

Physical properties and tensile strength evolution of gypsum materials under different water content conditions

Original

Physical properties and tensile strength evolution of gypsum materials under different water content conditions / Wang, C., Wei, S., Zhang, D., Yang, Y., Yu, B., Pan, Y.. - In: CONSTRUCTION AND BUILDING MATERIALS. - ISSN 0950-0618. - 364:(2023). [10.1016/j.conbuildmat.2022.129865]

Availability:

This version is available at: 11583/2997252 since: 2025-02-06T14:17:27Z

Publisher:

Elsevier Ltd

Published

DOI:10.1016/j.conbuildmat.2022.129865

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

Elsevier postprint/Author's Accepted Manuscript

© 2023. This manuscript version is made available under the CC-BY-NC-ND 4.0 license
<http://creativecommons.org/licenses/by-nc-nd/4.0/>. The final authenticated version is available online at:
<http://dx.doi.org/10.1016/j.conbuildmat.2022.129865>

(Article begins on next page)

1 Physical properties and tensile strength evolution of gypsum materials under 2 different water content conditions

3 Chongyang Wang¹², Sijiang Wei³, Dongming Zhang^{12*}, Yushun Yang⁴, Beichen Yu¹², Yisha Pan⁵

4 1. State Key Laboratory of Coal Mine Disaster Dynamics and Control, Chongqing University, Chongqing 400030,
5 China.

6 2. School of Resources and Safety Engineering, Chongqing University, Chongqing 400030, China.

7 3. School of Energy Science and Engineering, Henan Polytechnic University, Jiaozuo 454000, Henan, China.

8 4. Faculty of Architecture and Civil Engineering, Huaiyin Institute of Technology, Huai'an 223001, Jiangsu,
9 China

10 5. School of Surveying, Mapping and Land Information Engineering, Henan Polytechnic University, Jiaozuo
11 454000, Henan, China.

12 * Correspondence: zhangdm@cqu.edu.cn.

13 **Abstract:** To investigate the physical properties and tensile strength evolution of gypsum materials under
14 different water content conditions, three groups of gypsum standard samples were prepared, including natural,
15 saturated and dehydrated gypsum samples. Electron microscopy, composition analysis, Brazilian splitting tests
16 and numerical simulation analysis were carried out. The results are as follows. The larger the water content of
17 the samples is, the higher the content of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, the more flocculation structures on the surface, and the
18 higher the longitudinal wave propagation speed. Saturation and dehydration change the composition of the
19 sample and cause a change in its tensile strength and failure mode. The fitting effect of the modified Hoek–Brown
20 criterion is obviously better than that of the modified Mohr–Coulomb criterion because it considers the influence
21 of uniaxial compressive strength. The Balmer criterion introduces the index b and greatly improves its regression
22 accuracy. The vertical compressive stress exhibits a concentrated distribution. At $x=0$, the peak value of the
23 vertical load appears at the two ends of contact between the specimen and the indenter and gradually decreases
24 from both ends to the centre. The horizontal stress is mainly tensile stress, and the value at $x=0$ is the theoretical
25 tensile strength. It is uniformly distributed on the line directly affected by the vertical load and decreases to both
26 ends of the specimen along the line. It is reduced to 0 at the left and right ends. The horizontal deformation arises
27 from both sides of the disk tip and then spreads downwards in an arc during loading. The horizontal strain in the
28 upper part of the disk is always larger than that in the lower part. The upper part reaches its tensile strength and
29 then fails due to the continuous loading.

30 **Keywords:** gypsum, tensile strength, strength criterion, stress distribution, numerical simulation.

31 1. Introduction

32 In the field of architecture and geotechnical engineering, the tensile strength of brittle materials is the main
33 parameter affecting the failure of engineering structures. As the main method of measuring the tensile strength
34 of materials, the Brazilian splitting test has been widely used in construction, civil engineering, geotechnical
35 engineering and other fields [1-6]. Saksala et al. conducted dynamic Brazilian disc tests on Kuru granite based
36 on viscoplastic and damage mechanics constitutive models and finite element numerical techniques. The results
37 show that the loading rate is linearly correlated with the indirect tensile strength of the Kulu granite disc [7]. Shen
38 et al. investigated the tensile properties of hardened fly ash high-strength concrete. The experimental results
39 showed that its tensile Young's modulus at an early age increased with increasing uniaxial tensile strength [8]. Li
40 et al. analysed the relationship between the tensile strength and fracture area of cement paste specimens through
41 laboratory tests. The results show that the tensile strength is negatively correlated with the addition of SCM and
42 the W/B ratio [9]. Verma et al. used AUSBIT's Brazilian disc test, successfully conducted on concrete and various
43 rocks, to evaluate the effectiveness of AUSBIT in controlling local dynamics using acoustic emission and digital

44 image correlation techniques [10]. Lochan et al. studied the tensile strength of glass fibre-reinforced polymer
45 (GFRP) reinforcing bars through laboratory tests and proposed the three-point flexure test as an alternative for
46 direct tensile testing of GFRP bars. It has been proven that flexural testing can provide an easy and reliable
47 method for obtaining the tensile strength of GFRP reinforcing bars [11]. Gaedicke et al. studied the effect of
48 porosity variation on the relationship between tensile strength and compressive strength through laboratory tests
49 on 16 kinds of mixed materials [12]. Mardoukhi et al. studied the dynamic tensile behaviour of two granitic rocks
50 using the Brazilian disc test and split Hopkinson compression bar. High strain rate Brazilian plate tests were
51 carried out at -30 °C, -50 °C and -70 °C on samples without cyclic thermal loading. The results show that rocks
52 with different microstructures respond differently to cyclic thermal loads. Lowering the test temperature will lead
53 to an increase in the tensile strength of the two rocks being studied [13, 14]. Zhao et al. carried out a Brazilian
54 disk split test on MSC cubes and analysed the effects of the water-cement ratio and stone powder content on the
55 tensile strength of MSC. Forecast models of the long-term tensile strength of MSC were proposed [15].

56 The above studies on the tensile properties of rock and other materials have achieved many meaningful results
57 [16-21]. Liang et al. discussed several kinds of common random distribution functions used to realize the
58 heterogeneity of elastic damage models and studied a probability model of random distribution function failure
59 processes under loading [22]. Yang et al. conducted Brazilian splitting and acoustic emission tests on seven
60 groups of samples with different inclination angles. The results show that the direction of bedding has an
61 important influence on the splitting strength and the final failure mode of shale samples. With the increase in
62 bedding angle, the splitting strength of shale samples decreases gradually [23]. Aggelis studied the different
63 characteristics of the signals emitted by different fracturing stages through acoustic emission monitoring of
64 several kinds of concrete during the experiment and proposed several AE indices, which could be used to classify
65 cracks according to their modes [24]. Jiang et al. used statistical analysis and hypothesis testing to study the
66 random distribution characteristics of the characteristic strength and deformation parameters of two kinds of
67 basalt and pointed out that dispersion is an inherent property of basalt, and only a certain number of repeated
68 tests can reliably evaluate its basic mechanical properties [25]. Du et al. carried out a failure process test under
69 the splitting load of a Brazilian disc of shale with different bedding inclination angles. By using digital image
70 correlation technology, they studied the initiation time, spatial location, propagation law and fracture mechanism
71 of microfractures in carbonized shale with different bedding directions [26]. To study the influence of the
72 interface direction of hard rock and soft rock on its failure form, Li et al. carried out a Brazilian splitting test with
73 multiple groups of loading angles on composite samples containing rock-cement mortar and studied the failure
74 mechanism of the interface using PFC^{2D} software [27]. Yang used PFC^{2D} software with a built-in single disk
75 discrete element model of the Brazilian splitting test, studied the radius ratio and the influence of eccentricity on
76 the disc deformation and strength characteristics, and concluded that the stiffness of a single disk and the
77 maximum tensile strength decrease with increasing radius ratio and increase with increasing eccentricity [28].

78 Finally, there are many studies on the influence of water content on rock properties [29-32]. Vasarhelyi
79 statistically analysed the simple compressive strength and shear Young's modulus of 35 British sandstones. The
80 test results of these sandstones under wet and dry conditions were analysed, and the petrophysical parameters
81 were obtained [33]. Zhou et al. carried out compression and tensile tests on sandstone samples with different
82 water contents. From the laboratory tests, reductions in the compressive and tensile strengths of sandstone under
83 static and dynamic states in different saturation processes were observed [34]. Erguler et al. determined the
84 uniaxial compressive strength, tensile strength and elastic modulus of several rock samples with different water
85 contents. The results show that the uniaxial compressive strength, elastic modulus and tensile strength decrease
86 by 90%, 93% and 90%, respectively, with increasing moisture content [35]. Daraei used the uniaxial compressive
87 strength test results of 67 samples from eight rock types under dry, natural and saturated conditions to illustrate
88 in detail the effect of water content on critical and failure strains [36]. Tang et al. tested the tensile strength of
89 compacted clayey soil with different water contents and dry densities, and the results showed that the tensile

90 strength of compacted clayey soil significantly depends on the water content [37].

91 Although the above scholars have performed much research on deformation, strength and failure
92 characteristics through tensile tests and the influence of water content on the mechanical properties of rock, there
93 are few studies on the mechanism of tensile strength evolution with the microstructure and composition of the
94 material under different water content conditions. In addition, studies on the stress and strain distribution of the
95 loading axial plane in the Brazilian splitting test are also insufficient. Different water-rich times cause different
96 water contents in roofs supporting water-rich tunnels and then change their physical and mechanical properties
97 [33]. Therefore, it is of great significance to study the evolution characteristics of the physical properties and
98 tensile strength of roof rock mass under different water content conditions. However, natural rock joints and
99 fissures exhibit with strong anisotropy. The homogeneity of gypsum-like rock is good after many procedures,
100 such as stirring and vibration. To exclude irrelevant variables, based on previous studies [38-42], this paper selects
101 high-strength gypsum powder to prepare gypsum rocks under different water content conditions and carries out
102 wave velocity tests, electron microscopy scanning, composition analysis, Brazilian splitting and other tests. The
103 physical properties and tensile strength evolution of gypsum samples under different moisture contents were
104 analysed. Three improved strength criteria were used to evaluate the tensile strength of the specimens. On this
105 basis, the plane stress and deformation distributions of the specimens were analysed to provide theoretical
106 reference for the failure and instability mechanism of water-rich rock masses in underground engineering.

107 **2. Materials and Methods**

108 **2.1 Sample Preparation**

109 Grade α high-strength gypsum powder, produced by Sichuan Hongtai Biochemical Co., Ltd., Nanchong, China,
110 was selected. Its main component is $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$. A block was prepared according to a ratio of water to paste
111 of 3:10, and a standard sample was processed. The processed samples were divided into three groups: natural,
112 water-saturated and dehydrated samples. The specific preparation and grouping process can be found in the
113 literature [38, 40]. We will not repeat it here.

114 **2.2 Testing device**

115 2.2.1 Ultrasound testing device

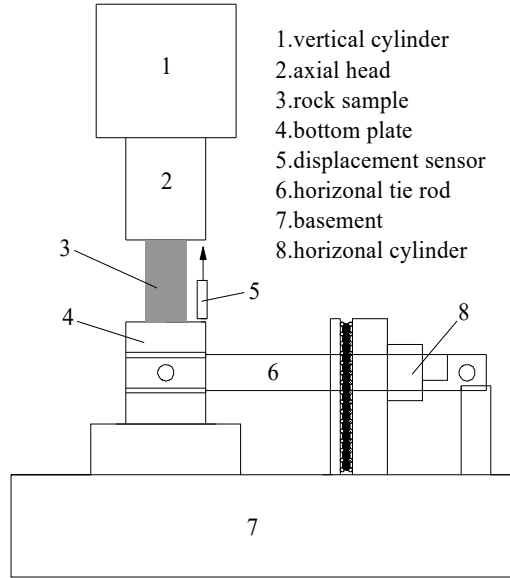
116 The samples were subjected to ultrasonic testing by using an UTA2001A ultrasonic inspection monitor, as
117 shown in Figure 1. The UTA2001A system is an intelligent measuring instrument with a microprocessor. Its
118 sampling frequency can be set to 10, 5, 2, and 1 MHz.



119
120 Figure 1. UTA2001A ultrasonic inspection monitor.

121 2.2.2 Mechanical testing device

122 The test was carried out on an RMT-150B electrohydraulic servo rock test system, as shown in Figure 2. The
123 maximum axial load of the test system was 1000 kN. It can carry out tests such as rock uniaxial compression,
124 indirect (direct) stretching and compression-shear tests.



(a)

(b)

Figure 2. Electro-hydraulic servo rock test system of RMT-150B. (a) Test system. (b) Schematic diagram of uniaxial compression.

2.3 Test Methods

1) Longitudinal wave velocity measurement: The sampling frequency was 10 MHz. The samples were sealed in fresh bags in time before and after the test to avoid water absorption or dehydration of the samples in air (the same below).

