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Abstract. We study the finite temperature equation of state by using an effective lagrangian in which a dilaton field reproduces the breaking of scale symmetry in QCD. We start by extending a previous investigation in the pure gauge sector, where the dynamics of the gluon condensate, expressed in terms of a dilaton lagrangian, is dominated below the critical phase transition temperature, while at greater temperatures the condensate evaporates in the form of quasi-free gluons. In this context, we study the role of the inclusion of thermal fluctuations of the dilaton field and compare our results with the lattice QCD data. Moreover, we take into account of the meson sector at zero chemical potential by means of an effective lagrangian which incorporates broken scale in addition to spontaneously broken chiral symmetry. Beyond the mean-field approximation, the relevance of the thermal fluctuations of the scalar glueball, other than the contribution of the σ and π meson fields is considering by following a general technique. In this framework, we investigate the thermodynamic nature of the phase transition.

1 Pure gauge sector

Severals effective Lagrangians in nuclear physics are constructed to replicate the behavior of QCD in the strong interaction regime, characterized by the breaking of chiral and scale symmetries [1–3]. Chiral symmetry breaking is linked to the generation of fermionic masses, whereas the breaking of scale invariance arises from quantum corrections, known as the scale or trace anomaly, and is likely related to the development of a gluon condensate. More precisely, the breaking of scale invariance in QCD is manifested in the non-conservation of the dilation current. In order to reproduce the QCD scale anomaly a dilaton field has been introduced, whose Lagrangian reads [4]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathcal{V}\,, \tag{1}$$

where

$$\mathcal{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln\frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln\chi\right), \tag{2}$$

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with $\chi = \phi/\phi_0$ and $B_0 = B\phi_0^4$. In the description of the pure gauge sector, we consider three distinct contributions to the thermodynamic potential [4]. The first one, $\langle \mathcal{V}(\chi)\rangle - P_{\rm dil}(\chi, T)$, arises from the gluon condensate, whose dynamics are expressed in terms of a dilaton Lagrangian (the angle brackets denote a thermal average). The second one, $P_{\rm q-free}(\chi, T)$, is due to the quasi-free gluons: when the value of the gluon condensate is large, at $T < T_C$, most of the gluons are frozen, while, for $T > T_C$, the condensate evaporates and gluons become quasi-free particles. The last one, $P_{\rm int}(\chi, T)$, is due to the perturbative gluon-gluon interaction. Taking into account the presence of thermal fluctuations, it is possible to divide the glueball into two distinct components, the mean field $\bar{\chi}$ and the fluctuating part Δ_χ, i.e., $\chi = \bar{\chi} + \Delta_\chi$, with $\langle \Delta_\chi \rangle = 0$. The thermal fluctuation for a generic meson field at zero chemical potential is given by the following equation:

$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i} \frac{1}{e^{\beta \epsilon_i} - 1}\, dk\,, \tag{3}$$

where n_i the degeneracy factor, $\epsilon_i^2 = k^2 + m_i^{*2}$ and m_i^* is the effective mass. The thermal average of a complicated functions of χ, $f(\chi)$, can be evaluated following the technique proposed in Ref.s [5, 6] by performing integrations with the following Gaussian weighting function

$$P_\chi(z) = \sqrt{\frac{1}{2\pi \langle \Delta_\chi^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\chi^2 \rangle}\right), \tag{4}$$

finding that

$$\langle f(\chi) \rangle = \int_{-\infty}^{\infty} P_\chi(z) f(\bar{\chi} + z)\, dz\,. \tag{5}$$

The thermodynamical potential in the mean field approximation is:

$$\frac{\Omega(\chi, T)}{V} = \langle \mathcal{V}(\chi) \rangle - P_{\rm dil}(\chi, T) - P_{\rm q-free}(\chi, T) - P_{\rm int}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle\,, \tag{6}$$

where

$$P_{\rm dil}(\chi, T) = \frac{1}{6\pi} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_\chi^{*2}}} \frac{1}{e^{\beta\sqrt{k^2 + m_\chi^{*2}}} - 1}\,, \tag{7}$$

$$P_{\rm q-free}(\chi, T) = -2(N_c^2 - 1) T \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-k/T}\right) \Theta\left(k - K(\chi)\right)\,, \tag{8}$$

