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Original
Feasibility Study on LGVF for Spacecraft Formation Flight Mission / Ruggiero, Dario; Ito, Takahiro; Capello, Elisa; Tsuda, Yuichi. - (2024), pp. 595-600. (2024 SICE Festival with Annual Conference Kochi City (JPN) 27-30 August 2024).

Feasibility Study on LGVF for Spacecraft Formation Flight Mission

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Abstract: Formation flight is a key feature for several new space missions concepts, and it consists of multiple spacecraft working as single rigid structure, improving flexibility and redundancy of the mission. Recent studies have shown the possibility to take advantage of Low Earth Orbit for experimental validation of new formation flight technologies. In this context, low-thrust proximity formations are investigated. The main challenge of the onboard system is to guarantee high level of autonomy, mission success and safety. Low-thrust imposes tight requirements for spacecraft relative motion. A novel approach is investigated, where each spacecraft move on a circular trajectory, defined Circular Relative Orbit, with respect to a reference orbit. Failure analysis is carried out to evaluate safety implication of tracking the proposed trajectory. A prediction model based on the presence of external disturbances is designed to improve safety distance in case of complete failure. Lyapunov Guidance Vector Field method is combined with the Artificial Potential Field, and investigated as guidance algorithm for formation flight in Circular Relative Orbit. Simulation campaign is carried out to evaluate the feasibility of the proposed approach for: formation gathering, collision avoidance, and formation reconfiguration.

Keywords: Guidance and Flight Control, Intelligent Systems, Autonomous Decentralized Systems

1. INTRODUCTION

Recently, several new space missions design have been based on formation flight technology. Having multiple spacecraft working as a single large rigid structure allows improved flexibility and redundancy of the mission. Space formation flight missions are generally categorized as space interferometers or telescopes with long focal length. Main missions are summarized in [1] and the references therein. The real challenge of formation flight mission is to guarantee high level of autonomy to deal with all the mission phases: formation gathering, keeping, reconfiguration and safety. Past researches on space formation flight are mainly focused on attitude stabilization and control, as in [2, 3]. Attitude is generally really sensitive and high accuracy needs to be guaranteed, as seen in [4, 5], where different strategies for autonomous attitude recovery from a critical perturbation is investigated. For what may concern position guidance and control, main contribution focuses on classical approaches, relying on leader-following and trajectory tracking, as seen in [6, 7]. In recent studies, [8-10], Low Earth Orbit (LEO) has been evaluated as suitable environment for testing new formation flight technologies, for practical and economic reasons. As pointed out in [8], having LEO formation flight, while dealing with disturbances and low-thrust, imposes tight requirements in how the formation can be achieved. In this context, proximity formations are evaluated. This leads to definition of the Circular Relative Orbit (CRO) trajectory. It consists of a circular trajectory along which the spacecraft moves cost-free around a reference point (in an ideal scenario). In order to guarantee the motion on the CRO trajectory and high formation accuracy, an effective guidance algorithm needs to be designed. Moreover, failure analysis in proximity of the CRO trajectory is carried out to evaluate safety implications. LEO environment's external disturbances improves safety, making the failed spacecraft able to leave the proximity of the formation in shorter time than the ideal case. Moreover, a prediction model of the failed spacecraft based on disturbances is included to identify a strategy to increase safety distance.

Main studies dealing with guidance and control methods addressing proximity and variable formations refer to optimal control and Artificial Potential Field (APF), as seen in [11, 12]. These algorithms are generally applied to leader-follower formations or using assignment algorithms. In [13, 14], the application of these algorithms is extended to a generalized formation approach, where a group of spacecraft gets into a regular polygon shaped formation around a reference point, and at a given distance from it. Having a generalized framework allows to describe each spacecraft motion with respect to a reference orbit, and without a fixed dependency with others. This framework simplifies formation adaptability to the number of elements since it is part of guidance algorithm computation. However, problem statement does not comply with actuators limitations or fuel consumption. For this reason, in this paper, the design of a guidance algorithm for a generalized formation approach is combined with fuel consumption and continuous low-thrust problem statement.