2) Composition analysis of the samples: A D8-ADVANCE X-ray diffractometer was used to analyse the composition of the samples in each group to test the composition change characteristics of the samples under different water content conditions.

3) Microscopic composition analysis: The sample fragments with different water contents were analysed by a JSM-6390LV scanning electron microscopy (SEM) system.

4) Brazil splitting test: The test was carried out on an RMT-150B electrohydraulic servo test system. The test adopted the displacement control mode, and the loading rate was 0.005 mm/s.

3. Test results and analysis

3.1 Sample Characteristics

3.1.1 Physical Properties

The density, moisture content, wave velocity and other physical properties of the samples in different states were tested, as shown in Table 1 [38, 43].

In Table 1, the water content (f_w) is the relative value. In this paper, the water content of the samples in the high-temperature dehydration group is 0, and the water content of the natural group and the saturated group is calculated based on high-temperature dehydration group.

Table 1. Physical properties of the samples

| Group | m (g) | f_w | ρ (g/cm ³) | v (m/s) |
|------------------|---------|-------|-----------------------------|-----------|
| Natural | 338.7 | 0.254 | 1.726 | 2967 |
| Water-saturation | 371.2 | 0.374 | 1.891 | 3017 |
| Dehydration | 270.2 | 0 | 1.377 | 1531 |

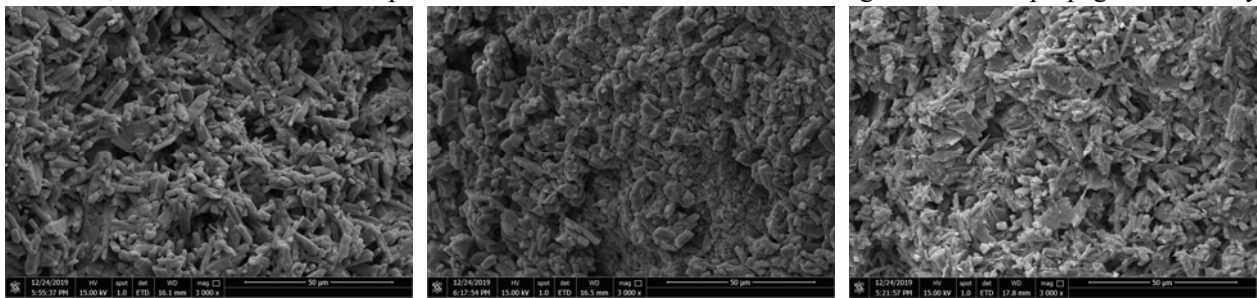
As seen in Table 1, the moisture content and longitudinal wave velocity of the samples in the saturated group are the largest, while those in the high-temperature dehydration group are the smallest. Due to the loss of crystal

150 water in the high-temperature environment, the internal structure of the samples was damaged, resulting in the
 151 samples in the high-temperature dehydration group having the lowest wave velocity. Compared with that of the
 152 natural group, the wave velocity of the high-temperature group was reduced by 48.4%. This may be due to the
 153 uneven heating of minerals inside the sample, resulting in uncoordinated deformation and expansion and then
 154 micromicroscopic cracks. At the same time, the cracking of crystallized water after heating causes corresponding
 155 physical and mechanical changes in the structure and material properties of the sample, resulting in a complex
 156 distribution of cracks and defects. When ultrasonic waves encounter cracks and defects in the propagation process,
 157 they must undergo refraction, diffraction and other phenomena. The weakening of the waveguide property and
 158 the increase in energy attenuation led to a decrease in the longitudinal wave velocity. The longitudinal wave
 159 velocity is closely related to the mechanical properties of samples after high-temperature dehydration.

160 3.1.2 Microscopic composition analysis

161 An FEI Quanta 250 FEG-SEM system was used to scan the microstructure of each sample used in this study.
 162 Figure 3 shows the scanning results with 3000× magnification [38, 39].

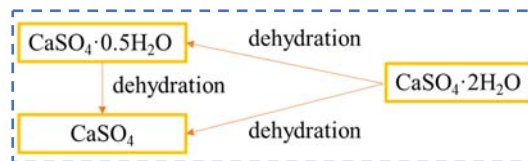
163 As seen from Figure 3, the interior of the remoulded sample under natural conditions is homogeneous and
 164 compact, with dense particles, smooth crystal fractures, sharp edges and corners, and partial lumps, as shown in
 165 Figure 3(a). The water-saturated sample contains more dihydrate gypsum, and the homogeneous compact edges
 166 and angles are not obvious in the sample, as shown in Figure 3(b). After dehydration at high temperature for 24
 167 h, a large number of arranged and disordered prismatic crystals formed inside the sample, and microcracks were
 168 distributed among the crystals. Some crystals exhibited fracture phenomena, the fractures were clearly visible,
 169 and the number of micropores increased significantly, as shown in Figure 3(c). Combined with the physical
 170 properties presented in Section 3.1.1, the larger the moisture content of the sample is, the more flocculent
 171 structures on its surface and the higher the longitudinal wave propagation speed. The smaller the sample moisture
 172 content is, the more disordered the prisms on the surface and the lower the longitudinal wave propagation velocity.



173
 174 (a) (b) (c)
 175 Figure 3. Scanning results at 3000 times magnification. (a) Natural. (b) Water saturation. (c) Dehydration

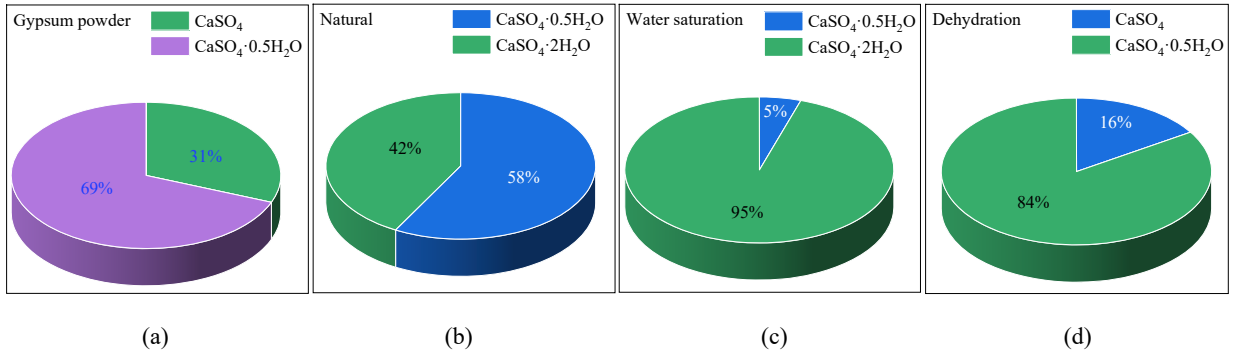
176 3.1.3 Mineral composition analysis

177 Gypsum is a monoclinic crystalline mineral mainly composed of $\text{CaSO}_4 \cdot n\text{H}_2\text{O}$, which exists in various forms,
 178 such as $\text{CaSO}_4 \cdot \text{H}_2\text{O}$, $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$ and $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, due to different water contents, as shown in Figure 4 [42].
 179 There are great differences in the physical properties of gypsum materials with different water contents. Therefore,
 180 it is of great significance to explore the composition of samples under different water content conditions.



181
 182 Figure 4. Schematic diagram of gypsum material composition changes under water absorption and dehydration.

183 X-ray diffraction tests were carried out on high-strength gypsum powder remoulded samples in different water
 184 cut states, and the main mineral crystal components were obtained, as shown in Figure 5.



185

186

187

188

189

190

191

192

193

194

195

196

197

198

199

200

201

202

203

204

205

206

207

Figure 5. Main mineral components of gypsum powder remoulded samples. (a) Gypsum powder. (b) Natural. (c) Water saturation. (d) Dehydration.

As seen from Figure 5, after casting high-strength gypsum powder into the remoulded sample, calcium sulfate disappeared, mainly manifested as CaSO_4 , the water content increased, and $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$ became $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$ and $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ by absorbing water. The main mineral composition of the saturated sample was the same as that of the natural sample, but the main difference is that the saturated sample contained a higher proportion of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$. After high-temperature dehydration at 220°C , $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ became CaSO_4 and $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$ by dehydration.

Combined with the analysis in Section 3.1.2, the flocculent structure on the surface of the water-saturated sample and natural sample is $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, while the rip-shaped crystal on the surface of the high-temperature dehydrated sample is $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$. Through saturation and dehydration, different structural components are generated in the sample, and these structural components have a great influence on the physical properties (longitudinal wave velocity, microstructure, etc.) of the sample.

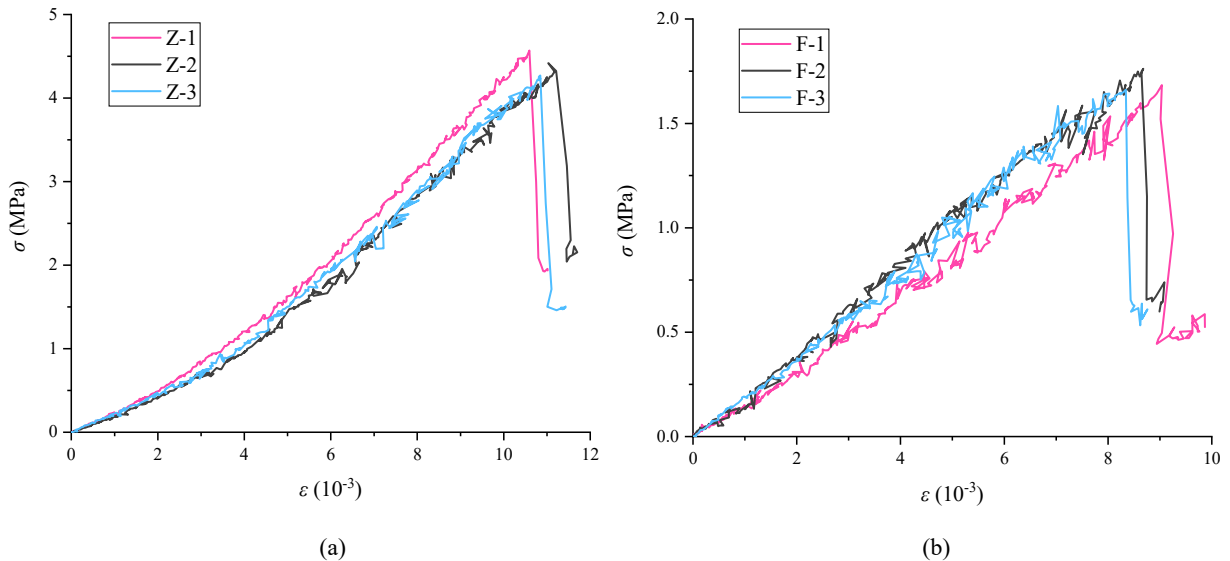
3.2 Stress–strain characteristics

The formula for calculating the tensile strength of each sample in different water content states is

$$\sigma_t = \frac{2P}{\pi Dt} \quad (1)$$

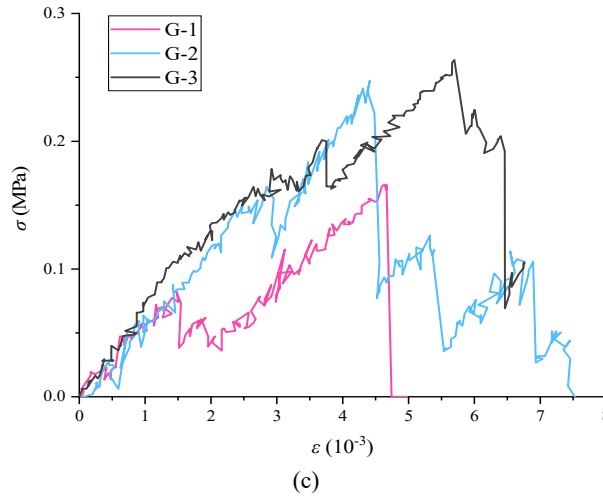
where σ_t is the tensile strength of rock, P is the failure load of the sample, D is the diameter of the sample, and t is the thickness of the sample.

According to Equation (1), the splitting tensile strength of each sample in different water cut states was calculated, and its stress–strain curve was drawn, as shown in Table 2 and Figure 6.



208

209



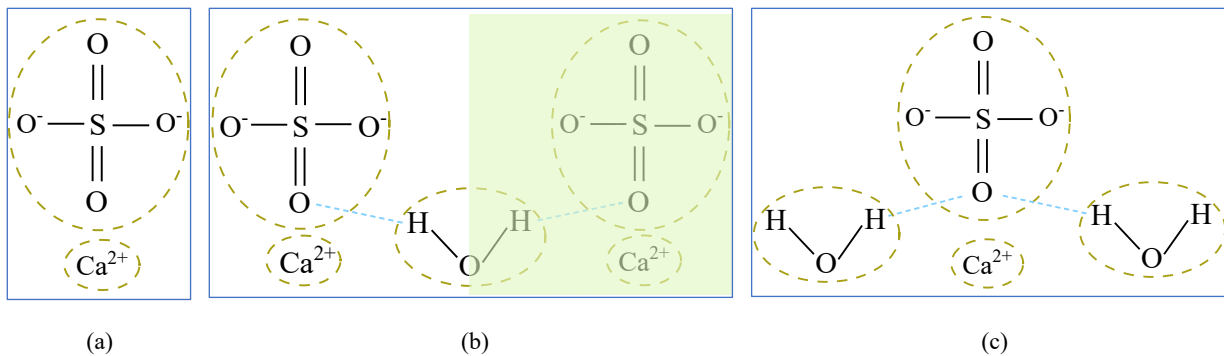
210
211
212 Figure 6. Stress-strain curves of each sample in Brazil splitting test. (a) Natural. (b) Water saturation. (c) Dehydration.

