDEs is proposed as a starting point of another optimal control problem. In particular, the model is continuously structured by age, thus the population has been distinguished by different ages. The question of the best vaccination policy among individuals of different ages, given a limited vaccine supply, has been addressed. Building on an approach already developed in literature, the IDEs system can be rewritten as an equation of Volterra first-type with respect to time and a Fredholm second-type with respect to the structure variable. Using the comparison principle and a rescaled version of the equation, the optimal policy is characterized at least with respect to time, [1].

[1] L. Almeida, R. Ducasse and E. Paparelli. "Structured SIR model under optimal age-dependent vaccination policy". In preparation (2024).

[2] T. Lorenzi, E. Paparelli and A. Tosin. "Optimal control of a phenotype-structured SIS model". In preparation (2024).

[3] T. Lorenzi, E. Paparelli and A. Tosin. "Modelling coevolutionary dynamics in heterogeneous SI epidemiological systems across scales". In: *Communications in Mathematical Sciences* 22.8 (2024), pp. 2131–2165.

[4] E. Paparelli, R. Giambó and H. Maurer. "Optimal control of an epidemiological Covid-19 model with state constraint". In: *Discrete and Continuous Dynamical Systems - B* (2024).