The proposed guidance algorithm is designed to make

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the spacecraft attracted to the CRO trajectory while dealing with formation flight. APF-based algorithm is considered because this method is computationally efficient, and it is possible to combine different APF function to comply with a complex problem statement. In this case, Lyapunov Guidance Vector Field (LGVF) method is investigated as possible solution. The LGVF-based guidance algorithm is based on the work done in [15], where the problem of keeping an Unmanned Aerial Vehicle hovering over a target on a circular trajectory is addressed, and in [16, 17], where it is used in combination with Interfered Fluid Dynamical System method for trajectory planning and collision avoidance maneuvers. Moreover, it is combined with the work done in [14], where an attractive-repulsive APF function is defined to get two systems at the desired relative distance, and reaching formation. The performance of the proposed guidance strategy are evaluated performing a Monte Carlo Simulation Campaign, with random initial conditions (in proximity of the CRO trajectory) and external disturbances. Simulation of formation gathering, collision avoidance and reconfiguration are carried out to prove the feasibility and the limits of the proposed approach.

The major contribution carried out in this study is: (i) Definition of a framework to study and design high autonomous guidance and control algorithms for formation flight. (ii) Evaluation of LGVF method in combination with APF-based formation function to address formation flight problem in CRO. (iii) Safety and feasibility analysis.

The paper is organized as follow. In Section 2, spacecraft model dynamics is described and related to the CRO trajectory. In Section 3, failure analysis is carried out. The proposed guidance algorithm is introduced in Section 4, while simulations results are discussed in Section 5. Finally, concluding remarks are given in Section 6.

2. SPACECRAFT MODEL

In this section, spacecraft model is discussed. The spacecraft formation is modelled as generalized motion with respect to a circular reference orbit. Equation of motion are derived from the propagation of the relative dynamics, usually known as Hill's or Clohessy-Wiltshire (CW) Equation, [18, 19] in the Local-Vertical-Local-Horizontal (LVLH) reference frame. The LVLH reference frame is centered in a virtual object orbiting the Earth on a circular reference orbit of radius r_o, with x-axis oriented as the orbital velocity direction, z-axis pointing to the center of the Earth, and y-axis completing the terns. The position dynamics of each spacecraft is modelled with respect to the LVLH reference frame, and independently from the others spacecraft, as

$$\begin{array}{rcl} \ddot{x}_{lvlh} & = & 2\omega\dot{z}_{lvlh} + u_x + w_{d,x} \\ \ddot{y}_{lvlh} & = & -\omega^2 y_{lvlh} + u_y + w_{d,y} \\ \ddot{z}_{lvlh} & = & 3\omega^2 z_{lvlh} - 2\omega\dot{x}_{lvlh} + u_z + w_{d,z} \end{array} \tag{1}$$

where $\omega = \sqrt{\mu/r_o^3}$ is the orbital angular rate, $p = [x_{lvlh},\ y_{lvlh},\ z_{lvlh}]^T$ and $v = [\dot{x}_{lvlh},\ \dot{y}_{lvlh},\ \dot{z}_{lvlh}]^T$ are spacecraft position and velocity, $u = [u_x,\ u_y,\ u_z]^T$ is the control input, and $w = [w_{d,x},\ w_{d,y},\ w_{d,z}]^T$ is the vector of external disturbances. All the terms are expressed in the LVLH reference frame. Each spacecraft state is assumed to be well known at each instant. LEO environment and its influence on formation are discussed in [8]. External disturbances are mainly related to atmospheric drag and J2 effect. For a triangular formation in LEO ($r_o \simeq 2000$ km) with separation distance around 100 m, disturbances can be included using a simplified model, given as constant acceleration of the order of magnitude $w \sim -10^{-6.5}$ m/s^2, and acting on the x-axis.