$$\begin{aligned} P_{\rm int}(\chi, T) = g^2 N_c (N_c^2 - 1) \Bigg\{ (-3) \left[\int \frac{d^3k}{(2\pi)^3} \frac{1}{k} N_B\left(\frac{k}{T}\right) \Theta\left(k - K(\chi)\right) \right]^2 \\ + \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{1}{k_1 k_2} N_B\left(\frac{k_1}{T}\right) N_B\left(\frac{k_2}{T}\right) \Theta\left(k_1 - K(\chi)\right) \\ \times \Theta\left(k_2 - K(\chi)\right) \left[\frac{9}{4} \Theta\left(|\boldsymbol{k}_1 + \boldsymbol{k}_2| - K(\chi)\right) - \frac{1}{4} \Theta\left(|\boldsymbol{k}_1 - \boldsymbol{k}_2| - K(\chi)\right) \right] \Bigg\}\,. \end{aligned} \tag{9}$$

At high temperatures, the primary contribution to thermodynamic observables should be attributed to quasi-free gluons, rather than to glueballs, which are dilaton field excitations. Conversely, quasi-free gluons are expected to be suppressed below the critical temperature T_c, the temperature at which the dilaton field exhibits discontinuity, thus below T_c, we assume they are frozen within the gluon condensate. Whereas, above T_c, the condensate evaporates, allowing gluons to become quasi-free particles. In order to take this behaviour into account, we introduce in the above last equation an infrared cut-off K in the gluon distribution function. This ensures that only gluons with momentum exceeding K contribute to the thermodynamics of the system. Furthermore, at extremely high temperatures, the influence of the dilaton field on the thermodynamic quantities should be negligible. To this end, an ultraviolet cutoff, equal to $3K$, will be incorporated into the expression of $P_{\rm dil}$. Following Ref. [4], we assume that K is a function of the gluon condensate, which is defined as the expectation value of the dilaton field:

$$K(\chi) = \frac{A}{(1-\chi)^{\alpha}} \,. \tag{10}$$

Consequently, if $\chi \to 1$, then $K \to \infty$, while if $\chi \to 0$, then $K \to A$. We use $A = 1.015$ GeV and $\alpha = 0.5$. In Eq.(9), $N_B(x) = (e^x - 1)^{-1}$ is the Bose–Einstein distribution and g^2 is the temperature dependent running coupling-constant

$$g^2(T) = \frac{48\pi^2}{11N_c \ln\left(\frac{T^2+S^2}{\Lambda^2}\right)} \,, \tag{11}$$

where S is a regulator, related to the existence of a minimal momentum K for the propagating gluons. In particular, we assume $S = 7.15$ GeV and $\Lambda = T_c$, where T_c is the value of the critical temperature computed in the model.

In order to realize consistent thermodynamics, it is necessary to ensure that Ω is a minimum with respect to variations in the mean field $\bar{\chi}$. The pressure is $P = -\Omega/V$. In Fig. 1, we present the behavior of P and I as a function of temperature, where $I = (\epsilon - 3P)/T^4$ is the trace anomaly and $\epsilon = T\frac{dP}{dT} - P$. The green curves shown in Fig.1 is obtained by solving the

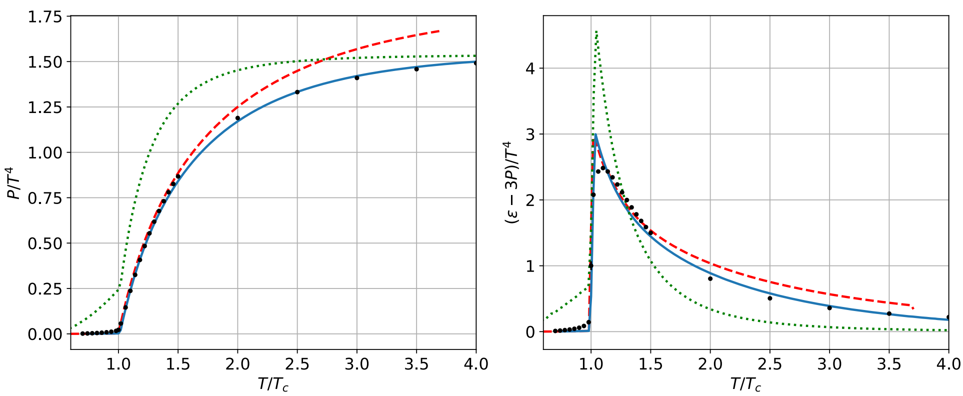

Figure 1. The left panel shows P/T^4, compared with the lattice data [7]. The red dashed curve represents a scenario where, as in Ref. [4], the thermal fluctuations related to the χ field are not considered while in the solid blue the thermal fluctuations are taken into account. The green dotted curves represents a result obtained by assuming that the constituent gluons are described by an Abelian vector field, as in Ref. [5]. The right panel shows the trace anomaly I.

