Dilute, ultracold atomic gases provide an excellent platform for the study of collective quantum properties, due to their manipulability and the relatively simple character of the interactions. In this context, binary immiscible mixtures of Bose-Einstein condensates display exotic topological excitations such as quantum massive vortices (i.e. vortices whose core is filled by a minority component). Quantum vortices are not only of fundamental interest in the context of superfluidity, but also have analogies in cosmology, superconductivity, nonlinear optics, and can be related to the quantum Hall effect. The emergence of a vortex mass is typical of a mixture, but might as well be due to finite temperature effects, or to impurities, and leads to intriguing phenomena. In the thesis we focus on the two-dimensional dynamics of massive vortices in mixtures of two different bosonic species, featuring contact interaction and with a hard-wall circular potential. We derive a point-like (PL) model for N_v massive vortices via a variational Lagrangian approach, and apply it to the study of the effect of the interspecies coupling on the massive vortices' dynamics. On this basis, we find and characterize some notable solutions for two-vortex trajectories in the case of imbalanced vortex masses. We validate our analytical results against numerical simulations of the (mean-field) Gross-Pitaevskii equations describing the mixture. Our characterization of the imbalanced vortex pair leads to the identification of fascinating dynamical regimes, which allow for an indirect measure of the microscopic vortex masses, given their positions and precession frequency. Subsequently, we extend the study of the vortex pair to include a time-dependent vortex mass, by considering a quantum tunneling of the infilling component. Via numerical simulations, we find that such system features a macroscopic dynamics resulting in a Bosonic Josephson Junction (BJJ). BJJs, whose dynamics presents analogies with superconducting Josephson Junctions, were studied and observed for coherent Bose gases in optical potentials. In the BJJs, the interaction character of the neutral atoms was shown to lead to novel effects such as the macroscopic quantum self-trapping. Remarkably, we find that our two-vortex system displays all the (nonlinear) phenomena characterizing a BJJ, and it is robust and stable over time. We also derive the corresponding Bose-Hubbard model of the BJJ and its mean-field approximation, giving some analytical expressions for the model's coefficients as a function of the significant system's parameters. Our work opens the path to exciting outlooks, such as the study of tunneling of the infilling component in vortex necklaces and lattices, the effect of disorder and asymmetries brought by potentially different vortex-core sizes, the inclusion of multiply quantized vortices, and the extension of the models to Fermi superfluids.