213 Meanwhile, by analysing the stress–strain curve of Figure 6, the tensile splitting failure process of natural,
214 saturated and high-temperature dehydrated specimens can be divided into four stages: the compaction stage,
215 elastic deformation stage, plastic deformation stage and failure stage. When the natural and saturated samples
216 reached the peak tensile strength, the stress–strain curve dropped instantaneously, and the samples showed
217 obvious brittle failure. However, the high-temperature dehydrated samples showed the characteristics of multiple
218 splits during the splitting process, and the plastic deformation stage time became significantly longer. Moreover,
219 when the load reached the peak value, the samples did not fail rapidly, showing certain plastic failure
220 characteristics.

221 Table 2. Statistical table of tensile strength of gypsum rocks

| Group | σ_t (MPa) | Group | σ_t (MPa) | Group | σ_t (MPa) |
|------------|------------------|------------------|------------------|-------------|------------------|
| Natural | 3.858 | Water saturation | 1.452 | Dehydration | 0.142 |
| | 3.939 | | 1.53 | | 0.264 |
| | 3.76 | | 1.467 | | 0.215 |
| Mean value | 3.85 | Mean value | 1.48 | Mean value | 0.207 |

222 This corresponds to the physical characteristics of the sample in Section 3.1. To further analyse the relationship
223 between the strength and composition of the sample, the molecular structure diagram of the sample under
224 different moisture content conditions is shown in Figure 7. The samples in the natural group contained more
225 $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$. As Figure 7 (b) shows, $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$ belongs to the monoclinic system, and a $[\text{SO}_4]^{2-}$
226 tetrahedron and Ca^{2+} are connected into a layered structure parallel to the $[1\ 0\ 0]$ and $[0\ 1\ 0]$ surfaces. Half of the
227 H_2O molecules are located in this channel and are connected with O^{2-} in $[\text{SO}_4]^{2-}$ by hydrogen bonds, making this
228 a relatively stable state. This is the reason for the maximum tensile strength of the natural specimens.



229
230

231 Figure 7. Molecular structure of samples under different moisture content (a) CaSO₄. (b) CaSO₄·0.5H₂O. (c) CaSO₄·2H₂O

232 According to Figure 5(c), the composition of the water saturated sample is the same as that of the natural
 233 sample and includes CaSO₄·0.5H₂O and CaSO₄·2H₂O. However, the content of CaSO₄·2H₂O in the water-
 234 saturated sample greatly increased. Figure 7 (c) shows that CaSO₄·2H₂O belongs to the monoclinic crystal system.
 235 A [SO₄]²⁻ tetrahedron and Ca²⁺ are connected into a bilayer structure parallel to the [0 1 0] surface, and two H₂O
 236 molecules are distributed between the bilayer. Among them, the O²⁻ molecules in the [SO₄]²⁻ tetrahedron and H₂O
 237 are connected by hydrogen bonds, demonstrating a relatively stable state. As a result, the tensile strength of the
 238 specimens in the water-saturated group was second only to that in the natural group, and the stress–strain curve
 239 characteristics of the specimens under Brazilian disc loading were similar to those in the natural group.

240 The dehydration group contained a large amount of CaSO₄. Figure 7 (a) shows that CaSO₄ belongs to the
 241 orthorhombic system, and the crystal structure is composed of [SO₄]²⁻ tetrahedra and Ca²⁺. Due to the loss of a
 242 large amount of free water and structural water in the oven, its physical and mechanical properties began to
 243 become extremely unstable, the tensile strength was greatly reduced, and it showed obvious plasticity during the
 244 loading process. This shows that high-temperature dehydration has an obvious deterioration effect on the sample.

245 3.3 Strength Characteristics

246 3.3.1 Mohr–Coulomb criterion

247 According to Mohr–Coulomb strength criterion [45],

$$248 \begin{cases} \tau = c + \sigma \tan \varphi \\ \sigma_t = \frac{2c \cos \varphi}{1 + \sin \varphi} \end{cases} \quad (2)$$

249 where τ is the shear stress, σ is the normal stress, c is cohesion, and φ is the angle of internal friction.

250 Using Equation (2) to solve the tensile strength of the sample requires mechanical parameters such as cohesion
 251 and the internal friction angle. Based on previous laboratory tests, the basic mechanical parameters of the gypsum
 252 rocks used in this paper are summarized, as shown in Table 3 [39].

253 Table 3. Physical and mechanical properties of gypsum rocks

| Group | c (MPa) | φ (°) | σ_c (MPa) |
|------------------|-----------|---------------|------------------|
| Natural | 13.18 | 27 | 44.53 |
| Water saturation | 11.39 | 15 | 22.2 |
| Dehydration | 8.50 | 28.7 | 9.14 |

254 Substituting the mechanical parameter results in Table 3 into Equation (2), it can be rewritten as

$$255 \begin{cases} \tau_1 = 13.18 + 27\sigma_1 \\ \tau_2 = 11.39 + 15\sigma_2 \\ \tau_3 = 8.50 + 28.7\sigma_3 \end{cases} \quad (3)$$

256 Equation (3) can be used to preliminarily calculate the tensile strength σ_t of each sample under different water
 257 content conditions as follows:

$$258 \begin{cases} \sigma_{t1} = \frac{2 * 13.18 * \cos 27^\circ}{1 + \sin 27^\circ} = 16.1 \text{ MPa} \\ \sigma_{t2} = \frac{2 * 11.39 * \cos 15^\circ}{1 + \sin 15^\circ} = 17.5 \text{ MPa} \\ \sigma_{t3} = \frac{2 * 8.5 * \cos 28.7^\circ}{1 + \sin 28.7^\circ} = 10.1 \text{ MPa} \end{cases} \quad (4)$$

260 According to Equation (4), the tensile strength calculated by the Mohr–Coulomb criterion differs greatly from

263 the actual value, and the average value of the former is 8.6 times that of the latter. Therefore, it is necessary to
 264 propose a modified Mohr–Coulomb strength criterion:

$$265 \quad \sigma_t' = \alpha \cdot \frac{2c \cos\varphi}{1 + \theta \sin\varphi} + \beta$$

266 (5)

267 In Equation (5), α , β and θ are the cohesion force correction coefficient, internal friction angle correction
 268 coefficient and intercept correction coefficient, respectively. Through the approximation of the formula and data,
 269 the value of each parameter can be calculated as follows:

$$270 \quad \begin{cases} \alpha = 0.38 \\ \beta = -5.2 \\ \theta = 0.1 \end{cases}$$

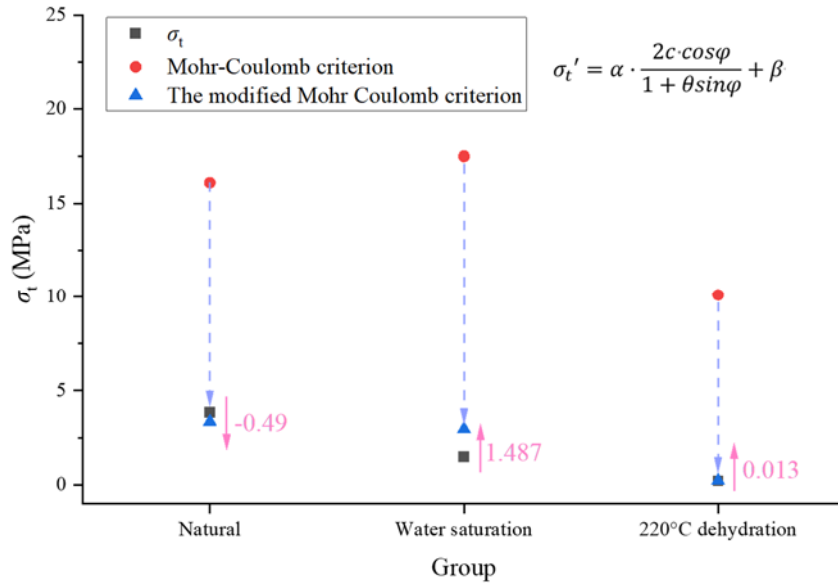
271 (6)

272 By substituting Equation (6) into Equation (5), the modified value σ_t' of the tensile strength of each sample
 273 under different water content conditions is

$$274 \quad \begin{cases} \sigma_{t1}' = 3.36 \text{ MPa} \\ \sigma_{t2}' = 2.97 \text{ MPa} \\ \sigma_{t3}' = 0.22 \text{ MPa} \end{cases}$$

275 (7)

276 To further compare the fitting degree of the modified Mohr–Coulomb strength with the experimental data, the
 277 tensile strength–water content curve was drawn, as shown in Figure 8.



278
 279 Figure 8. Tensile strength–water content curve under Mohr–Coulomb strength criterion.

280 As seen from Figure 8, the accuracy of the modified Mohr–Coulomb strength criterion is significantly
 281 improved compared with that before the modification. Compared with the original value, the average value of
 282 the modified Mohr–Coulomb strength criterion is 18% different, which can reflect the change law of the cohesion,
 283 internal friction angle and tensile strength of gypsum rock samples under different conditions to a certain extent.

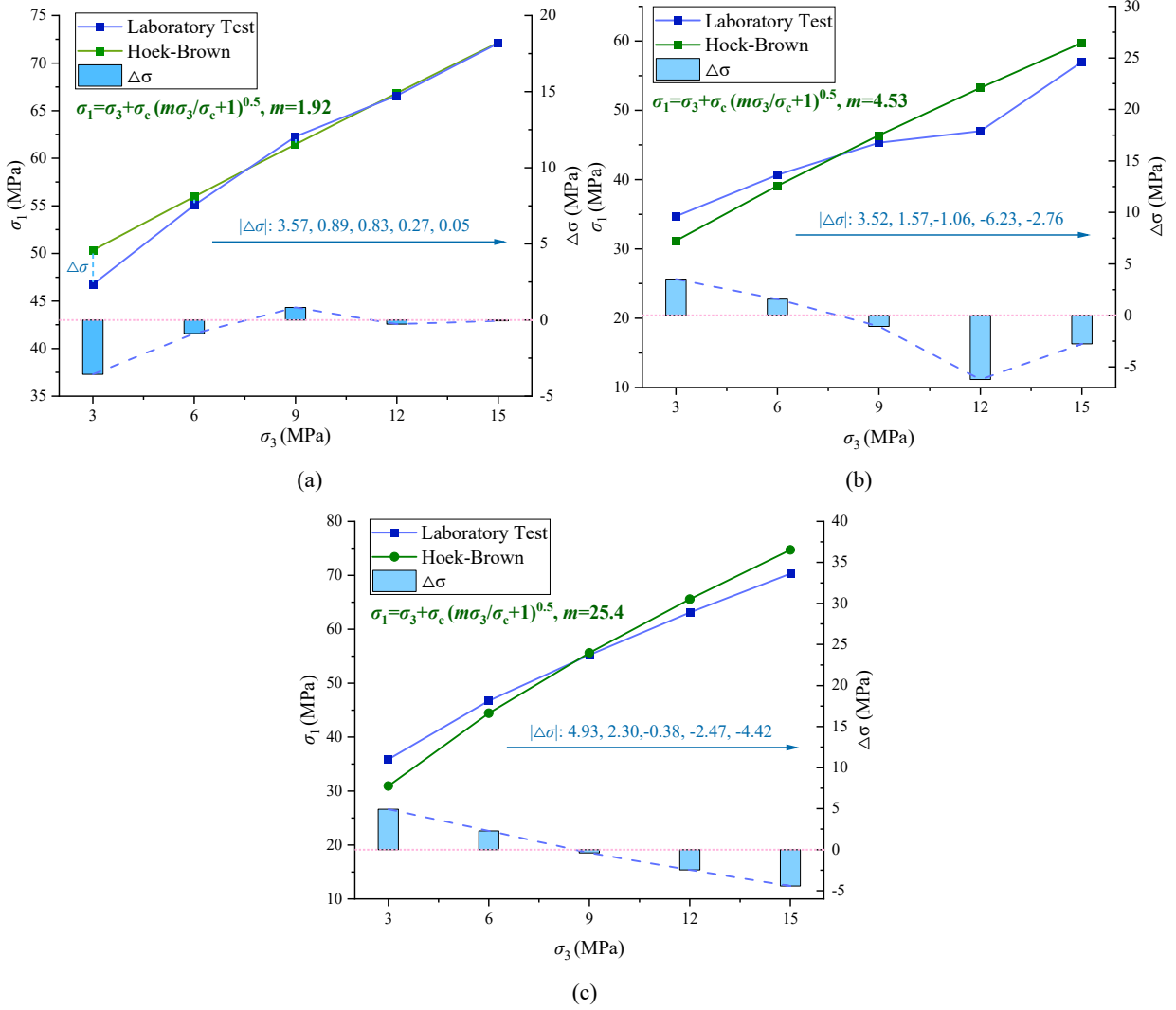
284 3.3.2 Hoek–Brown criterion

285 According to the Hoek–Brown criterion [46, 47],

$$286 \quad \sigma_1 = \sigma_3 + \sigma_c \sqrt{\frac{l\sigma_3}{\sigma_c} + 1}$$

287 where σ_1 and σ_3 are the maximum and minimum principal stresses at rock failure, respectively; l is the
 288 empirical parameter; and σ_c is the uniaxial compressive strength of rock.

