Knowledge Transfer between Oscillators and Real Vibrating Structures to Enrich Dynamic Monitoring Datasets

Knowledge Transfer between Oscillators to Enrich Dynamic Monitoring Datasets

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Abstract. Transfer Learning (TL) techniques can be exploited in engineering structures to overcome the main limit of the data-driven approaches for Dynamic Monitoring, i.e. the lack of a labelled dataset for some structural configuration of the monitored systems. A variety of methods can be implemented, but those that enable heterogeneous TL based on Domain Adaptation have proven to be particularly useful, as they allow knowledge to be transferred within a population composed of a wider range of structures. Among them, the Kernelized Bayesian Transfer Learning (KBTL) can be used to improve the knowledge of a less monitored structure exploiting the knowledge of a more monitored one. In this paper the KBTL is assumed to transfer information from an oscillator to a spatial frame of which few observations are available. This is done by comparing the KBTL performance with those of a Support Vector Machine model generated from data representative only of the spatial frame. Both models are trained on datasets composed by natural frequencies of the two systems estimated at different temperature ranges.

Keywords: Dynamic Monitoring, Transfer Learning, Domain Adaptation, Frames, SDOF Oscillator.

Introduction

The lack of labelled data is the main limit that is present nowadays for the application of supervise machine learning algorithms, such as classification model in the field of Vibration Based Structural Health Monitoring (VBSHM) for damage detection [1]. Indeed, most of the structures on which a dynamic monitoring system is installed is currently under a healthy condition, hence information about different damage states cannot be extrapolated from the dynamic response to the environmental excitation recorded during the monitoring activities. To overcome this limitation, researchers are analysing and evaluating the possibility to use

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transfer learning techniques to enable the development of useful models for the damage detection.

VBSHM analyses the dynamic response of the structure under investigation through the identification of diagnostic characteristics, such as natural frequencies and mode shapes. The VBSHM can be developed following two different approaches [2]: (i) data-driven approach, for which the diagnostic characteristics are directly extrapolated from the structural dynamic response to environmental or earthquake excitations; (ii) model-driven approach, for which the diagnostic characteristic are obtained from simulation developed on the mathematical model, properly calibrated, of the structure. However, both methods present some limitations. In data-driven method the lack of labelled data representative of a damaged condition is evident while is not a limitation in the model-driven method, in which any kind of damage can be simulated. On the other hand, in model-driven method features are collected from simplified mathematical models, which do not account for all the factors that act on the real structure. Therefore, to take advantage from the model-driven approach, it is necessary to have a calibrated model, which can be obtained after an extensive survey campaign. In this view, hybrid approaches could be picked to compensate the restriction of the two methods by contemporarily processing features arising from data-driven and model-driven approach.

Along these paths of action, it is possible to identify two different applications of transfer learning algorithms as support for VBSHM: (i) Transfer Learning (TL) from simulated features to experimental features, and (ii) TL from experimental features to other experimental features. Case (i) allows enrichment of an experimental dataset exploiting data simulated from the calibrated model, and case (ii) allows enrichment of an experimental dataset exploiting a second experimental dataset, when two similar monitored structures are available. The several structures and models considered make up a population, within a transfer of knowledge from the richer domain to the sparsest one can be attempted. Indeed, a population is a collection of different systems within which some similarities exist. The main factors that introduce differences within a population are material, geometry, and topology, and on the base of these discrepancy a population can be defined as homogeneous or heterogeneous [3], [4].

In this paper, the problem of transfer of knowledge between oscillators, namely a SDOF oscillator and a spatial frame, is addressed. This is done with a Domain Adaptation (DA) approach [5], specifically the Kernelized Bayesian Transfer Learning (KBTL) [6], which allows to exploit data of the SDOF oscillator to enrich the dataset of the frame one. Its performances are here compared with those of a Support Vector Machine (SVM) [7] inferred from only data of the spatial frame to highlights the effectiveness of the DA. This paper is developed with simulated data for both the structures, since it represents a first step in understanding how this method can improve the knowledge of a poorly monitored structure, if information from a quite different systems is available. Consequently, this study can be read as a first attempt in understanding how differences within a population affect algorithm performance.