In this paper, we are mainly focusing on free motion ($u = w = 0$), as particular solution of Equation (1). The CRO trajectory is given as

$$\begin{array}{rcl} x_{lvlh} & = & -R\cos(\omega t + \phi) \\ y_{lvlh} & = & \sqrt{3}R\sin(\omega t + \phi)/2 \\ z_{lvlh} & = & R\sin(\omega t + \phi)/2 \end{array} \tag{2}$$

where $\phi \geq 0$ is a phase angle, and $R > 0$ is the CRO trajectory radius. The combination of Equations (1) and (2), with $u = w = 0$, leads to $\ddot{x} = 0$. The radius R depends on the type of formation and the desired separation distance $d > 0$: for a linear 2-spacecraft formation $R = d/2$, for a triangular 3-spacecraft formation $R = d/\sqrt{3}$.

3. FAILURE ANALYSIS

In this section, spacecraft trajectory propagation is considered to evaluate safety and collision risk. Worst case scenario is expressed by a complete failure: the failed spacecraft is in free motion, and other spacecraft have no knowledge about its state. Spacecraft dynamics, Equation (1), has an analytical solution for free motion, and it is expressed by

$$x(t) \quad = \quad H(t)x_0. \tag{3}$$

where $x_0 = [x_{lvlh},\ y_{lvlh},\ z_{lvlh},\ \dot{x}_{lvlh},\ \dot{y}_{lvlh},\ \dot{z}_{lvlh}]^T$ is the initial condition, and $H(t)$ is defined in [18]. Considering the ideal case (with no external disturbances), the trajectory propagation of the failed spacecraft corresponds with the actual state. Moreover, if the spacecraft is on the CRO trajectory, according to its definition, it will keep is motion on the CRO trajectory safely.

If the initial condition x_0 is not part of the CRO trajectory, the uncertainty on the spacecraft state is propagated according to

$$P(t) \quad = \quad H^T(t)P_0H(t). \tag{4}$$

where P_0 is the covariance error on the initial state. In Figure 1, the uncertainty on the failed spacecraft position is reported as function of time, assuming an error with respect to the CRO trajectory of the order of magnitude of m for position and mm/s for velocity. The error for x_{lvlh} increase rapidly, and this implies that without

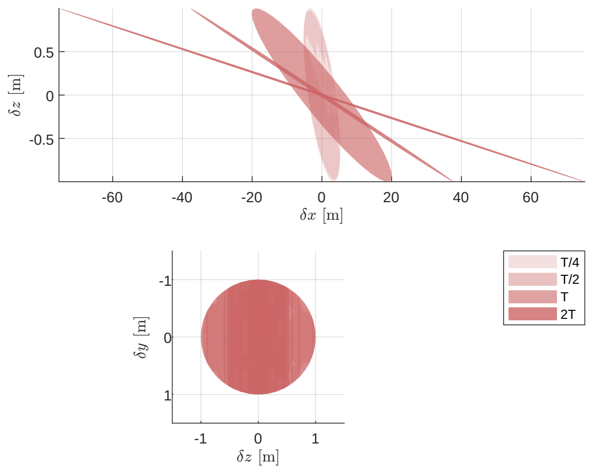

Fig. 1 Uncertainty on failed spacecraft position.

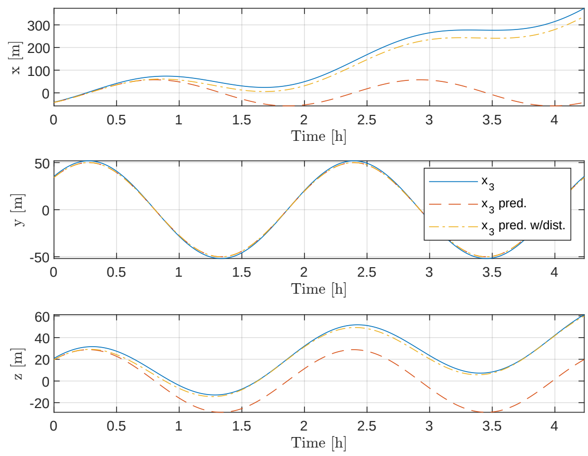

Fig. 2 Prediction model.

a good knowledge of the initial state x_0 is impossible to predict accurately the failed spacecraft trajectory.

