Proceedings of the ASME 2011 30th International Conference on Ocean, Offshore and Arctic Engineering
OMAE2011
June 19-24, 2011, Rotterdam, The Netherlands

OMAE2011-50014

RESPONSE OF A SPAR PLATFORM UNDER THE OCCURRENCE OF EXTREME WAVES

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ABSTRACT

In this note, the response of a Spar Platform is studied when subject to very high waves. The novelty of this paper consists in adopting the Quasi Determinism (QD) theory in its first formulation for the determination of the response of the structure. In fact, one of the peculiarities of QD theory is that it can be adopted under any boundary condition.

The Spar Platform has been modeled as a two-dimensional 3-degree-of-freedom (3dof) rigid cylindrical floating body, moored to the seabed, floating in deep water. Further, it is supposed to be a slender body under the action of extreme waves neglecting any diffraction effect. Moreover, such a system is considered to be nonlinear, because of the nonlinear damping derived from the drag force and because of the stiffness of the moorings, which in the model have been studied as nonlinear springs. The non linear coupled set of differential equations of motion have been integrated in order to obtain the response of the structure by means of numerical methods commonly adopted in Literature.

Thus, the response under the occurrence of very high wave crests is studied by using Quasi Determinism theory and then the results compared and validated with those obtained through Monte- Carlo simulations.

INTRODUCTION

In the last decades, floating structures are used more and more often either as an extension of land towards the sea or for supporting offshore facilities. In the former case, the technical community usually refers to artificial floating islands, called Very Large Floating Structures (VLFS), often composed of several smaller floating elements (cellular floating bodies); such structures can hold infrastructures and services, like ports, airports, buildings and so forth. In the second case, deepwater technology often adopts floating bodies (a single cell) in oil extracting field. Examples of the latter kind of structures can be seen in Spar Platforms, Tension Leg Platforms (TLPs) and Floating Production Storage and Offloading (FPSO) Units. Floating structures are usually used when environmental and wave climate allow the use of these structures and generally they are preferred in deep water field, i.e. in the cases when gravity and base supported structures could not be convenient. More generally, floating bodies offer a wide range of potential applications, because of their flexibility: they can be set coastline, in a port or in deep water, in shallow or intermediate water or in deep water.

Cellular floating structures show several pros:

- The relative ease and rapidity in the construction (actually, these elements can be built in dry docks and then assembled in situ);
- They can be easily moved;
- They are relatively economic, especially in deep water;
- They can have an almost constant distance from the free surface displacement;
- They are eco-compatible and they respect the marine eco-system;
- They are seismically isolated.

For all these reasons, some studies about the dynamics of these structures and the hydrodynamics around them have been carried out in some of the most important laboratories worldwide, especially in those countries (Netherlands, Japan and etc) for which the management of the land and the increase of the settlement densities is a crucial issue, or where the exploitation of oil fields in deep water is necessary.

Studies about the dynamics and hydrodynamics of Spar platforms have been carried out both in time and space domains (see [1], [2], [3]) and sometimes also experimental data recorded in platforms

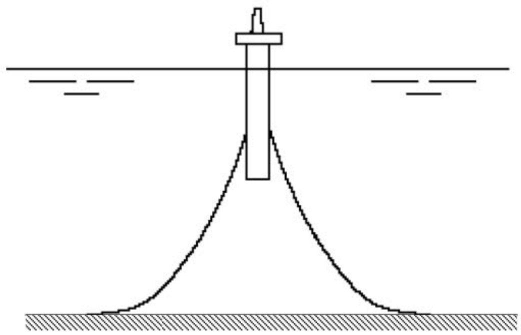

Figure 1. Model of a floating platform anchored to the seabed by means of catenaries.

installed in the Gulf of Mexico have been used (see [4]). In general, Authors have used classical solving techniques, such as for example Monte-Carlo Simulations (for a state of art about this topic, see also [5]) or equivalent linearization techniques (see [6] and [7]).