289 According to Equation (8), the $\sigma_1 - \sigma_3$ curves of the triaxial test in the laboratory under the Hoek–Brown
 290 criterion are drawn, as shown in Figure 9.



291
292

293
294

295 Figure 9. Curves of $\sigma_1 - \sigma_3$ under Hoek–Brown criterion. (a) Natural. (b) Water saturation. (c) Dehydration.

296 Figure 9 shows that the $\sigma_1 - \sigma_3$ curve under the Hoek–Brown criterion has a high correlation with the
 297 laboratory curve fitting, the maximum residual value does not exceed 7 MPa, and the floating proportion does
 298 not exceed 15%. This indicates that the Hoek–Brown criterion can well reflect the variation characteristics of the
 299 strength parameters of gypsum rocks under triaxial compression.

300 In the Hoek–Brown criterion, the formula for calculating tensile strength is

$$301 \quad \sigma_t = \frac{2\sigma_c}{l + \sqrt{l^2 + 4}} \quad (9)$$

302 Substituting the l values calculated in Equation (8) and Figure 11 into Equation (9), the tensile strength and
 303 fitting parameters of the samples under the three conditions can be calculated as follows:

$$304 \quad \begin{cases} \sigma_{t1} = 18.98MPa \\ \sigma_{t2} = 4.68MPa \\ \sigma_{t3} = 0.36MPa \end{cases}, \begin{cases} l_1 = 1.92 \\ l_2 = 4.53 \\ l_3 = 25.4 \end{cases} \quad (10)$$

305
306 According to Equation (10), the tensile strength calculated by the Hoek–Brown criterion is also very different
 307 from the actual value, and the average value of the former is 4.3 times that of the latter. Based on the modified
 308 Mohr–Coulomb criterion, a modified Hoek–Brown criterion is also proposed, namely,
 309

310

$$\sigma_t' = \alpha \frac{2\sigma_c}{l + \sqrt{l^2 + 4}} + \beta = \alpha\sigma_t + \beta$$

311

(11)

312

313

314

In Equation (11), α and β are the uniaxial strength correction coefficient and intercept correction coefficient, respectively. Through the approximation of the formula and data, the value of each parameter can be calculated as follows:

315

$$\begin{cases} \alpha = 0.19 \\ \beta = 0.18 \end{cases}$$

316

(12)

317

318

By substituting Equation (12) into Equation (11), the modified value σ_t' of the tensile strength of each sample under different water content conditions is

319

$$\begin{cases} \sigma_{t1}' = 3.79MPa \\ \sigma_{t2}' = 1.07MPa \\ \sigma_{t3}' = 0.25MPa \end{cases}$$

320

(13)

321

322

323

According to Equation (13), compared with the original value, the average value calculated by the modified Hoek–Brown criterion is 8% different, is better than the modified Mohr–Coulomb criterion and can accurately reflect the variation in the strength parameters of samples in different states.

324

3.3.3 Balmer criterion

325

According to the Balmer criterion [48],

326

$$\sigma_1 = \sigma_c \left(1 + \frac{\sigma_3}{\sigma_t}\right)^b$$

327

(14)

328

329

where σ_c and σ_t are the uniaxial compressive strength and tensile strength of rock, respectively, and b is the test constant. According to Table 3, the test data are substituted into Equation (14), namely,

330

$$\text{Balmer criterion: } \begin{cases} 46.75 = \sigma_c \left(1 + \frac{3}{\sigma_t}\right)^b \\ 62.29 = \sigma_c \left(1 + \frac{9}{\sigma_t}\right)^b \\ 72.09 = \sigma_c \left(1 + \frac{15}{\sigma_t}\right)^b \end{cases}, \begin{cases} 34.71 = \sigma_c \left(1 + \frac{3}{\sigma_t}\right)^b \\ 45.33 = \sigma_c \left(1 + \frac{9}{\sigma_t}\right)^b \\ 56.98 = \sigma_c \left(1 + \frac{15}{\sigma_t}\right)^b \end{cases}, \begin{cases} 35.86 = \sigma_c \left(1 + \frac{3}{\sigma_t}\right)^b \\ 55.23 = \sigma_c \left(1 + \frac{9}{\sigma_t}\right)^b \\ 70.29 = \sigma_c \left(1 + \frac{15}{\sigma_t}\right)^b \end{cases}$$

331

(15)

332

According to Equation (15), σ_c and σ_t can be calculated by fitting:

333

$$\begin{cases} \sigma_{t1} = 3.88 \\ \sigma_{t2} = 1.45 \\ \sigma_{t3} = 0.22 \end{cases}, \begin{cases} b_1 = 0.28 \\ b_2 = 0.37 \\ b_3 = 0.48 \end{cases}$$

334

(16)

335

336

337

338

According to Equation (16), the error of the average value calculated by the Balmer criterion compared with the original value is only 0.14%, which is the highest accuracy among the three criteria. This indicates that the Balmer criterion is most suitable to reflect the variation law of the strength parameters of samples in different states.

339

3.3.4 Comparison of the three criteria

340

341

Figure 10 shows compares the tensile strength values predicted by the three criteria. The upper right corner of Figure 10 shows the residual diagram of the calculated results of each criterion and the original value.

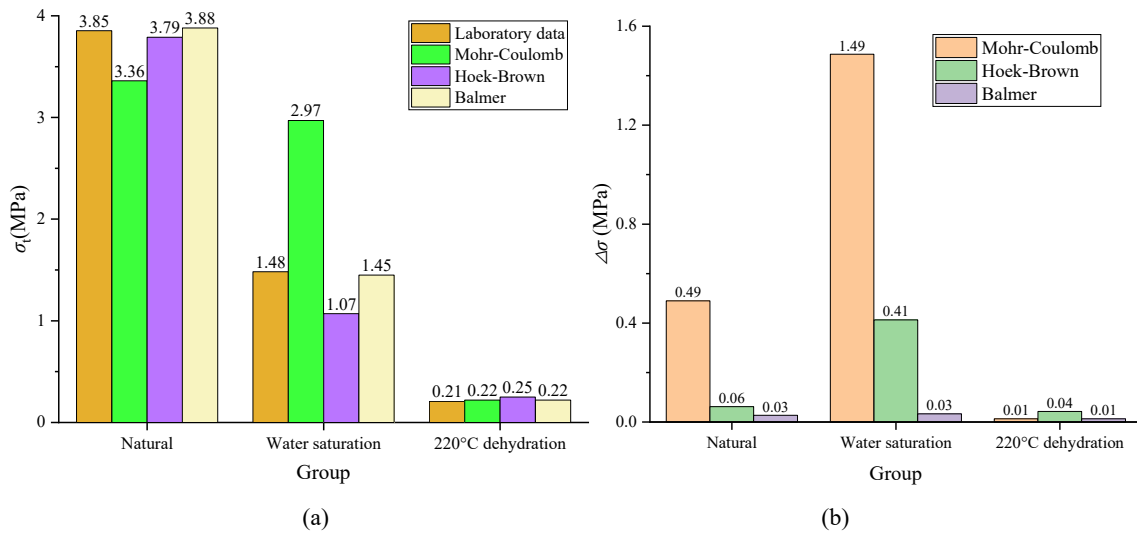


Figure 10. Comparison of predicted values of the three tensile strength criteria. (a) Predicted results. (b) $\Delta\sigma$.

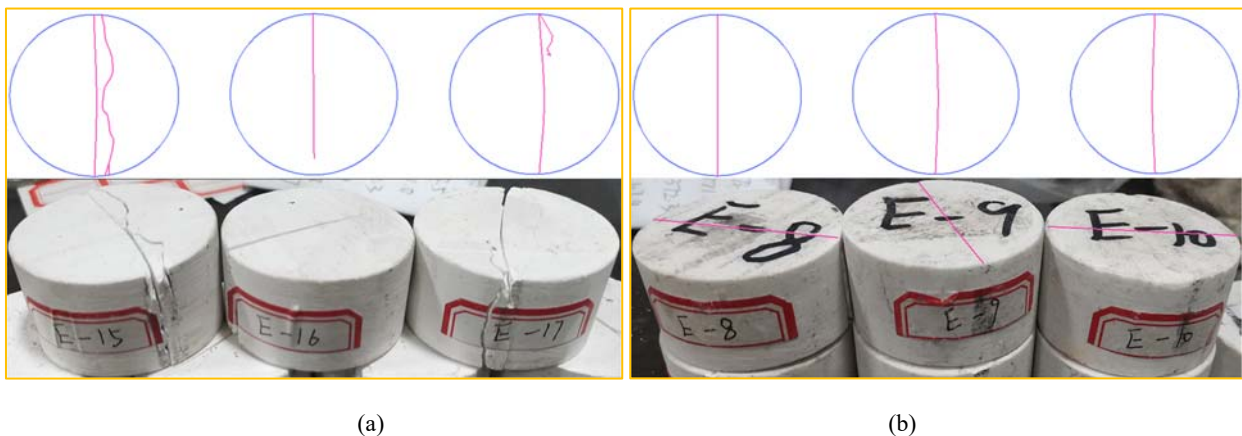
Figure 10 shows that the modified Mohr–Coulomb criterion is quite different from the actual value in evaluating the mechanical properties of the samples in the natural and saturated groups and is more accurate in evaluating the samples in the high-temperature dehydration group. This is because the Mohr–Coulomb criterion mainly considers the mechanical parameters in the conventional triaxial compression test, such as the internal friction angle and cohesion of the sample, and relies more on the triaxial test results of the sample. The more accurate the triaxial test results are, the closer the calculated tensile strength value is to the actual value.

Compared with the Mohr–Coulomb criterion, the modified Hoek–Brown criterion has significantly improved the fitting effect when evaluating the mechanical properties of the natural and saturated samples. This is because, in addition to the triaxial test results, the Hoek–Brown criterion also considers the uniaxial compressive strength of the samples, which makes the calculation results more accurate and the fitting accuracy higher than that with the Mohr–Coulomb criterion.

In addition to considering the results of a single triaxial test, the Balmer criterion also introduces exponential b , which changes the criterion equation from a polynomial to an exponential function, improves the adaptability of the equation and greatly improves its regression accuracy.

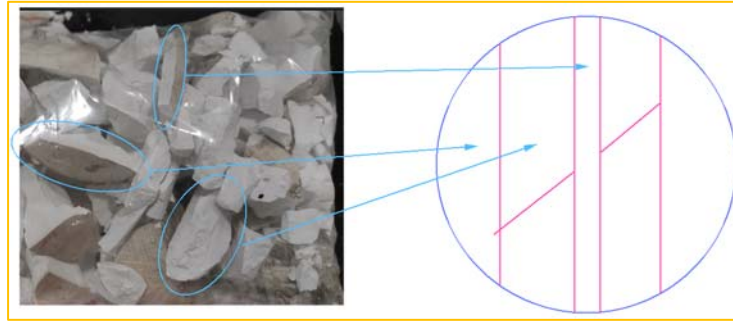
3.4 Failure characteristics

Rock failure is caused by fracture development, including microcrack initiation, propagation and macroscopic crack formation [49, 50]. To study the difference in the failure characteristics of samples with different moisture contents after the splitting test in Brazil, the plane crack distribution after failure was plotted, as shown in Figure 11.



(a)

(b)



(c)

Figure 11. Failure characteristics of specimens after splitting test. (a) Natural. (b) Water saturation. (c) Dehydration.

Figure 11 shows that the specimens in the natural group exhibit obvious brittle failure characteristics, with cracks first appearing from the top of the disk and then gradually extending downwards. The samples in the natural group were not dehydrated or saturated, so their molecular structure was relatively stable, and the mutual attraction between adjacent parts was also strong. In the case of tensile failure of the disc, the middle adjacent part experiences secondary failure due to a large friction force, resulting in multiple cracks.

