Summary

This thesis presents doctoral research focused on the analysis of electromagnetic integral equations discretized using boundary element methods (BEMs), with particular emphasis on stability, computational efficiency, and achievable accuracy.

Computational Electromagnetics (CEM) is the scientific discipline dedicated to developing accurate and efficient solvers for Maxwell's equations. The high predictive power of electromagnetic theory makes simulators crucial for advancements in engineering, and growing technological demands drive the need for efficient and reliable solvers that can handle multi-frequency scenarios. Boundary integral equations, characterized by high accuracy and a reduced number of degrees of freedom relative to volume-based methods, are frequently preferred in CEM, with applications ranging from the design of antennas and microwave devices, to the simulation of the electromagnetic compatibility of electronic equipment and medical applications.

Among medical applications, a notable example is the use of non-invasive electroencephalographic (EEG) measurements for real-time neuroimaging, particularly valuable as a preliminary step in the surgical treatment of focal epilepsy. EEG simulations require high-fidelity, real-time models that can handle a large number of degrees of freedom to accurately represent the head's anatomical complexity. In the contribution "Giunzioni, Merlini, Adrian, Andriulli, *On a Calderón preconditioner for the symmetric formulation of the electroencephalography forward problem without barycentric refinements*, (Journal of Computational Physics, 2023)" we proposed a stabilization strategy for the symmetric formulation—a common integral equation used in EEG forward modeling—aimed at preconditioning the system with respect to both the number of unknowns and the material contrast between the skull and the inner tissues. Notably, our strategy does not require any expensive discretization of operators with dual basis functions on barycentric refinements and gives rise to a symmetric, positive-definite system of linear equations, which allows the application of the conjugate gradient method, an iterative method that exhibits a smaller computational cost compared to other Krylov subspace methods applicable to non-symmetric problems.

Another challenge in electromagnetic simulations is the low-frequency instability, which arises from the decoupling between electric and magnetic field components. In the modeling of penetrable media using the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) equation, this issue is coupled with the strong dependency of the conditioning

of the solution of the system to the conductivity of the scatterer, creating challenges for simulating eddy currents in electrically large-to-mid-sized objects characterized by skin depths which are comparable to the size of the geometry. Moreover, when extremely low-frequency fields are into play, numerical cancellations intrinsic to the floating-point arithmetic can strongly affect the accuracy of the scattered electromagnetic field. All these problems can be handled by treating the quasi-Helmholtz components of the system separately, as shown in "Giunzioni, Scazzola, Merlini, Andriulli, *Low-Frequency Stabilizations of the PMCHWT Equation for Dielectric and Conductive Media: On a Full-Wave Alternative to Eddy-Current Solvers*, (arXiv preprint arXiv:2408.01321, 2024)", where a preconditioning strategy based on quasi-Hemholtz projectors for the PMCHWT equation applied to lossy media has been proposed. The aim of this research is to overcome the use of eddy-currents solvers, which rely on approximations of the physics and, under certain conditions, offer lower accuracy than full-wave solvers, especially in multi-scale scenarios with both electrically large and small objects.

After addressing low-frequency instabilities, we turned to the high-frequency regime, where the frequency increases while the ratio between wavelength and mesh size remains constant. We analyzed some common integral equations for electromagnetic scattering by an infinitely extended cylinder, including electric and magnetic field integral equations and a combined field, preconditioned, resonance-free integral equation. A spectral analysis of the operator matrices made it possible to identify a discrepancy between the eigenvalues of the integral operators and the ones of the matrices discretized by means of boundary element methods, which, for some operators, becomes more and more significant in the high-frequency limit. Further analysis showed that this spectral discrepancy could degrade the accuracy of BEM solutions, even in the ideal conditions of infinite accuracy and infinite precision. Surprisingly and contrary to the common beliefs about BEMs, this led us to the identification of a form of "pollution" affecting some of the equations considered in the high-frequency scenario, where the solution accuracy decreases with increasing frequency. This important discovery opens the possibility to new axes of research on further investigation of the phenomenon and on possibilities of mitigating or overcoming this effect.