In this note, the dynamics of a floating moored body subject to very high sea waves is studied. For the first time, the Quasi-Determinism (QD) Theory (Boccotti, [8], [9], [10], [11], [12]) has been adopted for studying in detail the dynamics of a floating system. Such a body has been modeled as a slender body under the action of very high waves, and so the soliciting forces have been computed by adopted the model described by Morison equation. Results in terms of forces and dynamic responses obtained through the QD theory have been validated by means of Monte-Carlo simulations of random sea waves.

DESCRIPTION OF THE MODEL

The model herein examined is a cylindrical floating body, which may represent a single cellular element of a Floating Island or a very simple Classical Spar Platform. It is supposed to be anchored to the seabed through moorings, which have been modeled as springs.

Figure 1 shows an example of the model. It is symmetric both in the geometry and the loading combinations: thus, it is possible to study the dynamics of the body in the plane domain instead of the space domain. The three degrees of freedom (DOFs) (y, z, θ) represent respectively surge, heave and pitch. The body is supposed to be rigid, so such coordinates refer to center of gravity of the platform, i.e. where the mass of the platform is supposed to be lumped.

As for the reference frame, the vertical axis has origin at the mean water level and the horizontal one is directed towards the direction of propagation of waves.

EQUATIONS GOVERNING THE DYNAMICS OF THE BODY

In the reference frame describe above, the equations of motion can be written as:

$$M_y \cdot \ddot{y} + C_y \cdot \dot{y}\left|\dot{y}\right| + F_{my}(t) = F_y(t)\,, \tag{1}$$

$$M_z \cdot \ddot{z} + W - B + F_{mz}(t) + K_h \cdot z = F_z(t) \tag{2}$$

and

$$J_\theta \cdot \ddot{\theta} + C_\theta \cdot \dot{\theta}\left|\dot{\theta}\right| + K_\theta \cdot \theta = M_\theta(t)\,, \tag{3}$$

in which the dotted variables mean derivation with respect to time.

W is the weight of the structure and B represents buoyancy. Coefficients C_y and C_θ are damping coefficients derived straightforwardly from Morison's equation, being such a body not fixed.

The stiffness of the moorings is supposed to be a linear function of the horizontal and vertical displacements; this makes the first two equations coupled.

$$F_{my} = \alpha_1 y + \beta_1 z + C_1 \tag{4}$$

and

$$F_{mz} = \alpha_2 y + \beta_2 z + C_2\,. \tag{5}$$

The mass matrix of the system is supposed to be diagonal.

COMPUTATION OF FORCES

Chakrabarti in [13] defined the motion regimes around slender, medium and large bodies and the appropriate methods for the calculation of forces. Of course, diffraction theories can be applied to large bodies, which can affect the wave motion around the structures; on the contrary, Morison model represents the appropriate method when dealing with slender bodies.

Forces estimated in a diffracted field and through Morison's equation differ significantly when the ratio between diameter (or significant dimension of the structure) (D) and wavelength (L_o) is greater than one as Brebbia and Walker [14] say. Kim & Chen [15] compared the results in terms of forces for a jacket fixed platform by using both the Morrison approximation (see reference [16]) and the diffraction theory. The same authors showed that both the methods give almost the same results when the ratio between D and L_o is small enough. In the following application, the platform has a diameter of D= 36 m, and even though in modest seas the diffraction theory should be used, during more severe events such a structure behaves as a slender body. Thus, the horizontal force can be computed by means of the Morison Equation.

$$F_y(t) = \int_{-d}^{\eta} \left[f_{in}(z,t) + f_{dg}(z,t) \right] \mathrm{d}z \tag{6}$$

where d is the water depth, η is the free surface displacement; the inertial component of the force is given by:

$$f_{in}(z,t) = C_{in}\rho_w \frac{\pi D^2}{4} a_y(z,t) \quad (7)$$

while the drag one is equal to:

$$f_{dg}(z,t) = C_{dg}\rho_w \frac{D}{2} v_y(z,t)\left|v_y(z,t)\right| \quad (8)$$

in which the hydrodynamic coefficients are equal to C_{in}=1.85 and C_{dg}=0.62, valid for value of the ratio between Reynolds number and Keulegan number greater than 10000.