Compared with that of the natural group, the plasticity of the water-saturated group is enhanced, and the failure characteristics of the samples are not obvious. The specimen pieces after failure are relatively complete and can be pieced together into a disk again. This is because the higher water content reduces the brittleness of the sample, and more H_2O molecules surround $CaSO_4$ ions, playing a "buffer" role. When the specimen is damaged, the impact effect is greatly weakened, and secondary damage to the specimen is avoided.

Compared with the tensile strengths of the natural group and the water-saturated group, the tensile strength of the samples in the dehydration group decreased significantly, only approximately 0.2 MPa. In a splitting test, the specimen quickly reached its tensile strength and became fragmented. After roughly piecing the sample together, the crack direction of the sample is still vertical, and the sample is split into prisms, hemispheres, hexahedra with a parallelogram cross section and other shapes. This is because long-term dehydration causes the sample to lose most of its bound water and free water, the bonding force between $CaSO_4$ ions is greatly reduced, and the load that the sample can bear is also reduced. Therefore, it is easy to break under an external force.

4. Discussion

4.1 Plane stress distribution of the Brazilian disc

A schematic diagram of the traditional Brazilian splitting test is shown in Figure 12. In Figure 12, BC is the theoretical fracture surface of the sample, DE is the vertical segment of the theoretical fracture surface, r_1 is the distance from point A to point B, and r_2 is the distance from point A to C.

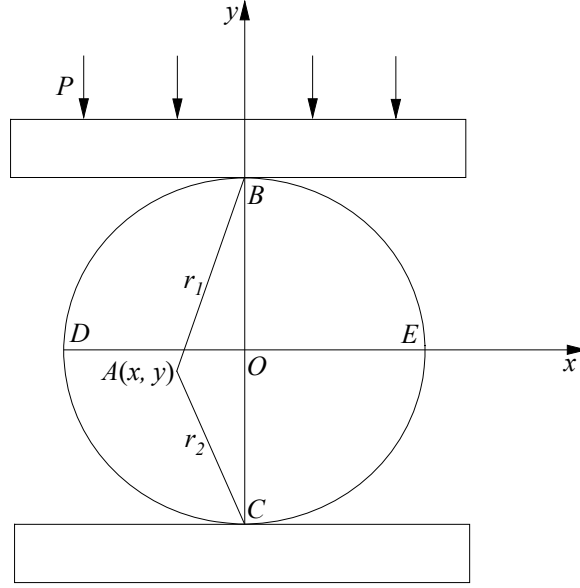


Figure 12. Schematic diagram of splitting test in Brazil.

Using the line of segment DE as the X-axis and the line of segment BC as the Y-axis, the XOY coordinate system is established, and the analytic formulas for stresses at A(x, y) are as follows [51]:

$$\begin{cases} \sigma_x = \frac{2P}{\pi t} \left\{ \frac{x^2(R+y)}{[x^2 + (R+y)^2]^2} + \frac{x^2(R-y)}{[x^2 + (R-y)^2]^2} \right\} - \frac{P}{\pi Rt} \\ \sigma_y = \frac{2P}{\pi t} \left\{ \frac{(R+y)^3}{[x^2 + (R+y)^2]^2} + \frac{(R-y)^3}{[x^2 + (R-y)^2]^2} \right\} - \frac{P}{\pi Rt} \\ \tau_{xy} = \frac{2P}{\pi t} \left\{ \frac{x(R+y)^2}{[x^2 + (R+y)^2]^2} + \frac{x(R-y)^2}{[x^2 + (R-y)^2]^2} \right\} \end{cases} \quad (17)$$

where R is the radius, t is the thickness, P is the concentrated loading force, σ_x and σ_y are the normal stresses, and τ_{xy} is the shear stress.

Equation (17) reflects the plane stress distribution of the Brazilian splitting disk sample. When $x=0$, Equation (17) can be simplified as follows:

$$\begin{cases} \sigma_x = -\frac{P}{\pi Rt} \\ \sigma_y = \frac{2P}{\pi t} \left\{ \frac{1}{R+y} + \frac{1}{R-y} \right\} - \frac{P}{\pi Rt} \\ \tau_{xy} = 0 \end{cases} \quad (18)$$

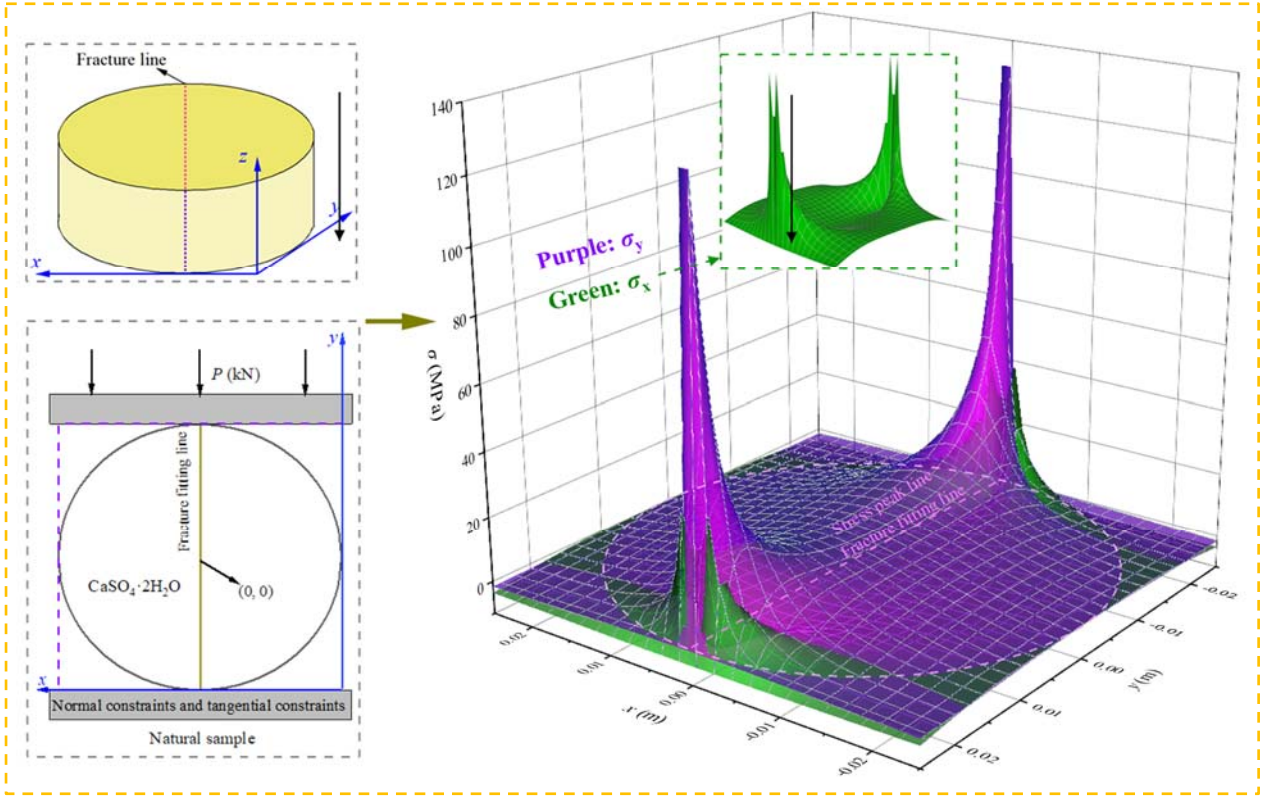
When $y=0$, Equation (17) can be written as

$$\begin{cases} \sigma_x = \frac{2P}{\pi t} \left\{ \frac{x^2 R}{(x^2 + R^2)^2} + \frac{x^2 R}{(x^2 + R^2)^2} \right\} - \frac{P}{\pi Rt} \\ \sigma_y = \frac{2P}{\pi t} \left\{ \frac{R^3}{(x^2 + R^2)^2} + \frac{R^3}{(x^2 + R^2)^2} \right\} - \frac{P}{\pi Rt} \\ \tau_{xy} = \frac{2P}{\pi t} \left\{ \frac{xR^2}{[x^2 + R^2]^2} + \frac{xR^2}{[x^2 + R^2]^2} \right\} \end{cases} \quad (19)$$

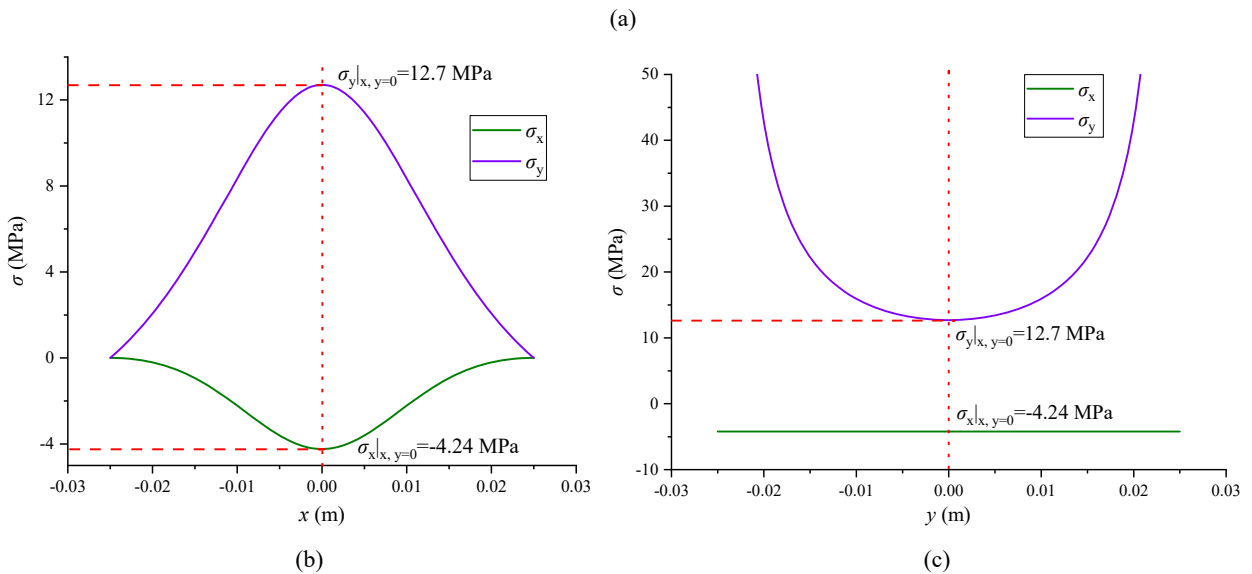
σ_x in Equation (18) is numerically equal to the tensile strength σ_t in Equation (1), which reflects the horizontal tensile stress at line segment BC ($x=0$). σ_y represents the vertical compressive stress at segment BC. $\tau_{xy} = 0$ means that at $x=0$, the shear stress is equal to zero. σ_x in Equation (19) reflects the horizontal tensile

409 stress value at line segment DE, which is $y=0$. σ_y reflects the vertical compressive stress at segment DE. $\tau_{xy} =$
 410 0 means that at $y=0$, the shear stress is equal to zero.

411 To explore the plane stress distribution law of the Brazilian splitting disk sample, the plane distribution curves
 412 of horizontal and vertical stress in Equations (17) to (19) are drawn by taking the natural sample as an example,
 413 as shown in Figure 13. At the same time, to further analyse the variation trend of the horizontal and vertical stress
 414 of the sample at $x=0$ and $y=0$, the two stress–displacement curves in Figure 13 were fused to obtain Figure 14.



415
 416



417
 418

419 Figure 13. Plane stress distribution of Brazilian split disk specimen. (a) Plane stress. (b) $y=0$. (c) $x=0$.

420 Figures 13 and 14 show that the vertical stress of the Brazilian disc sample presents the characteristics of a
 421 concentrated distribution. Along the X-axis direction, the vertical stress inside the sample increases first and then
 422 decreases with increasing x , and the vertical stress reaches its maximum at $x=0$. When $x=0$, along the Y-axis
 423 direction, the vertical stress in the sample first decreased and then increased with increasing y , and the vertical
 424 stress peaked at the line $(0, \pm 0.025)$. From the above analysis, the vertical stress in the Brazilian disc sample is

425 compressive stress and concentrated at $x=0$, where the vertical load P directly acts. At $x=0$, the peak value of the
 426 vertical load appeared at both ends of the line, that is, the two ends of contact between the specimen and the
 427 indenter. From both ends of the specimen to the centre, the vertical stress gradually decreases because the gradient
 428 stress is generated under the action of external forces.