1. Domain Adaptation with Kernelized Bayesian Transfer Learning

Domain Adaptation is a TL approach which allows the share of knowledge among different domains, therefore among different structures and mathematical model in dynamic monitoring applications. These algorithms are based on two essential ingredients domains, and tasks. A domain $D = \{\mathcal{X}, P(X)\}$ is an object that consists of a feature space $\mathcal{X}$ and a marginal probability distribution $P(X)$ over a N finite sample of feature data $X = \{\boldsymbol{x}_i\}_{i=1}^{N} \in$

$\mathcal{X}$. While a task $\mathcal{T} = \{\mathcal{Y}, f(\cdot)\}$ is the combination of a label space $\mathcal{Y}$ and a predictive function $f: \mathcal{X} \rightarrow \mathcal{Y}$.

The Kernelized Bayesian Transfer Learning (KBTL) is a supervised learning algorithm used to share knowledge within a population admitting features inconsistency. This model has been introduced by Gönen & Margolin in 2014 [6] to share knowledge across multiple datasets from different systems generating a linear classification model in a shared latent subspace. This model assumes T domains $\{D_t\}_{t=1}^{T}$ with inconsistent feature spaces $\{\mathcal{X}_t\}_{t=1}^{T}$. Each domain has an associated task $\{\mathcal{T}_t\}_{t=1}^{T}$ with consistent label spaces $\mathcal{Y}_t = \mathcal{Y}_k \ \forall\, t, k \in 1{:}\,T$, hence a global and unique label space $\mathcal{Y}$ shared among the T systems can be assumed. Each domain has N_t finite features observations $X_t = \{\boldsymbol{x}_{t,i} \in \mathcal{X}_t\}_{i=1}^{N_t}$ at which correspond a finite set of label observations $Y_t \in \mathcal{Y}$. For each domain/task pair, there is a specific kernel function $k_t(\cdot,\cdot)$ that defines the correlation among the features observations, therefore each domain is represented by its own kernel matrix $K_t = k_t(X_t, X_t{}')$.

Conceptually, the algorithm can be summarized in two main phases: (i) projection of dataset into a shared latent subspace $\mathcal{H}$ of dimension $\mathcal{R}$, using a kernel-based dimensionality reduction model for each domain, and (ii) deduction of a linear discriminative classifier in the shared latent subspace. The first phase has been designed to manage the features inconsistency and to reduce the distance between datasets. Indeed, the representation of each domain in the shared latent subspace is obtained as $\{H_t = A_t^{\intercal} K_t \in \mathbb{R}^{\mathcal{R} \times N_t}\}_{t=1}^{T}$, where $\{A_t \in \mathbb{R}^{N_t \times \mathcal{R}}\}_{t=1}^{T}$ is the optimal linear projection matrix learned by the algorithm for each domain and $\{K_t \in \mathbb{R}^{N_t \times N_t}\}_{t=1}^{T}$ the domain kernel matrix. The second stage is then composed by the classification part that calculates the predicted outputs in the shared latent subspace $\mathcal{H}$ with a linear classifier $f_t = H_t^{\intercal}\boldsymbol{\omega} + \mathbf{1}b \ \forall\, t \in 1{:}\,T$, where the weights vector $\{\boldsymbol{\omega} \in \mathbb{R}^{\mathcal{R} \times 1}\}$ and the bias $\{b \in \mathbb{R}^{1 \times 1}\}$ are the classifier parameters learned by the algorithm and shared among all the domains. The KBTL can be used for binary classification or multi-class classification, the content of this paper is developed with the multi-class classification model. Since the KBTL is a hierarchical Bayesian model, some slight differences in the hypotheses exist if binary classification is performed instead of a multi-class classification. For the details about the different hypotheses the authors refer to [6], [8].

The KBTL is a hierarchical model constructs from priors, hyper-parameters, latent variables, and model parameters. The priors are, the matrix of priors $\{\Lambda_t \in \mathbb{R}^{N_t \times \mathcal{R}}\}$ of the task-specific projection matrix A_t, the vector of priors $\{\boldsymbol{\eta} \in \mathbb{R}^{\mathcal{R} \times 1}\}$ of the weights vector $\boldsymbol{\omega}$ and the prior $\{\gamma \in \mathbb{R}^{1 \times 1}\}$ for the bias parameter b. Each of these three priors are defined from a set of two hyper-parameters $\{\alpha_\lambda, \beta_\lambda\}$ for Λ_t, $\{\alpha_\eta, \beta_\eta\}$ for $\boldsymbol{\omega}$ and $\{\alpha_\gamma, \beta_\gamma\}$ for b. Other hyper-parameters are the variance of the shared latent subspace σ_h^2 and the non-negative margin ν used to find a margin among the classes. Note that, for multi-class problem with C classes, each class has its own set of classifier parameters $\{b_c \in \mathbb{R}^{1 \times 1}, \boldsymbol{\omega}_c \in \mathbb{R}^{\mathcal{R} \times 1}\}_{c=1}^{C}$. Indeed, the model is built in a *one-vs-all* manner, which means that each class has its own classifier that separates it from the other classes. This implies that, for each class the probability that a given observation belongs to it is computed, then the label assigned to that observation will be the one for which the highest probability has been computed.