Under the hypothesis of small pitch oscillations, the vertical force is given by integrating the overpressure at the bottom of the cylinder

$$F_z(t) = \int_\Omega \Delta p \, \mathrm{d}\Omega \,. \quad (9)$$

The moment is generated by horizontal forces with respect to the centre of gravity of the structure.

Given the symmetry of the plane model, cylindrical waves have been studied, without taking into account effects due to directional spreading. These waves are supposed to be generated by wind and a typical spectrum for wind waves has been considered, i.e. mean JONSWAP spectrum (see also [17] for further details):

$$E(\omega) = \alpha g^2 \omega^{-5} \exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \cdot \exp\left\{\ln\gamma \exp\left[-\frac{\left(\omega-\omega_p\right)^2}{2\sigma^2\omega_p^2}\right]\right\} \quad (10)$$

where ω_p is the peak frequency of the spectrum, Phillips' parameter is $\alpha = 0.01$, and the parameters characterizing mean JONSWAP spectrum are

$$\gamma = 3.3; \quad \sigma\begin{cases} = 0.07 & \text{se } \omega \le \omega_p \\ = 0.09 & \text{se } \omega > \omega_p \end{cases}. \quad (11)$$

According to the sea states theory (for a complete state of art, see [18] and [19]) for an irrotational wave motion, the free surface displacement can be considered as the summation of elementary components

$$\eta(y,t) = \sum_{i=1}^{N} a_i \cos(k_i y - \omega_i t + \varepsilon_i) \quad (12)$$

in which N is a very high number, the amplitudes are a_i infinitesimal and all of the same order, the angular frequencies ω_i are different one another and the phases ε_i are uniformly distributed in $[0, 2\pi]$. The angular frequencies are related to the wave numbers k_i through the linear dispersion rule, which can be written in deep water (i.e. when the ratio d/L_o is greater than 0.5) as

$$k_i = \omega_i^2 / g \,. \quad (13)$$

The velocity potential function can be written as

$$\phi(y,z,t) = g\sum_{i=1}^{N} a_i \omega_i^{-1} \frac{\cosh[k_i(d+z)]}{\cosh(k_i d)} \cdot \sin(k_i y - \omega_i t + \varepsilon_i). \quad (14)$$

Thus the component of horizontal velocity, horizontal acceleration and overpressure can be calculated as:

$$v_y(y,z,t) = \partial\phi / \partial y \,, \quad (15)$$

$$a_y(y,z,t) = \partial^2\phi / (\partial y \partial t) \quad (16)$$

and

$$\Delta p(y,z,t) = -\rho \; \partial\phi/\partial t \,. \quad (17)$$

QUASI-DETERMINISM THEORY

The Quasi-Determinism (QD) Theory has been developed by Boccotti during the 80's. The main result of the theory is that the free surface displacement and the velocity potential function tend to deterministic functions in case of occurrence of high waves or crests in a random sea field. For this reason, the kinematic components (velocity and acceleration) can be computed deterministicly and so the deterministic forces acting on a structure when a high wave or a high crest occurs can be determined through Eqs. (7), (8) and (9).

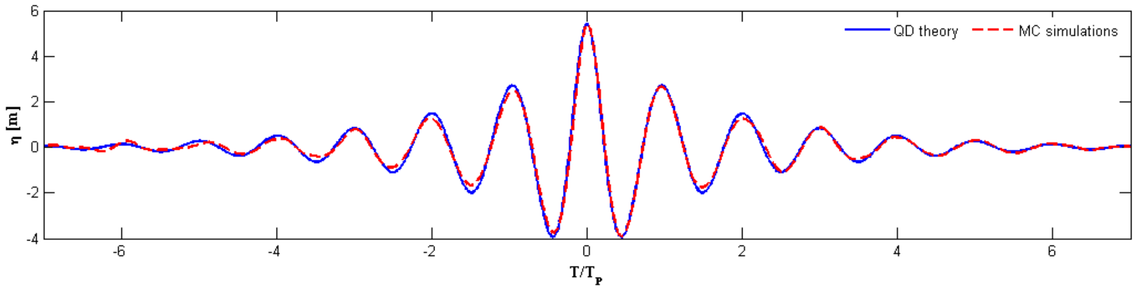

Figure 2. Profile of the free surface displacement computed by means of the Quasi Determinism (QD) Theory compared with the forces of the average ensemble of the thousandth part of highest random crests generated through Monte-Carlo simulations.