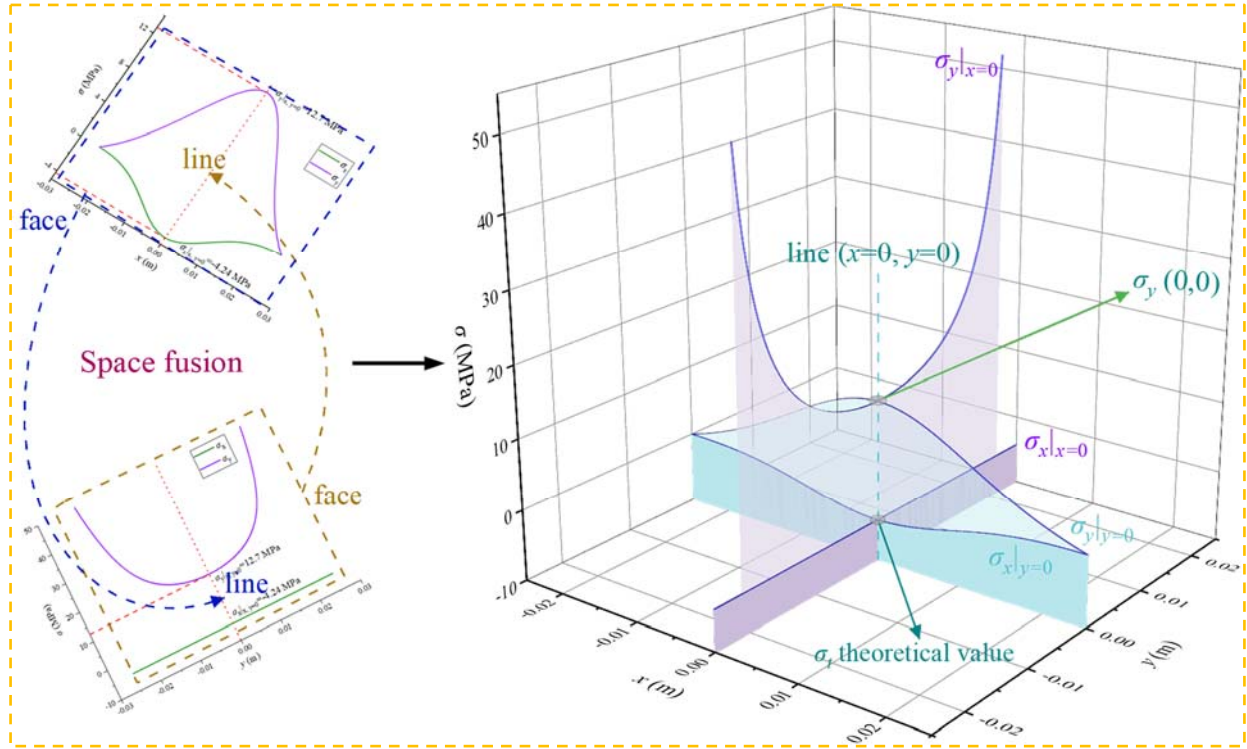


Figure 14. Stress-displacement curves of disk plane when $x, y=0$.

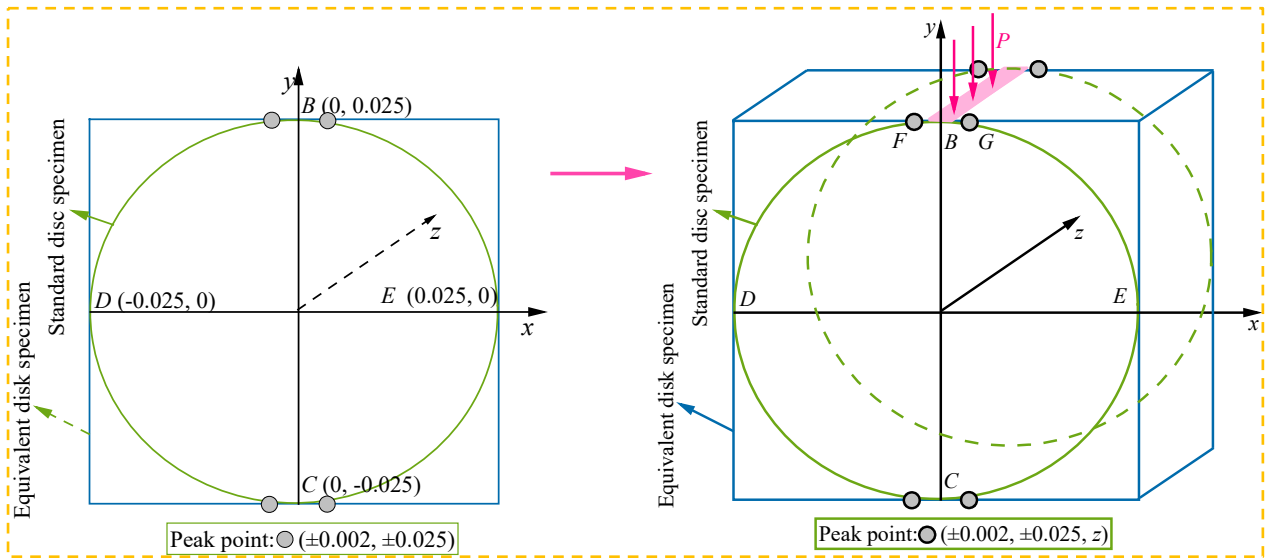
429
 430
 431 By observing the horizontal stress in Figures 13 and 14, the variation in the horizontal stress in the sample is
 432 relatively complex. At $x=0$, the horizontal stress of the sample is stable to 4.24 MPa, which is the theoretical
 433 tensile strength σ_t . For the convenience of plotting, the diameter and thickness of the sample are $\Phi = 0.05$ m and
 434 $d = 0.03$ m, resulting in a difference between the conclusion and the laboratory results, which does not affect the
 435 analysis of the spatial distribution of stress in the sample. At $y=0$, along the X-axis direction, the horizontal stress
 436 inside the sample increases first and then decreases with the increase in x . From the above analysis, the horizontal
 437 stress in the Brazilian disc sample is tensile stress, which is concentrated at $x=0$ and uniformly distributed on the
 438 line directly affected by the vertical load P , and decreases from this line to both ends of the specimen, decreasing
 439 to 0 at the left and right ends of the specimen $(\pm 0.025, 0)$.

440 Figures 13 and 14 show an interesting phenomenon. The horizontal stress distribution in Figure 13 (the figure
 441 in the green dashed box) shows that there are only points $(0, \pm 0.025)$ at both ends of the $x=0$ line that maintain
 442 the steady state of horizontal stress. When the two points are extended in any direction of X, the horizontal stress
 443 will be greatly increased. For example,

$$\begin{cases} \lim_{x=0, y \rightarrow -0.025^+} \sigma_x = -4.24 \text{ MPa} \\ \lim_{x=0, y \rightarrow -0.025^-} \sigma_x = 30 \text{ MPa} \\ \sigma_x(x = \pm 0.002, y = \pm 0.025) = 40 \text{ MPa} \end{cases}$$

(20)

445
 446 Equation (20) shows many strange phenomena; for example, the horizontal stress curve is not differentiable at
 447 $(0, 0.025)$ and oscillates abnormally at the end of the disk. Points $(\pm 0.002, \pm 0.025)$ are not in the range of the
 448 standard disk sample but in the boundary of the equivalent disk sample. To intuitively understand the causes of
 449 the horizontal stress peak, schematic drawings of the standard disc sample and equivalent disc sample are shown
 450 in Figure 15.



451

452

Figure 15. Standard disc sample and equivalent disc sample.

453

454

455

In Figure 15, the green circle is the section of the Brazilian disk sample, and the blue rectangle is the domain of stress represented by Equation (17). For the Brazilian splitting test, the part outside the green cylinder is not of practical significance, but the cause of its stress change is worth exploring.

456

457

458

459

460

461

462

As shown in Figure 15, the Z-axis is introduced to make the plane figure three-dimensional. Equation (17) expresses the cube with local forces on the top. When the cube does not exist, there is no constraint around the disk sample; in arc \widehat{DF} and \widehat{EG} sections prone to deformation, it is not easy to produce a stress concentration. However, when the cube exists, it is equivalent to adding constraints around the disk, leading to the horizontal stress peak at $(\pm 0.002, \pm 0.025)$ due to stress concentration. This is the reason why the left and right limits of the horizontal stress curve in Equation (20) are not equal at point $(0, \pm 0.025)$ and the stress is concentrated at point $(\pm 0.002, \pm 0.025)$.

463

4.2 Plane strain distribution of the Brazilian disc

464

465

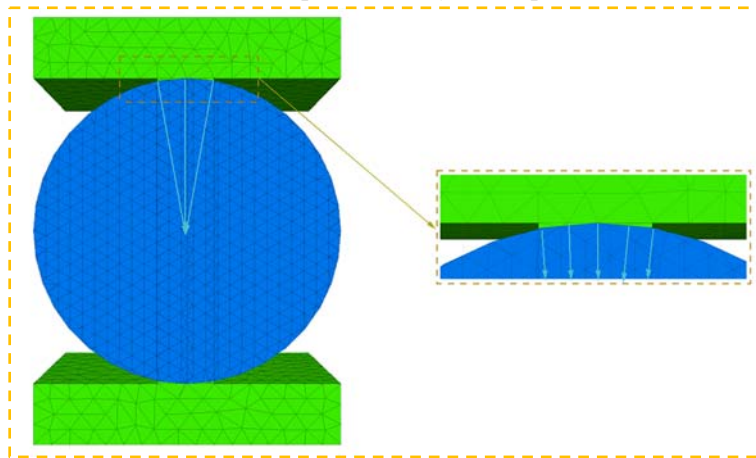
466

467

In Section 3.3 and Section 4.1, we analysed the strength characteristics and stress distribution characteristics of the sample in the splitting test, but the analysis is from the perspective of stress. To further explore the deformation mechanism of the sample in the Brazilian splitting test, 3DEC software was used to simulate the Brazilian splitting test by taking the natural sample as an example.

468

The established model was divided into three parts, namely, 1 sample and 2 indenters, as shown in Figure 16.



469

470

Figure 16. 3DEC model.

471

472

473

In Figure 16, the indenter is rectangular in shape, and the size is $50 \text{ mm} \times 25 \text{ mm} \times 10 \text{ mm}$. The sample size is $\Phi 50 \text{ mm} \times d 25 \text{ mm}$, which is consistent with the theoretical sample size in Section 4.1. The contact end between the indenter and the specimen is arc-shaped to ensure that the specimen can be tightly attached.

474 The constitutive parameters of the model are as follows: the indenter is set as linear elastic constitutive, the
 475 default is isotropic, and sliding and tensile failure are not allowed. The sample is set as the Mohr–Coulomb
 476 constitutive. According to the data in Table 3 and the previous test results, the strength parameter values are set,
 477 as shown in Table 4 (34).

478 Table 4. Mechanical parameters of coal seam roof and floor

| Group | c (MPa) | φ (°) | E (GPa) | E_{50} (GPa) |
|------------------|-----------|---------------|-----------|----------------|
| Natural | 13.18 | 27 | 3.87 | 2.95 |
| Water saturation | 11.39 | 15 | 2.09 | 1.36 |
| Dehydration | 8.50 | 28.7 | 1.15 | 0.58 |

479 The boundary conditions of the model are as follows: the three-way velocity of the fixed lower indenter is 0
 480 to fix the model; the tangential velocity of the upper indenter is fixed to 0 to prevent the upper indenter from
 481 generating displacement outside the normal direction during loading and then generating shear stress on the
 482 surface of the sample (SZX, SZY, etc.). After the boundary conditions are applied, the prestressing force is set to
 483 0.1 MPa, namely,

$$484 \text{ In situ stress } \langle s_{xx}, s_{yy}, s_{zz}, s_{xy}, s_{xz}, s_{yz} \rangle = \langle 0, 0, 0.1e6, 0, 0, 0 \rangle$$

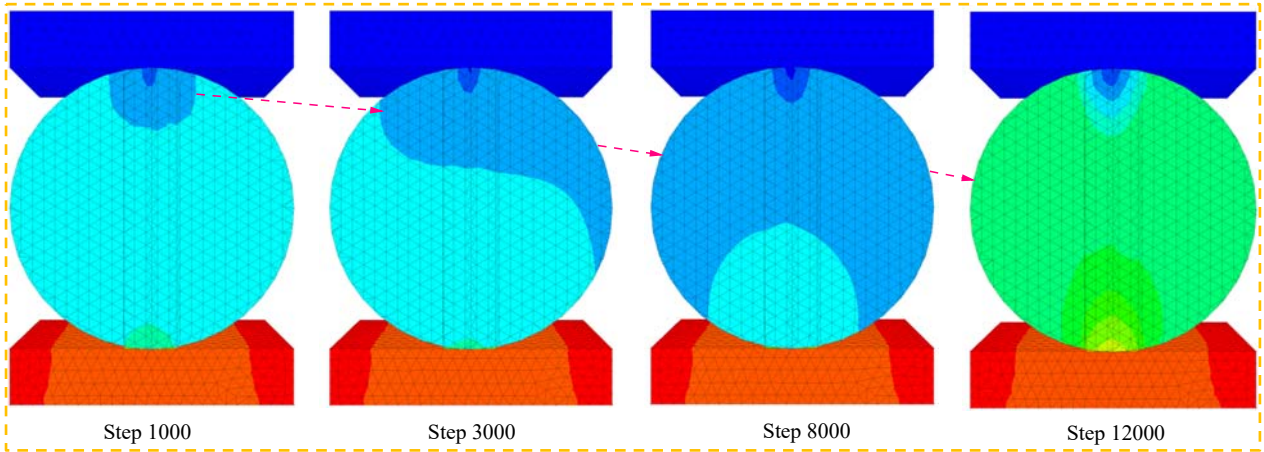
485 Since the force on the block under the action of the testing machine is not completely uniform, a stress gradient
 486 is added from the centre of the block to each surface (in the model, the centre of the sample is the origin), namely,

$$487 \text{ zgrad } \langle s_{xxz}, s_{yyz}, s_{zzz}, s_{xyz}, s_{xzz}, s_{yzz} \rangle = \langle 5, 5, 5, 0, 0, 0 \rangle$$