field equation, derived from the following Lagrangian $\mathcal{L}_{\mathcal{G}}$, defined as in Ref. [5]

$$\mathcal{L}_{\mathcal{G}} = \mathcal{L} + \mathcal{L}_{\mathcal{A}} \,, \tag{12}$$

where $\mathcal{L}$ is the same of (1) and $\mathcal{L}_{\mathcal{A}}$ is

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{4}\boldsymbol{A}_{\mu\nu}\cdot\boldsymbol{A}^{\mu\nu} + \frac{1}{2}G^2\phi^2\boldsymbol{A}_{\mu}\cdot\boldsymbol{A}^{\mu} \,. \tag{13}$$

Following a standard procedure, the field equation of the dilaton field, as well as the masses of the glueball and gluon field, m^*_χ and m^*_A, respectively, can be solved in a self-consistent way.

2 Meson sector at zero chemical potential

The present investigation can be extended by considering the meson sector at zero chemical potential through the introduction of an effective Lagrangian that incorporates broken scale in addition to spontaneously broken chiral symmetry [8]

$$\tilde{\mathcal{L}} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tilde{\mathcal{V}}\,, \tag{14}$$

where

$$\begin{aligned}\tilde{\mathcal{V}} =& \mathcal{V}(\chi) - \frac{1}{2}B_0\delta\chi^4\ln\frac{\sigma^2+\pi^2}{\sigma_0^2} + \frac{1}{2}B_0\delta\chi^2\left[\frac{\sigma^2+\pi^2}{\sigma_0} - \frac{\chi^2}{2}\right]\\ &-\frac{1}{4}\epsilon_1'\chi^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) - \chi\right] - \frac{3}{4}\epsilon_1'\,;\end{aligned} \tag{15}$$

here $\delta = 4/33$, $\epsilon_1' = \sigma_0^2 m_\pi^2$ is the explicit symmetry breaking term and $\mathcal{V}(\chi)$ is defined as in Eq.(2). The approach adopted in the previous section can be extended to the case of σ and $\boldsymbol{\pi}$ fields, by decomposing them into mean field and fluctuating parts, where the mean value of the $\boldsymbol{\pi}$ field is zero. As in the previous case, we can define the following Gaussian weight functions [6]

$$P_\sigma(z) = \sqrt{\frac{1}{2\pi\langle\Delta_\sigma^2\rangle}}\exp\left(-\frac{z^2}{2\langle\Delta_\sigma^2\rangle}\right), \tag{16}$$

and

$$P_{\boldsymbol{\pi}}(y) = \sqrt{\frac{2}{\pi}}\left(\frac{3}{\langle\Delta_{\boldsymbol{\pi}}^2\rangle}\right)^{(3/2)}\exp\left(-\frac{3y^2}{2\langle\Delta_{\boldsymbol{\pi}}^2\rangle}\right), \tag{17}$$

in order to evaluate the average of a complicated function $O(\bar{\sigma}+\Delta_\sigma,\boldsymbol{\pi}^2)$, over the fluctuating fields σ and $\boldsymbol{\pi}$, in the following manner

$$\langle O(\bar{\sigma}+\Delta_\sigma,\boldsymbol{\pi}^2)\rangle = \int_{-\infty}^{\infty}dzP_\sigma(z)\int_0^\infty dyy^2P_{\boldsymbol{\pi}}(y)O(\bar{\sigma}+z,y^2)\,. \tag{18}$$