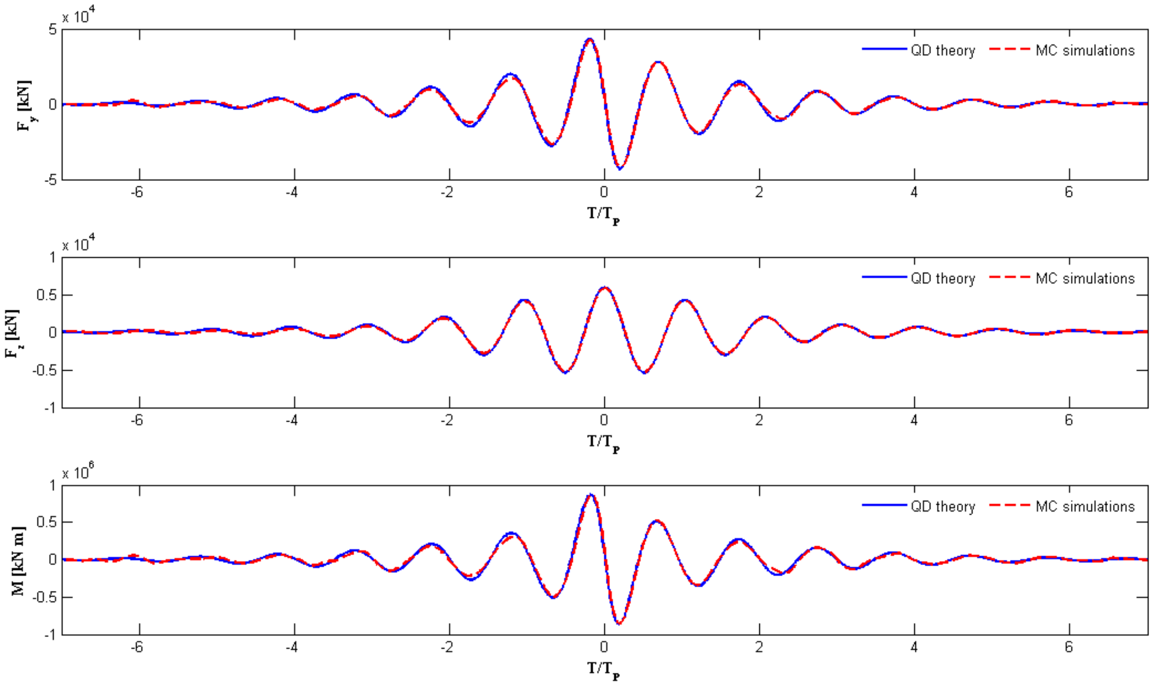

Figure 3. Profiles of the forces and moment computed by means of the Quasi Determinism (QD) Theory compared with the forces of the average ensemble of the thousandth part of highest random crests generated through Monte-Carlo simulations.

The theory has been expressed in two different formulations. The former, Boccotti ([8], [9]) (see also Tromans et al., [under the name of 'New Wave' Theory), describes the mechanics of waves when in a fixed point and time instant a very high wave crest occurs; of course it has to be very high with respect to the sea storm it belongs to. In its second formulation, the theory shows what happens when a very high wave (crest-to-trough) occurs (Boccotti, [10], [11], [12]). Extensions to the QD theory have been obtained by Arena [21] and Arena & Fedele [22], who dealt with second order effects in homogeneous field, Arena & Romolo [23] who studied the interaction between very high wave groups with uniform and stationary currents and Nava et al. [24] who combined both the second order effects and the presence of currents.