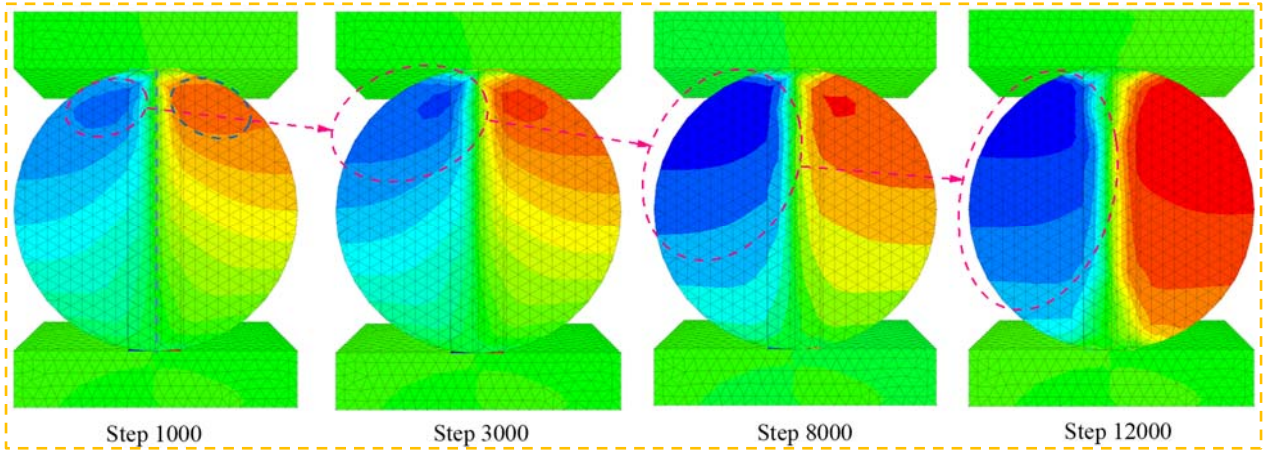
488 According to the above steps, the stress distribution applied to the media is shown in Equation (21), where
 489 σ_{xx}^0 is the stress at the origin of coordinates.

$$490 \sigma_{zz} = \sigma_{zz}^0 + (s_{zzx}) * x + (s_{zzy}) * y + (s_{zzz}) * z \quad (21)$$

492 After the initial stress is applied, the displacement test is carried out on the sample, and the upper indenter is
 493 loaded with the displacement control method at a speed of -0.003 mm/s. In the loading process, the displacement
 494 cloud diagram of the sample is shown in Figure 17.



495
 496



(b)

Figure 17. Cloud diagram of displacement distribution.(a) Vertical displacement.(b) Horizontal displacement.

Figure 17(a) shows that during the loading process, the deformation of the sample in the vertical direction first arises from the top tip and spreads downwards in a "ripple" shape with the prolongation of the loading time. This is consistent with the conventional compression test, and the deformation propagates from near the indenter to far away from the indenter. As seen from Figure 14, at $x=0$ and $y=0$, the vertical stress of the sample is 3 times the horizontal stress, while the compressive strength of the general rock material is more than 8 times the tensile strength, which eventually leads to tensile failure of the sample.

Figure 17(b) shows that during the loading process, the deformation of the sample in the horizontal direction first arises from both sides of the upper tip and spreads downwards with the extension of the loading time. Taking the red dotted circle in Figure 17 as an example, the colour depth represents the size of the deformation. The darker the colour is, the greater the deformation. At the beginning of loading, the expansion deformation appeared on both sides of the upper part of the sample along the centreline, concentrated in the oval area on both sides, and then extended to both ends and the lower side until it propagated to the bottom. While the original deformation propagates downwards, the new deformation also gradually arises and continues to spread, resulting in gradient displacement. According to the above analysis, in the process of the splitting test in Brazil, the strain in the upper part is always greater than that in the lower part. Under the continuous loading of the indenter, the upper part of the sample reaches its tensile strength first, resulting in failure.

This does not conflict with the horizontal stress distribution laws described in Figure 13 and Equation (17). Equation (17) illustrates the distribution law of disc stress in the steady state, while Figure 17 (b) shows the variation in displacement inside the specimen during loading. During the loading process, the horizontal deformation of the upper part of the sample was always greater than that of the lower part because the sample did not reach a stable state. After the loading is stopped and equilibrium is calculated, the horizontal displacement in the sample along both sides of the centreline shows a stable distribution.

5. Conclusion

By laboratory tests, theoretical analysis and numerical simulation, the physical and mechanical properties of specimens under different moisture content conditions were investigated in this paper. On this basis, the plane stress and strain distributions of Brazilian split specimens were analysed in depth. The conclusions are as follows.

1) The longitudinal wave velocity, microscopic characteristics and composition of the samples are closely related to the water content. The larger the water content of the sample is, the higher the content of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$, the more flocculent structures on its surface, and the higher the longitudinal wave propagation speed. The natural sample contains more $\text{CaSO}_4 \cdot 0.5\text{H}_2\text{O}$, and its tensile strength is the highest. The content of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ in the saturated samples increases, which leads to a decrease in tensile strength. The dehydrated samples lose most of their free water and structural water, resulting in the most obvious deterioration in their properties.

532 2) The modified Hoek–Brown criterion considers the influence of the uniaxial compressive strength of the
 533 sample, and the fitting effect is obviously improved compared to that of the modified Mohr–Coulomb criterion.
 534 Thus, the Balmer criterion introduces exponential parameter b , which changes the criterion equation from a
 535 polynomial to an exponential function, greatly improves its regression accuracy.

536 3) The samples of the natural group are prone to secondary failure in tensile testing, resulting in multiple cracks.
 537 In the saturated group samples, H₂O molecules dissociate around CaSO₄ ions, which plays a role in "buffering",
 538 and secondary damage is avoided. The samples in the dehydration group lose most of their bound water and free
 539 water, and the binding force between CaSO₄ ions is greatly reduced. Thus, fracture occurs easily under the action
 540 of an external force.

541 4) The vertical stress of the Brazilian disk sample is compressive stress, which mainly shows concentrated
 542 distribution characteristics. The horizontal stress is mainly tensile stress, and the value at $x=0$ is the theoretical
 543 tensile strength. The horizontal stress is uniformly distributed on the line directly affected by vertical load P ,
 544 decreases from this line segment to both ends of the specimen, and reduces to 0 at the left and right ends of the
 545 specimen.

546 5) The horizontal deformation of the sample first originates from both sides of the disk tip and spreads
 547 downwards in an arc with the extension of the loading time. The horizontal strain in the upper part of the disk is
 548 always greater than that in the lower part. Under the continuous loading of the indenter, the upper part of the
 549 sample reaches its tensile strength first, resulting in failure.

550 6. Variable annotations

| Variate | Annotation | Variate | Annotation |
|------------|----------------------------------|-------------|--------------------------------------|
| m | Quality (g) | σ_x | Horizontal stress (MPa) |
| f_w | Water content | σ_y | Vertical stress (MPa) |
| ρ | Density (g/cm ³) | τ_{xy} | Shear stress (MPa) |
| v | Longitudinal wave velocity (m/s) | σ_1 | The maximum principal stresses (MPa) |
| σ_t | Tensile Strength (MPa) | σ_3 | The minimum principal stresses (MPa) |
| c | Cohesion (MPa) | σ_c | Uniaxial compressive strength (MPa) |
| φ | Internal friction angle (°) | σ | Normal stress (MPa) |
| E | Elasticity modulus (GPa) | l | Empirical parameter |
| E_{50} | Modulus of deformation (GPa) | b | Test constant. |

551 7. Statements

552 7.1 Funding

553 Supported by Scientific Research Foundation of State Key Lab of Coal Mine Disaster Dynamics and Control
 554 (Project Approval Number: 2011DA105287-zd201804) and National Natural Science Foundation of China (No.
 555 51974104).

556 7.2 Competing Interests

557 The authors have no relevant financial or non-financial interests to disclose.

558 7.3 Author Contributions

559 **Wang Chongyang:** Data curation, Writing- Original draft preparation. **Zhang Dongming:** Writing-
 560 Reviewing and Editing. **Wei Sijiang:** Funding acquisition, Investigation. **Yang Yushun:** Supervision, Validation.
 561 **Yu Beichen:** Conceptualization, Methodology. **Pan Yisha:** Methodology.

562 7.4 Data Availability

563 The datasets generated during and analyzed during the current study are available from the corresponding
 564 author on reasonable request.

565 **References**

- 566 [1] Y. Gan, S. Wu, Y. Ren, G. Zhang, Evaluation indexes of granite splitting failure based on RA and AF of AE
567 parameters. *Rock and Soil Mechanics*, 41(7) (2020): 2324-2332. <https://doi.org/10.16285/j.rsm.2019.1460>.
- 568 [2] H. Deng, J. Li, M. Zhu, R. Wang, X. Yuan, S. Luo, Research on effect of disc thickness-to-diameter ratio on
569 splitting tensile strength of rock. *Chinese Journal of Rock Mechanics and Engineering*, 31(4) (2012): 792.
- 570 [3] Y. Han, Z. Wang, Y. Tang, Mechanical behavior of different rocks in the splitting test. *Journal of China*
571 *University of Mining and Technology*, 49(5) (2020):863. <https://doi.org/10.13247/j.cnki.jcmt.001192>.
- 572 [4] W. Yao, K. Xia, X. Li, Non-local failure theory and two-parameter tensile strength model for semi-circular
573 bending tests of granitic rocks. *International Journal of Rock Mechanics and Mining Sciences*. 110 (2018),9-
574 18. <https://doi.org/10.1016/j.ijrmms.2018.07.0020>
- 575 [5] F. Nascimento, A. Guimaraes, C. Castro, Comparative study on permanent deformation in asphalt mixtures
576 from indirect tensile strength testing and laboratory wheel tracking. *Construction and Building Materials*.
577 305 (2021). <https://doi.org/10.1016/j.conbuildmat.2021.124736>.
- 578 [6] N. Wu, J. Fu, Z. Zhu, B. Sun, 2020. Experimental study on the dynamic behavior of the Brazilian disc sample
579 of rock material. *International Journal of Rock Mechanics and Mining Sciences*. 130 (2020).
580 <https://doi.org/10.1016/j.ijrmms.2020.104326>.
- 581 [7] T. Saksala, M. Hokka, V. Kuokkala, J. Makinen, 2013. Numerical modeling and experimentation of dynamic
582 Brazilian disc test on Kuru granite. *International Journal of Rock Mechanics and Mining Sciences*. 59
583 (2013),128-138. <https://doi.org/10.1016/j.ijrmms.2012.12.018>.
- 584 [8] D. Shen, X. Shi, S. Zhu, X. Duan, J. Zhang, Relationship between tensile Young's modulus and strength of
585 fly ash high strength concrete at early age. *Construction and Building Materials*. 123 (2016), 317-326.
586 <https://doi.org/10.1016/j.conbuildmat.2016.06.145>.
- 587 [9] Y. Li, J. Li, Relationship between fracture area and tensile strength of cement paste with supplementary
588 cementitious materials. *Construction and Building Materials*. 79 (2015), 223-228. <https://doi.org/10.1016/j.conbuildmat.2015.01.052>.
- 590 [10] R. Verma, G. Nguyen, M. Karakus, A. Taheri, Capturing snapback in indirect tensile testing using AUSBIT-
591 Adelaide University Snap-Back Indirect Tensile test. *International Journal of Rock Mechanics and Mining*
592 *Sciences*. (2021). <https://doi.org/10.1016/j.ijrmms.2021.104897>.
- 593 [11] P. Lochan, M. Polak, Determination of tensile strength of GFRP bars using flexure tests. *Construction and*
594 *Building Materials*. 314 (2021). <https://doi.org/10.1016/j.conbuildmat.2021.125630>.
- 595 [12] C. Gaedicke, A. Torres, K. Huynh, A. Marines, A method to correlate splitting tensile strength and
596 compressive strength of pervious concrete cylinders and cores. *Construction and Building Materials*. 125
597 (2016), 271-278 <https://doi.org/10.1016/j.conbuildmat.2016.08.031>.
- 598 [13] A. Mardoukhi, Y. Mardoukhi, M. Hokka, V. Kuokkala, Effects of heat shock on the dynamic tensile behavior
599 of granitic rocks. *Rock Mechanics and Rock Engineering*. 50(5) (2017),1171-1182. <https://doi.org/10.1007/s00603-017-1168-4>.
- 601 [14] A. Mardoukhi, Y. Mardoukhi, M. Hokka, V. Kuokkala, Effects of test temperature and low temperature
602 thermal cycling on the dynamic tensile strength of granitic rocks. *Rock Mechanics and Rock Engineering*.
603 54(1) (2021),443-454. <https://doi.org/10.1007/s00603-020-02253-6>.
- 604 [15] S. Zhao, X. Ding, M. Zhao, C. Li, S. Pei, Experimental study on tensile strength development of concrete
605 with manufactured sand. *Construction and Building Materials*. 138 (2017), 247-253. <https://doi.org/10.1016/j.conbuildmat.2017.01.093>.
- 607 [16] Y. Ren, V. Vavrycuk, S. Wu, Y. Gao, Accurate moment tensor inversion of acoustic emissions and its
608 application to Brazilian splitting test. *International Journal of Rock Mechanics and Mining Sciences*. 141
609 (2021), <https://doi.org/10.1016/j.ijrmms.2021.104707>.