The thermodynamical potential in the mean field approximation is

$$\begin{aligned}\frac{\tilde{\Omega}(\chi,\sigma,\boldsymbol{\pi},T)}{V} =& \langle\tilde{\mathcal{V}}\rangle + \frac{T}{2\pi^2}\int_0^\infty dkk^2\left[\ln(1-e^{-\beta\omega_\sigma}) + 3\ln(1-e^{-\beta\omega_\pi})\right]\\ &-\frac{1}{2}m_\chi^{*2}\langle\Delta_\chi^2\rangle - \frac{1}{2}m_\sigma^{*2}\langle\Delta_\sigma^2\rangle - \frac{1}{2}m_{\boldsymbol{\pi}}^{*2}\langle\Delta_{\boldsymbol{\pi}}^2\rangle\\ &-P_{\rm dil}(\chi,T) - P_{\rm q-free}(\chi,T) - P_{\rm int}(\chi,T) + \frac{1}{4}[B_0(1-\delta)+\epsilon_1']\,.\end{aligned} \tag{19}$$

Here a constant term has been added so that $\langle\tilde{\mathcal{V}}\rangle$ is zero in the vacuum and the terms in the second line are subtracted, thus avoiding double counting.

In Fig.2, we present the behavior of the mean fields $\bar{\chi}$ and $\bar{\sigma}/\sigma_0$ as a function of temperature.

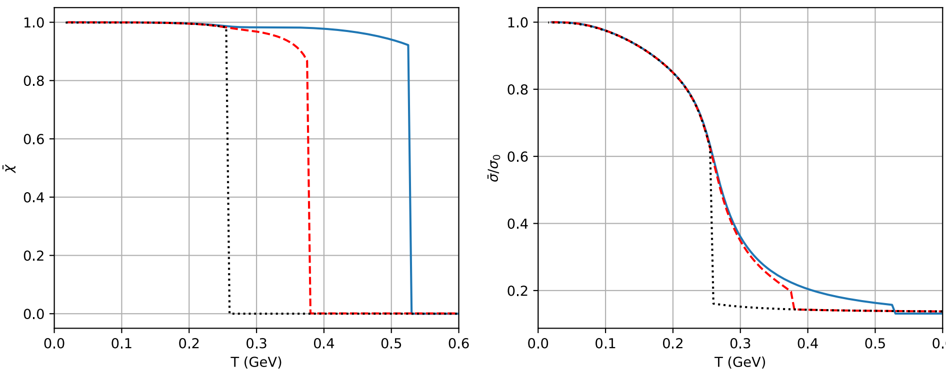

Figure 2. In the left panel, the mean glueball field $\bar{\chi}$ is presented as a function of temperature. The blue solid curve is derived by considering only the thermal fluctuations on the σ and $\boldsymbol{\pi}$ fields. The red dashed curve is obtained by incorporating fluctuations in the χ field, while the black dotted curve is obtained by including gluons in the same way of Ref. [4]. In the right panel, we report the mean sigma field $\bar{\sigma}/\sigma_0$ as a function of temperature.

3 Conclusions and Outlook

In this study, we extend a previous work in which the equation of state at finite temperature was investigated by using an effective Lagrangian in which a dilaton field reproduces the breaking of the scale symmetry in QCD. This is based on an evaporation model in which the gluons are frozen at low temperatures in the non-perturbative condensate, while at high temperature they escape from the condensate and behave as almost free particles. We have shown that it is possible to reproduce the main results of the EOS lattice QCD for thermodynamic quantities. By taking into account the thermal fluctuations in the dilaton field χ, we are able to reproduce more accurately the pure gauge lattice data, particularly at high temperatures ($T > 2.0\ T_c$), as shown in Fig. 1. Furthermore, the incorporation of the thermal fluctuations and the use of the rigorous technique introduced in Ref. [6], allows us to avoid the main problems encountered in Ref. [4], i.e., the appearance of an imaginary part in the thermodynamical potential, previously assumed as the presence of a system instability. In this framework, the model shows a first order transition, obtained by minimizing the thermodynamical potential for $T_c = 0.270$ GeV.