In this note, the QD theory has been used to determine the forces on the floating body described above and then the dynamic response has been calculated. In particular, it has been adopted the first formulation of the theory, in case of cylindrical 2D waves. Provided that the free surface displacement is a random, Gaussian and ergodic process, if at a certain point (y_o) and time instant t_o a very high wave crest H_c with respect to the standard deviation σ of the sea state it belongs to

$$\eta\left(y_o,t_o\right)=H_C \tag{18}$$

with

$$H_C/\sigma\to\infty \tag{19}$$

then, according to the first formulation of the theory, conditions in Eq.s (18) and (19) are necessary and sufficient for the occurrence of a very high sea wave group. Thus, the free surface displacement function tends deterministicly in the closeness of (y_o)

$$\overline{\eta}(y_o+Y,t_o+T)=\frac{\Psi(Y,T)}{\Psi(0,0)}H_C \tag{20}$$

where $\Psi(Y,T)$ is the space-time covariance

$$\Psi\left(Y,T\right)\equiv\left\langle\eta\left(y_o,t\right)\eta\left(y_o+Y,t+T\right)\right\rangle. \tag{21}$$

Similarly, the deterministic velocity potential function is given by

$$\overline{\phi}(y_o+Y,z,t_o+T)=\frac{\Phi(X,Y,z,T)}{\Psi(0,0,0)}H_C \tag{22}$$

where

$$\Phi\left(Y,z,T\right)\equiv\left\langle\eta\left(y_o,t\right)\phi\left(y_o+Y,z,t+T\right)\right\rangle. \tag{23}$$

By applying the sea state theory, Eq. (20) can be written as

$$\overline{\eta}(y_o+Y,t_o+T)=H_C\int_0^\infty E(\omega)\cos\left(kY-\omega T\right)\mathrm{d}\omega\Big/\int_0^\infty E(\omega)\mathrm{d}\omega \tag{24}$$

and the velocity potential function in Eq. (22) as

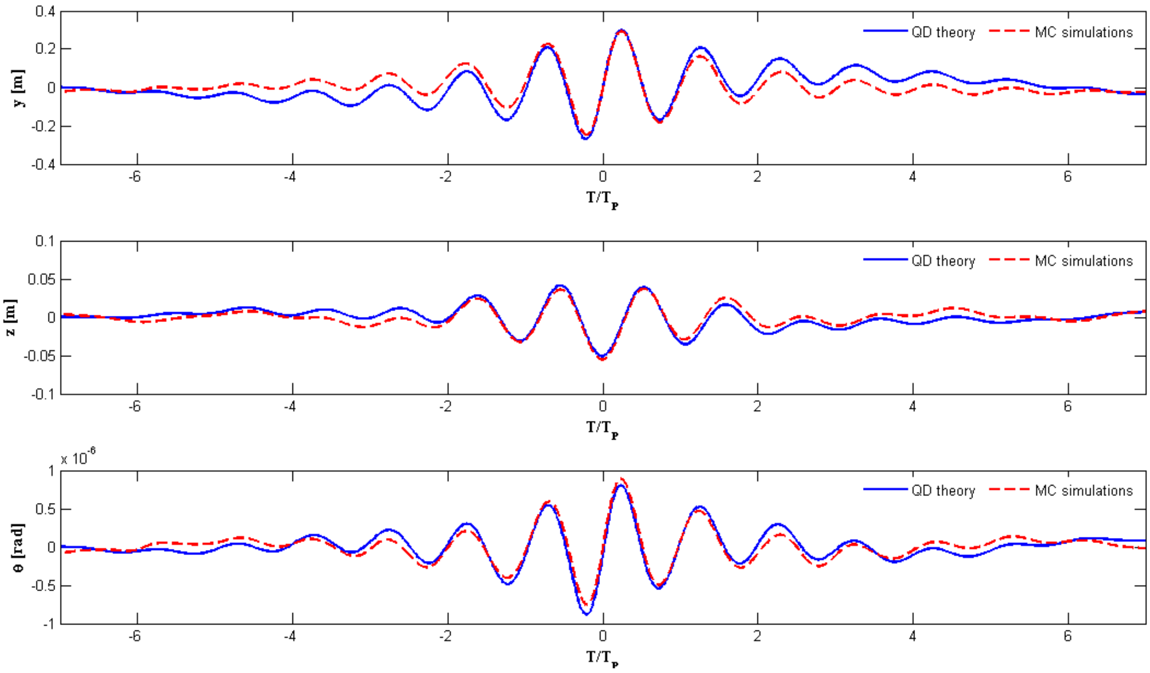

Figure 4. Profiles of the dynamic responses computed by means of the Quasi Determinism (QD) Theory compared with the forces of the average ensemble of the thousandth part of highest random crests generated through Monte-Carlo simulations