- 610 [17] P. Hou, Y. Xue, F. Gao, F. Dou, S. Su, C. Cai, C. Zhu, Effect of liquid nitrogen cooling on mechanical
611 characteristics and fracture morphology of layer coal under Brazilian splitting test. *International Journal of*
612 *Rock Mechanics and Mining Sciences*. 151 (2022). <https://doi.org/10.1016/j.ijrmms.2021.105026>.
- 613 [18] Y. Ma, H. Huang, DEM analysis of failure mechanisms in the intact Brazilian test. *International Journal of*
614 *Rock Mechanics and Mining Sciences* (2018). <https://doi.org/102.109-119>. [10.1016/j.ijrmms.2017.11.010](https://doi.org/10.1016/j.ijrmms.2017.11.010).
- 615 [19] N. Erarslan, D. Williams, Experimental, numerical and analytical studies on tensile strength of rocks.
616 *International Journal of Rock Mechanics and Mining Sciences*. 49 (2012), 21-30. [https://doi.org/](https://doi.org/10.1016/j.ijrmms.2011.11.007)
617 [10.1016/j.ijrmms.2011.11.007](https://doi.org/10.1016/j.ijrmms.2011.11.007).
- 618 [20] N. Erarslan, Z. Liang, D. Williams, Experimental and numerical studies on determination of indirect tensile
619 strength of rocks. *Rock Mechanics and Rock Engineering*. 45 (2012), 739–751. [https://doi.org/10.](https://doi.org/10.1007/s00603-011-0205-y)
620 [1007/s00603-011-0205-y](https://doi.org/10.1007/s00603-011-0205-y).
- 621 [21] O. Mahabadi, B. Cottrell, G. Grasselli, An example of realistic modelling of rock dynamics problems:
622 fem/dem simulation of dynamic brazilian test on barre granite. *Rock Mechanics and Rock Engineering*. 43(6)
623 (2010), 707-716. <https://doi.org/10.1007/s00603-010-0092-7>.
- 624 [22] Z. Liang, C. Tang, Y. Zhang, T. Ma, On probability model of physico-mechanical parameters of quasi-brittle
625 materials and associated mechanical failure behaviors. *Chinese Journal of Rock Mechanics and Engineering*,
626 27(4) (2008): 718-727. <https://doi.org/10.3321/j.issn:1000-6915.2008.04.010>.
- 627 [23] Z. Yang, B. He, L. Xie, C. Li, J. Wang, Strength and failure modes of shale based on Brazilian test. *Rock*
628 *and Soil Mechanics*, 36(12) (2015): 3447. <https://doi.org/10.16285/j.rsm.2015.12.015>.
- 629 [24] D. Aggelis, Classification of cracking mode in concrete by acoustic emission parameters. *Mechanics*
630 *Research Communications*, 38(3) (2011):153-157. <https://doi.org/10.1016/j.mechrescom.2011.03.007>.
- 631 [25] Q. Jiang, J. Cui, X. Feng, G. Liu, B. Wang, T. Liu, Y. Lv, Stochastic statistics and probability distribution
632 estimation of mechanical parameters of basalt. *Rock and Soil Mechanics*, 38(3) (2017): 784. [https://](https://doi.org/10.16285/j.rsm.2017.03.022)
633 doi.org/10.16285/j.rsm.2017.03.022.
- 634 [26] M. Du, P. Pan, W. Ji, Z. Zhang, Y. Gao, Time-space laws of failure process of carbonaceous shale in Brazilian
635 split test. *Rock and Soil Mechanics*, 37(12) (2016): 3437-3446. <https://doi.org/10.16285/j.rsm.2016.12.012>.
- 636 [27] X. Li, L. Luo, C. Li, Experimental study of directivity effect of rock interface under Brazilian splitting.
637 *Chinese Journal of Engineering*, 39(9) (2017):1295-1304. [https://doi.org/10.13374/j.issn2095-9389.2017.](https://doi.org/10.13374/j.issn2095-9389.2017.09.001)
638 [09.001](https://doi.org/10.13374/j.issn2095-9389.2017.09.001).
- 639 [28] S. Yang, Y. Li, Y. Huang, W. Tian, Particle flow analysis of macroscopic and microscopic mechanical
640 properties of Brazilian disc containing a hole under splitting test. *Journal of China University of Mining and*
641 *Technology*, 48(5) (2019): 984-992. <https://doi.org/10.13247/j.cnki.jcumt.001055>.
- 642 [29] D. Zhang, R. Gamage, M. Perera, C. Zhang, W. Wanniarachchi, (2017). Influence of water saturation on the
643 mechanical behaviour of low-permeability reservoir rocks. *Energies*. 10(2), 236; [https://doi.org/10.3390/](https://doi.org/10.3390/en10020236)
644 [en10020236](https://doi.org/10.3390/en10020236).
- 645 [30] Z. You, Y. Ma, Z. Wang, J. Ma, Tensile strength variation of a silty clay under different temperature and
646 moisture conditions. *Cold Regions Science and Technology*. 189. [https://doi.org/10.1016/j.coldregions.2021.](https://doi.org/10.1016/j.coldregions.2021.103314)
647 [103314](https://doi.org/10.1016/j.coldregions.2021.103314).
- 648 [31] T. Kim, C. Hwang, Modeling of tensile strength on moist granular earth material at low water content.
649 *Engineering Geology*. 69(3-4),233-244. [https://doi.org/10.1016/S0013-7952\(02\)00284-3](https://doi.org/10.1016/S0013-7952(02)00284-3).
- 650 [32] H. Li, C. Tang, Q. Cheng, S. Li, X. Gong, B. Shi, (2019). Tensile strength of clayey soil and the strain
651 analysis based on image processing techniques. *Engineering Geology*. 253, 137-148.
652 <https://doi.org/10.1016/j.enggeo.2019.03.017>.
- 653 [33] B. Vasarhelyi, (2003). Some observations regarding the strength and deformability of sandstones in dry and
654 saturated conditions. *Bulletin of Engineering Geology and the Environment*. 62, 245–249. [https://doi.org/10.](https://doi.org/10.1007/s10064-002-0186-x)
655 [1007/s10064-002-0186-x](https://doi.org/10.1007/s10064-002-0186-x).

- 656 [34] Z. Zhou, X. Cai, W. Cao, X. Li, C. Xiong, (2016). Influence of Water Content on Mechanical Properties of
657 Rock in Both Saturation and Drying Processes. *Rock Mechanics and Rock Engineering*. 49, 3009–3025.
658 <https://doi.org/10.1007/s00603-016-0987-z>.
- 659 [35] Z. Erguler, R. Ulusay. Water-induced variations in mechanical properties of clay-bearing rocks. *International*
660 *Journal of Rock Mechanics and Mining Sciences*.46(2), 355-370. [https://doi.org/10.1016/j.ijrmmms.2008.](https://doi.org/10.1016/j.ijrmmms.2008.07.002)
661 [07.002](https://doi.org/10.1016/j.ijrmmms.2008.07.002).
- 662 [36] A. Daraei, S. Zare, (2018). Effect of Water Content Variations on Critical and Failure Strains of Rock. *KSCE*
663 *Journal of Civil Engineering*. 22, 3331–3339. <https://doi.org/10.1007/s12205-018-0592-7>.
- 664 [37] C. Tang, X. Pei, D. Wang, B. Shi, J. Li, (2015). Tensile Strength of Compacted Clayey Soil. *Journal of*
665 *Geotechnical and Geoenvironmental Engineering*. 141(4). [https://doi.org/10.1061/\(asce\)gt.1943-5606.](https://doi.org/10.1061/(asce)gt.1943-5606.0001267)
666 [0001267](https://doi.org/10.1061/(asce)gt.1943-5606.0001267).
- 667 [38] S. Wei, C. Wang, Y. Yang, M. Wang, Physical and mechanical properties of gypsum-like rock materials.
668 *Advances in Civil Engineering*, (2020), 1–17, <https://doi.org/10.1155/2020/3703706>.
- 669 [39] C. Wang, Study on Low-Frequency Dynamic Physico-Mechanical Properties of Gypsum Rocks. MSc Thesis,
670 *Henan Polytechnic University*. (2021).
- 671 [40] C. Wang, S. Wei, Y. Pan, S. Zhang, Experimental study on fatigue life of gypsum-like rock under uniaxial
672 compression with different loading frequencies. *Pure and Applied Geophysics*, 179 (2022), [https://doi.org/](https://doi.org/10.1007/s00024-022-02966-5)
673 [10.1007/s00024-022-02966-5](https://doi.org/10.1007/s00024-022-02966-5).
- 674 [41] C. Wang, S. Wei, M. Wang, C. Xu, S. Zhang, Rockburst characteristics of gypsum-like rocks after fatigue
675 cyclic loading. *Arabian Journal of Geosciences*, 15(15) (2022). [https://doi.org/10.1007/s12517-022-10636-](https://doi.org/10.1007/s12517-022-10636-y)
676 [y](https://doi.org/10.1007/s12517-022-10636-y).
- 677 [42] Y. Pan, C. Wang, Y. Wang, Mechanical degradation mechanism of rock under seismic disturbance stress.
678 *Quarterly Journal of Engineering Geology and Hydrogeology*, 55 (2022). [https://doi.org/10.1144/qjegh](https://doi.org/10.1144/qjegh2022-007)
679 [2022-007](https://doi.org/10.1144/qjegh2022-007).
- 680 [43] S. Chen, Z. Du, Z. Zhang, H. Zhang, Z. Xia, F. Feng, (2020), Effects of chloride on the early mechanical
681 properties and microstructure of gangue-cemented paste backfill, *Construction and Building Materials*,
682 235(2): 117504. <https://doi.org/10.1016/j.conbuildmat.2019.117504>.
- 683 [44] M. Dolezelova, L. Scheinherrova, J. Krejsova, A. Vimrova, Effect of high temperatures on gypsum-based
684 composites. *Construction and Building Materials*. 168 (2018) 82-90. [https://doi.org/10.1016/j.conbuildmat.](https://doi.org/10.1016/j.conbuildmat.2018.02.101)
685 [2018.02.101](https://doi.org/10.1016/j.conbuildmat.2018.02.101).
- 686 [45] X. Shi, Y. Meng, G. Li, Comparative analyses of several rock strength criteria. *Rock and Soil Mechanics*,
687 32(1) (2011): 209-215. <https://doi.org/10.16285/j.rsm.2011.s1.022>.
- 688 [46] E. Hoek, E. Brown, Empirical strength criterion for rock masses. *Journal of Geotechnical and*
689 *Geoenvironmental Engineering*, 106 (GT9) (1980), 1013-1035. [https://doi.org/10.1016/0022-1694\(80\)](https://doi.org/10.1016/0022-1694(80)90029-3)
690 [90029-3](https://doi.org/10.1016/0022-1694(80)90029-3).
- 691 [47] E. Hoek, Strength of rock and rock masses. *International Society of Rock Mechanics News Journal*, 2(2)
692 (1994): 4-16.
- 693 [48] L. Wu, Study on rock mass strength criterion. MSc Thesis, *Chang 'an University*. (2004).
- 694 [49] S. Chen, Z. Xia, F. Feng, D. Yin, (2021). Numerical study on strength and failure characteristics of rock
695 samples with different hole defects. *Bulletin of Engineering Geology and the Environment*. 80(2): 1523-
696 1540. <https://doi.org/10.1007/s10064-020-01964-y>
- 697 [50] J. Jiang, X. Feng, C. Yang, G. Su, Failure characteristics of surrounding rocks along the radial direction of
698 underground excavations: An experimental study. *Engineering Geology*. 281 (2021). [https://doi.org/10.](https://doi.org/10.1016/j.enggeo.2020.105984)
699 [1016/j.enggeo.2020.105984](https://doi.org/10.1016/j.enggeo.2020.105984).
- 700 [51] N. Muskhelishvili, Some basic problems of mathematical theory of elasticity. *Axel Springer*. (1977).