The trajectory propagation model is defined as virtual spacecraft. It is built starting from Equation (1), with $u = w_d = 0$, and the initial condition is taken as a point on the CRO trajectory. The other two spacecraft moves in formation with the virtual spacecraft. A simulation campaign is carried out to evaluate the minimum distance between the failed spacecraft and the other spacecraft in formation considering random initial condition in proximity of the CRO trajectory. Results prove that this approach leads to high collision risk trajectory. However, in the real case, the failed spacecraft is subjected to external disturbances, and it leaves faster the proximity of the formation. Considering a trajectory propagation model including the presence of the disturbances (only $u = 0$), and where the initial condition is taken as a point on the CRO trajectory, makes the prediction of motion more accurate. In particular, in Figure 2 an example of failure propagation is reported: blue continuous line represents the real motion of the failed spacecraft, while dotted lines the prediction models evolution. For LEO environment the presence of external disturbances is more effective than the orbital perturbation. For this reason, the model including the disturbances (in yellow) is more effective than ideal model propagation (in red). This allows to design a collision avoidance maneuver exploiting the formation keeping with the virtual spacecraft. In the general case, this would attract the other two spacecraft out of the CRO trajectory. For this reason, the formation with the virtual spacecraft is evaluated using its projection on the CRO trajectory.

4. GUIDANCE ALGORITHM

In this section, the guidance algorithm is discussed. The APF method is based on the definition of a potential field as function of the system state $U = U(x)$. The guidance algorithm is designed to drive the system to the minimum of the function U. Typical approach consists of tracking a reference velocity, which is oriented as the gradient of the potential function U, and given as

$$v_{ref} = -\nabla U. \tag{5}$$

The main advantage of this approach is that function U may be defined combining more terms with different purpose. In order to deal with the proposed formation flight problem, the potential function U is required to attract each spacecraft to the desired CRO trajectory, and to handle the formation. For this reason, it is defined as the combination of

$$v_{ref} = v_{ref,1} + v_{ref,2}. \tag{6}$$

The definition of the potential function term to attract the spacecraft to the desired CRO trajectory is based on the LGVF technique, [15]. The LGVF method consists of the definition of a velocity field in analogy with the fluid dynamics. The final formulation for in plane motion is given as

$$\begin{aligned} u^* &= \delta\left[-x^*(r^2 - R^2) + 2y^* rR\right], \\ v^* &= \delta\left[-y^*(r^2 - R^2) - 2x^* rR\right], \\ w^* &= \delta\left[-\lambda r(z^* - H)\right], \end{aligned} \tag{7}$$

where $\lambda > 0$, and

$$\begin{aligned} r &= \sqrt{x^{*2} + y^{*2}}, \\ \delta &= \frac{\omega R}{r\sqrt{(r^2+R^2)^2 + \lambda(z^{*2} - H^2)^2}}. \end{aligned} \tag{8}$$

The velocity field is rotated from the CRO plane to the LVLH reference frame using a rotation matrix. This is expressed by

$$v_{ref,1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/3) & -\sin(\pi/3) \\ 0 & \sin(\pi/3) & \cos(\pi/3) \end{bmatrix} \begin{bmatrix} u^* \\ v^* \\ w^* \end{bmatrix}, \tag{9}$$

and the rotational velocity field is graphically represented in Figure 3.