$$\bar{\phi}(y_o+Y,z,t_o+T)=gH_C\int_0^{\infty}\omega^{-1}E(\omega)\frac{\cosh\left[k(d+z)\right]}{\cosh kd}\cos(kY-\omega T)\mathrm{d}\omega\Big/\int_0^{\infty}E(\omega)\mathrm{d}\omega \tag{25}$$

The kinematic components of horizontal velocity and acceleration and overpressure can be calculated by means of Eqs. (15), (16) and (17), using the deterministic potential in in Eq. (25).

APPLICATION

The QD theory has been used in order to obtain the forces on the floating cylinder as shown in Figure 1. The diameter of the cylinder is equal to D=36 m, it is sunk for 50 m. The centre of gravity is 35.5 m deep under the mean sea level and the water depth is d=100 m. A mean JONSWAP spectrum is herein adopted with a significant wave height of H_s= 5.5 m and peak period of T_p= 10 s. The highest crest is supposed to be H_c=5.40 m. The free surface displacement and the kinematic components have been calculated in the time domain $(-7T_p, 7T_p)$ in order to determine the forces appearing in Eqs. (1), (2) and (3). The dynamic response corresponding to the group of high waves has been obtained through integration of the equations of motion Eqs. (1), (2) and (3) by means of a numerical algorithm, based on Runge-Kutta methods (IV order, constant time step).

The validation of the results obtained by means of QD theory has been carried out through Monte-Carlo simulations. In particular, 200 sea states have been generated using the same mean JONWAP spectrum adopted in the QD theory. The total number of crest-to-trough waves generated has been 186736 waves (around 950 waves for each sea state). The algorithm adopted for the numerical simulations is based on Fast Fourier Transform (FFT) method. The main advantage is that this method is not time consuming; however, in order to get an appropriate sampling rate, it is convenient to generate shorter time samples. Also in this case, free surface displacement and kinematic components have been generated in order to determine the forces on the structure. Thereafter, the integration of the differential equation of motion allowed the determination of the random dynamic responses.

After, the data were post-processes. In particular the numerical samples were sorted on the basis of the crest heights and so the thousandth part of highest crest were averaged. This averaging process involved the free surface displacement, the forces and the responses of the 186 waves having higher crests. The amplitude of the crest of the average ensemble is 5.4 m. The comparison between QD theory and numerical simulations in terms of forces and responses is shown on Figures 2, 3 and 4.

In Figure 2 the agreement between the profiles of the free surface displacement obtained by means of QD theory and the average ensemble of the thousandth part of highest crest obtained through Monte Carlo simulations is shown. In Figure 3 the QD forces are compared with the forces of the average ensemble: the profiles of forces and moment totally agree, both in qualitative and quantitative terms. One can achieve the same conclusions if looking at Figure 4, in which the comparison between QD theory and Monte-Carlo simulations has done in terms of the dynamic responses.

CONCLUDING REMARKS

In this note, for the first time in the scientific literature, the Quasi Determinism (QD) Theory has been adopted for determining the dynamic response of a simple floating system having three degrees of freedom. Results have been validated though Monte Carlo simulations of forces and dynamic responses. QD theory reveals a perfect agreement with the results obtained through the simulations.

Results shown in this paper can be used by designers, at least during the first stages of their work: in fact, the QD theory seems to be a reliable instrument for determining the dynamic response due to high waves in sea. QD theory, in fact, provides results almost identical to the ones obtained through the numerical simulation at a much lower computational cost.

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