Afterthat, we also extend this technique to the meson sector at zero chemical potential. In order to obtain the vacuum pion mass, we have chosen the explicit symmetry breaking parameter $\epsilon_1' = \sigma_0^2 m_\pi^2$. We took $\sigma_0 = 0.110$ GeV and $m_\pi = 0.138$ GeV. The thermal fluctuations, as defined in Ref. [6], were employed for the σ, $\boldsymbol{\pi}$ fields and the dilaton field χ. Regarding Fig. 2, the introduction of fluctuations in the dilaton field χ results in a reduction of the critical temperature, from $T_c = 0.530$ GeV to $T_c = 380$ GeV. The fluctuations in the χ field also have a significant impact on the masses: in the absence of fluctuations, the masses of the σ and $\boldsymbol{\pi}$ fields vanish after T_c, however, in the presence of fluctuations, they continue to increase linearly with increasing temperature. Fig. 2 shows that at high temperatures the solution for $\chi \sim 1$ becomes unstable and χ become equal to zero. The fourth panel of Fig. 3 shows how the minimum of the potential shifts from a value around $\chi \sim 1$ at low temperatures to $\chi = 0$ at higher temperatures. The two minima of the potential have the same value at $T = 0.530$ GeV. This feature can be interpreted as a first-order phase transition at a critical temperature T_c, which restores scale symmetry.

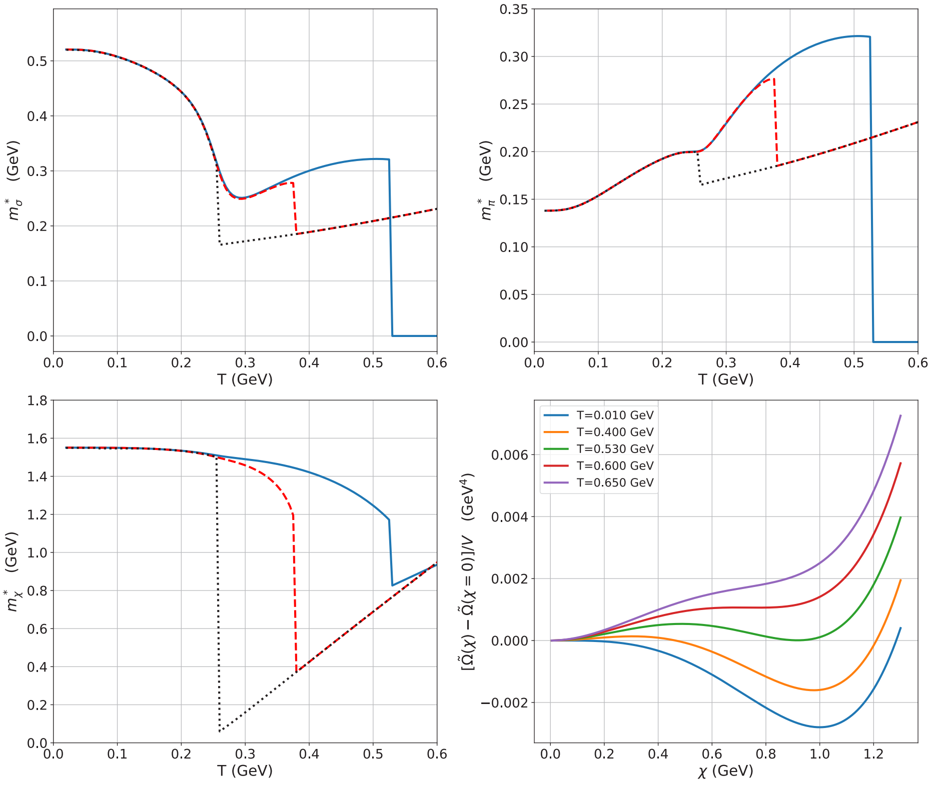

Figure 3. The effective masses m^*_σ, m^*_π and m^*_χ are shown as a function of the temperature in the top-left, top-right and bottom-left panels respectively. The blue solid curve is derived by considering only the thermal fluctuations on the σ and $\boldsymbol{\pi}$ fields. The red dashed curve is obtained by incorporating also the fluctuations in the χ field, while the black dotted curve corresponds to the gluons considered in the same way of Ref. [4]. The lower right-hand panel displays the behavior of the thermodynamic potential $\tilde{\Omega}$ as a function of χ for different values of temperature. In this last panel, only fluctuations in the σ and $\boldsymbol{\pi}$ fields are considered (the potential is normalised to zero at the origin).

A further extension of this work will be the study of a chiral Lagrangian which takes into account of the nucleons and quark degrees of freedom, by considering investigations on high energy astrophysical phenomena and relatististic heavy ion collisions [9, 10].

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