When the spacecraft is in proximity of the CRO ($r \simeq R$, $z^* \simeq H$), the reference velocity is simply given by

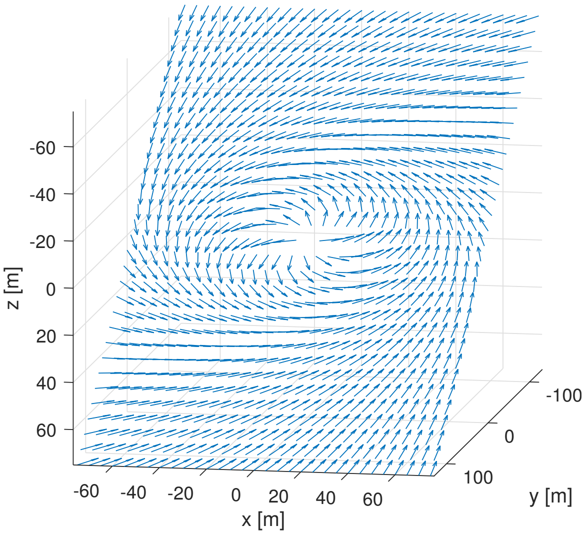

Fig. 3 LGVF velocity field.

$$v_{ref,1}\Big|_{(r\simeq R, z^*\simeq H)} = \begin{bmatrix} 0 & 0 & 2\omega \\ 0 & 0 & \omega/\sqrt{3} \\ -\omega/2 & 0 & 0 \end{bmatrix} p. \quad (10)$$

Assuming the closed-loop system is tracking the velocity field such to have $v = v_{ref,1}$, and the ideal dynamics ($w_d = 0$), Equation (1) leads to

$$\begin{aligned} \ddot{x}_{lvlh} &= -\omega^2 x_{lvlh}, \\ \ddot{y}_{lvlh} &= -\omega^2 y_{lvlh}, \\ \ddot{z}_{lvlh} &= -\omega^2 z_{lvlh}. \end{aligned} \quad (11)$$

For each $R > 0$, Equation (11) is solved by an harmonic function corresponding to the desired CRO trajectory, Equation (2). Therefore, following the proposed guidance allows to track the desired optimal trajectory.

The definition of the potential function term to address the formation is based on the equilibrium shaping method, described in [14]. Considering a formation of N spacecraft, the formation term $v_{ref,2}$ for the i-th spacecraft is given as

$$v_{ref,2} = -\sum_{j\neq i}^{N-1} x_{ij} b \left(e^{-\frac{d^2}{c}} - e^{-\frac{x_{ij}^T x_{ij}}{c}} \right), \quad (12)$$

where $b, c > 0$ are constant gains, $d > 0$ is the desired separation distance, and x_{ij} is the vector pointing from the i-th to j-th spacecraft. The formation term consists of a repulsive-attractive field as function of the relative position of each spacecraft. The reference velocity is $v_{ref,2} = 0$ when the spacecraft are at the desired separation distance.

A simple feedback controller is designed to track the velocity field, and it is given as

$$u = -K(v - v_{ref}). \quad (13)$$

5. SIMULATION RESULTS

In this section, feasibility study on CRO trajectory guidance based on LGVF and APF method is carried out by means of simulation campaign. This paper addresses the problem of spacecraft formation flight in CRO. Formation is defined using relative motion with respect of a virtual point orbiting on a circular reference orbit. In this way, each spacecraft is modelled independently using Equation (1). Having the spacecraft moving on a CRO around the reference frame allows to keep the formation with thrust-free motion (with no disturbances), and satisfying low-thrust requirements. LGVF guidance is implemented to make the spacecraft attracted to the CRO trajectory, and it is given by Equation (10). LGVF guidance is combined with the APF-based function given in Equation (12) to address spacecraft separation. Simulation campaign is carried out to evaluate the performance of the proposed approach for: formation gathering, collision avoidance, and formation reconfiguration. Simulation are carried out considering random initial condition in proximity of the CRO trajectory, with an error of m for position and mm/s for velocity, and considering a maximum control input of $|u| \leq 10^{-5}$ m/s^2. Reference trajectory is given as circular LEO orbit ($r_o \simeq 2000$ km). Orbital factors, such as orbital inclination angle, eccentricity and radius, affects formation flight. Orbital factors are taken into account considering variable external disturbances with order of magnitude $10^{-6.5}$ m/s^2.