2. Case Study

In this section, first steps towards the transfer of knowledge between different systems are presented. The dynamic characteristics used as features of the problem are the natural frequencies of two oscillators, namely a SDOF oscillator and a 3D frame. Due to their differences in plan and elevation development as their dynamic response, they are considered as a *heterogeneous population*. To understand if with the KBTL algorithm is possible to transfer knowledge between so different systems, the different classes of the classification problem do not consist of distinct types of damage, but of different temperature states. Thanks to this application it has been possible to demonstrate the efficiency of the transfer of knowledge between these two rather different systems as a starting point for future application of transfer learning between real full-scale monitoring systems, installed on complex and different structures such as those of Cultural Heritage.

For this problem of classification three different classes of temperature are considered, to each of which a different label is assigned. The three classes covered the range of the positive temperatures. On the base of the range of temperatures for which data are available from both the structures, the following classes are considered:

- Class 1, for a temperature of 3°C.
- Class 2, for a temperature of 8°C.
- Class 3, for a temperature of 13°C.

Hence, the label space $\mathcal{Y}$ shared by the two domains is $\{'1',' 2', '3'\}$.

2.1 Source Domain

For this application, the Source Domain (SD) is composed by the fundamental natural frequency of a SDOF oscillator.
The SDOF oscillator is characterized by a steel cubic mass of 25 kg placed at the top of an empty squared column 0.50 m high fixed to the ground. The cubic mass side is equal to 0.15 m, the column side to 0.44 m and column thickness to 0.01 m.

The frequency observations considered are related to the three different temperatures. Each observation is obtained by imposing a variation in the elastic modulus. The value to be assigned to the elastic modulus is chosen in accordance with the $E(T)$ law obtained from the experimental campaign developed on the physical oscillator. Moreover, to obtain more observations for each temperature, the elastic modulus $E(T)$ is slightly altered with a gaussian noise of variance equal to σ. Where σ (the 20% of the experimental standard deviation) is chosen to generate an uncertainty in the frequency-temperature law proportional to the distance between the minimum and maximum value of frequency that was measured at the fixed temperature. Consequently, for each of the three-temperature class declared above 60 observations of the fundamental frequency are simulated and the final source domain is obtained as shown in Fig. 1 (a), where the three different classes are represented in blue, green and purple respectively.

2.2 Target Domain

The Target Domain (TD) consists of the first three natural frequencies simulated from the FEM of a multiple oscillator, namely a laboratory spatial frame.

This frame is equipped with three square plan floors, carried by four pillars positioned at the vertices and anchored at the slab base. Both the piers and the slab are composed of aluminium. The floor configured as squared plates measuring 400 x 400 mm and 5 mm thick, while the pillars are 900 mm high and with a rectangular section of 20 x 3 mm. Pillars and slabs are connected to each other by means of galvanized steel L-profiles, measuring 20 x 20 mm, 2 mm thick. While the connections of the pillars to the ground are carried out the same components but of 30x30 mm. The reference system is considered to have the X axis as the direction of the columns strong inertia, the Y axis as the direction of the weak inertia and the Z axis the vertical one.

The KBTL algorithm ([6], [8]) is applied here to test whether it can be exploited to infer a classifier model that generalizes well for unseen target data when few observations of the target structure are available for its training. Therefore, working with a FEM as a target structure implies that it is possible to consider few observations for the algorithm training while also being capable to simulate enough observations to test the inferred model. Consequently, a total of 60 observations of the first three natural frequencies are obtained through FEM simulations inducing an elastic modulus alteration of all the twelve columns following the same procedure described for the SD. These first three frequencies are related to the first mode along Y, the first mode along X and the second mode along Y. Then, the simulated dataset is randomly split into training and test dataset as it is shown in Fig. 1 (b), where data with larger diameter compose the training set, while the remaining ones the test set.

