The cavity method for in and out of equilibrium systems on diluted graphs

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This work explores the cavity method from statistical physics as applied to systems of variables interacting on diluted graphs. The cavity approximation, originally conceived for studying disordered systems like spin glasses, has evolved into a versatile framework, providing message-passing algorithms such as belief propagation. Here we consider both static (equilibrium) and dynamic (non-equilibrium) scenarios, exploiting the effectiveness of the cavity approach in its analytical as well as algorithmic variations.

The first chapter introduces the mathematical formulation of the cavity method as originated in physics and then re-invented in the fields of communications and computer science.

In chapter two we study the closest vector problem, a prototypical task in discrete optimization with applications in cryptography and the theory of computational complexity. By means of a mapping to a spin glass model, the cavity method provides semi-analytical results for an infinite-size system. These are then compared with the outcome of approximate optimization on finite instances. Of particular interest is the interplay between phase transitions in the geometry of the space of solutions and the performance of optimization algorithms.

In the third chapter, devoted to non-equilibrium dynamics, we introduce and develop the Matrix Product Belief Propagation (MPBP) algorithm for computing observables of Markov processes on graphs. We consider how reweighting a process, an operation which arises naturally when considering inverse problems and atypical trajectories, can lead to major computational challenges. By leveraging the matrix product state approximation from quantum mechanics, MPBP reduces computational complexity while maintaining high accuracy for dynamical models such as epidemic spreading and Glauber dynamics.

Finally, chapter four contains ongoing work on an extension of MPBP able to describe the steady state of stochastic processes on graphs. This is done by exploiting the Uniform Matrix Product State (UMPS) formalism originally formulated for homogeneous one-dimensional quantum systems.