In Figure 4, simulation results for the formation gathering scenario are reported. Spacecraft initial conditions are set randomly in proximity of the CRO. In each scenario, the algorithm is able to drive the formation in the desired conditions in reasonable time. Once, triangular formation is achieved, spacecraft keep is motion on the CRO trajectory without affecting the formation. For what may concern collision risk, in Figure 5, the minimum distance between the failed spacecraft and the others is evaluated considering the formation flight with the virtual spacecraft moving on the CRO trajectory. This leads to high collision risk trajectory both for ideal motion ($w = 0$) and in presence of disturbances. The collision avoidance maneuver including the prediction model based on external disturbances is shown in Figure 6, the capability of the framework to guarantee collision avoidance in presence of one spacecraft failure is shown. Results show that, even if the virtual spacecraft projection is considered, the divergence of the virtual spacecraft leads to high perturbation in the other two spacecraft relative position. This is related to the variation of the speed with the spacecraft moves the CRO trajectory, which is related to the required force to stay on this trajectory. Anyway, a minimum distance of 10 m from collision is guaranteed. Finally, the capability of the proposed approach to perform formation reconfiguration is shown in Figure 7.

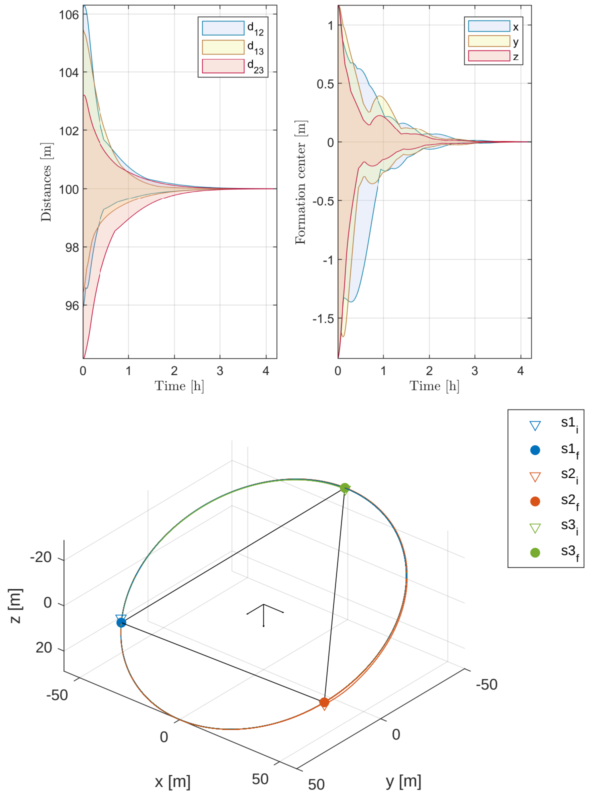

Fig. 4 Simulation campaign: formation gathering.

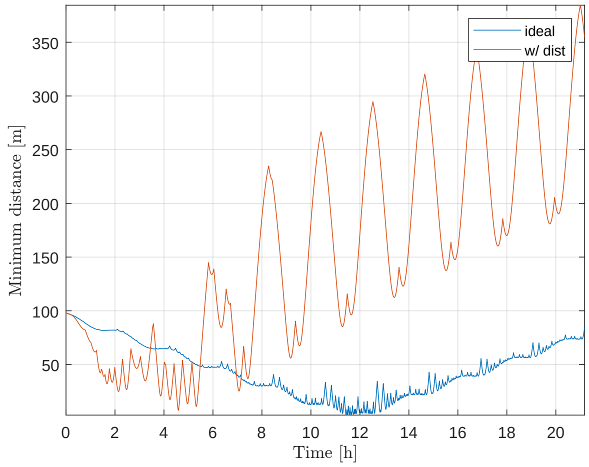

Fig. 5 Simulation campaign: Minimum distance from failed spacecraft.