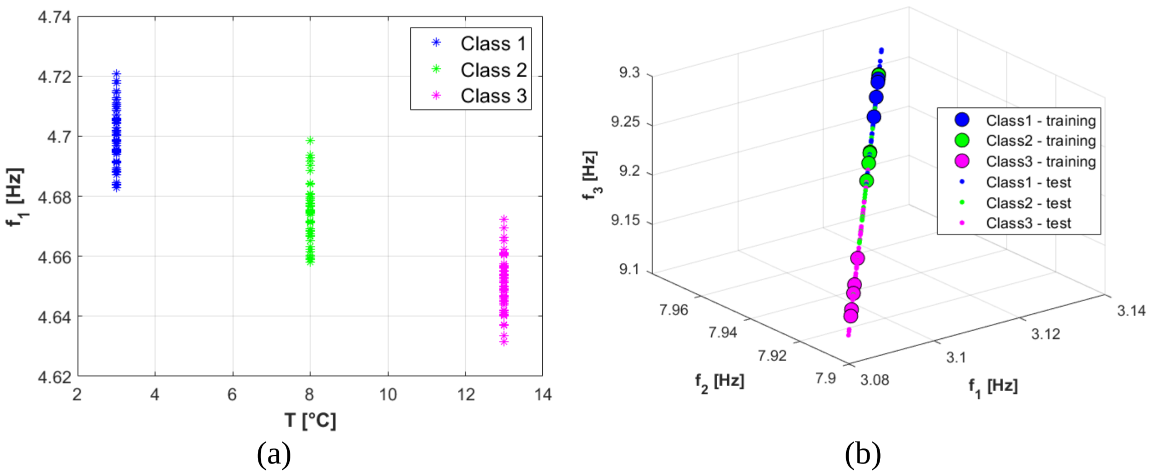


Fig. 1. (a) Source Domain for the algorithm training and, (b) Target Domain split into training and test dataset.

3. Comparison between SVM model and KBTL model

To demonstrate the possibility to transfer knowledge from the SDOF oscillator to the spatial frame, the performances in terms of accuracy of the KBTL model are compared to those of a SVM model. In Table 1 the number of observations used for the training phase and the test phase of the two algorithms are called up.

The SVM algorithm used is the one implemented in the *MATLAB Classification Learner Toolbox*. For its training only data representative of the spatial frame is provided, and a two-fold cross-validation is used to reduce possible overfitting problems. Then, the test phase is performed on the remaining data of the frame itself. The two figures of the training (Fig. 2 (a)) and test (Fig. 2 (b)) phase are composed by two subplots, the one on the left shows the labels predicted by the algorithm, while the one on the right the labels to be predicted by the algorithm. With this model an accuracy of 93.3% during the training phase and of 63.6 % during the test phase are reached. These accuracies could be improved by using a transfer learning method, based on Domain Adaptation, to achieve an improvement in the model predictive ability.

Hence, the KBTL is applied to improve the accuracy in predicting the TD unseen data by exploiting the knowledge of the SD. To develop the KBTL model, the authors have chosen a dimension $\mathcal{R}$ of the shared latent subspace $\mathcal{H}$ equal to 2, to admit a graphical representation of the model to directly visualized possible overfitting problems. A linear function is used to obtain the kernel matrices. After a tuning procedure the following set of hyperparameter was chosen: $(\alpha_\lambda, \beta_\lambda) = (10^{-3}, 10^{-3})$, $(\alpha_\eta, \beta_\eta) = (10^{-3}, 10^{-3})$ and $(\alpha_\gamma, \beta_\gamma) = (10^{-3}, 10^{-3})$. Then, 2000 iterations are performed, a standard deviation of the shared latent subspace σ_h equal to 6.16 is used and the non-negative margin ν is fixed equal to 0. In Fig. 2 (c) the training phase is reported, where the SD is represented with the star and the TD with the triangle. In this figure two subplots of the latent subspace are visible, the one on the left shows the data coloured as are predicted by the algorithm, i.e. *'predicted labels'*; while the one on the right shows the data coloured as should be predicted by the algorithm, i.e. *'correct labels'*. During this training phase an accuracy of 93.3 % is reached. Then, in Fig. 2 (d) the test phase of KBTL model is reported, the figure is composed as the training one, '*predicted labels'* on the left and *'correct labels'* on the right, and in this phase an accuracy of 71.5 % is reached.