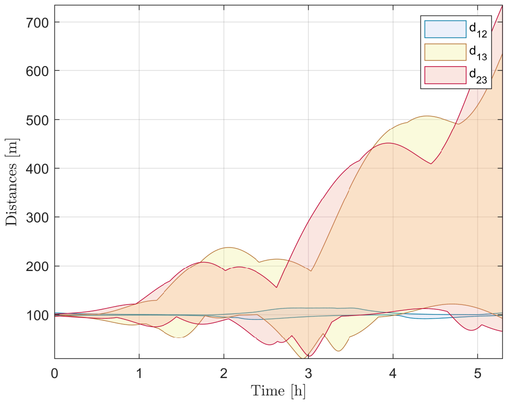

Fig. 6 Simulation campaign: collision avoidance.

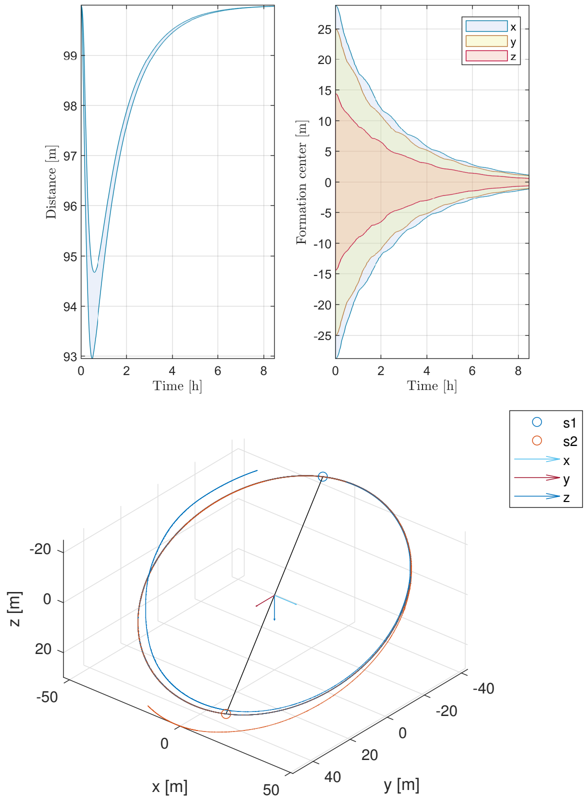

Fig. 7 Simulation campaign: formation reconfiguration.

6. CONCLUDING REMARKS

In this paper, a feasibility study on low-thrust proximity formation flight guidance is addressed. Problem formulation is expressed as tracking a CRO trajectory, allowing force-free motion, while achieving high formation accuracy. Therefore, LGVF guidance is designed to track the CRO trajecotry, and it is combined with APF-based function to get the required spacecraft separation. This allows to have a low computational cost guidance algorithm that, in combination with a feedback controller, is employed for different maneuvers. Simulation study is carried out to prove the effectiveness of the method for formation gathering in CRO, collision avoidance in case of one spacecraft failure, and reconfiguration. Even if the algorithm shows good performance for formation keeping and reconfiguration, it's not really effective for collision avoidance. In particular, having low-thrust control may lead to formation drifting (and instability) or high collision risk trajectory. For this reason, a different solution or an intelligent policy to determine how the formation should change in case of failure should be investigated in future work. Moreover, LGVF approach leads to the tracking of the CRO trajectory if the closed-loop system allows velocity field tracking. Under the assumption of bounded control input, this can be guaranteed only when the spacecraft is in proximity of the CRO trajectory. Future work should address guidance and control design in order to guarantee closed-loop system stability for in-CRO-plane motion and under bounded control input.

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