Table 1. Observations for training and test phase of the models: SVM and KBTL.

Model	Domain	Training			Test		
		y = 1	y = 2	y = 3	y = 1	y = 2	y = 3
SVM	Target	5	5	5	55	55	55
	Source	0	0	0	0	0	0
KBTL	Target	5	5	5	55	55	55
	Source	60	60	60	0	0	0

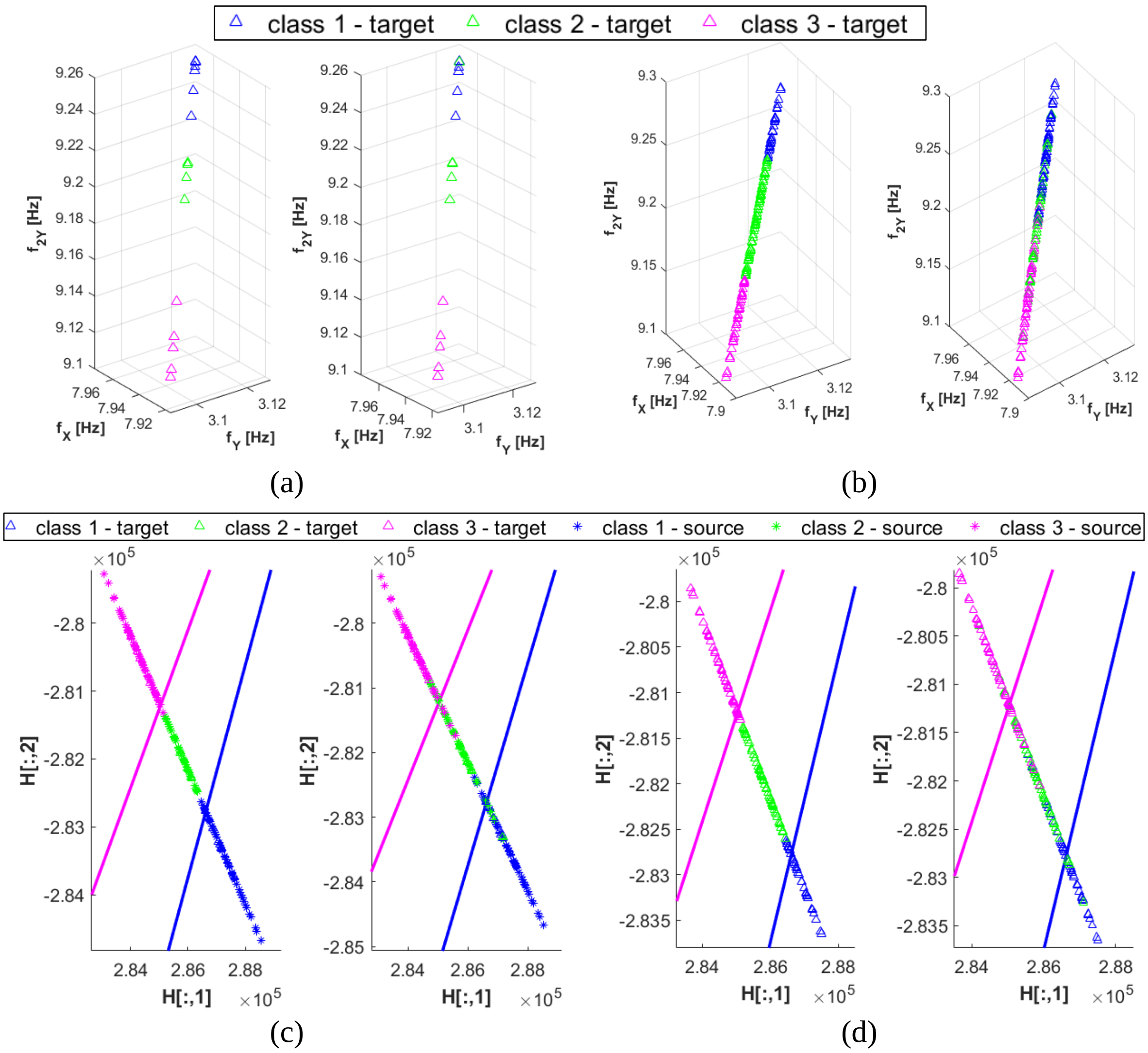


Fig. 2. (a) Training and (b) Test of SVM model (c) Training and (d) Test of the KBTL model. Each plot shows predictions on the left and labels to be predicted on the right. TD and SD are represented by the triangle △ and star *, respectively.

4. Results

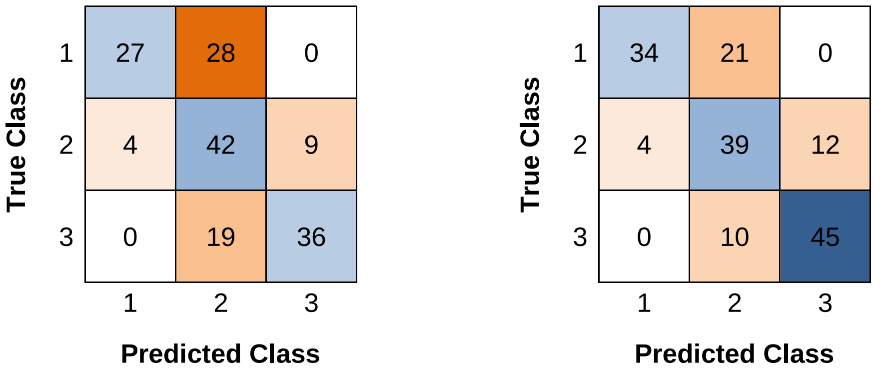
In Table 2 the comparison between the accuracies obtained with the SVM and those obtained with the KBTL is shown. From this table it is possible to identify an increment in the test accuracy of the 7.9 % moving from a simple SVM model to the KBTL model. Since the classification part of the KBTL model can be traced back to a simple linear classifier, such as SVM, the comparison of the performances of the KBTL with those of a SVM can be interpreted as the evaluation of the efficiency of the DA procedure. Hence, the increment in the accuracy of the test phase can be read as the increment induced by the process of domain adaptation. In this way it is shown how the knowledge of the SDOF oscillator can be useful in implementing that of the spatial frame, despite the fact that the two systems present differences in structural developments and dynamic responses. At the same time, it can be observed from Table 3 how the accuracies of the different classes are more homogenous in the KBTL model with respect to those of the SVM model. With the KBTL model, two classes reach a quite good accuracy, higher 70.9%, and only one results as lower; while with the SVM model only class 2 reaches a good accuracy while all the others are strongly lower (≤ 65.45%). Therefore, the KBTL not only provides higher accuracy in the test phase, but also results in better homogeneity in the accuracies of individual classes, so there is not one class preponderating over the others. However, also in KBTL model one class, i.e. class 1, remains an underdog with an accuracy of 61.82%, which is prevalently confused with class 2. A different way of representing the performance of a model is the confusion matrix, therefore in Fig. 3 the confusion matrix of the test phase of the two models are reported, where for each class it is possible to visualize the correct and incorrect predictions, and for the incorrect ones it is possible to identify the wrong class assigned by the model to the data.

Table 2. Accuracies of the two models.

Model	Training	Test
SVM	93.3%	63.6%
KBTL	93.3%	71.5%

Table 3. Test accuracies of the two models for each temperature class.

Model	class 1 - test	class 2 - test	class 3- test
SVM	49.10%	76.36%	65.45%
KBTL	61.82%	70.91%	81.82%



(a) (b)

Fig. 3. Confusion Matrix of the test phase of the (a) SVM model and (b) KBTL model.

5. Conclusions

In this paper an application of transfer of knowledge has been performed through DA to enrich datasets as support for future dynamic monitoring applications. The effectiveness of the method is demonstrated through an application based on simulated dataset representative of two oscillators, namely a SDOF oscillator and a spatial frame. Thanks to this application it is possible to observed how an increment in the predictive capability for the target structure can be reached exploiting data of the source one through the application of the KBTL. This application leads to a non-negligible improvement in the prediction accuracy of the target structure of the 7.9%. Consequently, this study represents a promising trial field for future applications of transfer learning approaches, based on domain adaptation, to enrich sparse dataset leveraging different but interconnected systems. A future work development will be the application of that algorithm to support the monitoring of real structures with remarkable differences in their planimetric and elevation development, as in their dynamic responses. An interesting application of such method could be in the field of cultural heritage monitoring, where it is generally unlikely to find identical geometries and materials.

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